# **Robust Methods for Identifying Optimal Reservoir Operation Strategies Using Deterministic and Stochastic Formulations**

Azadeh Ahmadi · Mohammad Karamouz · Ali Moridi

Received: 27 September 2008 / Accepted: 16 December 2009 / Published online: 5 January 2010 © Springer Science+Business Media B.V. 2009

Abstract Water allocation in a competing environment is a major social and economic challenge especially in water stressed semi-arid regions. In developing countries the end users are represented by the water sectors in most parts and conflict over water is resolved at the agency level. In this paper, two reservoir operation optimization models for water allocation to different users are presented. The objective functions of both models are based on the Nash Bargaining Theory which can incorporate the utility functions of the water users and the stakeholders as well as their relative authorities on the water allocation process. The first model is called GA-KNN (Genetic Algorithm-K Nearest Neighborhood) optimization model. In this model, in order to expedite the convergence process of GA, a KNN scheme for estimating initial solutions is used. Also KNN is utilized to develop the operating rules in each month based on the derived optimization results. The second model is called the Bayesian Stochastic GA (BSGA) optimization model. This model considers the joint probability distribution of inflow and its forecast to the reservoir. In this way, the intrinsic and forecast uncertainties of inflow to the reservoir are incorporated. In order to test the proposed models, they are applied to the Satarkhan reservoir system in the north-western part of Iran. The models have

A. Ahmadi

Department of Civil Engineering, Isfahan University of Technology, Isfahan, Iran e-mail: aahmadi@cc.iut.ac.ir

M. Karamouz (⊠) School of Civil Engineering, University of Tehran, Tehran, Iran e-mail: karamouz@ut.ac.ir

M. Karamouz Polytechnic Institute of NYU, Brooklyn, NY, USA e-mail: mkaramou@poly.edu

A. Moridi Water Engineering Research Center, Tarbiat Modares University, Tehran, Iran e-mail: moridi@cic.aut.ac.ir unique features in incorporating uncertainties, facilitating the convergence process of GA, and handling finer state variable discretization and utilizing reliability based utility functions for water user sectors. They are compared with the alternative models. Comparisons show the significant value of the proposed models in reservoir operation and supplying the demands of different water users.

**Keywords** Reservoir operation • Optimization model • Genetic Algorithm • Bayesian Decision Theory • Inflow uncertainty • Water allocation

### 1 Introduction

Real-time operation of reservoir systems requires specific operating rules. These rules are guides for water conservation and release policies which conserve water for future use and maintain flood control capabilities. Inflow to a reservoir is the most important source of uncertainty in the reservoir operation and development of operating policies for a system. Deterministic models have been widely used for modeling complex and large-scale water resources systems. In these models, the uncertainty is sometimes implicitly incorporated using long time generated synthetic time series of uncertain inflows that include the worst case scenario of droughts and floods experienced during the historical record. However, stochastic reservoir operation models can be formulated to incorporate uncertainty in inflows and the forecast model explicitly.

Forecasts of streamflow have been incorporated in reservoir operation models, and some investigators have explicitly recognized forecast uncertainty. Karamouz and Houck (1987) developed an algorithm that consisted of a deterministic dynamic program with a regression analysis to consider the inflow uncertainty implicitly. Stedinger et al. (1984), Trezos and Yeh (1987) and Kelman et al. (1990) have tried to incorporate forecast uncertainty in the reservoir operation. Eiger and Shamir (1991) developed a model for optimal multi-period operation of a multi- reservoir system with uncertain inflows and water demands formulated and solved by the finite generation algorithm. Uncertainties are considered in chance constraints and the stochastic variables are assigned discrete probability distributions. Karamouz and Vasiliadis (1992) proposed a Bayesian Stochastic Dynamic Programming (BSDP) method, which includes inflow, storage and forecast as state variables, and used Bayesian Decision Theory (BDT) to incorporate new information by updating the prior probabilities to posterior probabilities, to generate reservoir operating rules.

Seifi and Hipel (2001) proposed a method for long-term reservoir operation planning with stochastic inflows. The problem is formulated as a two-stage stochastic linear program and solved with interior-point optimization algorithm. Chang et al. (2002) developed an optimal regulation program; grey fuzzy stochastic dynamic programming (GFSDP), for reservoir operation. Karamouz and Mousavi (2003) developed a Fuzzy Stochastic Dynamic Programming (FSDP) model, where a form of the fuzzy Markov chain was defined, and then fuzzy transition probabilities were calculated to determine the expected value of the objective function. Celeste et al. (2008) proposed a procedure to incorporate streamflow uncertainty by means of stochastic and deterministic optimization models. A monthly operation model by an explicit stochastic programming approach is first solved and then its information is used to guide the daily operation, which is solved by deterministic optimization.

Classical optimization techniques are useful tools for solving many reservoir operation problems; however, the computational requirements are too burdensome in many instances. Computational burden and representation of the problem within an optimization solver has been a hurdle in solving many complex multiple reservoir operation problems characterized by a large number of decision variables (Teegavarapu and Simonovic 2001). There are numerous techniques that can guarantee global optimal solutions, for optimal reservoir operation in water resources literature. Genetic Algorithms (GA) have been widely used in recent years for optimal planning and operation of water resources systems. East and Hall (1994) used GA to maximize the benefits from power generation and irrigation water supply subject to constraints on storage and releases from a four-reservoir system using GA. Wardlaw and Sharif (1999) evaluated GAs for optimal reservoir system operation with a view to presenting fundamental guidelines for implementation of the approach to practical problems. Pelikan et al. (2000) proposed an algorithm that uses an estimation of the joint distribution of promising solutions in order to generate new candidate solutions in GA. The proposed algorithm is called the Bayesian Optimization Algorithm (BOA). Burn and Yulianti (2001) have shown the capabilities of GAs for identifying solutions for classical waste-load allocation problems. They demonstrated that GA is an effective solution technique for solving a number of optimization problems. The approach can handle discrete decision variables and can efficiently identify the trade-off relationship that exists for a multiobjective optimization problem. Kerachian and Karamouz (2006) used an algorithm combining a water quality simulation model and a stochastic conflict resolution GA-based optimization technique for determining optimal reservoir operating rules. In their model, the basic structure of SDP was used in incorporating the inflow state transition probabilities.

GAs employ a random, yet directed search for locating the globally optimal solution. They are superior to gradient descent techniques as the search is not biased towards the locally optimal solution. One characteristics of GAs is the ability to converge to an optimum solution (local or global), after locating the region containing the optimum. A shortcoming of GA is the long convergence time for optimization problems especially with many decision variables. In that case, sometimes it is impossible to reach the global optimum in a reasonable run time.

One of the goals of recent research on GA is to prevent it from getting stuck at a local optimum by approaching adaptive GA. Srinivas and Patnaik (1994) have recommended the use of adaptive probabilities of crossover and mutation to realize the maintaining diversity in the population and sustaining the convergence capacity of the GA. Herrera and Lozano (2003) have developed an adaptive GA based on fuzzy techniques to include suitable exploitation/exploration relationships for avoiding the premature convergence problem. Mei-yi et al. (2004) proposed an adaptive GA with diversity-guided mutation which combines adaptive probabilities of crossover and mutation.

In this study, due to considering the uncertainties of inflow and forecasted flow to the reservoir, the number of decision variables is high. In order to overcome the computational complexity and reduce the run time of the GA model, the KNN (K Nearest Neighborhood) model is used to implement an adaptive GA. In this method a flexible learning processes of the system's past performance is provided through the so called "Nearest Neighborhood". This way, a feasible solution could be generated based on what was observed/repeated in the past considering the present system's attributes to search the space for the region containing the global optimum. It offers a unique opportunity to come up with an initial estimate of the system's behavior in an environment like GA to expedite the model convergence process. The KNN model is a nonparametric estimation of probability densities and regression functions through weighted average of the dependent variables. Karlsson and Yakowitz (1987) developed this method for time series analysis and estimating parameters. Soon after successful applications were used in hydrologic engineering (Galeati 1990; Kember and Flower 1993; Todini 2000; Araghinejad et al. 2006; Bannayan and Hoogenboom 2008). Ostfeld and Salomons (2005) developed a methodology for calibration of a water quality model using the combination of a hurdle-race and a hybrid GA-KNN. The hurdle race is used for accepting/rejecting a proposed set of model parameters during simulation; the KNN for approximating the objective function response surface and the GA for linking both.

All of the above techniques and methods are well cited in literature; however, there are only a few real world case studies in the literature that have combined different aspects of these planning tools. In this paper, these techniques are combined in an integrated fashion in two deterministic and stochastic models for the reservoir operation. The objective function is reliability based and it is treated in the context of the Nash Bargaining Theory for resolving conflict among water users/stakeholders [please refer to Karamouz et al. (2003) for more details about Nash Bargaining Theory].

In this study, the two developed models (deterministic and stochastic models) for reservoir operation are applied to the Satarkhan reservoir which is located in the north-western part of Iran. In the deterministic model, a KNN estimator is used with the Genetic Algorithm to derive reservoir operating policies for each month. This is an adaptive approach to facilitate the model convergence process. The objective of the stochastic model is to develop an optimal water release scheme from the reservoir considering the uncertainty of inflow and its forecast. The stochastic model uses statistical descriptions of the streamflow and forecast in a Bayesian Genetic Algorithm framework to obtain the operating policies. These models are applied to the case study and the results are compared with the alternative models and with each other utilizing 17 years of historical and 50 years of generated monthly inflow time series. This paper is organized as follows: In Section 2, the methodology of the proposed models and the model formulations are explained. In Section 3, the case study is introduced; the characteristics of the reservoir, the assumptions and parameters in implementation of each model are given. The results are discussed in Section 4 followed by a summary and conclusion.

#### 2 Methodology

In this study, two reliability based deterministic and stochastic optimization models solved by Genetic Algorithm (GA) for reservoir operation have been developed. Genetic Algorithm is an adaptive method trying to imitate the biological and genetic process and can successfully be applied to optimization problems.

GA is a population of individuals, named chromosomes. Each chromosome represents a potential solution to a problem. This solution is evaluated by its fitness function. As each chromosome represents a potential solution, the fitness of each chromosome as a candidate solution, must be evaluated by a random search process to form the decision space. Additionally, fitness is a function of the problem constraints, and must be satisfied. Through successive generation, fitness should progressively be improved towards an optimum solution. The new population is generated using the genetic operators including Selection, Crossover, and Mutation. More details of genetic algorithms can be obtained in the works of Michalewicz (1992) and Gen and Chang (2000). Genetic algorithms usually consist of the following steps:

- 1. Encoding of the decision variables and placing them in a chromosome.
- 2. Creating an initial population (first generation).
- 3. Determination of fitness for every chromosome in the current population (fitness evaluation).
- 4. Setting the probability for mutation and crossover.
- 5. The selection of better chromosomes for matching and running a cross over operator for shuffling the selected chromosomes.
- 6. Performing mutation for selected chromosomes.
- 7. Repeat steps 3–6 to obtain the optimal or near optimal solutions.

In general, GA starts with a population of chromosomes and later modifies them through genetic operators to produce better fitting chromosomes. The main fields of application of GAs include problems with high complexity and non-linear behavior. In the varying length GA (VLGA) model, proposed by Kerachian and Karamouz (2006), the initial value for each new gene is equal to the average value of the corresponding genes in the previous years that are obtained from the last optimization sequence.

In this study, a VLGA model is extended in both deterministic and stochastic models. In the deterministic model, in the first stage a KNN model is used as a simulation model to estimate the initial value for new chromosomes from the last optimization sequence. In this model, the chromosome length is increased sequentially based on KNN estimates with respect to the reservoir inflow, the reservoir storage and the water demands in each month. In the second stage, the KNN model is used to develop the reservoir operating rules utilizing the optimization policies obtained from the deterministic model.

In the stochastic model a Bayesian Stochastic GA (BSGA) optimization model is developed which uses the general framework of the Bayesian Stochastic Dynamic Programming (BSDP) model, proposed by Karamouz and Vasiliadis (1992). The BSGA model uses Bayesian Decision Theory (BDT) to update prior to posterior probabilities. The operating policy designated by this model is a set of rules specifying the storage levels at the beginning of the next month based on reservoir storage and inflow of each month, and the forecasted flow for the next month.

The Nash Bargaining Theory is used as the objective functions of both optimization models that includes the players' preference in a reliability form (presented by a utility function), as well as their disagreement point and the individual risk taking attitudes in the decision process. For more details about the Nash Bargaining Theory, the readers are referred to Karamouz et al. (2003). Different sectors are considered including environmental, agricultural, industrial, and domestic water supply sectors and regional water authority.

The deterministic and stochastic models are developed using 17 years of historical data and tested with a generated time series of inflow to the reservoir. In order to compare the two models, the performance indices such as reliability, resiliency and vulnerability proposed by Hashimoto et al. (1982) and used by others (see Karamouz et al. 2003) are utilized. The performance indices help to assess the operation of the reservoir under different modeling conditions.

# 2.1 Deterministic Model

There are many concepts and methods to find the consensus among players (water users). One might consider the problem as a multi-objective optimization problem with the objectives of the different decision-makers. Conflict situations can also be modeled as social choice problems in which the rankings of the decision-makers are taken into account in the final decision. Another way of resolving conflicts was offered by Nash, who considered a certain set of conditions the solution has to satisfy, and proved that exactly one solution satisfies his "fairness" requirements.

