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Water Resources Sustainability and Optimal Cropping Pattern in Farming Systems; A Multi-Objective Fractional Goal Programming Approach

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Abstract Water resources sustainability has the main contribution to the existence and durability of the farming systems and strongly depends on the cropping pattern practices. A comprehensive cropping pattern planning takes in to account the high level of interrelation of the environmental, economic and social aspects of farming systems. In order to assess the sustainability of water resources and determine an optimal pattern of cropping in a rural farming system, this paper introduces two ratios of "net return/water consumption" and "labor employment/water consumption" and attempts to simultaneously optimize them as the sustainability indicators. To this purpose, a multi-objective fractional goal programming (MOFGP) procedure is considered as the main approach of the study to be accomplished by several other single and multi-objective linear and fractional programming models. The results show that the FP models are more significant to contribute in assessing the sustainability indicators compared to the LP models, and the MOFGP solution is considered better, compared to the single objective FP solutions. The results will be illustrated quantitatively.

Keywords Water resources sustainability · Optimal cropping pattern · Multi-objective fractional goal programming (MOFGP)

1 Introduction

Water shortage is a worldwide problem and is more severe in arid and semi-arid regions. This problem becomes even more severe by increasing the water demands

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due to the population growth, improving the living standards and the small-scale climatic changes (WorldBank 1992; Mariolakos 2007). The only feasible solution to this problem is to make efficient use of water in agriculture and to increase productivity of limited water resources. Recently, enhancing the irrigation efficiency and water productivity has been investigated in several studies (e.g., Onta et al. 1991; Mainuddin et al. 1997; Raju and Kumar 1999; Haouari and Azaiez 2001; Sethi et al. 2002; Benli and Kodal 2003; Tsakiris and Spiliotis 2006; Sethi et al. 2006; Sahoo et al. 2006; Liu et al. 2009; Kilic and Anac 2010; Montazar et al. 2010). It is also believed that with appropriate water management practices in crop planning, up to 50% of available water can be saved (e.g., Shangguan et al. 2002). However, different agricultural, environmental and socio-economic criteria should be taken in to account to find an appropriate water management and consequently crop planning practices in farming systems. These criteria are generally conflicting and inconsistent. For example, maximizing the net return in a farming system requires more withdrawal of water resources, while the sustainability of the system entails reducing the water consumption.

In cases when several objective functions (conflicting and incommensurable) exist, the optimal solution for one function is not necessarily optimal for the other functions, and hence one may introduce the notion of the best compromise solution, also known as nondominated solution, efficient solution, noninferior solution, Pareto's optimal solution (Stancu-Minasian and Pop 2003). Therefore, a compromise solution among such conflicting criteria needs to be defined. Multiple criteria decision-making (MCDM) methods such as goal programming (GP) have frequently been used to simultaneously optimize several objectives in crop planning (e.g., Sarker and Quaddus 2002; Tsakiris and Spiliotis 2006; Sharma and Jana 2009; Vivekanandan et al. 2009) and water resources management (e.g., Al-zahrani and Ahmad 2004; Bravo and Gonzalez 2009). A recent progress in this context is developing fractional programming (FP) models with multiple objectives. In this case, each objective takes the form of a ratio that has a linear numerator and denominator. Thus, fractional programming deals with a situation where a ratio between physical and/or economical functions, for example cost/time, cost/volume, cost/profit, or other quantities that measure the efficiency of a system, is minimized (Stancu-Minasian and Pop 2003).

In many practical applications, optimization of ratios of criteria gives more insight in to the situation than the optimization of each criterion (Craven 1988). Using ratios in the formulation of a problem assures that only the solutions with better achievements per unit of resource would be selected and also combining the objectives in ratios facilitates the management of solutions (Lara and Stancu-Minasian 1999). In fact, these ideas call for a technically efficient use of resources as a necessary condition of sustainability under a preventive framework in order to achieve the maximum level of output allowed by a level of inputs or to use the minimum levels of inputs to achieve a desired level of outputs (Lara and Stancu-Minasian 1999). Hence, as stated by Monteith (1990), the question for an operational strategy in this approach is not maximizing 'per se' but maximizing outputs and minimizing inputs. In other words, this means maximizing the desired outputs and minimizing the undesired outputs and using non-renewable and scarce inputs. Such types of problems, the objectives of which are ratio functions and conflicting in nature, are inherently multi-objective fractional programming problems and there exist several methodologies to solve these problems (Chakraborty and Gupta 2002). From among these methodologies, the goal programming approach is a more generalized one (e.g., Kornbluth and Steuer 1981; Gómez et al. 2006).

