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Water Resources Management and Planning under Uncertainty: an Inexact Multistage Joint-Probabilistic Programming Method

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Abstract In this study, an inexact multistage joint-probabilistic programming (IMJP) method is developed for tackling uncertainties presented as interval values and joint probabilities. IMJP improves upon the existing multistage programming and inexact optimization approaches, which can help examine the risk of violating joint-probabilistic constraints. Moreover, it can facilitate analyses of policy scenarios that are associated with economic penalties when the promised targets are violated within a multistage context. The developed method is applied to a case study of water-resources management within a multi-stream, multi-reservoir and multi-period context, where mixed integer linear programming (MILP) technique is introduced into the IMJP framework to facilitate dynamic analysis for decisions of surplus-flow diversion. The results indicate that reasonable solutions for continuous and binary

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variables have been generated. They can be used to help water resources managers to identify desired system designs against water shortage and for flood control, and to determine which of these designs can most efficiently accomplish optimizing the system objective under uncertainty.

Keywords Dynamics · Inexact optimization · Multistage · Joint probability · Planning · Scenario analysis · Uncertainty · Water resources

1 Introduction

For decades, the growing population, developing economy, varying natural conditions, shrinking water availabilities, and deteriorating quality of water resources have exacerbated the issue of water allocation. Serious water shortage could arise under disadvantageous river-flow conditions, leading to complexities in identifying desired plans for resources allocation. On the other hand, flooding is becoming one of the most destructive types of natural hazards, particularly under changing climatic conditions. These have led to a variety of adverse impacts on the social–economic development and human life. Furthermore, uncertainties that exist in many system parameters and their interrelationships could intensify the complexities of water allocation and flood control due to the temporal and spatial variations in water availabilities and storage capacities. Decision supports for effectively planning water resources management under various uncertainties and complexities are desired.

Previously, many researchers tried to tackle these difficulties through stochastic mathematical programming (SMP) approaches (Loucks et al. 1981; Pereira and Pinto 1985, 1991; Tsakiris 1988; Kelman et al. 1990; Abrishamchi et al. 1991; Efremides and Tsakiris 1994; Martin 1995; Ferrero et al. 1998; Huang and Loucks 2000; Watkins et al. 2000; Seifi and Hipel 2001; Azaiez 2002; Luo et al. 2003, 2007; Wang et al. 2003; Li et al. 2006; Tsakiris et al. 2007; Li and Huang 2008). Among them, multistage stochastic programming with recourse (MSP) was an effective technique that could handle uncertainties expressed as probability distributions as well as permit revised decisions in each time stage based on information of sequentially realized uncertain events (Birge and Louveaux 1997; Dupačová 2002). In MSP, the uncertain information was often modeled as a multilayer scenario tree. The primary advantage of scenario-based stochastic programming was the flexibility it offered in modeling the decision processes and defining the scenarios, particularly if the state dimension was high (Birge 1985). For example, Watkins et al. (2000) proposed a scenario-based multistage stochastic programming model for planning water supplies from highland lakes, where dynamics and uncertainties of water availability (and thus water allocation) could be taken into account through generation of multiple representative scenarios. Li et al. (2006) developed an interval-parameter multistage stochastic linear programming method for supporting water resources decision making, which could deal with uncertainties expressed as discrete random variables and interval values. However, these methods were incapable of accounting for the risk of violating joint-probabilistic constraints within a multi-reservoir system; moreover, the reservoir's storage capacity was neglected. In fact, spills associated with the storage limitations are critical for reservoir operations.