The following assumptions are underlying the Nash theory:

- In the case when the decision-makers are unable to reach an agreement, all decision-makers will get low objective function values.
- The solution has to provide at least the disagreement outcome to all decisionmakers. The feasibility condition requires that the decision-makers cannot get more than the amount available.
- No decision-maker would agree to an outcome that is worse than the amount he/she would get anyway without the agreement.
- The solution has to be non-dominated. It shows there is no better possibility available for all.
- The solution must not depend on unfavorable alternatives. It shows if certain possibilities become infeasible but the solution remains feasible, then the solution must not change.
- Increasing linear transformation should not alter the solution. If any of the decision-makers change the unit of their objective, then a linear transformation is performed on the criteria-space.
- If two decision-makers have equal positions in the definition of the conflict then they must get equal objective values at the solution. It shows a certain kind of fairness stating that if two decision-makers have the same outcome possibilities and same disagreement outcome, then there is no reason to distinguish among them in the final solution.

The most common solutions are the Nash solution (Nash 1950), and the nonsymmetric Nash solution (Harsanyi and Selten 1972). Varian (1995) demonstrated a brief discussion of the Game theory and Nash solution of the economic based problems. Richards and Singh (1997) used the Nash Bargaining Theory to reach a compromise for water allocations. Ganji et al. (2007) used the game theory to consider the associated preference among different consumers due to limited water. In these researches, the uncertainties of the inflow and forecasted flow to the reservoir are not considered. In this study, this shortcoming has been realized and a state transition probability is considered to incorporate both natural inflow and forecast model uncertainties in a stochastic model. In order to evaluate the results, a deterministic optimization model is also developed to provide the optimal monthly release from the reservoir during the planning horizon based on the Nash objective function. In the model formulation, the objective function is the multiplication of the utility functions of different water users seeking different reliability levels considering their disagreement points. The disagreement point is the minimum acceptable share by each player. It is an element of the payoff space, which is assigned as d = (da, di, dd, de, ds), and reflects the lowest level of acceptable payoff for players. The model formulation is as follows:

Maximize 
$$Z = \prod_{t=1}^{12^*Y} \begin{bmatrix} (fa(A_t) - da)^{wa} \times (fi(In_t) - di)^{wi} \times (fd(D_t) - dd)^{wd} \\ \times (fe(E_t) - de)^{we} \times (fs(S_{t+1}) - ds)^{ws} \end{bmatrix}$$
 (1)

Subject to:

$$A_t = \frac{ra_t}{Da_t} \qquad t = 1, \dots, 12 \times Y \tag{2}$$

$$In_t = \frac{ri_t}{Di_t} \tag{3}$$

$$D_t = \frac{rd_t}{Dd_t} \tag{4}$$

$$E_t = \frac{re_t}{De_t} \tag{5}$$

$$R_t = ra_t + ri_t + rd_t + re_t \quad \forall t \tag{6}$$

$$S_{t+1} = S_t + I_t - R_t - L_t \qquad t = 1, \dots, 12 \times Y$$
(7)

$$0 \le R_t \le R^{Max}(S_t) \quad \forall t \tag{8}$$

$$S^{Min} \le S_{t+1} \le S^{Max} \tag{9}$$

where:

fa()/da	Utility function/disagreement point related to the reliability of allocated water to agricultural (a) demand
fi()/di	Utility function/disagreement point related to the reliability of allocated water to industrial (i) demand
fd()/dd	Utility function/disagreement point related to the reliability of allocated water to domestic (d) demand
fe()/de	Utility function/disagreement point related to the reliability of allocated water to environmental (e) demand
fs()/ds	Utility function/disagreement point related to the reservoir storage (s)
$Da_t, Di_t, Dd_t, De_t$	Demands of agricultural, industrial, domestic and environmen- tal sectors during time period <i>t</i> , respectively (MCM)
$ra_t, ri_t, rd_t, re_t$	Allocated water to agricultural, industrial, domestic and environmental sectors during time period <i>t</i> , respectively (MCM)

$A_t, In_t, D_t, E_t$	Reliability of meeting the demand for agricultural, industrial,
	domestic and environmental sectors during time period $t$ ,
	respectively
wa, wi, wd, we	Relative authority or weight of the decision-makers/
	stakeholders which are in charge of agricultural, industrial,
	domestic and environmental demands, and water storage
Y	Time horizon of optimization model (years)
$S_t$	Reservoir storage at the beginning of time period $t$ (MCM)
$R_t$	Total release during time period t (MCM)
$I_t$	Inflow in time period t (MCM)
$L_t$	Total loss during time period t due to evaporation and infil-
	tration (MCM)
$R^{Max}(S_t)$	Allowable release during time period $t$ considering reservoir
	storage (MCM)
$S^{Min}$	Minimum storage of the reservoir (MCM)
$S^{Max}$	Maximum storage of the reservoir (MCM)

Considering the computational complexity of the problem, in this study, the VLGA model proposed by Kerachian and Karamouz (2006) is extended to include a K Nearest Neighborhood (KNN) estimator for generating the initial solution. This model is referred to as GA-KNN optimization of the reservoir operation model. In this model, the number of genes (chromosome length) is sequentially increased to effectively lead to the initial feasible solutions for a near global optimal solution. Figure 1 shows the schematic of a chromosome of the GA-KNN model. The gene values are the monthly release from the reservoir and the allocated water to each sector. A small record of inflow is selected first and the optimal monthly allocated water to each water user is obtained using a GA optimization model. Then the KNN model estimates the new chromosomes based on the optimum solution of the previous sequence. In other words, the KNN algorithm is used to find the previously observed similar months with respect to the inflow to the reservoir, the reservoir storage at the beginning of each month and the water demand during each month. Then it generates the initial solutions for each year. Figure 2 shows the flowchart of the GA-KNN model.

The K Nearest Neighborhood method is a non-parametric regression methodology, which uses the similarity (neighboring in the sense of numerical closeness) between observations of predictors and K similar sets of past observations to obtain the best estimate for a dependent variable. K vectors of the past observations are

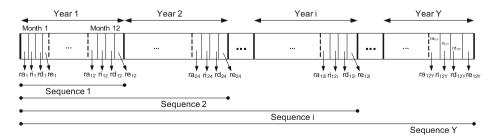
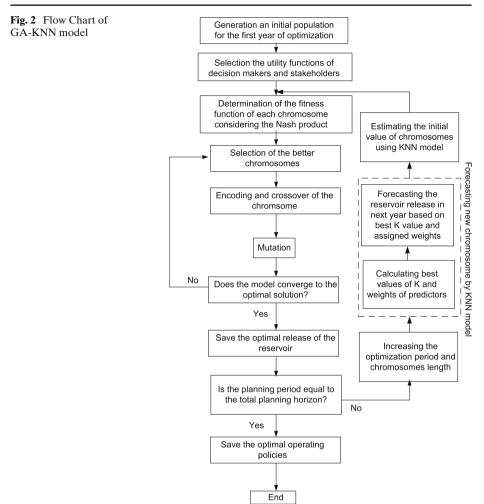


Fig. 1 Schematic of a chromosome of GA-KNN model



recognized that have the minimum Euclidean norm from the present condition among all candidates. The distance between the current and past observed condition is calculated by the Euclidian distance (Karlsson and Yakowitz 1987) between current and historical predictors. A short summary of this method is as follows:

For each forecast  $Z_t$  at time t, let  $P_t^m$  be an *m*-dimensional attribute vector of past records. To estimate a dependent variable,  $(Z_t)$ , the KNN method calculates the distance between current vectors and the set of K past nearest Neighborhoods of  $P_t^m$ . Those  $P_t^m$  are selected that have the minimum norm  $||P_t^m - P_r^m||$  among all candidates, where  $P_r^m$  is an *m*-dimensional attribute vector of the rth nearest data. KNN models choose the most common values of  $Z_r$  among the K training examples nearest to  $Z_t$ . The most widely used distance to identify neighborhoods is the Euclidean norm, which for an m-dimensional feature vector is calculated as:

$$Dist_{t,r} = \|P_t^m - P_r^m\| = \sqrt{w_1 \left(p_t^1 - p_r^1\right)^2 + w_2 \left(p_t^2 - p_r^2\right)^2 + \dots + w_m \left(P_t^m - P_r^m\right)^2}$$
(10)

🖉 Springer

where  $Dist_{t,r}$  is the Euclidean norm from past observations  $(P_t^m)$  to the rth nearest data. *m* is total number of predictors,  $w_m$  is the weight of each predictor in calculating the distance between the current attribute vectors and the Neighborhoods according to their distance. The estimate is then obtained as a weighted average of the nearest neighborhoods, in the way that greater weight is given to closer Neighborhoods. The contribution of each Neighborhood may be determined according to the inverse of its rank. A kernel function proposed by Lall and Sharma (1996) defines the weights, which lead to a K-NN regression estimate of:

$$Z_{t} = \frac{1}{\sum_{j=1}^{K} \frac{1}{j}} \sum_{j=1}^{K} \left(\frac{1}{j}\right) Z_{j}$$
(11)

where j is the order of the nearest neighborhoods in which the nearest have the lowest order (j = 1 to K), and  $Z_i$  is the magnitude of the nearest neighborhood j.

In this study, the Euclidean norm for a three-dimensional feature vector (the monthly inflow to the reservoir, the reservoir storage at the beginning of each month and the water demand during each month) is calculated as:

$$Dis_{t,r} = \sqrt{w_I (I_t - I_r)^2 + w_S (S_t - S_r)^2 + w_D (D_t - D_r)^2}$$
(12)

where:

 $I_r$ ,  $I_t$  are the inflows to the reservoir,  $S_t$ ,  $S_r$  are the reservoir storage volumes, and  $D_t$ ,  $D_r$  are the water demands in the current sequence (t) and in previous sequences (r), respectively. The weights at an increment of 0.01 and the number of neighborhoods (from 1 to number of the data – 1) that produce the lowest mean square error of estimating are found by cross validation of the calibrated data set. First, the KNN model is coupled with the GA optimization model for generating the initial solution in each sequence. This method is used for estimating reservoir water release in each month. In the second stage, the KNN model is also used for development of reservoir operating rules utilizing the results of the optimization model.

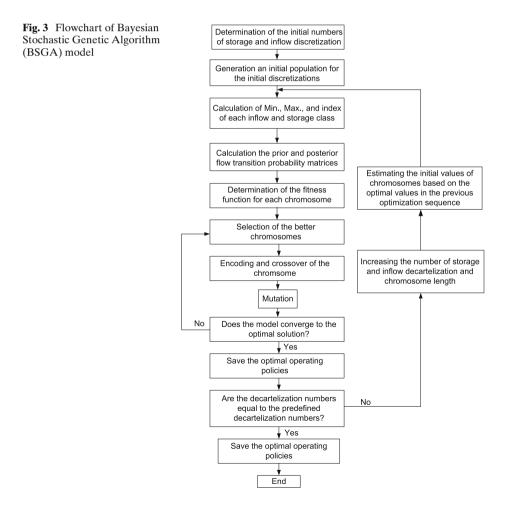
#### 2.2 Bayesian Stochastic Genetic Algorithm (BSGA)

In the stochastic model, the joint probability distribution of observed inflow and forecasted inflow to the reservoir is considered. The Bayesian Stochastic Genetic Algorithm, BSGA, considers the intrinsic and forecast uncertainties of inflow by updating transition probabilities as new flow data becomes available utilizing Baye's theorem (BT). In the BSGA model, the forecast characteristics are also included as state variables. The policies are generated for each month, based on the reservoir storage and the inflow and forecasted inflow discretization. The BSGA model determines the reservoir storage at the end of period  $t(S_{t+1})$  whereas the reservoir storage at the beginning of time period t is  $S_t$ , the representative of streamflow into the reservoir during time period t is  $I_t$  and the forecasted inflow for the next time period t + l is  $H_t$ . In the BSGA model, each chromosome contains the operating policies capturing the natural and the forecast uncertainties of inflow to the reservoir. The

Nash Bargaining Theory is also used to bring consensus among the water users the same way it was used in the deterministic model. Figure 3 shows the flowchart of the BSGA model. The results of the BSGA model are compared with the Bayesian Stochastic Dynamic Programming (BSDP) model which has similar framework for the water release from the reservoir. In the BSGA model, in addition to water releases form the reservoir, water allocation to each sector is also determined based on the Nash Bargaining Theory. Due to a high number of state and decision variables especially for handling finer state variable discretization, the optimization problem with computational complexity is solved with the use of the genetic algorithm (GA) method.