In this study, a rural region situated in the eastern part of the city of Isfahan, central Iran, was selected to investigate appropriate cropping pattern and water resources management scenarios in regard to its farming system sustainability. This region is perfectly rural with relatively high population density, of which almost 70% is involved in agricultural sector (Amini Fasakhodi 2009). Limited irrigation water, which is mainly from groundwater in the region, cannot meet the requirements of common cropping pattern. Additionally, the aquifers are depleting (Fig. 1) due to extensive withdrawal of groundwater as reported by RWOI (2007a) and generally low level of precipitation. The rapid fall of groundwater level creates a "chain reaction" of physical and ecological consequences that can lead to serious socio-economic repercussions such as immigration and suburbia phenomenon. Conservation of water level in the groundwater reservoirs of the region has also been emphasized and recommended in the previous studies, for example, the study done by Sogreah Ingéneries Consultant (1974). This study, however, was carried out at the time when the water table and climate had a far better condition. Thus, there is an imminent need for a more efficient pattern of cropping on the regional scale to meet the objectives of land utilization, maximization of labor employment and income of farmers based on the available water resources. Therefore, the socio-economic aspects of the farming system were considered in terms of maximizing the net return and labor employment opportunities. In order to connect these socio-economic aspects to the water resources as a main environmental aspect of the farming system, two ratios were defined and formulated in the form of "net return/water consumption" and "employment/water consumption". These ratios hence were considered as two fractional objectives of the study, and a multi-objective fractional goal programming (MOFGP) procedure was formulated to simultaneously optimize them based on a set of constraints related to some other production resources availabilities. In addition to MOFGP model as the main approach of the study, some other single and multi-objective linear and fractional models were also formulated and solved in order to compare the relevant cropping patterns in terms of their potential contributions to assessing the sustainability indicators. Efficiency of water consumption is one of the most important and widespread issues about farming systems sustainability. Defining

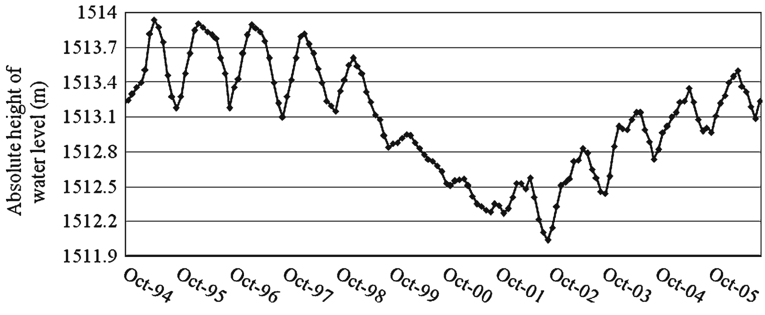

Fig. 1 Dropping of water table level in the aquifer which the study area is included, measured monthly (RWOI 2007a)

and optimizing such ratios via appropriate patterns of cropping elucidate a better management of the socio-economic aspects of a farming system by reducing the use of scarce water resources. Thus, the associated environmental impacts of farming activities can also be reduced through more efficient use of water resources.

2 Materials and Methods

2.1 Linear Fractional Programming

Mathematical Programming (MP) has been widely used to investigate different aspects of agricultural systems in recent decades. Among the different MP techniques, Fractional Programming (FP), which is similar to Data Envelopment Analysis (DEA; Charnes et al. 1978) according to its mathematical background, is a well-known technique for optimizing the efficiency of several decision-making units. Nonetheless, in the contexts of agricultural systems, FP can be considered as a natural way of approaching the issues related to the sustainability of the systems (Lara and Stancu-Minasian 1999). In FP the goal is to optimize the ratio between physical and/or economic functions (Gómez et al. 2006), which are linear combinations of decision variables. In a general form, the mathematical structure of a single objective linear fractional program with n decision variables and m constraints can be written as (Goedhart and Spronk 1995):

$$\begin{aligned} &\text{Max} \quad g = \left(c^T x + \alpha\right) / \left(d^T x + \beta\right) \\ &st: \quad x \in S = \{x \in R^n | Ax \leq b; \quad x \geq 0; \quad b \in R^m\} \end{aligned} \tag{1}$$

The numerator and denominator of the goal fraction are real functions defined on R^n, with the decision variables vector x, technical coefficients vectors c, d and scalar constants α, β. The right hand side (RHS) vector, b, of the constraints is defined on R^m, so technical coefficients, A, form an $m \times n$ matrix.

To find the optimum solution for this problem, a new variable (y) need to be introduced under an additional assumption in which the denominator of the above quotient is strictly positive throughout the feasible set of solutions (see the details in Charnes and Cooper 1962).

$$y = x.t \text{ and } t = \left(1/d^T x + \beta\right). \tag{2}$$

Using this transformation, the original fractional problem is changed to an ordinary linear programming problem with an additional constraint as follows:

$$\begin{aligned} &\text{Max} \quad g = c^T y + \alpha \\ &\text{st:} \quad Ay - bt \leq 0; \quad d^T y + \beta.t = 1; \quad y, t \geq 0 \end{aligned} \tag{3}$$

Based on this transformation, if $(y', t')^T$ is an optimum solution of the problem, then $x' = y'/t'$ will be an optimum solution of the original fractional problem (Charnes and Cooper 1962; Goedhart and Spronk 1995).

2.2 Multi-Objective Fractional Goal Programming (MOFGP)

In order to simultaneously optimize two fractional objectives, a multi-objective fractional programming procedure is needed. To achieve the purpose, we adopted the multi-objective fractional goal programming (MOFGP) procedure as a main approach of the study. To do this, a vector of decision variables, x, needs to meet a set of r linear constraints so that each of the fractional objectives obtains a desired value. In order to include multiple goals in the formulation of the optimization problem, deviational variables are used. These variables, which are not negative, measure the difference between the desired values and the obtained actual results for each of the objectives. With a predefined priority order of the goals, the optimization problem can be formulated as follows (Gómez et al. 2006):

$$\begin{aligned} &\text{Min} \quad \sum\nolimits_m w_m.n_m \\ &\text{s.t.} \quad x \in S \\ &\frac{c_m^t x + \alpha_m}{d_m^t x + \beta_m} + n_m - p_m = u_m \quad ; \quad n_m, p_m \geq 0 \end{aligned} \tag{4}$$

where c_m, $d_m \in R^n$, a_m, $b_m \in R$, w_m is the weight of the mth goal and n_m, p_m are the negative and positive deviational variables for the same goal. The desired value of fractional goal for the mth goal (u_m) is calculated by solving a single objective fractional program problem as explained in the previous section. Due to the both of study fractional objectives must be maximized, the deviational variables to be minimized are the negative ones. By multiplying Eq. 4 by $d_m^t x + \beta_m$ and assuming that it is always positive in the decision space, the problem can be further simplified as:

$$\begin{aligned} &\text{Min} \quad \sum\nolimits_m w_m n'_m \\ &\text{s.t.} \quad x \in X_S \\ &c_m^t x + \alpha_m - \left(d_m^t x + \beta_m\right) u_m + n'_m - p'_m = 0 \\ &n'_m, p'_m \geq 0 \end{aligned} \tag{5}$$