Chance-constrained programming (CCP) method can reflect the reliability of satisfying (or risk of violating) system constraints under uncertainty (Charnes et al. 1972; Charnes and Cooper 1983). It does not require that all of the constraints be totally satisfied. Instead, the constraints can be satisfied in a proportion of cases with given probabilities (Loucks et al. 1981). The CCP methods contain two categories: individual probabilistic constraints (IPC) and joint probabilistic constraints (JPC). In the IPC problems, each individual constraint is satisfied at a probability level; consequently, probability for all constraints will be less than probability level for individual constraint (Zhang et al. 2002). In the JPC problems, in comparison, the whole set of uncertain constraints are enforced to be satisfied at least a probability level; this allows an increased robustness in controlling system risk in the optimization process. There have been many applications of CCP to water resources management (ReVelle et al. 1969; Loucks et al. 1981; Fujiwara et al. 1988; Dupačová et al. 1991; Morgan et al. 1993; Srinivasan and Simonovic 1994; Rangarajan 1995; Huang 1998; ReVelle 1999; Edirisinghe et al. 2000). In general, although the CCP can deal with uncertainties (of constraints) presented as probability distributions, three limitations may exist: (1) it has difficulties in handling independent uncertainties in the objective coefficients (Infanger 1993); (2) when uncertainties exist in the left-hand sides, the resulting nonlinear model would be associated with a number of difficulties in global-optimum acquisition; (3) for many practical problems, the quality of information that can be obtained for various uncertainties is mostly not satisfactory enough to be presented as probability distributions (Huang and Loucks 2000). Interval-parameter programming (IPP) is an alternative for dealing with uncertainties existing in the left- and right-hand sides and in the objective function that cannot expressed as distribution functions (Huang et al. 1995). Nevertheless, no previous study was reported on the development of multistage joint-probabilistic programming associated with inexact optimization as well as the relevant application to water resources management.

Therefore, the objective of this study is to develop an inexact multistage joint-probabilistic programming (IMJP) method in response to the above challenges. Techniques of multistage stochastic programming with recourse (MSP), joint-probabilistic constraint programming (JPC), and interval-parameter programming (IPP) will be incorporated within a general framework. IMJP will be able to deal with uncertainties expressed as not only probability distributions but also interval values. It can also help examine the risk of violating joint probabilistic constraints. A case study will then be provided for demonstrating how the developed method will support the planning of water resources management within a multi-stream, multi-reservoir and multi-period context.

The developed IMJP method will improve upon the existing stochastic programming approaches with advantages in uncertainty reflection, policy investigation, risk assessment, and dynamic analysis (Li et al. 2006, 2007; Li and Huang 2008; Guo et al. 2008). Compared with the inexact two-stage programming methods (Li et al. 2007; Li and Huang 2008; Guo et al. 2008), the IMJP can incorporate more dynamic and uncertain information within its modeling framework. The dynamics of uncertainties (and thus decisions) can be taken into account through generation of a multilayer scenario tree, and this allows corrective actions to be undertaken dynamically for the pre-regulated policies and can thus help minimize the expected recourse cost; in comparison, these two-stage optimization approaches do not require a tree structure, where uncertainties are considered to be discrete and mutually independent. In comparison with the interval-parameter multistage linear programming (IMSLP) method as advanced by Li et al. (2006), the IMJP is advantageous in tackling uncertainties presented as joint probabilities within a multi-stream and multi-reservoir system; moreover, a surplus-flow-diversion plan is considered to avoid flooding event; nevertheless, in IMSLP, the reservoir's storage capacity is neglected. In fact, spills associated with the storage limitations are critical for reservoir operations. Moreover, all of these previous two-stage and multistage optimization methods are incapable of addressing the risk of violating system constraints under uncertainty. Generally, four special characteristics of the developed method can make the IMJP unique compared with the existing optimization techniques: (1) it can handle uncertainties in the model's left-hand sides and objective function presented as interval values and those in the right-hand sides as joint probability distributions; (2) it can reflect the dynamics of system uncertainties and decision processes under a complete set of scenarios within a multistage context; (3) it can help examine the reliability of satisfying (or the risk of violating) the system constraints under uncertainty; moreover, with joint probabilistic constraints, it possesses an increased robustness in tackling the system risk in the optimization process; (4) it can facilitate analyses of policy scenarios that are associated with different levels of economic penalties when the regulated policy targets are violated.