## 2.2.1 Optimization Model

In this model, the Nash objective function is used for optimizing the water allocation to each sector/water user while maintaining the reservoir storage level. In the model formulation, the objective function is the multiplication of utility functions of the



sectors (agricultural, industrial, domestic, and environmental) subtracted from their point of disagreement.

$$Maximize \ Z = E\left\{\prod_{m=1}^{12} \left[ \left( \prod_{r=1}^{nu} \left( f_{r,m}(R_{m,r,i,j,g}) - d_{r,m} \right)^{w_r} \right) \cdot \left( fs_m(S_{m+1,i,j,g}) - ds_m \right)^{w_s} \right] \right\}$$
(13)

Subject to:

$$E\left\{\prod_{m=1}^{12}\left[\left(\prod_{r=1}^{nu}\left(f_{r,m}(R_{m,r,i,j,g})-d_{r,m}\right)^{w_{r}}\right)\cdot\left(fs_{m}(S_{m+1,i,j,g})-ds_{m}\right)^{w_{s}}\right]\right\}$$
$$=\prod_{m=1}^{12}\left[\sum_{n=1}^{12N+m}\sum_{i=1}^{n_{i}}\sum_{g=1}^{n_{g}}\sum_{g=1}^{n_{g}}\left(F_{n,m,i,j,g}\right)-\sum_{n=1}^{12N+m-1}\sum_{i=1}^{n_{i}}\sum_{g=1}^{n_{g}}\left(F_{n,m,i,j,g}\right)\right](14)$$

$$F_{n,m,i,j,g} = \begin{cases} B_{m,i,j,g} + \sum_{h=1}^{n_j} \left( \phi_{m+1} \left[ I_{m+1}^h \left| H_{m+1}^g, I_m^j \right] \right. \\ \left. \cdot \sum_{k=1}^{n_g} \xi_{m+1} \left[ H_{m+1}^k \left| I_m^h \right] \cdot \left( F_{n,m+1,i,j,g} \right) \right) & m = 2,...,12 \\ B_{m,i,j,g} + \sum_{h=1}^{n_j} \left( \phi_{m+1} \left[ I_{m+1}^h \left| H_{m+1}^g, I_m^j \right] \right. \\ \left. \cdot \sum_{k=1}^{n_g} \xi_{m+1} \left[ H_{m+1}^k \left| I_m^h \right] \cdot \left( F_{n,12,i,j,g} \right) \right) & m = 1 \end{cases}$$

$$(15)$$

$$B_{m,i,j,g} = \left(\prod_{r=1}^{nu} \left( f_{r,m}(R_{m,r,i,j,g}) - d_{r,m} \right)^{w_r} \right) \cdot \left( fs_m(S_{m+1,i,j,g}) - ds_m \right)^{w_s}$$
(16)

$$\phi_{m+1} \left[ I_{m+1}^{h} \left| H_{m+1}^{g}, I_{m}^{j} \right] = \frac{\lambda_{m+1} \left[ H_{m+1}^{g} \left| I_{m+1}^{h} \right] \cdot \rho_{m+1} \left[ I_{m+1}^{h} \left| I_{m}^{j} \right] \right]}{\xi_{m+1} \left[ H_{m+1}^{g} \left| I_{m}^{j} \right]}$$
(17)

$$\xi_{m+1} \left[ H_{m+1}^{g} \left| I_{m}^{j} \right] = \sum_{l=1}^{n_{j}} \left[ \lambda_{m+1} \left[ H_{m+1}^{g} \left| I_{m+1}^{l} \right] \cdot \rho_{m+1} \left[ I_{m+1}^{l} \left| I_{m}^{j} \right] \right]$$
(18)

$$\sum_{r=1}^{ma} R_{m,r,i,j,g} = S_i - S_{m+1,i,j,g} + I_j - L_{m,i,j}$$
(19)