The linear form of Eq. 5 is equivalent to the fractional form of Eq. 4 and the following relationship exists between their deviational variables.

$$n'_m = n_m\left(d_m^t x + \beta_m\right); \quad p'_m = p_m\left(d_m^t x + \beta_m\right) \tag{6}$$

For the purpose of searching the solutions in the S that verify all the goals at a given priority level, as shown by Caballero and Hernández (2006) existence or non-existence of such solutions can be deduced by solving Eq. 5.

2.3 Data Sources and Model Description

2.3.1 Study Area

The study area, a portion of the Zayande-Roud river basin in central Iran, is a rural region situated between the north latitudes of 32° 19′ 06″ to 32° 31′ 59″ and east longitudes of 51° 45′ 40″ to 52° 06′ 32″ covering an area of 340.55 km^2 about

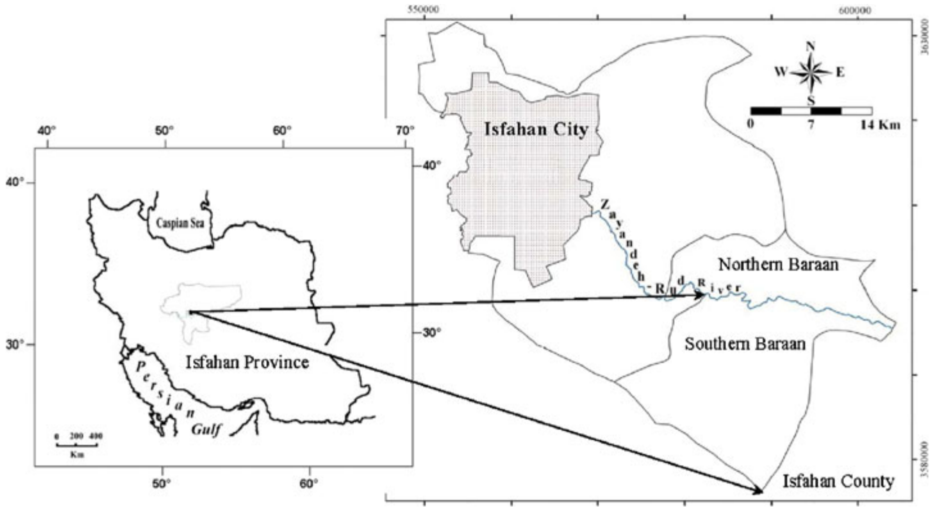

Fig. 2 The map of study area, Baraan rural district, Isfahan county, Isfahan province, Iran

30 km south-east of the provincial capital Isfahan (Fig. 2). The area, named south Baraan, is bounded on the north by Zayandeh-Roud, the largest river in central Iran. It is located in the semi-arid climate, with mean annual temperature of 16°C and annual rainfall ranging between 72.75 and 115.5 mm. The mean elevation of the region is 1550 m above the mean sea level. This rural area covers 22 villages with a total population of 15,210 (about 4,048 households), of which about 10,500 are occupied in agriculture. So, farming is still the main way of life in this region and the driving force for its development (Amini Fasakhodi 2009). The area comprises two distinct morphological units, arable alluvial plain in the northern part along the riverside and the uncultivable mountainous in the southern part. The tableland alluvial plain (about 14,000 ha) includes the most fertile soils of the region from which 12,000 ha currently is under cultivation. Farming practices are usually based on irrigation in two cropping seasons, spring and winter. The major crops in the spring are rice and silage maize whereas for the winter season are wheat, barley and onion, and the common annual crop is alfalfa (which planted in spring season).

2.3.2 Overview of Applied Data Set

The summary of database used for this study is given in Tables 1 and 2. The net return, indicating the marginal revenue of the crops per unit area of farming, was calculated taking into account the potential crop yield, the market price and the cost of production. The corresponding data were collected from the region's center of agricultural services, by interviewing the experts and completing the standard cropping cost–benefit questionnaire (Ministry of Jihad-E-Agriculture 2007). Labor requirements data, for production of the crops per unit area of farming, were also estimated for the planting, crop protection and harvesting periods in a cropping season

Table 1 Coefficients matrix and RHS for attributes and constraints

Objectives and constraints	Activities (main crops of the region)						
	x_1 Wheat	x_2 Barley	x_3 Rice	x_4 Maize	x_5 Alfalfa	x_6 Onion	
Net return ($\times 10^6$ Rs)	8.74	7.01	18.98	29.96	8.77	19.11	Max.
Employment (man-day)	22.39	19.39	71.1	37.29	84.2	137.3	Max.
Total water use ($\times 10^2$ m^3)	48	40.6	151.93	63.24	104.2	60.2	Min.
Land use (ha)	1	1	1	1	1	1	$\leq$12000
Seasonality (Rotation)	+1	+1	−1	−1	−1	+1	$\geq$0
Capital ($\times 10^6$ Rs)	6.06	5.59	23.02	21.04	16.53	35.49	$\leq$160000

separately and added together to calculate the labor force technical coefficients for crops in total cropping season duration (Table 1).