2 Methodology

Firstly, a multistage stochastic linear programming with recourse model can be formulated as follows:

$$\text{Max} \quad f = \sum_{t=1}^{T} C_t X_t - \sum_{t=1}^{T} \sum_{k=1}^{K_t} p_{tk} D_{tk} Y_{tk} \tag{1a}$$

subject to:

$$A_{rt} X_t \le B_{rt}, \quad r = 1,\ 2, \cdots, m_1; t = 1, 2, \cdots, T \tag{1b}$$

$$A_{it} X_t + A'_{itk} Y_{tk} \le \widetilde{w_{itk}}, \quad i = 1, 2, \cdots, m_2; t = 1, 2, \cdots, T; k = 1, 2, \cdots, K_t \tag{1c}$$

$$x_{jt} \ge 0, \quad x_{jt} \in X_t,\ j = 1, 2, \cdots, n_1; t = 1, 2, \cdots, T \tag{1d}$$

$$y_{jtk} \ge 0, \quad y_{jtk} \in Y_{tk},\ j = 1, 2, \cdots, n_2; t = 1, 2, \cdots, T; k = 1, 2, \cdots, K_t \tag{1e}$$

where p_{tk} is probability of occurrence for scenario k in period t, with $p_{tk} > 0$ and $\sum_{k=1}^{K_t} p_{tk} = 1$; D_{tk} are coefficients of recourse variables (Y_{tk}) in the objective function; A'_{itk} are coefficients of Y_{tk} in constraint i; $\widetilde{w_{itk}}$ is random variable of constraint i, which is associated with probability level p_{tk}; K_t is number of scenarios in period t, with the total being $K = \sum_{t=1}^{T} K_t$. In model (1), the decision variables are divided into two subsets: those that must be determined before the realizations of random variables are disclosed (i.e., x_{jt}), and those (recourse variables) that can be determined after the realized random-variable values are available (i.e., y_{jtk}).

Obviously, model (1) can deal with uncertainties in the right-hand sides presented as random variables with known probability distributions when coefficients in the left-hand sides and in the objective function are deterministic. However, randomness in other right-hand-side parameters may also need to be tackled. For example, for water resources management within a multi-reservoir system, uncertainties presented in terms of joint probabilities may exist in storage capacities (i.e., the storage capacities may be fixed with a level of probability, which represents the admissible risk of violating the uncertain capacity constraints). Such uncertainties could lead to complexities in water allocation where interactive and dynamic relationships exist within a multistage context. The technique of joint probabilistic constraints (JPC) can be used for dealing with such complexities (Miller and Wager 1965; Charnes and Cooper 1983). A general JPC formulation can be expressed as (Miller and Wager 1965):

$$\text{Min } c^T x \tag{2a}$$

subject to:

$$\mathrm{P}\left(T_s x \geq \varepsilon_s, s = 1, 2, ..., m_3\right) \geq q \tag{2b}$$

$$Ax \geq b \tag{2c}$$

$$x \geq 0 \tag{2d}$$

Obviously, in JPC, the entire set of uncertain constraints are enforced to be satisfied with at least a joint probability of q (Zhang et al. 2002; Lejeune and Prekopa 2005); thus, an increased robustness in controlling the system risk can be accomplished, compared with the IPC problems (where each individual constraint is satisfied at a probability level). Model (2) is generally nonlinear and possibly non-convex due to the existence of joint probabilities for multiple random variables (ε_s). By letting the random variables take a set of individual probabilistic constraints, the JPC problem can be equivalently formulated as a linear programming model as follows (Lejeune and Prekopa 2005):

$$\text{Min } c^T x \tag{3a}$$

subject to:

$$T_s x \geq F_s^{-1}\left(q_s\right), s = 1, 2, \cdots, m_3 \tag{3b}$$

$$\sum_{s=1}^{m_3} \left(1 - q_s\right) \leq 1 - q \tag{3c}$$

$$Ax \geq b, \tag{3d}$$

$$x \geq 0, \tag{3e}$$

where q_s ($s = 1, 2, \ldots, \mathrm{m}_3$) are random variables constrained to be larger than or equal to q, and F_s^{-1} refer to inverse probability distributions of the random variables