$$S_{\min} \le S_{m+1,i,j,g} \le S_{\max} \tag{20}$$

$$\sum_{r=1}^{nu} R_{m,r,i,j,g} \le R^{\max}\left(S_i\right) \tag{21}$$

# where:

iIndex of characteristic reservoir storagejIndex of characteristic inflowgIndex of characteristic inflow $n_i$ The number of reservoir storage discretizati $n_j$ The number of forecasted inflow discretization $n_g$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $w_r, ws$ Relative authority or weight of the cstakeholders which are in charge of the $r^{th}$ $w_{r,i,j,g}$ Water allocation to user $r$ in month $m$ when t $s_{m+1,i,j,g}$ Storage volume at the end of month $m$ , w $m_{m,i,j,g}$ Value of the first two terms of the Nash pp $reservoir storage index at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to the rolation to its j and the index of forem_{index} at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to the rolation to its j and the index of foreN_iNumber of iterations to achive the stationary$	Index of characteristic reservoir storage Index of characteristic inflow Index of characterestic forecasted inflow discretization The number of forecasted inflow discretization The number of sectors/water users Expected value Utility function/ disagreement point related to the <i>r</i> th water user in month <i>m</i> Utility function/ disagreement point related to the water storage in month <i>m</i> $s_m()/ds_m$ Utility function/ disagreement point related to the water storage in month <i>m</i> $r_r$ , wsRelative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{dh}$ water user and water storage, respectively $m_{r,i,jg}$ Water allocation to user <i>r</i> in month <i>m</i> when the storage index at the beginning of the month is <i>i</i> the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m_{r,i,jg}$ Storage volume at the end of month <i>m</i> , when the storage index at the beginning of the month is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m_{i,j,jg}$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month <i>m</i> is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m_{i,j,jg}$ Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) $m_{i,j}$ Total loss during month <i>m</i> due to evaporation and infiltrations to achive the stationary state transition probability matrix (year) $m_{i,j}$ M	r	Index of sector/water user
jIndex of characteristic inflowgIndex of characterestic forecasted inflow $n_i$ The number of reservoir storage discretization $n_j$ The number of forecasted inflow discretization $n_g$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $wr, ws$ Relative authority or weight of the c $stakeholders which are in charge of the r^{dh}wr, wsRelative authority or weight of the cstakeholders which are in charge of the r^{dh}water storage, respectivelyR_{m,r,i,j,g}Storage volume at the end of month m, we index at the beginning of the month is i the index of forecasted flow is g (MCM)S_{m+1,i,j,g}Value of the first two terms of the Nash pureservoir storage corresponding to the reservoir storage index at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to the reservoir storage index at the beginning of month m (MCM)L_{m,i,j}Total loss during month m due to e infiltration when the storage index at the beginning of the flow is j (MCM)NNumber of iterations to achive the stationary$	Index of characteristic inflowIndex of characteristic inflowindex of characterestic forecasted inflowindex of characterestic forecasted inflowindex of characteristic forecasted inflowindex of characteristic forecasted inflow discretizationindex of the number of inflow discretizationindex of the number of forecasted inflow discretizationindex of the number of sectors/water usersindex inflowindex inflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflowinflow		
gIndex of characterestic forecasted inflow $n_i$ The number of reservoir storage discretizati $n_j$ The number of inflow discretization $n_g$ The number of forecasted inflow discretizati $nu$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $w_r, ws$ Relative authority or weight of the costakeholders which are in charge of the $r^{th}$ $w_r, ws$ Relative authority or weight of the costakeholders which are in charge of the $r^{th}$ $w_{r,i,j,g}$ Water allocation to user $r$ in month $m$ when to at the beginning of the month is $i$ the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m$ , widex at the beginning of the Nash pr reservoir storage index at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the r index i at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the month is $i$ and the index of the flow is $j$ (MCM) $N$ Number of iterations to achive the stationary	Index of characterestic forecasted inflowiThe number of reservoir storage discretizationjThe number of inflow discretizationgThe number of forecasted inflow discretizationuThe number of sectors/water usersii)Expected valueiii)Utility function/ disagreement point related to the <i>r</i> th wateruser in month mUtility function/ disagreement point related to the waterism()/dsmUtility function/ disagreement point related to the waterism()/dsmUtility function/ disagreement point related to the waterism()/dsmUtility function/ disagreement point related to the waterism()/dsmWater allocation to user <i>r</i> in charge of the <i>r</i> <sup>th</sup> water user andwater storage, respectivelyWater allocation to user <i>r</i> in month <i>m</i> when the storage index at the beginning of the month is <i>i</i> the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM)m+1,i,jgStorage volume at the end of month <i>m</i> , when the storage index at the beginning of the month is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM)imi, jgValue of the first two terms of the Nash product when the reservoir storage index at the beginning of month <i>m</i> is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> iiReservoir storage corresponding to the inflow index <i>j</i> (MCM)iiMonthly inflow volume corresponding to the inflow index <i>j</i> (MCM)iiTotal loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM)iiNumber of iterations to		
$n_i$ The number of reservoir storage discretizati $n_j$ The number of inflow discretization $n_g$ The number of forecasted inflow discretizati $nu$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $mu$ Utility function/ disagreement point related $m_r, ws$ Relative authority or weight of the constance of the $r^{th}$ $w_r, ws$ Relative authority or weight of the constance of the $r^{th}$ $w_{r,i,j,g}$ Water allocation to user $r$ in month $m$ when the total the beginning of the month is $i$ the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, will inflow is j and the index of forecasted flow is gB_{m,i,j,g}Value of the first two terms of the Nash preservoir storage index at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to the reservoir storage index at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to the reservoir storage index at the beginning of month m (MCM)I_m, i, jTotal loss during month m due to endNNumber of iterations to achive the stationary$	iThe number of reservoir storage discretizationjThe number of inflow discretizationgThe number of forecasted inflow discretizationuThe number of sectors/water users $C_i$ Expected value $C_i$ Expected value $C_i$ Utility function/ disagreement point related to the rth water $v_{m}()/d_{r,m}$ Utility function/ disagreement point related to the water $v_{m,n}()/d_{sm}$ Utility function/ disagreement point related to the water $v_{m,ri,jg}$ Relative authority or weight of the decision-makers/ $v_{m,ri,jg}$ Water allocation to user r in month m when the storage index $m+1.i,jg$ Storage volume at the end of month m, when the storage $m+1.i,jg$ Storage volume at the end of month m, when the storage $m+1.i,jg$ Value of the first two terms of the Nash product when the reservoir storage corresponding to the reservoir storage index at the beginning of month m is i, the index of the inflow is j and the index of forecasted flow is g $m_{i,j,jg}$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of month m is i, the index of the inflow is j and the index of the reservoir storage index i at the beginning of month m (MCM) $m_{i,j,j}$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of the month is i and the index of the flow is j (MCM) $m_{i,j}$ Total loss during month m flow is j (MCM) $m_{i,j}$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of the month is i and the index of the		
$n_j$ The number of inflow discretization $n_g$ The number of forecasted inflow discretizat $nu$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related user in month $m$ $fs_m()/ds_m$ Utility function/ disagreement point related storage in month $m$ $w_r, ws$ Relative authority or weight of the c stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when t at the beginning of the month is $i$ the index and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, wiindex at the beginning of the month is i, tinflow is j and the index of foreB_{m,i,j,g}Value of the first two terms of the Nash prreservoir storage index at the beginning of rindex i at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to the rindex i at the beginning of month m (MCM)I_jTotal loss during month m due to erinfiltration when the storage index at the beginning to theinfiltration when the storage index at the brNNumber of iterations to achive the stationary$	jThe number of inflow discretizationgThe number of forecasted inflow discretizationuThe number of sectors/water users $C_i^{(m)}$ Expected value $C_i^{(m)}$ Utility function/ disagreement point related to the rth water $is_m()/ds_m$ Utility function/ disagreement point related to the water $is_m()/ds_m$ Utility function/ disagreement point related to the water $is_m()/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Utility function/ disagreement point related to the water $is_m(i)/ds_m$ Relative authority or weight of the decision-makers/ $is_m(i)/ds_m$ Relative authority or weight of the decision-makers/ $is_m(i)/ds_m$ Relative authority or weight of the storage index at the beginning of the month $is$ $i$ the index of the inflow is $j$ $m+1,i,jg$ Storage volume at the end of month $m$ , when the storage $m+1,i,jg$ Value of the first two terms of the Nash product when the $i$ Reservoir storage corresponding to the inflow is $g$ $i$ Reservoir storage corresponding to the inflow index $j$ $i$ Monthly inflow volume corresponding to the inflow index $j$ $i$	-	
$n'_g$ The number of forecasted inflow discretizat $nu$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $g_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $g_{r,m}()/d_{r,m}$ Utility function/ disagreement point related $w_r, ws$ Relative authority or weight of the $c$ $w_{r, ws}$ Relative authority or weight of the $c$ $w_{r, i, j, g}$ Water allocation to user $r$ in month $m$ when t $at$ the beginning of the month is $i$ the index $c$ $and$ the index of forecasted flow is $g$ (MCM) $S_{m+1, i, j, g}$ Storage volume at the end of month $m, w$ $mindex at the beginning of the month is i, tmindex at the beginning of the month is i, tmindex of the inflow is j and the index of foreS_iReservoir storage corresponding to the reservoir storage corresponding to the reservoir storage index at the beginning of month m (MCM)I_jTotal loss during month m due to endmonth is i and the index of the flow is j (MCM)L_{m,i,j}Total loss during month m due to endNNumber of iterations to achive the stationary$	$g_g$ The number of forecasted inflow discretization $u$ The number of sectors/water users $C_i^{k}$ Expected value $U_{r,m}()/d_{r,m}$ Utility function/ disagreement point related to the <i>r</i> th water $v_{sm}()/ds_m$ Utility function/ disagreement point related to the water $v_{sm}()/ds_m$ Utility function/ disagreement point related to the water $v_{sm}()/ds_m$ Utility function/ disagreement point related to the water $v_{sm}(r, ws)$ Relative authority or weight of the decision-makers/ $v_{stakeholders}$ which are in charge of the $r^{th}$ water user and $w_{ter}$ storage, respectively $w_{m,r,i,jg}$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1,i,jg$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $v_{aue}$ of the inflow is $j$ and the index of forecasted flow is $g$ $v_{m,i,j}$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ is $i$ , the index of the inflow index $j$ (MCM) $v_{am,i,j}$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $v_{m+1}[I_{m+1}^{h} H_{m+1}^{g}, I_{m}^{j}]$ Posterior state transition from current forecast flow $(g$ in-		
$nu$ The number of sectors/water users $E\{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related user in month $m$ $fs_m()/ds_m$ Utility function/ disagreement point related storage in month $m$ $w_r, ws$ Relative authority or weight of the $c$ stakeholders which are in charge of the $r^{dr}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when t at the beginning of the month is $i$ the index and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ inflow is $j$ and the index of forecasted flow is $f$ $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the r index $i$ at the beginning of month $m$ (MCM) $I_j$ Total loss during month $m$ due to end infiltration when the storage index at the beginning to the flow is $j$ $N$ Number of iterations to achive the stationary	$u$ The number of sectors/water users $C_{i}$ Expected value $C_{i,m}$ Utility function/ disagreement point related to the <i>r</i> th water user in month <i>m</i> $s_m()/ds_m$ Utility function/ disagreement point related to the water storage in month <i>m</i> $v_r, ws$ Relative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $v_m, r, i, j_g$ Water allocation to user <i>r</i> in month <i>m</i> when the storage index at the beginning of the month is <i>i</i> the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m+1, i, j_g$ Storage volume at the end of month <i>m</i> , when the storage index at the beginning of the month is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $w_{m,i,j,g}$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month <i>m</i> is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> $m_{m,i,j,g}$ Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> or the inflow index <i>j</i> (MCM) $m_{m,i,j}$ Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) $m_{m+1}[I_{m+1}^h H_{m+1}^g, I_m^f]$ Posterior state transition from current forecast flow ( <i>g</i> in- probability matrix (year)		
$E{\}$ Expected value $f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related user in month $m$ $fs_m()/ds_m$ Utility function/ disagreement point related storage in month $m$ $w_r, ws$ Relative authority or weight of the or stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the at the beginning of the month is $i$ the index $f$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the $(MCM)$ $L_{m,i,j}$ Total loss during month $m$ due to e infiltration when the storage index at the brown of the flow is $j$ (MCM)NNumber of iterations to achive the stationary	C{}Expected value $k_{rm}()/d_{r,m}$ Utility function/ disagreement point related to the <i>r</i> th water user in month <i>m</i> $s_m()/ds_m$ Utility function/ disagreement point related to the water storage in month <i>m</i> $r, ws$ Relative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $w_{m,r,i,j,g}$ Water allocation to user <i>r</i> in month <i>m</i> when the storage index at the beginning of the month is <i>i</i> the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m+1.i,j,g$ Storage volume at the end of month <i>m</i> , when the storage index at the beginning of the month is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $w_{n,i,j,g}$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month <i>m</i> is <i>i</i> , the index of the inflow is <i>g</i> index <i>i</i> at the beginning of month <i>m</i> (MCM) $m_{n,i,j}$ Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) $m_{n,i,j}$ Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) $m_{n,i,j}$ Posterior state transition from current forecast flow ( <i>g</i> in- probability matrix (year)		
$f_{r,m}()/d_{r,m}$ Utility function/ disagreement point related user in month $m$ $fs_m()/ds_m$ Utility function/ disagreement point related storage in month $m$ $w_r, ws$ Relative authority or weight of the $c$ stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when t at the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , t inflow is $j$ and the index of forecasted flow i $g$ (MCM) $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of $r$ index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the $r$ infiltration when the storage index at the the month is $i$ and the index of fore $month m$ (MCM) $N$ Number of iterations to achive the stationary	$i_{rm}()/d_{r,m}$ Utility function/ disagreement point related to the <i>r</i> th water user in month <i>m</i> $S_m()/ds_m$ Utility function/ disagreement point related to the water storage in month <i>m</i> $r_r$ , wsRelative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $B_{m,r,i,jg}$ Water allocation to user <i>r</i> in month <i>m</i> when the storage index at the beginning of the month is <i>i</i> the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m+1.i,j,g$ Storage volume at the end of month <i>m</i> , when the storage index at the beginning of the month is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m_{m+1,i,j,g}$ Value of the first two terms of the Nash product when the reservoir storage corresponding to the reservoir storage index <i>i</i> at the beginning of month <i>m</i> (MCM) $m_{m,i,j}$ Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month <i>s j</i> (MCM) $m_{m,i,j}$ Total loss during month <i>m</i> due to evaporation and infiltration sto achive the stationary state transition probability matrix (year) $m+1[I_{m+1}^h H_{m+1}^s, I_m^m]$ Posterior state transition from current forecast flow (g in-		
user in month $m$ $fs_m()/ds_m$ Utility function/ disagreement point relate storage in month $m$ $w_r, ws$ Relative authority or weight of the $c$ stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when t at the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , t inflow is $j$ and the index of forecasted flow i $s, t$ inflow is $j$ and the index of forecasted flow i $s, t$ inflow is $j$ and the index of forecasted flow i $s, t$ inflow is $j$ and the index of forecasted flow i $s, t$ index at the beginning of the Nash pr reservoir storage index at the beginning of $r$ index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the $r$ (MCM) $L_{m,i,j}$ Total loss during month $m$ due to e infiltration when the storage index at the br month is $i$ and the index of the flow is $j$ (MC N	$s_m(j)/ds_m$ user in month $m$ $s_m(j)/ds_m$ Utility function/ disagreement point related to the water storage in month $m$ $v_r, ws$ Relative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $w_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m_{m,i,j,g}$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m+1[I_{m+1}^h H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow $(g$ in- probability matrix (year)		•
$fs_m()/ds_m$ Utility function/ disagreement point related storage in month $m$ $w_r, ws$ Relative authority or weight of the $c$ stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the at the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i,$ the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of $reindex of the inflow is j and the index of foreS_iS_iReservoir storage corresponding to the reindex i at the beginning of month m (MCM)I_jMonthly inflow volume corresponding to therotal loss during month m due to einfiltration when the storage index at the brown is j (MCNNNumber of iterations to achive the stationary$	$S_m()/ds_m$ Utility function/ disagreement point related to the water storage in month $m$ $r, ws$ Relative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $m,r,i,jg$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Posterior state transition from current forecast flow ( $g$ in- probability matrix (year)	$J_{r,m}()/a_{r,m}$	
storage in month $m$ $w_r, ws$ Relative authority or weight of the $c$ stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the at the beginning of the month is $i$ the index $f$ $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , the inflow is $j$ and the index of forecasted flow is $g$ $B_{m,i,j,g}$ Value of the first two terms of the Nash preservoir storage index at the beginning of $r$ $S_i$ Reservoir storage corresponding to the $r$ $I_j$ Monthly inflow volume corresponding to the $r$ $I_j$ Total loss during month $m$ due to er $N$ Number of iterations to achive the stationary	storage in month m $v_r, ws$ Relative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $w_{m,r,i,j,g}$ Water allocation to user r in month m when the storage index at the beginning of the month is i the index of the inflow is j and the index of forecasted flow is g (MCM) $m+1.i,j,g$ Storage volume at the end of month m, when the storage index at the beginning of the month is i, the index of the inflow is j and the index of forecasted flow is g (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month m is i, the index of the inflow is j and the index of forecasted flow is g (MCM) $m,i,j,j$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of the month is i and the index of the flow is j (MCM) $m,i,j$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of the month is i and the index of the flow is j (MCM) $m,i,j$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of the month is i and the index of the flow is j (MCM) $m+1[I_{m+1}^h H_{m+1}^g, I_m^h]$ Posterior state transition from current forecast flow (g in-	$f_{s}$ ()/ $d_{s}$	
$w_r, ws$ Relative authority or weight of the or stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the at the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM). $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , the inflow is $j$ and the index of forecasted flow is $f$ . $B_{m,i,j,g}$ Value of the first two terms of the Nash provide reservoir storage index at the beginning of $r$ index of the inflow is $j$ and the index of fore $S_i$ $S_i$ Reservoir storage corresponding to the $r$ index $i$ at the beginning of month $m$ (MCM) $I_j$ Total loss during month $m$ due to exist infiltration when the storage index at the begin ingender $m$ of the flow is $j$ (MCM)NNumber of iterations to achive the stationary.	$r, ws$ Relative authority or weight of the decision-makers/ stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $m,r,i,j,g$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ $i$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $j$ Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Posterior state transition from current forecast flow ( $g$ in-	$\int s_m()/us_m$	
stakeholders which are in charge of the $r^{th}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when t at the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , t inflow is $j$ and the index of forecasted flow i $S_{m,i,j,g}$ $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of $r$ index of the inflow is $j$ and the index of fore $S_i$ $S_i$ Reservoir storage corresponding to the $r$ index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the month is $i$ and the index of the flow is $j$ (MC NNNumber of iterations to achive the stationary	stakeholders which are in charge of the $r^{th}$ water user and water storage, respectively $m,r.i,j.g$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1.i,j.g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j.g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ $m,i,j.g$ Value of the first two terms of the Nash product when the reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $m,i,j$ Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Posterior state transition from current forecast flow ( $g$ in-		
$R_{m,r,i,j,g}$ water storage, respectively $R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when that the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m, w$ index at the beginning of the month is $i$ , the inflow is $j$ and the index of forecasted flow is $g$ $B_{m,i,j,g}$ Value of the first two terms of the Nash preservoir storage index at the beginning of $r$ index of the inflow is $j$ and the index of fore $S_i$ Reservoir storage corresponding to the $r$ index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the infiltration when the storage index at the beginner $s_i$ infiltration when the storage index at the beginner $s_i$ infiltration when the storage index at the beginner $s_i$ infiltration when the storage index at the beginner $s_i$ infiltration $s_i$ infiltration storage index at the beginner $s_i$ infiltration $s_i$ infiltration $s_i$ index of the flow is $j$ (MCN)NNumber of iterations to achive the stationary	$m_{m,i,j,g}$ water storage, respectively $m_{m+1,i,j,g}$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ $i$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $j$ Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,n,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,n,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,n,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,n,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,n,i,j$	$W_r, WS$	