The irrigation water requirements (IWR) of the crops were collected and obtained monthly from two data bases, Farshi et al. (1997) and Alizadeh and Kamali (2007). These data (Table 2) are based on the climatological circumstances and crop calendar of the region. The Penman-Monteith method was used to estimate the potential evapotranspiration (ET_0). Based on the calculated evapotranspiration, the seasonal IWR of crops per unit area of farming was then estimated by adding the monthly IWR of corresponding crops, in order to calculate the total water consumption.

The monthly water resources availability (groundwater and surface water) in the study area (Table 2) were calculated using the records of the regional water organization of the Isfahan (RWOI 2006, 2007b).

2.3.3 The Model

The structure of the study model was formulated as below:

$$\text{Eff.} \quad \left\{ \frac{\sum_i N_i.x_i}{\sum_i W_i.x_i}, \frac{\sum_i Em_i.x_i}{\sum_i W_i.x_i} \right\} \tag{7}$$

Table 2 Coefficients matrix and RHS (right hand sides) for monthly water constraints and water availabilities

Water use ($\times 10^2$ m^3)	Activities (main crops of the region)						RHS
	x_1 Wheat	x_2 Barley	x_3 Rice	x_4 Maize	x_5 Alfalfa	x_6 Onion	
Apr. ($k = 1$)	13.8	13.8	0	0	9.9	12.9	$\leq$97656.78
May ($k = 2$)	15.6	11.3	0	0	3.4	17.1	$\leq$119834.4
June ($k = 3$)	2.2	0	12.32	0	15.8	5.1	$\leq$85770
Jul. ($k = 4$)	0	0	32.28	10.62	16.5	0	$\leq$90216.3
Aug. ($k = 5$)	0	0	33.67	19.4	15.7	0	$\leq$103979.3
Sep. ($k = 6$)	0	0	35.38	20.01	12.6	0	$\leq$96897
Oct. ($k = 7$)	0	0	28.06	13.21	8	4.1	$\leq$62574
Nov. ($k = 8$)	0.9	0.4	4.22	0	4.5	4	$\leq$50488.6
Dec. ($k = 9$)	1.4	1.2	0	0	1.7	2.8	$\leq$26368
Mar. ($k = 12$)	8.1	8.1	0	0	6.1	7.4	$\leq$58863.2

s. t.

$$\sum_i (x_i)_s \leq A \qquad \forall s$$

$$\sum_i \mathrm{IWR}_{ik}.x_i \leq (\eta_a.\mathrm{SW}_k + \eta_b.\mathrm{GW}_k) \quad \forall k$$

$$\sum_i (x_i)_{S1} - \sum_i (x_i)_{S2} \geq 0$$

$$\sum_i c_i.x_i \leq C$$

$$x_i \geq 0; \qquad (\mathrm{i} = 1, 2, \ldots, 6); \qquad (\mathrm{k} = 1, 2, ..., 12) \tag{8}$$

Nomenclature of the subscripts, variables and parameters of the model is:

i	= 1, 2, ..., 6 crop type index
s	= 1, 2 cropping season index
k	= 1, 2, ..., 12 month index
x_i	Allocated land to ith crop (ha)
N_i	Net return of ith crop (10^6 Rs/ha)
Em_i	Labor requirement during the cropping season for ith crop (man-day/ha)
A	Total cultivable area in the region (ha)
c_i	Per unit area cost of production for ith crop (10^6 Rs/ha)
C	Total available capital in the rural region for farming activities (10^6 Rs)
W_i	Net water requirement for ith crop during the cropping season (10^2 m^3/ha)
IWR_{ik}	Net irrigation water requirement for ith crop during kth month (10^2 m^3/ha)
SW_k	Available surface water in the region during the kth month (10^2 m^3)
GW_k	Available groundwater in the region during the kth month (10^2 m^3)
η_a	Irrigation efficiency of surface water at the region (%)
η_b	Field water application efficiency of groundwater at the region (%).

Total available capital, C parameter, was obtained based on the existing pattern of cropping in the region, by solving a calibrated LP model for maximization of the net return in the objective function. In a calibrated LP, the decision variables x_i take the given values as less than or equal to the allocated lands to each of the crops in the existing pattern of cropping and hence considered as constraints. Contrarily, the unknown availability of some production resources in the right hand size (RHS) of the relevant constraints, such as the total capital availability (C) in the present problem, were taken into consideration as decision variables where obtained by solving the model. The net return, N_i, was calculated by taking into account the current market price (10^6 Rs/ton), yield (ton/ha) and cost of production (10^6 Rs/ha) of ith crop.