(ε_s). Consequently, the JPC technique can be incorporated with the above MSP framework to deal with uncertainties presented as joint probabilities; this leads to a hybrid multistage joint-probabilistic programming (MJP) model as follows:

$$\text{Max} \quad f = \sum_{t=1}^{T} C_t X_t - \sum_{t=1}^{T} \sum_{k=1}^{K_t} p_{tk} D_{tk} Y_{tk} \tag{4a}$$

subject to:

$$A_{rt} X_t \leq B_{rt}, \quad r = 1,\ 2, \cdots,\ m_1; t = 1,\ 2, \cdots,\ T \tag{4b}$$

$$A_{it} X_t + A'_{itk} Y_{tk} \leq \widetilde{w_{itk}}, \quad i=1,\ 2,\ \cdots, m_2;\ t=1,\ 2,\ \cdots, T;\ k=1, 2, \cdots, K_t \tag{4c}$$

$$A_{st} X_t + A'_{st} Y_{tk} \geq F_{st}^{-1}(q_s),\ s=1,\ 2,\ \cdots,\ m_3;\ t=1,\ 2,\ \cdots, T;\ k = 1, 2, \cdots, K_t \tag{4d}$$

$$\sum_{s=1}^{m_3} (1 - q_s) \leq 1 - q \tag{4e}$$

$$x_{jt} \geq 0, \quad x_{jt} \in X_t,\ j = 1,\ 2, \cdots,\ n_1;\ t = 1,\ 2,\ \cdots, T \tag{4f}$$

$$y_{jtk} \geq 0, \quad y_{jtk} \in Y_{tk},\ j = 1,\ 2, \cdots,\ n_2;\ t = 1,\ 2, \cdots, T;\ k = 1, 2, \cdots,\ K_t \tag{4g}$$

Model (4) can not only address uncertainties in its right-hand sides presented as probability distributions but also reflect the reliability of satisfying (or risk of violating) system constraints. However, uncertainties that exist in the left-hand sides and the objective function may also need to be reflected. In many practical problems, the quality of information that can be obtained is often not satisfactory enough to be presented as probabilistic distributions; besides, even if such distributions are available, reflection of them in large-scale stochastic optimization models could be extremely challenging (Birge and Louveaux 1997; Huang and Loucks 2000; Li et al. 2006). Therefore, for uncertainties in left-hand sides and cost/revenue parameters in the objective function, an extended consideration is the introduction of interval-parameter programming (IPP) technique into the MJP framework. This leads to an inexact multistage joint-probabilistic programming (IMJP) model as follows:

$$\text{Max} \quad f^{\pm} = \sum_{t=1}^{T} C_t^{\pm} X_t^{\pm} - \sum_{t=1}^{T} \sum_{k=1}^{K_t} p_{tk} D_{tk}^{\pm} Y_{tk}^{\pm} \tag{5a}$$

subject to:

$$A_{rt}^{\pm} X_t^{\pm} \leq B_{rt}^{\pm}, \quad r = 1,\ 2, \cdots,\ m_1;\ t = 1,\ 2, \cdots,\ T \tag{5b}$$

$$A_{it}^{\pm} X_t^{\pm} + A'^{\pm}_{itk} Y_{tk}^{\pm} \leq \widetilde{w_{itk}^{\pm}}, \quad i=1,\ 2,\cdots, m_2;\ t=1,\ 2,\cdots, T;\ k=1, 2,\cdots, K_t \tag{5c}$$

$$A_{st}^{\pm} X_t^{\pm} + A'^{\pm}_{st} Y_{tk}^{\pm} \leq F_{st}^{-1}(q_s), \quad s=1,\ 2,\cdots,\ m_3;\ t=1,\ 2,\cdots, T;\ k=1, 2,\cdots, K_t \tag{5d}$$