$R_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the at the beginning of the month is $i$ the index $a$ and the index of forecasted flow is $g$ (MCM). $S_{m+1,i,j,g}$ Storage volume at the end of month $m$ , we index at the beginning of the month is $i$ , the inflow is $j$ and the index of forecasted flow is $j$ . $B_{m,i,j,g}$ Value of the first two terms of the Nash part reservoir storage index at the beginning of $reservoir$ storage corresponding to the re- index $i$ at the beginning of month $m$ (MCM). $I_j$ Monthly inflow volume corresponding to the (MCM). $L_{m,i,j}$ Total loss during month $m$ due to e infiltration when the storage index at the beginning of the stationary.NNumber of iterations to achive the stationary.	$P_{m,r,i,j,g}$ Water allocation to user $r$ in month $m$ when the storage index at the beginning of the month is $i$ the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $j$ Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m+1[I_{m+1}^h   H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow $(g$ in- motability matrix (year)		
at the beginning of the month is $i$ the index $i$ $S_{m+1,i,j,g}$ Storage volume at the end of month $m$ , w $S_{m+1,i,j,g}$ Storage volume at the end of month $m$ , windex at the beginning of the month is $i$ , tinflow is $j$ and the index of forecasted flow i $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of $r$ $B_{m,i,j,g}$ Reservoir storage corresponding to the reservoir storage corresponding to the reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the reservoir storage index at the beginning of month $m$ (MCM) $L_{m,i,j}$ Total loss during month $m$ due to eNNumber of iterations to achive the stationary	at the beginning of the month is <i>i</i> the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m+1,i,j,g$ Storage volume at the end of month <i>m</i> , when the storage index at the beginning of the month is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i> (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month <i>m</i> is <i>i</i> , the index of the inflow is <i>j</i> and the index of forecasted flow is <i>gi</i> Reservoir storage corresponding to the reservoir storage index <i>i</i> at the beginning of month <i>m</i> (MCM) <i>j</i> Monthly inflow volume corresponding to the inflow index <i>j</i> (MCM) <i>m,i,j</i> Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) <i>m+1</i> $I_{m+1}^h   H_{m+1}^g, I_m^j$ Posterior state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast flow ( <i>g</i> in- mother state transition from current forecast	D	
$S_{m+1,i,j,g}$ and the index of forecasted flow is $g$ (MCM) $S_{m+1,i,j,g}$ Storage volume at the end of month $m$ , w index at the beginning of the month is $i$ , t inflow is $j$ and the index of forecasted flow i $B_{m,i,j,g}$ Value of the first two terms of the Nash pr reservoir storage index at the beginning of $r$ index of the inflow is $j$ and the index of fore $S_i$ Reservoir storage corresponding to the $r$ index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month $m$ due to e infiltration when the storage index at the beginner infiltration storage index at the brown of the flow is $j$ (MC NNNumber of iterations to achive the stationary	and the index of forecasted flow is $g$ (MCM) $m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ $i$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) $j$ Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m+1[I_{m+1}^h   H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( $g$ in-	$R_{m,r,i,j,g}$	
$S_{m+1,i,j,g}$ Storage volume at the end of month $m$ , with index at the beginning of the month is $i$ , the index of forecasted flow is $j$ and the index of forecasted flow is $j$ and the index of the Nash processor storage index at the beginning of $r$ index of the inflow is $j$ and the index of forecasted flow is $j$ and the index of the normal storage index at the beginning of $r$ index of the inflow is $j$ and the index of forecasted flow is $j$ (MCM) $L_{m,i,j}$ Total loss during month $m$ due to exist infiltration when the storage index at the brown the storage index at the flow is $j$ (MCN)NNumber of iterations to achive the stationary interval storage index is $j$ (MCN)	$m+1,i,j,g$ Storage volume at the end of month $m$ , when the storage index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ $i$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m+1[I_{m+1}^h   H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( $g$ in-		
index at the beginning of the month is $i$ , t inflow is $j$ and the index of forecasted flow i Value of the first two terms of the Nash pr reservoir storage index at the beginning of r index of the inflow is $j$ and the index of fore $S_i$ $S_i$ Reservoir storage corresponding to the r index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month $m$ due to e infiltration when the storage index at the beginning of the flow is $j$ (MCN)NNumber of iterations to achive the stationary	index at the beginning of the month is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ (MCM) $m,i,j,g$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ $i$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m,i,j$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m+1[I_{m+1}^h   H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( $g$ in-		
$B_{m,i,j,g}$ inflow is j and the index of forecasted flow i $B_{m,i,j,g}$ Value of the first two terms of the Nash preservoir storage index at the beginning of r index of the inflow is j and the index of fore $S_i$ Reservoir storage corresponding to the r index i at the beginning of month m (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month m due to er infiltration when the storage index at the beginning of the flow is j (MCN)NNumber of iterations to achive the stationary	$i_{m,i,j,g}$ inflow is $j$ and the index of forecasted flow is $g$ (MCM) $v_{m,i,j,g}$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) Monthly inflow volume corresponding to the inflow index $j$ (MCM) $w_{m,i,j}$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $w_{m+1}[I_{m+1}^h H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( $g$ in-	$S_{m+1,i,j,g}$	
$B_{m,i,j,g}$ Value of the first two terms of the Nash preservoir storage index at the beginning of reservoir storage index at the beginning of reservoir storage corresponding to the reservoir at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month $m$ due to exist the storage index at the beginning of the flow is $j$ (MCN)NNumber of iterations to achive the stationary	$i_{m,i,j,g}$ Value of the first two terms of the Nash product when the reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) Monthly inflow volume corresponding to the inflow index $j$ (MCM) $m_{m,i,j}$ Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) $m_{m+1}[I_{m+1}^h   H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( $g$ in-		
$intrologicalreservoir storage index at the beginning of rS_ireservoir storage corresponding to the rS_iReservoir storage corresponding to the rI_jMonthly inflow volume corresponding to the(MCM)L_{m,i,j}Total loss during month m due to erinfiltration when the storage index at the beginning of the flow is j (MCN)NNumber of iterations to achive the stationary$	reservoir storage index at the beginning of month $m$ is $i$ , the index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) Monthly inflow volume corresponding to the inflow index $j$ (MCM) Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) Number of iterations to achive the stationary state transition probability matrix (year) Posterior state transition from current forecast flow ( $g$ in-		
$S_i$ index of the inflow is $j$ and the index of fore Reservoir storage corresponding to the re- index $i$ at the beginning of month $m$ (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month $m$ due to er infiltration when the storage index at the brown the storage in	index of the inflow is $j$ and the index of forecasted flow is $g$ Reservoir storage corresponding to the reservoir storage index $i$ at the beginning of month $m$ (MCM) Monthly inflow volume corresponding to the inflow index $j$ (MCM) Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) Number of iterations to achive the stationary state transition probability matrix (year) $m_{n+1}[I_{m+1}^{h} H_{m+1}^{g}, I_{m}^{j}]$ Posterior state transition from current forecast flow ( $g$ in-	$\mathbf{B}_{m,i,j,g}$	
$S_i$ Reservoir storage corresponding to the reindex i at the beginning of month m (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month m due to e infiltration when the storage index at the b month is i and the index of the flow is j (MCN)NNumber of iterations to achive the stationary	iReservoir storage corresponding to the reservoir storage index i at the beginning of month m (MCM) Monthly inflow volume corresponding to the inflow index j (MCM) $m,i,j$ Total loss during month m due to evaporation and infiltration when the storage index at the beginning of the month is i and the index of the flow is j (MCM) $M$ Number of iterations to achive the stationary state transition probability matrix (year) $m+1[I_{m+1}^h   H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow (g in-		reservoir storage index at the beginning of month $m$ is $i$ , the
$I_j$ index i at the beginning of month m (MCM) $I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month m due to e infiltration when the storage index at the b month is i and the index of the flow is j (MC N umber of iterations to achive the stationary	index <i>i</i> at the beginning of month <i>m</i> (MCM) Monthly inflow volume corresponding to the inflow index <i>j</i> (MCM) Total loss during month <i>m</i> due to evaporation and infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) Number of iterations to achive the stationary state transition probability matrix (year) $m_{+1}[I_{m+1}^{h} H_{m+1}^{g}, I_{m}^{j}]$ Posterior state transition from current forecast flow ( <i>g</i> in-		index of the inflow is <i>j</i> and the index of forecasted flow is <i>g</i>
$I_j$ Monthly inflow volume corresponding to the (MCM) $L_{m,i,j}$ Total loss during month $m$ due to exist infiltration when the storage index at the the month is $i$ and the index of the flow is $j$ (MCN)NNumber of iterations to achive the stationary	$ \begin{array}{ll} & \text{Monthly inflow volume corresponding to the inflow index } j \\ & \text{(MCM)} \\ & \text{Total loss during month } m \text{ due to evaporation and infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & \text{Monthly infiltration when the storage index at the beginning of the month is } i \text{ and the index of the flow is } j (\text{MCM}) \\ & Monthly infiltration when the storage index at the beginning of the monthly infiltration \\ & \text{Monthly infiltration when the storage index at the beginning of the monthly infiltration \\ & \text{Monthly infiltration when the storage index at the beginning of the monthly infiltratin \\ & \text{Monthly infiltration when the storage in$	$S_i$	Reservoir storage corresponding to the reservoir storage
$L_{m,i,j}$ (MCM) $L_{m,i,j}$ Total loss during month $m$ due to explicit infiltration when the storage index at the brown month is $i$ and the index of the flow is $j$ (MCN)NNumber of iterations to achive the stationary	(MCM) Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) Number of iterations to achive the stationary state transition probability matrix (year) Posterior state transition from current forecast flow (g in-		index <i>i</i> at the beginning of month <i>m</i> (MCM)
$L_{m,i,j}$ (MCM) $L_{m,i,j}$ Total loss during month $m$ due to explicit infiltration when the storage index at the brown month is $i$ and the index of the flow is $j$ (MCN)NNumber of iterations to achive the stationary	(MCM) Total loss during month $m$ due to evaporation and infiltration when the storage index at the beginning of the month is $i$ and the index of the flow is $j$ (MCM) Number of iterations to achive the stationary state transition probability matrix (year) Posterior state transition from current forecast flow (g in-	$I_i$	Monthly inflow volume corresponding to the inflow index <i>j</i>
infiltration when the storage index at the bmonth is i and the index of the flow is j (MCNN	infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) Number of iterations to achive the stationary state transition probability matrix (year) $M_{m+1}[I_{m+1}^h H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( <i>g</i> in-	,	
infiltration when the storage index at the bmonth is i and the index of the flow is j (MCNN	infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) Number of iterations to achive the stationary state transition probability matrix (year) $M_{m+1}[I_{m+1}^h H_{m+1}^g, I_m^j]$ Posterior state transition from current forecast flow ( <i>g</i> in-	$L_{m,i,i}$	
month is i and the index of the flow is j (MCNNNumber of iterations to achive the stationary	$ \begin{array}{c} \text{month is } i \text{ and the index of the flow is } j \text{ (MCM)} \\ \text{Number of iterations to achive the stationary state transition} \\ \text{probability matrix (year)} \\ \text{Posterior state transition from current forecast flow } (g \text{ in-}) \end{array} $	,,,,	e .
N Number of iterations to achive the stationary	Number of iterations to achive the stationary state transition probability matrix (year) $_{m+1}[I_{m+1}^{h} H_{m+1}^{g}, I_{m}^{j}]$ Posterior state transition from current forecast flow (g in-		
	probability matrix (year) $_{m+1}\left[I_{m+1}^{h}\middle H_{m+1}^{g}, I_{m}^{j}\right]$ Posterior state transition from current forecast flow (g in-	Ν	• • •
	$_{m+1}\left[I_{m+1}^{h}\middle H_{m+1}^{g},I_{m}^{j}\right]$ Posterior state transition from current forecast flow (g in-		•
	$m+1[m+1]n_{m+1}, n_m]$ Tosterior state transition from current forecast now (g in	$\phi = \left[I^{h} \mid H^{g} \mid I^{j}\right]$	
$\psi_{m+1}[r_{m+1}, r_{m+1}, r_m]$ is determined in the current for dev) and a previous actual flow (index) to the	dex) and a previous actual flow (index) to the current actual	$\varphi_{m+1}[1_{m+1}]1_{m+1}, 1_{m}]$	
		$\varepsilon [H^k   h]$	
flow ( <i>h</i> index) in month $m + 1$		$S_{m+1} \lfloor I_{m+1} \rfloor \lfloor I_m \rfloor$	
flow ( <i>h</i> index) in month $m + 1$ $\xi_{m+1} [H_{m+1}^k   I_m^h]$ Predictive probability function of the foreca	$_{n+1}[H_{m+1}^k I_m^h]$ Predictive probability function of the forecast flow (k range)	$\int \mathbf{u}^g  \mathbf{u} $	
$\xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] \qquad \qquad \text{flow } (h \text{ index}) \text{ in month } m+1 \\ \text{Predictive probability function of the forecal conditioned to a previous actual flow } (h \text{ range}) \Big]$	$_{n+1}[H_{m+1}^k I_m^h]$ Predictive probability function of the forecast flow (k range) conditioned to a previous actual flow (h range) in month m+1	$\lambda_{m+1} [\Pi_{m+1}   \Pi_{m+1}]$	
$ \begin{aligned} & \xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] \\ & \xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] \\ & \lambda_{m+1} \Big[ H_{m+1}^g \big  I_{m+1}^l \Big] \end{aligned} \qquad \begin{aligned} & \text{flow } (h \text{ index) in month } m+1 \\ & \text{Predictive probability function of the forecas} \\ & \text{conditioned to a previous actual flow } (h \text{ range} I h) \\ & \text{Likelihood function of the current forecas} \end{aligned} $	$ \begin{array}{l} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \end{array} $		
$\xi_{m+1}[H_{m+1}^k I_m^h]$ $\xi_{m+1}[H_{m+1}^g I_{m+1}^l]$ flow ( <i>h</i> index) in month <i>m</i> + 1 Predictive probability function of the forecase conditioned to a previous actual flow ( <i>h</i> range Likelihood function of the current forecase given that the actual flow belong to the <i>l</i> range	$ \begin{array}{l} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \end{array} $	$\rho_{m+1} [I_{m+1}^{\iota}   I_m^{\prime}]$	
$ \begin{aligned} & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g   I_{m+1}^l \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} I_{m+1}^l   I_{m+1}^j \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} I_{m+1}^l   I_m^j \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} I_{m+1}^l   I_m$	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \end{array} $	~	
$ \begin{aligned} & \xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \text{Fredictive probability function of the forecal conditioned to a previous actual flow } (h \text{ range}) \\ & \lambda_{m+1} \Big[ H_{m+1}^g \big  I_{m+1}^l \Big] & \text{Likelihood function of the current forecass given that the actual flow belong to the l range} \\ & \rho_{m+1} \Big[ I_{m+1}^l \big  I_m^j \Big] & \text{Prior flow transition from a previous actual is the current inflow } (l \text{ range}) \end{aligned} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \end{array} $		
$ \begin{split} & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^g   I_m^h \end{bmatrix} & \text{Predictive probability function of the forecase conditioned to a previous actual flow } (h \text{ range}) \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g   I_{m+1}^l \end{bmatrix} & \text{Likelihood function of the current forecase given that the actual flow belong to the l range of the current inflow (l range) \\ & S_{\min} & \text{Minimum volume of the reservoir (MCM)} \end{split} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \end{array} $	S <sub>max</sub>	
$ \begin{split} & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^g   I_m^h \end{bmatrix} & \text{Predictive probability function of the forecase conditioned to a previous actual flow } (h \text{ range}) \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g   I_{m+1}^l \end{bmatrix} & \text{Likelihood function of the current forecase given that the actual flow belong to the l range of the current inflow (l range) \\ & S_{\min} & \text{Minimum volume of the reservoir (MCM)} \end{split} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \end{array} $	$R^{Max}(S_m)$	
$ \begin{split} & flow (h \text{ index}) \text{ in month } m+1 \\ & flow (h \text{ index}) \text{ in month } m+1 \\ & flow (h \text{ index}) \text{ in month } m+1 \\ & Predictive probability function of the forecal conditioned to a previous actual flow (h range) \\ & \lambda_{m+1} [H^g_{m+1}   I^j_m] \\ & Likelihood function of the current forecast given that the actual flow belong to the l range \\ & \rho_{m+1} [I^l_{m+1}   I^j_m] \\ & Prior flow transition from a previous actual the current inflow (l range) \\ & S_{min} \\ & S_{max} \\ & R^{Max}(S_m) \\ & Allowable release during the month m constants \\ \end{split}$	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{m+1} \begin{bmatrix} I_{m+1}^{l} & I_{m+1}^{l} \end{bmatrix} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \\ & \text{Maximum volume of the reservoir (MCM)} \\ & \text{Allowable release during the month } m \text{ considering reservoir } \end{array} $		storage (MCM)
$\phi_{m+1}[I_{m+1}^h H_{m+1}^g, I_m^j]$ Posterior state transition from current fore		N $\phi_{m+1} [I_{m+1}^{h}   H_{m+1}^{g}, I_{m}^{j}]$ $\xi_{m+1} [H_{m+1}^{k}   I_{m}^{h}]$	infiltration when the storage index at the beginning of the month is <i>i</i> and the index of the flow is <i>j</i> (MCM) Number of iterations to achive the stationary state transition probability matrix (year) Posterior state transition from current forecast flow ( <i>g</i> index) and a previous actual flow ( <i>j</i> index) to the current actual flow ( <i>h</i> index) in month $m + 1$ Predictive probability function of the forecast flow ( <i>k</i> range) conditioned to a previous actual flow ( <i>h</i> range) in month $m+1$
		$\varepsilon [\mu k   \mu]$	
flow ( <i>h</i> index) in month $m + 1$		$\xi_{m+1} \left[ H_{m+1}^{\kappa} \right] I_m^m \right]$	
flow ( <i>h</i> index) in month $m + 1$ $\xi_{m+1} [H_{m+1}^k   I_m^h]$ Predictive probability function of the foreca	$_{n+1}[H_{m+1}^k I_m^h]$ Predictive probability function of the forecast flow (k range)		
$\xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] \qquad \qquad \text{flow } (h \text{ index}) \text{ in month } m+1 \\ \text{Predictive probability function of the forecal conditioned to a previous actual flow } (h \text{ range}) \Big]$	$_{n+1}[H_{m+1}^k I_m^h]$ Predictive probability function of the forecast flow (k range) conditioned to a previous actual flow (h range) in month m+1	$\lambda_{m+1} \left[ H^s_{m+1}   I^l_{m+1} \right]$	
$ \begin{aligned} & \xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] \\ & \lambda_{m+1} \Big[ H_{m+1}^g \big  I_{m+1}^l \Big] \end{aligned} \qquad \begin{array}{l} & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \text{Predictive probability function of the forecas} \\ & \text{conditioned to a previous actual flow } (h \text{ range} n) \\ & \text{Likelihood function of the current forecas} \end{aligned} $	$ \begin{array}{l} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \end{array} $		given that the actual flow belong to the <i>l</i> range
$\xi_{m+1}[H_{m+1}^k I_m^h]$ $\xi_{m+1}[H_{m+1}^g I_{m+1}^l]$ flow ( <i>h</i> index) in month <i>m</i> + 1 Predictive probability function of the forecast conditioned to a previous actual flow ( <i>h</i> range Likelihood function of the current forecast given that the actual flow belong to the <i>l</i> range	$ \begin{array}{l} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \end{array} $	$\rho_{m+1} [I_{m+1}^l   I_m^j]$	Prior flow transition from a previous actual flow ( <i>j</i> range) to
$ \begin{aligned} & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g   I_{m+1}^l \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} I_{m+1}^l   I_{m+1}^j \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} I_{m+1}^l   I_m^j \end{bmatrix} \\ & \lambda_{m+1} \begin{bmatrix} I_{m+1}^l   I_m$	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \end{array} $		the current inflow ( <i>l</i> range)
$ \begin{aligned} & \xi_{m+1} \Big[ H_{m+1}^k \big  I_m^h \Big] & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \text{Fredictive probability function of the forecas} \\ & \text{conditioned to a previous actual flow } (h \text{ range}) \\ & \lambda_{m+1} \Big[ H_{m+1}^g \big  I_{m+1}^l \Big] & \text{Likelihood function of the current forecass} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \rho_{m+1} \Big[ I_{m+1}^l \big  I_m^j \Big] & \text{Prior flow transition from a previous actual is} \\ & \text{the current inflow } (l \text{ range}) \end{aligned} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \end{array} $		
$ \begin{split} & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^g   I_m^h \end{bmatrix} & \text{Predictive probability function of the forecase conditioned to a previous actual flow } (h \text{ range}) \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g   I_{m+1}^l \end{bmatrix} & \text{Likelihood function of the current forecase given that the actual flow belong to the l range of the current inflow (l range) \\ & S_{\min} & \text{Minimum volume of the reservoir (MCM)} \end{split} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \end{array} $	$S_{\max}$	
$ \begin{split} & \xi_{m+1} \begin{bmatrix} H_{m+1}^k   I_m^h \end{bmatrix} & \text{flow } (h \text{ index}) \text{ in month } m+1 \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^g   I_m^h \end{bmatrix} & \text{Predictive probability function of the forecase conditioned to a previous actual flow } (h \text{ range}) \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g   I_{m+1}^l \end{bmatrix} & \text{Likelihood function of the current forecase given that the actual flow belong to the l range of the current inflow (l range) \\ & S_{\min} & \text{Minimum volume of the reservoir (MCM)} \end{split} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \end{array} $	$R^{Max}(S_m)$	Allowable release during the month <i>m</i> considering reservoir
$ \begin{split} & flow (h \text{ index}) \text{ in month } m+1 \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^k &   I_m^h \end{bmatrix} \\ & \text{Fredictive probability function of the forecase conditioned to a previous actual flow (h range) \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g &   I_{m+1}^l \end{bmatrix} \\ & \text{Likelihood function of the current forecase given that the actual flow belong to the l range \\ & \rho_{m+1} \begin{bmatrix} I_{m+1}^l &   I_m^j \end{bmatrix} \\ & \text{Prior flow transition from a previous actual for the current inflow (l range) } \\ & \text{Smax} \\ & \text{Maximum volume of the reservoir (MCM)} \\ & \text{Allowable release during the month } m \text{ constant} \end{split} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{m+1} \begin{bmatrix} I_{m+1}^{l} & I_{m+1}^{l} \end{bmatrix} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \\ & \text{Maximum volume of the reservoir (MCM)} \\ & \text{Allowable release during the month } m \text{ considering reservoir } \end{array} $		storage (MCM)
$ \begin{split} & flow (h \text{ index}) \text{ in month } m+1 \\ & \xi_{m+1} \begin{bmatrix} H_{m+1}^k &   I_m^h \end{bmatrix} \\ & \text{Fredictive probability function of the forecase conditioned to a previous actual flow (h range) \\ & \lambda_{m+1} \begin{bmatrix} H_{m+1}^g &   I_{m+1}^l \end{bmatrix} \\ & \text{Likelihood function of the current forecase given that the actual flow belong to the l range \\ & \rho_{m+1} \begin{bmatrix} I_{m+1}^l &   I_m^j \end{bmatrix} \\ & \text{Prior flow transition from a previous actual for the current inflow (l range) } \\ & \text{Smax} \\ & \text{Maximum volume of the reservoir (MCM)} \\ & \text{Allowable release during the month } m \text{ constant} \end{split} $	$ \begin{array}{ll} & \text{Predictive probability function of the forecast flow (k range)} \\ & \text{conditioned to a previous actual flow (h range) in month } m+1 \\ & \text{Likelihood function of the current forecast flow (g range)} \\ & \text{given that the actual flow belong to the } l \text{ range} \\ & \text{m+1} \begin{bmatrix} I_{m+1}^{l} & I_{m+1}^{l} \end{bmatrix} \\ & \text{Prior flow transition from a previous actual flow (j range) to} \\ & \text{the current inflow (l range)} \\ & \text{Minimum volume of the reservoir (MCM)} \\ & \text{Maximum volume of the reservoir (MCM)} \\ & \text{Allowable release during the month } m \text{ considering reservoir } \end{array} $		