Problem Constraints Set of the inequalities (8) in the model refers to the system constraints of the problem illustrated as follows:

Land Availability Constraints The sum of lands allocated to various crops in each season must be less than or equal to the total cultivable area in the region during each cropping season, namely

$$x_1 + x_2 + x_6 \leq 12000 \text{ (ha)} \quad \text{for winter } (s = 1) \text{ season,} \tag{9}$$

$$x_3 + x_4 + x_5 \leq 12000 \text{ (ha)} \quad \text{for spring } (s = 2) \text{ season} \tag{10}$$

Monthly Water Requirement Constraints The irrigation water requirements of all crops must be fully satisfied during all the seasons from the available surface water and groundwater resources. The water requirement constraints should be such that the crop water requirements in each month in the study area should be less than equal to that month cumulative water availability for both groundwater and surface resources. Therefore, the monthly water requirement constraints are (referred to the data provided in Table 2):

$$13.8x_1 + 13.8x_2 + 9.9x_5 + 12.9x_6 \leq 97656.78 \tag{11}$$

$$15.6x_1 + 11.3x_2 + 3.4x_5 + 17.1x_6 \leq 119834.4 \tag{12}$$

$$2.2x_1 + 12.32x_3 + 15.8x_5 + 5.1x_6 \leq 85770 \tag{13}$$

$$32.28x_3 + 10.62x_4 + 16.5x_5 \leq 90216.3 \tag{14}$$

$$33.67x_3 + 19.4x_4 + 15.7x_5 \leq 103979.3 \tag{15}$$

$$35.38x_3 + 20.01x_4 + 12.6x_5 \leq 96897 \tag{16}$$

$$28.06x_3 + 13.21x_4 + 8x_5 + 4.1x_6 \leq 62574 \tag{17}$$

$$0.9x_1 + 0.4x_2 + 4.22x_3 + 4.5x_5 + 4x_6 \leq 50488.6 \tag{18}$$

$$1.4x_1 + 1.2x_2 + 1.x_5 + 2.8x_6 \leq 62574 \tag{19}$$

$$8.1x_1 + 8.1x_2 + 6.1x_5 + 7.4x_6 \leq 58863.2 \tag{20}$$

Seasonality Constraint During a farming year, as a planning horizon, all of the lands along the region are not completely allocated to the mentioned crops, so that some of lands left on fallow in the spring season, which water resources encounter with some deficiencies. So, the crop rotation or seasonality of the farming activities will be

$$x_1 + x_2 - x_3 - x_4 - x_5 + x_6 \geq 0 \tag{21}$$

Capital Constraint The total amount of money that can be spent for farming activities must be less than or equal to the total available capital at the region, namely

$$6.06x_1 + 5.59x_2 + 23.02x_3 + 21.04x_4 + 16.53x_5 + 35.49_{x6} \leq 16000 \tag{22}$$

This constraint refers to the restriction of the available capital at the region for these activities, which calculated based on the existing farming situation of the region by solving a calibrated LP model.

Non-Negativity Constraints It is possible not to allocate any area for a crop in an allocation zone, but it is impossible to allocate a negative size of an area for a crop. Therefore, decision variables of the model cannot take negative values.

$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \tag{23}$$

3 Model Applications, Results and Discussion

Based on the above set of system constraints, several linear (A_1, B_1 and C_1 scenarios) and fractional (A_2, B_2 and C_2 scenarios) models with single and multiple objectives in the objective function were formulated and solved using the LINDO package for windows as detailed below. All objectives are considered for a farming year as a planning horizon is divided to two seasons.

3.1 The Objective Functions Formulation

3.1.1 LP Formulation for Maximization of Net Return (A_1 Scenario)

The economic objective like net return maximization is commonly aspired to by every decision maker. However, such objectives are more desired in farming systems and farmers always prefer a cropping pattern which can provide them with more financial returns which can be formulated (referred to the Table 1) as:

$$\text{Max} \quad 8.74x_1 + 7.01x_2 + 18.98x_3 + 29.96x_4 + 8.77x_5 + 19.11x_6 \tag{24}$$

3.1.2 LP Formulation for Maximization of Labor Employment (B_1 Scenario)

Labor-intensive cropping pattern to minimize unemployment as well as under-employment in the agricultural sector, especially in rural areas of under-developed or developing countries, can be considered as a way for promotion of social situation in farming systems which mathematically can be expressed (referred to the Table 1) as:

$$Max \quad 22.39x_1 + 19.39x_2 + 71.1x_3 + 37.29x_4 + 84.2x_5 + 137.3x_6 \tag{25}$$

3.1.3 LGP Formulation for Maximization of Net Return and Labor Employment Simultaneously (C_1 Scenario: LGP)

Based on the pay-off matrix (Table 4), the objective function values (O.F.V.s) of the above two LP models are 207141and 605454 respectively. By considering of these values as aspiration levels for net return (24) and labor employment (25) objectives, the corresponding linear goal programming (LGP) formulation will be:

$$\text{Min} \quad n_1 + n_2 \tag{26}$$

Subject to

$$\left.\begin{array}{l}(24) + n_1 - p_1 - 207141 = 0 \\ (25) + n_2 - p_2 - 605454 = 0\end{array}\right\} \text{(Goal constraints);}$$
$$\begin{array}{ll}(9) - (23) & \text{(System constraints);} \\ n_1, n_2, p_1, p_2 \geq 0 & \text{(System constraints);}\end{array} \tag{27}$$

n_1, n_2, p_1, and p_2 are respectively negative and positive deviational variables with regard to the under- and overachievements of the goals (24) and (25) from their aspiration levels.

3.1.4 FP Formulation for Maximization of "Net Return/Water Consumption" (A_2 Scenario)

In this scenario the economic objective of net return maximization is considered from the sustainability point of view and hence remodeled related to the total amount of water consumption in a farming year as a most determinant environmental resource. So the problem can be formulate as a linear fractional programming to optimize the ratio of net return/water consumption. Such ratio, in fact maximizes the profit in lieu of unit of water use. Mathematically (referred to the Table 1 technical coefficients):

$$\text{Max} \quad \frac{8.74x_1 + 7.01x_2 + 18.98x_3 + 29.96x_4 + 8.77x_5 + 19.11x_6}{48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6} \tag{28}$$

Subject to

System constraints: (9)–(23).