$$\sum_{s=1}^{m_3}(1-q_s) \le 1-q \tag{5e}$$

$$x_{jt}^{\pm} \ge 0, \quad x_{jt}^{\pm} \in X_t^{\pm}, \ j=1,\ 2,\cdots,\ n_1;\ t=1,\ 2,\cdots,T \tag{5f}$$

$$y_{jtk}^{\pm} \ge 0, \quad y_{jtk}^{\pm} \in Y_{tk}^{\pm}, \ j=1,\ 2,\cdots,\ n_2;\ t=1,\ 2,\cdots,T;\ k=1,2,\cdots,\ K_t \tag{5g}$$

where superscripts '-' and '+' represent lower and upper bounds of the interval parameters, respectively; an interval can defined as a number with known lower and upper bounds but unknown distribution information (Huang et al. 1995). Then, a two-step solution method is proposed for solving the IMJP model. The submodel corresponding to f^+ can be formulated in the first step when the system objective is to be maximized; the other submodel (corresponding to f^-) can then be formulated based on the solution of the first submodel. Let $x_{jt}^{\pm}$ $(j=1,\ 2,\ldots,\ j_1)$ be variables with positive coefficients in the objective function, $x_{jt}^{\pm}$ $(j=j_1+1,\ j_1+2,\ldots,\ n_1)$ be variables with negative coefficients, $y_{jtk}^{\pm}$ $(k=1,\ 2,\ \ldots,\ K_t$ and $j=1,\ 2,\ldots,\ j_2)$ be recourse variables with positive coefficients in the objective function, and $y_{jtk}^{\pm}$ $(k=1,\ 2,\ \ldots,\ K_t$ and $j=j_2+1,\ j_2+2,\ldots,\ n_2)$ be recourse variables with negative coefficients. Thus, the first submodel is (assume that $\mathbf{B}^{\pm}>0$ and $f^{\pm}>0$):

$$\text{Max} \quad f^+ = \sum_{t=1}^{T}\left(\sum_{j=1}^{j_1} c_{jt}^+ x_{jt}^+ + \sum_{j=j_1+1}^{n_1} c_{jt}^+ x_{jt}^-\right) - \sum_{t=1}^{T}\sum_{k=1}^{K_t} p_{tk}\left(\sum_{j=1}^{j_2} d_{jtk}^- y_{jtk}^- + \sum_{j=j_2+1}^{n_2} d_{jtk}^- y_{jtk}^+\right) \tag{6a}$$

subject to:

$$\sum_{j=1}^{j_1}\left|a_{rjt}\right|^- \text{Sign}\left(a_{rjt}^-\right)x_{jt}^+ + \sum_{j=j_1+1}^{n_1}\left|a_{rjt}\right|^+ \text{Sign}\left(a_{rjt}^+\right)x_{jt}^- \le b_{rt}^+,\ \forall r,\ t \tag{6b}$$

$$\sum_{j=1}^{j_1}\left|a_{ijt}\right|^- \text{Sign}\left(a_{ijt}^-\right)x_{jt}^+ + \sum_{j=j_1+1}^{n_1}\left|a_{ijt}\right|^+ \text{Sign}\left(a_{ijt}^+\right)x_{jt}^- + \sum_{j=1}^{j_2}\left|a'_{ijtk}\right|^+ \text{Sign}\left(a'^{+}_{ijtk}\right)y_{jtk}^- + \sum_{j=j_2+1}^{n_2}\left|a'_{ijtk}\right|^- \text{Sign}\left(a'^{-}_{ijt}\right)y_{jtk}^+ \le \widetilde{w_{itk}^+},\ \forall i,\ t;\ k=1,\ 2,\cdots,\ K_t \tag{6c}$$