Equation 13 is the Nash product; and Eqs. 14, 15, and 16 show the expected value of the Nash product considering allocated water to each sector. This expected value is calculated considering the first-order Markov process for inflow and forecasted flow, which is similar to the BSDP model when the algorithm reaches the stationary condition. The increase in the value of cumulative  $F_{n,m,i,j,g}$  reaches a constant value due to the stationary state condition of the state transition probability matrix. As shown in Eq. 14, this constant value is equal to the expected value of the objective function. Equations 17 and 18 present the probabilities (prior and posterior) calculated using the Bayesian Theory.

In the proposed model, the decision variables are the operating rules that determine the storage index at the end of month m, when the storage index at the beginning of the month is i, the index of the flow is j, and the index of forecasted flow is g. The number of genes in each chromosome is calculated based on the following equation:

$$NG = n_i \times n_j \times n_g \times nu \times M \tag{22}$$

where M is total month of time horizon.

#### 2.2.2 Model Characteristics

The intrinsic uncertainties are represented as the transition probabilities of inflow at time m to inflow at time m + 1. Furthermore, the Bayesian framework to update the prior probabilities is used in incorporating the uncertainty of the forecast flow. See Karamouz and Vasiliadis (1992) for a detailed discussion on this issue. The BSDP performance is limited by the number of the state variable discretization. The shortcoming has been realized in the development of a GA based Bayesian stochastic model for the finer state variable discretization. The ability of GA with varying chromosome length has been discovered through the work of Kerachian and Karamouz (2006), but that study was limited to extend the numbers of storage and inflow discretization by varying the number of time steps in the modeling (i.e. from one season per year to 12 months per year). In this study, the length of chromosomes is extended to include higher numbers of reservoir storage and inflow discretization for 12 months and to initiate with less chromosome length. In order to overcome the computational complexity of the problem, the discretization of reservoir storage and inflow has been sequentially increased. This way, the number of genes (chromosome length) is sequentially increased to expeditiously lead the initial feasible solutions to the near global optimum solution. The proposed GA with varying chromosome length as shown in Fig. 4 has the following steps:

- 1- A random initial population of reservoir storage at the end of each month for different intervals of monthly inflow, forecasted flow and reservoir storage at the beginning of each month is generated. In this step, the numbers of reservoir and inflow discretization are considered to be low. Therefore, the optimization model solves a small problem containing a few number of genes in each chromosome.
- 2- The optimal reservoir storage at the end of each month and monthly water release from the reservoir are obtained using the GA optimization method.
- 3- As presented in Fig. 4, the number of discretization of reservoir storage and inflow are doubled separately and the initial value of the chromosome for the

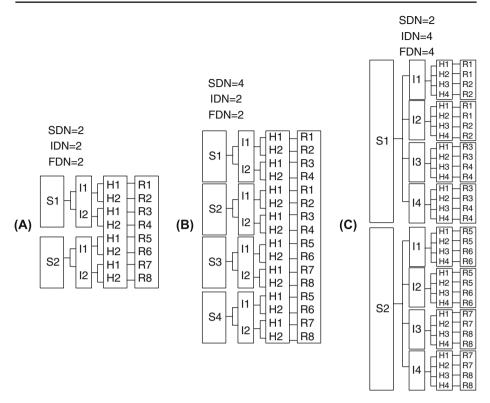


Fig. 4 Schematic of sequential increase of the chromosome lengh by increasing flow and storage discretizations

next sequence is generated based on the chromosome value of the last generation. When the number of discretization of the reservoir storage is doubled, the value of genes related to each discretization of reservoir storage is duplicated (transition from state A to state B) and when the number of inflow and forecasted flow discretization is doubled, the value of each gene is duplicated for each inflow and forecasted flow discretization (transition from state A to state C). This means, the chromosome length has increased sequentially based on increasing the reservoir storage and inflow discretizations and through updating the Bayesian probabilities. The optimization model considers the initial value for the chromosome in the next sequence based on the optimum solution of the previous sequence.

- 4- Optimization processes for the last population is done and the optimal reservoir storage at the end of each month is obtained up to a predefined maximum number of iterations.
- 5- Step 3 and 4 are repeated until the desired number of discretization is reached.

In this study, the maximum inflow intervals are considered as 8. Afterwards, increasing inflow discretization is stopped but the reservoir storage discretization is continuously increased until reaching a maximum predefined number such as 64 or 128.

#### 3 Case Study

Aharchay river basin in the north-western part of Iran is located between 47° 20′ and 47° 30′ north longitude and 38°20′ and 38°45′ east latitude. The only available surface water storage facility in the study area is the Satarkhan Dam as shown in Fig. 5. In order to evaluate the proposed models; they are applied to the Satarkhan reservoir.

The maximum capacity of the reservoir is about 131 MCM (million cubic meters) with an average annual inflow of 82 MCM. A schematic of the Satarkhan dam characteristics which shows the various water levels of reservoir and its related storage is presented in Fig. 6.