Based on the procedure described in the "Section 2.1" for linearization of the above ratio objective function and the structure of transformed constraints, by considering of variable transformations $t = 1/(48x_1 + 40.6x_2 + 151.9x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6)$ and $y_t = t.x_i$, the equivalent linear model will be:

$$Max \quad 8.74y_1 + 7.01y_2 + 18.98y_3 + 29.96y_4 + 8.77y_5 + 19.11y_6 \tag{29}$$

s.t.

$$\begin{array}{l}
y_1 + y_2 + y_6 - 12000t \leq 0 \\
y_3 + y_4 + y_5 - 12000t \leq 0 \\
13.8y_1 + 13.8y_2 + 9.9y_5 + 12.9y_6 - 97656.78t \leq 0 \\
\vdots \quad \text{transformed } (12) - (19) \\
8.1y_1 + 8.1y_2 + 6.1y_5 + 7.4y_6 - 58863.2t \leq 0 \\
y_1 + y_2 - y_3 - y_4 - y_5 + y_6 \geq 0 \\
6.06y_1 + 5.59y_2 + 23.02y_3 + 21.04y_4 + 16.53y_5 + 35.49y_1 - 16000t \leq 0 \\
48y_1 + 40.6y_2 + 151.93y_3 + 63.24y_4 + 104.2y_5 + 60.2y_6 = 1 \\
y_1, y_2, y_3, y_4, y_5, y_6, t \geq 0
\end{array} \tag{30}$$

Finally, the solution for pattern of cropping will be obtain through $x_i = \frac{y_1}{t}$.

3.1.5 FP Formulation for Maximization of "Labor Employment/Water Consumption" (B_2 Scenario)

The social objective of labor employment maximization also from the sustainability point of view and related to the total of water consumption as a most determinant environmental resource, remodeled as a linear fractional programming to optimize the ratio of labor employment/water consumption. The relevant FP model (referred to the Table 1) is:

$$Max \quad \frac{22.39x_1 + 19.39x_2 + 71.1x_3 + 37.29x_4 + 84.2x_5 + 137.3x_6}{48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6} \tag{31}$$

Subject to

System constraints: (9)–(23).

Similar to the previous linearization description, the equivalent LP model for this problem is also as bellow:

$$Max \quad 22.39y_1 + 19.39y_2 + 71.1y_3 + 37.29y_4 + 84.2y_5 + 137.3y_6 \tag{32}$$

Subject to

Set of constraints: (30).

Variable transformations of this model are the same of the previous one and calculation of the final solution for pattern of cropping is also the same.

3.1.6 FGP Formulation for Simultaneous Maximization of "Net Return/Water Consumption" and "Labor Employment/Water Consumption" (C_2 Scenario: MOFGP)

Optimizing of two above ratios together, as the sustainability indicators in a farming system, leads to a multi-objective fractional programming (MOFP) problem as the main approach of this study which can mathematically be expressed as:

$$\text{Eff.} \left\{ \frac{8.74x_1 + 7.01x_2 + 18.98x_3 + 29.96x_4 + 8.77x_5 + 19.11x_6}{48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6}, \right.$$
$$\left. \frac{22.39x_1 + 19.39x_2 + 71.1x_3 + 37.29x_4 + 84.2x_5 + 137x_6}{48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6} \right\} \tag{33}$$

Subject to

System constraints: (9)–(23).

The reasons behind the formulation of this kind of objectives (fractional objectives) for maximization are: (1) This type of ratios convey more information about the sustainability situation of the farming system and consequently water resources. (2) Maximization of this type of ratios will lead to the maximum or near maximum value of the net return or labor employment accompanied by the minimum or near minimum value of the water resources withdrawal simultaneously. Towards the sustainability of the whole system, this is more advantageous than maximizing the net return (labor employment) or minimizing the water consumption separately.

In this paper, the GP methodology has been considered in order to formulation and solving of this problem. So, the C_2 scenario presents the multi-objective fractional goal programming (MOFGP) as the main approach of the study in order to assessing the sustainability of water resources in a farming system for better management.

The objective function values (O.F.V.s) of the single objective FP models for "net return/water consumption" (A_2 scenario) and "labor employment/water consumption" (B_2 scenario) are 0.348 and 0.787 respectively, as depicted in the pay-off matrix (Table 4). Physically these ratios mean that the earned benefit and employment in lieu of consuming a unit (1 m^3) of water resources are 0.348 (10^6 Rs) and 0.787 (man-day), respectively. By considering of these values as aspiration levels for "net return/water consumption" (28) and "labor employment/water consumption" (31) objectives, the fractional goal programming model for C_2 scenario will based on the Eq. 4 described in material and methods section be:

$$\text{Min} \quad n_1 + n_2 \tag{34}$$

Subject to

$$\left.\begin{array}{l}(28) + n_1 - p_1 - 0.348 = 0\\(31) + n_2 - p_2 - 0.787 = 0\end{array}\right\} \text{(Goal constraints);}$$
$$(9) - (23) \qquad \text{(System constraints);} \tag{35}$$
$$n_1, n_2, p_1, p_2 \geq 0 \qquad \text{(Non-negativity constraints).}$$

n_1, n_2, p_1, and p_2 are respectively negative and positive deviational variables with regard to the under- and overachievements of the fractional goals (28) and (31) from their aspiration levels.