$$\sum_{j=1}^{j_1}\left|a_{sjt}\right|^- \text{Sign}\left(a_{sjt}^-\right)x_{jt}^+ + \sum_{j=j_1+1}^{n_1}\left|a_{sjt}\right|^+ \text{Sign}\left(a_{sjt}^+\right)x_{jt}^- + \sum_{j=1}^{j_2}\left|a'_{sjt}\right|^+ \text{Sign}\left(a'^{+}_{sjtk}\right)y_{jtk}^- + \sum_{j=j_2+1}^{n_2}\left|a'_{sjtk}\right|^- \text{Sign}\left(a'^{-}_{sjtk}\right)y_{jtk}^+ \le \left(\varepsilon_s^+\right)^{(q_s)},\ \forall s,\ t;\ k=1,\ 2,\cdots,\ K_t \tag{6d}$$

$$\sum_{s=1}^{m_3} (1 - q_s) \leq 1 - q \tag{6e}$$

$$x_{jt}^{+} \geq 0,\ \forall t;\ j = 1,\ 2, \cdots,\ j_1 \tag{6f}$$

$$x_{jt}^{-} \geq 0,\ \forall t;\ \ j = j_1 + 1,\ \ j_1 + 2, \cdots,\ n_1 \tag{6g}$$

$$y_{jtk}^{-} \geq 0,\ \forall t;\ \ j = 1,\ 2, \cdots,\ j_2;\ k = 1,\ 2, \cdots,\ K_t \tag{6h}$$

$$y_{jtk}^{+} \geq 0,\ \forall t;\ \ j = j_2 + 1,\ \ j_2 + 2, \cdots,\ n_2;\ k = 1, 2, \cdots,\ K_t \tag{6i}$$

Solutions of $x_{jt\,\mathrm{opt}}^{+}$ $(j = 1,\ 2,\ldots,\ j_1)$, $x_{jt\,\mathrm{opt}}^{-}$ $(j = j_1 + 1,\ j_1 + 2,\ldots,\ n_1)$, $y_{jtk\,\mathrm{opt}}^{-}$ $(j = 1,$ $2,\ldots,j_2$ and $k = 1,\ 2,\ \ldots,\ K_t)$ and $y_{jtk\,\mathrm{opt}}^{+}$ $(j = j_2 + 1,\ j_2 + 2,\ldots,\ n_2$ and $k = 1,\ 2,\ \ldots,$ $K_t)$ can be obtained through solving submodel (6). Based on the above solutions, the second submodel corresponding to f^{-} can be formulated as follows:

$$\begin{aligned} \text{Max} \quad f^{-} = & \sum_{t=1}^{T} \left(\sum_{j=1}^{j_1} c_{jt}^{-} x_{jt}^{-} + \sum_{j=j_1+1}^{n_1} c_{jt}^{-} x_{jt}^{+} \right) \\ & - \sum_{t=1}^{T} \sum_{k=1}^{K_t} p_{tk} \left(\sum_{j=1}^{j_2} d_{jtk}^{+} y_{jtk}^{+} + \sum_{j=j_2+1}^{n_2} d_{jtk}^{+} y_{jtk}^{-} \right) \end{aligned} \tag{7a}$$

subject to:

$$\sum_{j=1}^{j_1} \left| a_{rjt} \right|^{+} \mathrm{Sign}\left(a_{rjt}^{+} \right) x_{jt}^{-} + \sum_{j=j_1+1}^{n_1} \left| a_{rjt} \right|^{-} \mathrm{Sign}\left(a_{rjt}^{-} \right) x_{jt}^{+} \leq b_{rt}^{-},\ \forall r,\ t \tag{7b}$$

$$\begin{aligned} & \sum_{j=1}^{j_1} \left| a_{ijt} \right|^{+} \mathrm{Sign}\left(a_{ijt}^{+} \right) x_{jt}^{-} + \sum_{j=j_1+1}^{n_1} \left| a_{ijt} \right|^{-} \mathrm{Sign}\left(a_{ijt}^{-} \right) x_{jt}^{+} + \sum_{j=1}^{j_2} \left| a'_{ijtk} \right|^{-} \mathrm{Sign}\left(a'^{-}_{ijtk} \right) y_{jtk}^{+} \\ & \quad + \sum_{j=j_2+1}^{n_2} \left| a'_{ijtk} \right|^{+} \mathrm{Sign}\left(a'^{+}_{ijtk} \right) y_{jtk}^{-} \leq \widetilde{w_{itk}^{-}},\ \forall i,\ t;\ k = 1, 2, \cdots,\ K_t \end{aligned} \tag{7c}$$