Seventeen years of historical monthly streamflow data in Orang hydrometric station, located just upstream of the Satarkhan Dam is considered as the inflow to the reservoir. The general streamflow forecasting model ARIMA (p,d,q)  $(P,D,Q)_w$  is used for forecasting the inflow to the Satarkhan reservoir. Where "p" and "q" are the orders of the non-seasonal autoregressive and the non-seasonal moving average components. "P" and "Q" are the orders of the seasonal autoregressive and the non-seasonal difference, and "D" is the order of the seasonal difference of season "w".

The forecasted time series of inflow is predicted based on the historical data of the Orang hydrometric station just upstream of the reservoir. The best ARIMA (Autoregressive Integrated Moving Average) model is selected based on the white noise test of independence, the chi-square goodness-of-fit test, and the analysis of autocorrelation functions (ACF) and partial autocorrelation functions (PACF) as well as other model testing criteria. The selected model is ARIMA  $(2,1,1)(2,1,2)_{12}$ .

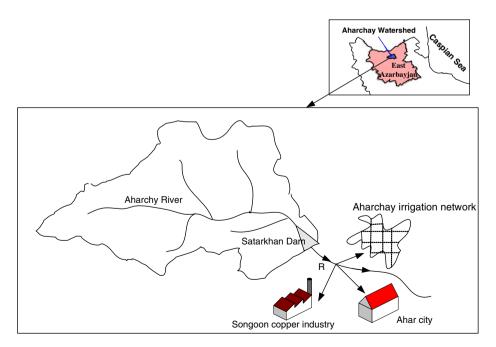


Fig. 5 Aharchay river and Satarkhan reservoir watershed

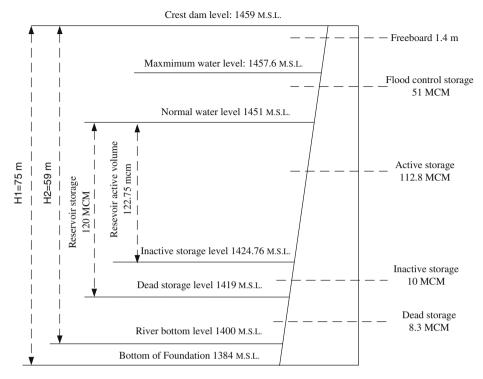


Fig. 6 Schematic of Satarkhan reservoir characteristics

It should be noted that for testing the operating rules, developed by different models, an ARMA (1,1) model has been fitted to the historical data to synthesize 50 years of monthly data. To test the optimization model, a forecast series is also needed. Therefore a 50 year forecast series is also predicted utilizing the 50 year synthesized data using the same ARIMA model that was identified before utilizing the historical data.

The Satarkhan dam supplies the domestic demands of Ahar City, the demand of Songoon copper industry and agricultural demand of 3,800 ha  $(1 \text{ ha} = 10,000 \text{ m}^2)$  to be expanded to over 6,000 ha in the future of pressure irrigation networks and instream flow requirements as the environmental water demand. Therefore, water demands downstream of the reservoir includes the domestic, industrial, agricultural and environmental demands that their monthly value are presented in Table 1. The lack of consensus among the water users and within a watershed tends to be pervasive and usually results in undue delays in the implementation of water resources development projects especially expansion of pressure irrigation networks. Currently, such conflicts could be resolved through participatory decision making. In this study, the proposed optimization models determine the monthly release from the reservoir and the amount of allocated water to each sector during the planning horizon by a compromise among users. Different water supply sectors in the study area are agricultural, industrial, domestic, environmental and Azarbayjan regional water authority. Utility functions indicate player's preference and individual risk taking attitudes in

Month	Domestic	Industrial	Agricultural	Environmental	Total
January	0.64	0.44	0.0	0.70	1.78
February	0.64	0.44	0.0	0.70	1.78
March	0.62	0.44	0.0	0.70	1.76
April	0.78	0.44	0.81	0.70	2.73
May	0.78	0.44	6.13	0.70	8.05
June	0.78	0.44	8.74	0.70	10.66
July	0.82	0.44	11.92	0.70	13.88
August	0.82	0.44	12.47	0.70	14.43
September	0.82	0.44	6.03	0.70	7.99
October	0.67	0.44	1.68	0.70	3.49
November	0.67	0.44	0.52	0.70	2.33
December	0.67	0.44	0.0	0.70	1.81

 Table 1
 The monthly demands of different sectors in the study area (MCM)

the decision process. The general format of the utility function based on the reliability of supplying the demand of different water users/stakeholders is illustrated in Fig. 7. In this figure, the utility value of each sector varies between zero and one when the reliability of supplying the water demand is in the range of a to d. The zero value indicates that the allocated resource has no value for the consumer, and the value of one represents 100% satisfaction. Social issues could be considered when a,b,c, and d parameters of each user's utility function are determined. In order to avoid eliminating the near optimal solution and to expedite the convergence process of the optimization model, the utility function decreases by water allocation with any excess water from water demands. Without considering parameter (d) presented in Fig. 7, the Nash product will be zero for supplying with any excess water from water demands. In the model, the total excess water is summed with environmental water allocation and then the utility function of environmental sector is calculated.

In order to determine the relative authority or weight of each sector, the AHP method is used. This method is based on a pair-wise comparison of the importance of different sectors (industrial sector, the Regional Water and Wastewater Company, the regional water authority, the agricultural sector, and the environmental sector) and the consistency of comparisons are verified. The pair-wise comparison matrix is

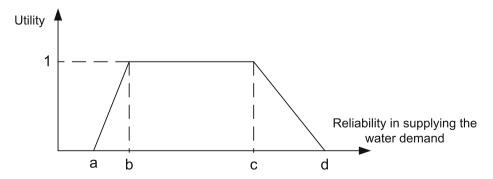


Fig. 7 Typical Utility function of allocated water of a water user

determined by sending questionnaires to different experts/agencies. The eigenvector of pair-wise comparison matrix is then used for estimating the relative weight (importance or priority) of different sectors.

The instream flow requirement (environmental demand) in the Aharchay River is the main concern of the environmental sector. The available data shows that about 10% of the historical inflow of the river in each month is considered as environmental demand in this study. Considering the importance of the instream flow requirements, the most favorable range is 9 to 10 MCM per year. The values of a,b,c, and d parameters for the environmental utility function are presented in the second row of Table 2.

The main objective of the agricultural sector is to increase the reliability of meeting agricultural demand. The most desirable range for water demand supply is considered as 80% to 100% of the demand and the lowest amount of agricultural sector utility is considered as 20% of the demand. This amount of water is necessary for irrigation of orchards in the study area. The utility function parameters for the reliability of supplying the agricultural water demand are presented in the third row of Table 2.

The main objective of the industrial sector is to increase the reliability of water to industrial demands. Supplying the industrial demand especially for the Songoon copper industry is one of the important functions of the Satarkhan dam. The most favorable range of water supplying reliability for the industrial sector is considered as 95% to 100%. The utility function parameters of this sector are given in the fourth row of Table 2.

The main objective of the regional Water and Wastewater Company is to supply the water demand for the City of Ahar. The desirable range of this sector is considered as 95% to 100% of the domestic demand and the lowest acceptable amount is about 90% of the demand where the remaining water demand can be supplied from groundwater. The utility function parameters of this sector are presented in the fifth row of Table 2.

The main objective of the Azarbayjan regional water authority is to maintain the storage level of the Satarkhan reservoir. The utility function of reservoir storage is defined considering the minimum and maximum allowable water storage levels in each month. The desirable variation of water level in Satarkhan reservoir is between the normal water level and the elevation of the water intake. The reservoir storages associated with these elevations are 120 and 45 MCM, respectively. The maximum and minimum allowable water storage in the reservoir is considered to be about 130 and 20 MCM respectively. The utility function of the objective for the reservoir storage is presented in the sixth row of Table 2.

Sector	а	b	с	d	w
Environmental	50	90	200	300	0.17
Agricultural	20	80	100	120	0.15
Industrial	80	95	100	130	0.19
Regional Water and Wastewater Company	90	95	100	130	0.32
Azarbayjan Regional Water Authority	20	45	120	130	0.17

 Table 2
 Utility function parameters for different sectors/water users

# 4 Results and Discussion

In this section, the results of applying the models to the case study are presented. A 17-year time series of streamflow (1982–1999) is used for evaluation of the model for water allocation from the Satarkhan reservoir to the domestic, industrial, agricultural and environmental demands. In all models, the performance indices are used to evaluate the results of models. These indices show how often the system does not fail (reliability), how quickly the system returns to a satisfactory state once a failure has occurred (resiliency) and how significant the consequences of failure are (vulnerability). Satisfactory state is a state that water demands for different sectors being supplied. The reliability can be defined as the number of data in a satisfactory state divided by the total number of data in time series. The resiliency can be expressed as the probability that if in an unsatisfactory state, the next state will be satisfactory. The vulnerability is the expected measure of the unsatisfactory state. It can be defined as the sum of positive different values between the threshold value and the unsatisfactory value divided by the number of times that an unsatisfactory value occurred. The threshold value in each month is the water demand [please refer to Karamouz et al. (2009) for more details about the formulation of the utilized indices in the case study]. In the first part of this section, the results of developing a deterministic model using the GA-KNN optimization model are presented and the performance of the model is compared with the alternative model such as the VLGA model. In the second part of this section, the results of the development of a BSGA are presented and compared with the results of BSDP model. In the discussion of the results section, the performances of developed models are evaluated.

# 4.1 Part 1-Deterministic Model

In order to generate the operating policies for a reservoir, a deterministic GA based model with the KNN estimator for initial solutions is developed. The results of the deterministic GA-KNN and VLGA models are presented in Table 3. In this table, the percentages of supplying the demands in the planning period are shown for both models. The results show that GA-KNN improves the reliability of supplying the demands by 7%.

Figure 8 shows the objective function variations of GA-KNN and VLGA models. As can be seen in this figure, the difference between the objective functions at the end of each optimization sequence and at the beginning of the new sequence is less in the GA-KNN model than VLGA model. Therefore, it takes less time to reach the optimal solution in each sequence using the GA-KNN model and it leads to produce

 Table 3
 Comparison of the results of supplying the water demand downstream of the reservoir for the deterministic models

Demand	VLGA			GA-KNN		
	Reliability %	Resiliency %	Vulnerability (MCM)	Reliability %	Resiliency %	Vulnerability (MCM)
Domestic	81	72	1.23	85	70	1.30
Industrial	90	83	1.31	93	79	1.43
Agricultural	54	43	1.57	57	42	1.59
Environmental	64	54	1.60	65	52	1.64

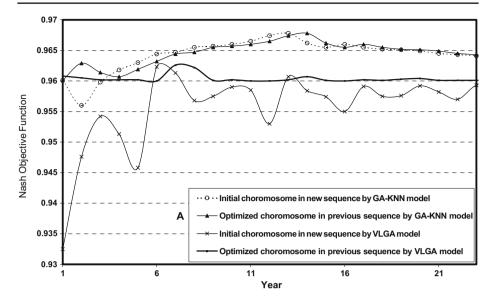


Fig. 8 The trend of GA-KNN and VLGA model convergence

more robust and reliable results while reducing the run time and the computational efforts significantly.

The GA-KNN model could be more effective in reducing the optimization run time in conditions with variable inflow and demand during each year of the planning horizon, because the GA-KNN model considers inflow and demand variations in generating the new gene values in each sequence.

#### 4.2 Part 2—Stochastic Model

In order to evaluate the performance of the stochastic model, the results of the BSGA and the BSDP optimization models are compared. The water releases from the reservoir utilizing BSDP and BSGA models and the water demand during the simulation period are presented in Fig. 9.

The reliability, resiliency, and vulnerability of supplying the water demand downstream of the reservoir for a 17-year simulation period are 69, 46%, 2.45 MCM in the BSGA model compared to 66, 46%, 2.45 MCM when using the BSDP model with equal numbers of inflow and reservoir storage discretization. The results of performance indices in supplying the demand of each sector are presented in Table 4.

The BSGA model seems more suitable for large scale reservoir systems than the BSDP model because it could handle more discrete storage levels than BSDP. The level of the reservoir storage in the BSDP model is 68 MCM compared with 74 MCM in the BSGA model which shows that the BSGA model better maintains the storage fluctuations during the time horizon of the operation. It shows the BSGA model with conflict resolution objective function has better performance in the reservoir operation for improving the reliability of supplying the water demand than the BSDP model.

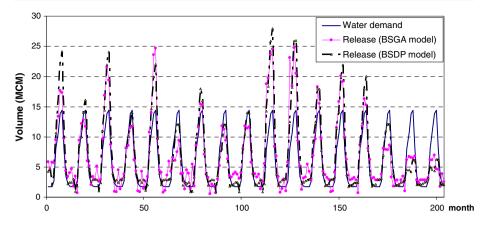


Fig. 9 Variation of monthly release by BSDP and BSGA models

4.3 Comparison of Results

In the previous section, the proposed deterministic and stochastic models to develop the operating policies were compared with the alternate models. In this section, in order to compare the proposed models with each other, a 50-year time series of reservoir inflow is generated.