Based on the Eq. 5 linearization procedure related to the above non-linear goal constraints, the equivalent final Linear GP model obtained as:

$$\text{Min} \quad n'_1 + n'_2 \tag{36}$$

Subject to

$$(24) - 0.348(48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6) + n'_1 - p'_1 = 0$$
$$(25) - 0.787(48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6) + n'_2 - p'_2 = 0$$
$$(9) - (23)$$
$$n'_1, n'_2, p'_1, p'_2 \geq 0. \tag{37}$$

There exist the following relationships between the deviational variables of two above linear and non-linear models.

$$\begin{array}{l}n'_1 = n_i(48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6)\\p'_1 = p_i(48x_1 + 40.6x_2 + 151.93x_3 + 63.24x_4 + 104.2x_5 + 60.2x_6)\end{array} i = 12 \tag{38}$$

3.2 Results and Discussion

By solving the described scenarios for economical, social and sustainability-oriented objectives with a similar structure of constraints (Eqs. 9–23), several patterns of cropping were obtained as detailed in Table 3, indicating the allocated lands to

Table 3 Extreme efficient points (cropping patterns) obtained from linear and fractional models in cases of single and multiple objectives

Efficient solutions	Activities					
	x_1	x_2	x_3	x_4	x_5	x_6
Current	5000	500	1000	3000	1000	1500
A_1	6546	0	260	4478	0	567
B_1	0	3703	825	1163	3012	1296
C_1	1497	2224	741	1247	3011	1279
A_2	0	4621	0	4407	0	1089
B_2	0	5365	440	2803	0	1380
C_2	0	4434	0	4339	0	1243

x_1 wheat, x_2 barley, x_3 rice, x_4 silage maize, x_5 alfalfa, x_6 onion, *current* existing pattern of cropping, A_1 LP model to optimize 'net return' objective, B_1 LP model to optimize 'labor employment' objective, C_1 LGP model to optimize 'net return' and 'labor employment' objectives simultaneously, A_2 FP model to optimize 'net return/water use' objective, B_2 FP model to optimize 'labor employment/water use' objective, C_2 FGP model to optimize 'net return/water use' and 'labor employment/water use' objectives simultaneously

different crops in each pattern. In order to assess the sustainability of each of these patterns, the ratios of "net return/water consumption" and "labor employment/water consumption" were defined and computed as the sustainability indicators. The amounts of net return, employment creation, water consumption and the above mentioned sustainability indicators were also calculated and presented in Table 4. The last two columns in this table show the measured values of the two fractional objectives.

In order to assess the advantage of fractional programming solutions (cropping patterns) over the linear programming ones for system sustainability, their measured indicators were compared in Table 5. As the results show, in the individual optimization of the economical objective, the sustainability indicators of "net return/water consumption" and "employment/water consumption", increased by

Table 4 Pay-off matrix, net return, employment, water use and measures of sustainability indicators of cropping patterns obtained from single and multi-objectives linear and fractional models

Efficient solutions	Net return	Employment (man-day)	Water use	Net return/ water use	Employment/ water use
Current	193500	594765	796450	0.243	0.747
A_1	207141	409908	671039	0.308	0.610
B_1	127647	605454	741109	0.172	0.817
C_1	130966	604956	744401	0.176	0.812
A_2	185240	403499	531875	0.348	0.758
B_2	156310	429310	545006	0.287	0.787
C_2	184828	418477	529241	0.349	0.790

x_1 wheat, x_2 barley, x_3 rice, x_4 silage maize, x_5 alfalfa x_6 onion, *current* Existing pattern of cropping; A_1 LP model to optimize 'net return' objective, B_1 LP model to optimize 'labor employment' objective, C_1 LGP model to optimize 'net return' and 'labor employment' objectives simultaneously, A_2 FP model to optimize 'net return/water use' objective, B_2 FP model to optimize 'labor employment/water use' objective, C_2 FGP model to optimize 'net return/water use' and 'labor employment/water use' objectives simultaneously

Table 5 Increase (decrease) of farming system sustainability indicators in FP optimization solutions compared to the LP solutions

Sustainability indicators	Economical objective (A_2/A_1)	Social objective (B_2/B_1)	Multiple objectives (C_2/C_1)
Net return/water use	12.98	66.8	98.3
Employment/water use	24.2	(3.6)	(2.7)

A_1; A_2; B_1; B_2; C_1; C_2: Are the same as the Tables 3 and 4
Values are percentage (difference of FP and LP measures divided by LP measures and multiplied by 100)

12.9% and 24.2% due to the solving of fractional programming model compared to the linear programming. In the individual optimization of the social employment objective and also the multiple objective programming context, though the "employment/water consumption" indicator slightly decreased by 3.6% and 2.7%, the other indicator "net return/water consumption" increased by 66.8, and approximately two times more (98.3%) in the case of fractional programming compared to linear one (Table 5). Figure 3 also shows these indicators (fractional objectives) for single and multiple objective models analogically in two cases of linear and fractional programming procedures.