$$\begin{aligned} & \sum_{j=1}^{j_1} \left| a_{sjt} \right|^{+} \mathrm{Sign}\left(a_{sjt}^{+} \right) x_{jt}^{-} + \sum_{j=j_1+1}^{n_1} \left| a_{sjt} \right|^{-} \mathrm{Sign}\left(a_{sjt}^{-} \right) x_{jt}^{+} + \sum_{j=1}^{j_2} \left| a'_{sjt} \right|^{-} \mathrm{Sign}\left(a'^{-}_{sjtk} \right) y_{jtk}^{+} \\ & \quad + \sum_{j=j_2+1}^{n_2} \left| a'_{sjtk} \right|^{+} \mathrm{Sign}\left(a'^{+}_{sjtk} \right) y_{jtk}^{-} \leq \left(\varepsilon_s^{-} \right)^{(q_s)},\ \forall s,\ t;\ k = 1, 2, \cdots,\ K_t \end{aligned} \tag{7d}$$

$$\sum_{s=1}^{m_3}(1-q_s) \le 1-q \tag{7e}$$

$$0 \le x_{jt}^{-} \le x_{jt\,\mathrm{opt}}^{+},\ \forall t;\ j=1,\ 2,\cdots,\ j_1 \tag{7f}$$

$$x_{jt}^{+} \ge x_{jt\,\mathrm{opt}}^{-},\ \forall t;\ j=j_1+1,\ j_1+2,\cdots,\ n_1 \tag{7g}$$

$$y_{jtk}^{+} \ge y_{jtk\,\mathrm{opt}}^{-},\ \forall t;\ j=1,\ 2,\cdots,\ j_2;\ k=1,2,\cdots,\ K_t \tag{7h}$$

$$0 \le y_{jtk}^{-} \le y_{jtk\,\mathrm{opt}}^{+},\ \forall t;\ j=j_2+1,\ j_2+2,\cdots,\ n_2;\ k=1,2,\cdots,\ K_t \tag{7i}$$

Solutions of $x_{jt\,\mathrm{opt}}^{-}$ ($j=1,2,\ldots,j_1$), $x_{jt\,\mathrm{opt}}^{+}$ ($j=j_1+1, j_1+2,\ldots,n_1$), $y_{jtk\,\mathrm{opt}}^{+}$ ($j=1,2,\ldots,j_2$ and $k=1,2,\ldots,K_t$), and $y_{jtk\,\mathrm{opt}}^{-}$ ($j=j_2+1, j_2+2,\ldots,n_2$ and $k=1,2,\ldots,K_t$) can be obtained through solving submodel (7). Therefore, combining solutions of submodels (6) and (7), we have solutions for the IMJP model as follows:

$$x_{jt\,\mathrm{opt}}^{\pm} = \left[x_{jt\,\mathrm{opt}}^{-},\ x_{jt\,\mathrm{opt}}^{+}\right],\ j=1,\ 2,\cdots,\ n_1;\ \forall t \tag{8a}$$

$$y_{jtk\,\mathrm{opt}}^{\pm} = \left[y_{jtk\,\mathrm{opt}}^{-}, y_{jtk\,\mathrm{opt}}^{+}\right],\ j=1,\ 2,\cdots,\ n_2;\ k=1,2,\cdots,\ K_t;\ \forall t \tag{8b}$$

$$f_{\mathrm{opt}}^{\pm} = \left[f_{\mathrm{opt}}^{-},\ f_{\mathrm{opt}}^{+}\right] \tag{8c}$$