In the deterministic model, the optimal release in each month is determined using the GA-KNN optimization model. Then, the operating rules to release water are developed using the KNN model with respect to the initial storage, the inflow to the reservoir, the monthly demand. The values of these variables at a given time with selected weights are substituted in the general form of KNN as shown in Eq. 12. The best weights of different independent variables including the inflow to the reservoir, the reservoir storage, and the water demand in each month are considered as 0.2, 0.3, and 0.5, respectively and the best value of K (number of nearest neighborhood) is considered as 40. Figure 10, illustrates the optimal releases which are the results of the operating rule generated by the KNN model. As shown in this figure, in the months with low water demands, the optimal policies resulting from the optimization model and the operating rules obtained from the KNN model are comparable. The correlation coefficient of two series of water release is about 88% for the dry season (March to September) and 78% for the wet season (October to February). Therefore, the operating rules are utilized to test the model performance with 50 years of generated time series of inflow to the reservoir.

Also the operating policies obtained from the stochastic model are utilized to simulate the optimal reservoir storage at the end of each month and, accordingly the

Demand	Reliability %	Resiliency %	Vulnerability (MCM)
Domestic	87	69	1.29
Industrial	93	79	1.41
Agriculture	59	41	1.55
Environmental	66	48	1.38

 Table 4
 Performance indices of suppling the demand with BSGA model

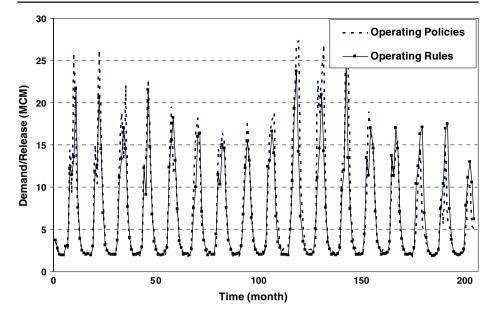


Fig. 10 Calibration of generated operating rules by KNN model

water releases from the reservoir in each month. The performance indices including the reliability, resiliency and vulnerability are given in Table 5.

As shown in this table, the stochastic model can make a gain of +4% in the reliability which means the total number of non-supplying demand months (unsatisfactory states) is 186 in the deterministic model and this is about 162 months in the stochastic model out of 600 months. The vulnerability values, the expected values of non-supplying water demand, are about 2.33 MCM and 2.06 MCM (3.3 and 2.9 percentage of the annual demand) in the stochastic and deterministic models, respectively. It shows the stochastic model considering uncertainties of the inflow and forecasted inflow to the reservoir has better performance in the reservoir operation for improving the reliability of supplying the water demand than the deterministic model.

The variation of reservoir storage for both models is presented in Fig. 11. As shown in this figure, the average water storages in the reservoir for the stochastic and deterministic models are about 89.7 MCM and 75.3 MCM with a standard deviation of 18.5 and 22.0, respectively. The results show that the stochastic model has better performance in maintaining the reservoir storage and more consistent behavior in reservoir operation.

 Table 5
 Performance indices of supplying the water demand for the deterministic and stochastic models

Model	Reliability %	Resiliency %	Vulnerability (MCM)
GA-KNN Model	69	47	2.06
BSGA Model	73	45	2.33

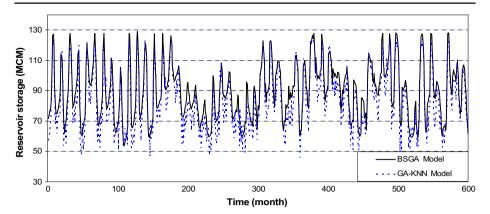


Fig. 11 The variation of reservoir storage for BSGA and GA-KNN models

This procedure can be easily applied to a problem with a longer planning horizon or more water users. As can be seen in the model formulation, it is independent of the number of users and each user can be considered as a player in the Nash Bargaining Theory. Therefore, applying the model to more users is easily possible. For more complex case studies with more reservoirs, based on the Nash Barging Theory, the total objective function can be the multiplication of objective functions of each reservoir considering their utility functions and weights. This study can be extended to include other sources of uncertainties and water quality issues.

#### **5** Summary and Conclusion

In this paper, in order to incorporate the effects of inflow and forecast uncertainties, a stochastic model is developed and compared with a deterministic model (with no accounted uncertainty) when they are applied to the Satarkhan reservoir in the north-western part of Iran. The objective functions of both models are based on the Nash Bargaining Theory.

In this study, the social supports in conflict resolution is not considered as a separate issue but it is implicitly considered, if it is assumed that the utility function is a reflection of the end user's benefit. For example the utility function of the agricultural sector has to reflect the interest of farmers. Furthermore adjustments could be made to the objection if it seems that the interest of agencies involved does not overlap with the interest of the public. This can be done by revising the utility function parameters of agencies/water users to something such as industrial with less social implications. In the deterministic model, the computational burden of a varying length GA optimization method is reduced by generating the feasible initial solutions using a KNN model as an estimator. The results show improvement in supplying the water demand and convergence of the GA optimization model. In the stochastic model, by coupling the Bayesian decision theory and stochastic genetic algorithm, the natural and forecast flow uncertainties can be captured. In order to overcome the computational complexity of the BSGA model, the discretization of reservoir storage and inflow increases sequentially up to a reasonable upper bound (8 for the inflow to the reservoir and 128 for the reservoir storage) by utilizing a varying length of chromosomes scheme. The results of the BSGA and BSDP models are compared and show that the proposed methodology has increased the efficiency of the developed reservoir operating policies while reducing the computation difficulties of the BSDP model (when increasing the numbers of the reservoir storage and the inflow discretization). This is perhaps the main advantage of the BSGA model compared to BSDP especially when water allocation to different sectors/water users is considered. Although the two models have different objective functions, the comparisons are considered as fair because in both models, the objectives are to meet the release targets.

The principle measure of the model effectiveness is related to meet the water demands in a simulated reservoir operation period. Namely, the performance of the proposed the deterministic and stochastic models is compared with a 50-year generated time series. The results show that the KNN model is an effective model for estimating the reservoir release with acceptable accuracy at a fraction of the run time of the stochastic model (a factor of 5 to 24 h) using a Pentium IV (2,800 MHz) computer. Nevertheless, the BSGA with Nash Bargaining Theory objective function increases the reliability of supplying the water demands during the planning horizon. BSGA is more robust than the GA-KNN model because it has considered the uncertainty of the forecasts in the development of the operating policies. The conclusions of improved performance of the proposed algorithms are based on a single case study and are therefore subject to further testing.

The development of both models allows the decision maker to have two effective tools with their own advantages. Utilizing both models will help cross check the results and make comparative analysis of the reservoir operation. If both models are indicating similar results, the reservoir operator/decision maker operates at a higher confidence level.

**Acknowledgements** This study was partially supported by a contract from Azarbayjan Regional Water Authority in Iran entitled "Drought management in the Aharchay watershed and the development of Satarkhan dam operating policies" with the University of Tehran.

#### References

- Araghinejad S, Burn DH, Karamouz M (2006) Long-lead probabilistic forecasting of streamflow using ocean-atmospheric and hydrological predictors. Water Resour Res 42:1–11. doi:10.1029/ 2004WR003853
- Bannayan M, Hoogenboom G (2008) Predicting realization of daily weather data for climate forecasts using the non-parametric nearest-neighbor re-sampling technique. Int J Climatol 28(10): 1357–1368. doi:10.1002/joc.1637
- Burn DH, Yulianti S (2001) Waste-load allocation using genetic algorithms. J Water Res Plan Manage 127(2):121–129. doi:10.1061/(ASCE)0733-9496(2001)127:2(121)
- Celeste AB, Suzuki K, Kadota A (2008) Integrating long- and short-term reservoir operation models via stochastic and deterministic optimization: case study in Japan. J Water Res Plan Manage 134(5):440-448
- Chang F, Hui S, Chen Y (2002) Reservoir operation using grey fuzzy stochastic dynamic programming. Hydrol Processes 16(12):2395–2408
- East V, Hall MJ (1994) Water resources system optimization using genetic algorithms. In: 1st int. conf. on hydroinformatics, Balkema, Rotterdam, The Netherlands, pp 225–231
- Eiger G, Shamir U (1991) Optimal operation f reservoirs by stochastic programming. Eng Opt 17:293–312
- Galeati G (1990) A comparison of parametric and non-parametric methods for runoff forecasting. Hydrol Sci J 35(1):79–94

- Ganji A, Khalili D, Karamouz M, Ponnambalam K, Javan M (2007) A fuzzy stochastic dynamic nash game analysis of policies for managing water allocation in a reservoir system. Water Resour Manage 22:51–66
- Gen MR, Chang L (2000) Genetic algorithm and engineering optimization. Chichester: Wiley
- Harsanyi JC, Selten R (1972) A generalized Nash solution for two-person bargaining games with incomplete information. Manage Sci 18:80–106
- Hashimoto TJ, Stedinger R, Loucks DP (1982) Reliability, resiliency, and vulnerability criteria for water resources performance evaluation. Water Resour Res 18(1):14–20
- Herrera F, Lozano M (2003) Fuzzy adaptive genetic algorithms: design, taxonomy, and future directions. Soft Comput 7(8):545–562. doi:10.1007/s00500-002-0238-y
- Karamouz M, Houck M (1987) Comparison of stochastic and deterministic dynamic programming for reservoir operating rule generation. Water Resour Bull 23(1):1–9
- Karamouz M, Mousavi SJ (2003) Uncertainty based operation of large scale reservoir systems: Dez and Karoon experience. J Am Water Resour Assoc 39(4):961–975
- Karamouz M, Vasiliadis HV (1992) Bayesian stochastic optimization of reservoir operation using uncertain forecasts. Water Resour Res 28(5):1221–1232
- Karamouz M, Szidarovszky F, Zahraie B (2003) Water resources systems analysis. Lewis, Boca Raton
- Karamouz M, Ahmadi A, Moridi A (2009) Probabilistic reservoir operation using Bayesian stochastic model and support vector machine. Adv Water Res 32:1588–1600. doi:10.1016/j.advwatres. 2009.08.003
- Karlsson M, Yakowitz S (1987) Nearest-neighbor methods for nonparametric rainfall-runoff forecasting. Water Resour Res 23(7):1300–1308
- Kelman J, Stedinger JR, Cooper LA, Hsu E, Yuan SQ (1990) Sampling stochastic dynamic programming applied to reservoir operation. Water Resour Res 26(3):447–454
- Kember G, Flower AC (1993) Forecasting river flow using nonlinear dynamics, stochastic. Stoch Hydrol Hydraul 7:205–212
- Kerachian R, Karamouz M (2006) Optimal reservoir operation considering the water quality issues: a stochastic conflict resolution approach. Water Resour Res 42:1–17. doi:10.1029/2005WR004575
- Lall U, Sharma A (1996) A nearest neighbor bootstrap for resampling hydrologic time series. Water Resour Res 32(3):679–693
- Mei-yi L, Zi-xing C, Guo-yun S (2004) An adaptive genetic algorithm with diversity-guided mutation and its global convergence property. J Cent South Univ Technol 11(3):323–327. doi:10.1007/ s11771-004-0066-6
- Michalewicz Z (1992) Genetic algorithms data structures evolutionary programs. Springer, New York
- Nash JF (1950) The bargaining problem. Econometrica 18:155-162
- Ostfeld A, Salomons S (2005) Hybrid genetic-instance based learning algorithm for CE-QUAL-W2 calibration. J Hydrol 310:122–142. doi:10.1016/j.jhydrol.2004.12.004
- Pelikan M, Goldberg DE, Cantu-Paz E (2000) Linkage problem, distribution estimation, and Bayesian networks. Evol Comput 8(3):311–340
- Richards A, Singh N (1997) Two level negotiations in bargaining over water, Game Theoretical Applications to Economics and Operations Research. Kluwer, Boston
- Seifi Å, Hipel KW (2001) Interior-point method for reservoir operation with stochastic inflows. J Water Res Plan Manage 127(1):48–57. doi:10.1061/(ASCE)0733–9496(2001)127:1(48)
- Srinivas M, Patnaik LM (1994) Adaptive probabilities of crossover and mutation in genetic algorithms. IEEE Trans Syst Man Cybern 24(4):656–667
- Stedinger JR, Sule BF, Loucks DP (1984) Stochastic dynamic programming models for reservoir operation optimization. Water Resour Res 20(11):1499–1505
- Teegavarapu RSV, Simonovic SP (2001) Optimal operation of water resource systems: trade-offs between modeling and practical solutions. Integr Water Resour Manage, IAHS Red Book, 272, IAHS, pp 257–263
- Todini E (2000) Real-time flood forecasting: operational experience and recent advanced. In: Marsalek J et al (eds) Flood issues in contemporary water management. Kluwer, Dordrecht, pp 261–270
- Trezos T, Yeh WW-G (1987) Use of stochastic dynamic programming for reservoir management. Water Resour Res 23(6):983–996
- Varian HR (1995) Coase, competition, and compensation. Japan World Econ 7(1):13–27
- Wardlaw R, Sharif M (1999) Evaluation of genetic algorithms for optimal reservoir system operation. J Water Res Plan Manage 125(1):25–33