Compared to the current situations and existing pattern of cropping (row 3 of Tables 3 and 4), the simultaneous improvement in both indicators of sustainability (calculated and reported in Table 6) has occurred only due to the fractional programming models. From the efficiency point of view, none of the points A_1, B_1 and C_1 LP solutions dominated the current situation according to both sustainability indicators (Table 6). Therefore, they were considered as technically non-efficient solutions. In contrast, A_2, B_2 and C_2 FP solutions are more efficient than the current situation. Although in single and even multiple linear programming frameworks, solutions such as A_1, B_1 and C_1 might be considered appropriate and eligible, their sustainability competence is very doubtful, as there exist solutions that are more efficient in term of net return and employment creation per unit of water resources consumption. Such solutions could be sought using fractional programming procedures as cited here. Additionally, in fractional programming context, C_2, which corresponds to the multi-objective fractional goal programming (MOFGP) procedure, dominates both A_2 and B_2 which correspond to the single objective fractional programming models

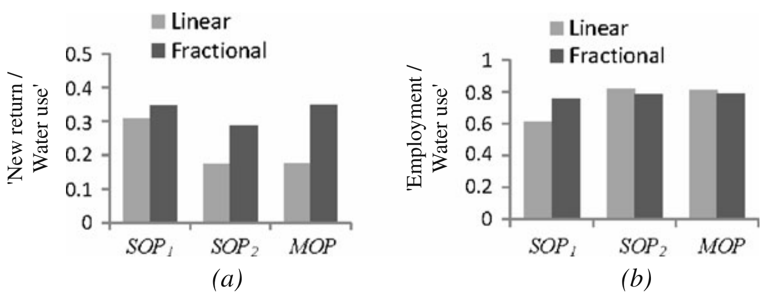

Fig. 3 Measures of sustainability indicators net return (**a**) and employment (**b**) per water consumption in linear and fractional models. SOP_1: Single objective programming models A1 and A2; SOP_2: Single objective programming models B1 and B2; MOP: Multi-objective programming models C1 and C2

Table 6 Dominance of FP solutions over the LP solutions compared to the existing pattern of cropping from the water resources sustainability indicators point of view (values are percentage)

Sustainability indicators	FP solutions			LP solutions		
	Economical objective (A_2/current)	Social objective (B_2/current)	Multiple objectives (C_2/current)	Economical objective (A_1/current)	Social objective (B_1/current)	Multiple objectives (C_1/current)
Net return/ water use	43.2	18.1	43.62	26.74	(29.2)	(27.57)
Employment/ water use	1.47	5.35	5.75	(18.34)	9.37	8.7

Current; A_1; A_2; B_1; B_2; C_1; C_2: Are the same as the Tables 3 and 4
Figures in parenthesis indicate non-dominance or decrease percentage of measures

through sustainability indicators. Results of this comparison are also presented in Table 7.

Sustainability of water resources in agricultural systems is a complex issue as it depends on various interdependent aspects. The adequate levels of net return and employment in farms are also the essential economic and social outputs required to ensure the sustainability and maintain the population of the farming system. Though economic development is in principle desirable, it often entails environmental depletion leading to a trade-off between environmental sustainability and economic development. To fully understand the linkages between agricultural production, income generation, employment creation and environmental sustainability, it is needed to examine the interdependence of such socio-economic attributes and how they affect sustainability.

Fractional programming (FP) approach, introduced and examined in this study, outperforms MCDM framework and is more suitable for studying sustainability problems. When the quantitative managing of the inputs and outputs of an agricultural system is at the core of concern, “ratios are a natural and more comprehensive way of dealing with the issues related with the sustainability of systems” (Lara and Stancu-Minasian 1999). In addition, ratios and the FP procedures facilitate the assessment of the solutions as explained and cited previously, compared to the LP and GP procedures. As the results showed, substitution of some excessive water consuming crops with the other less water consuming and also socio-economically beneficial ones in the cropping pattern leads the farming system of the region to a more sustainable situation. In this way, durability of the region’s water resources and consequently long lasting development of farming system could explicitly be supported and encouraged.

Table 7 Dominance of multi-objective over the single objective programming from the water resources sustainability measures point of view in FP solutions

Sustainability indicators	Multi-objectives programming compared to	
	Single economical objective (C_2/A_2)	Single social objective (C_2/B_2)
Net return/water use	0.28	4.22
Employment/water use	21.6	0.38

A_2; B_2; C_2: Are the same as the Tables 3 and 4
Values are percentage

4 Conclusions

This paper presents a regional scale problem about water resources management and consequently cropping pattern planning. Cropping pattern planning involves a complex set of interrelated environmental and socio-economic criteria, which are inherently conflicting and inconsistent. In order to consider and include the water resources sustainability in the cropping pattern planning, we are concerned with a special type of multi-objective programming problem where objective functions are of linear fractional structure. This kind of problem has many applications. In this paper, in addition to using the LP models to optimize the net return and labor employment objectives separately, and a linear GP model to optimize these objectives simultaneously, two fractional objectives in the cases of "net return/water consumption" and "employment/water consumption" ratios were also defined and optimized as sustainability indicators. To optimize these fractional objectives separately, two single objective linear fractional programming models were developed and a multi-objective fractional goal programming (MOFGP) procedure was also formulated for optimizing these sustainability indicators simultaneously as the main study approach. The advantages and appropriateness of FP models in contributing to the sustainability indicators compared to the LP models, and also MOFGP model cropping pattern solution compared to the single objective FP model solutions were discussed in detail and clarified quantitatively. Such patterns of cropping on the regional scale can achieve the socio-economic objectives of maximization of labor employment and income of farmers based on the available water resources, keeping them in farming systems especially in rural areas and additionally, reducing the associated environmental impacts of farming activities by more efficient use and management of water resources. Implementation of these managerial efforts in the whole region needs extensional contributions to make farmers adopt the changes.

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