3 Application to Water Resources Management

An authority is charged with delivering water to a municipality to meet demands for regional socio-economic development; meanwhile, it is responsible for flood control and environmental protection. In the study region, there are two streams and two reservoirs that supply water to the municipality (Fig. 1). Due to spatial and temporal variations of the relationships between water demand and supply, the desired water-allocation patterns would vary among different time periods. The authority wants to know how much water can be allocated to the municipality under varying stream inflows. If the targeted water is delivered, revenues will be generated for each unit of water allocated; however, if the targeted water is not delivered,

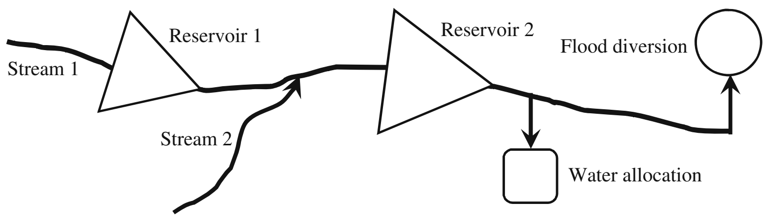

Fig. 1 Schematic of water resources management system

penalties will be generated from the shortfalls. In general, the penalties are associated with the acquisition of water from higher-priced alternatives and/or the negative consequences generated from the curbing of regional development plans when the promised water is not delivered (Loucks et al. 1981).

Uncertainties exist in many system components such as stream flows, reservoir capacities, water-allocation targets, and benefit/cost coefficients. The random characteristics of various processes and conditions, the errors in acquiring the modeling parameters, and the imprecision of the related system constraints are possible sources of the uncertainties. For example, in such a multi-reservoir system, uncertainties presented in terms of joint probabilities may exist in storage capacities (i.e., the storage capacities may be fixed with a level of probability, which represents the admissible risk of violating the uncertain capacity constraints). Moreover, the system is associated with multiple streams and multiple reservoirs where joint probabilities exist in terms of water availabilities and storage capacities. Such uncertainties can lead to interactive and dynamic complexities in terms of water allocation and diversion over a multistage context. Therefore, the developed IMJP method is effective for supporting water resources management under such complexities.

On the other hand, when the inflow levels are continuously high while the demands are confined by the maximum limitation, more surplus would be generated; when the surplus exceeds the reservoir's storage capacity, spill would occur that might potentially lead to flooding events. Losses can hardly be avoided when a flooding event occurs. As a result, a sound surplus-flow-diversion plan is desired for releasing the reservoirs' water to avoid overflow and thus reduce such losses. Mixed integer linear programming (MILP) is a useful technique for tackling this issue, where desired water diversion plans can be obtained through using binary variables to indicate whether a particular action needs to be undertaken (Li et al. 2007). Therefore, through introducing MILP into the IMJP framework, an inexact multistage joint-probabilistic integer programming (IMJIP) model for water resources management within a multi-reservoir system can be formulated as follows:

$$\begin{aligned}\text{Max} \quad f^{\pm} = {} & \sum_{t=1}^{T} \mathrm{NB}_t^{\pm} X_t^{\pm} - \sum_{t=1}^{T} \sum_{k_1=1}^{K_1^t} \sum_{k_2=1}^{K_2^t} p_{tk_1} p_{tk_2} \mathrm{PE}_t^{\pm} Y_{tk_1k_2}^{\pm} \\ & - \sum_{t=1}^{T} \sum_{k_1=1}^{K_1^t} \sum_{k_2=1}^{K_2^t} p_{tk_1} p_{tk_2} \left(\mathrm{FC}_t^{\pm} Z_{tk_1k_2}^{\pm} + \mathrm{VC}_t^{\pm} W_{tk_1k_2}^{\pm} \right) \end{aligned} \tag{9a}$$

subject to:

$$\begin{aligned} S_{(t+1)k_1}^{\pm} = {} & S_{tk_1}^{\pm} + \widetilde{Q_{tk_1}^{\pm}} - \left[A_1^a e_{1t}^{\pm} \left(\frac{S_{tk_1}^{\pm} + S_{(t+1)k_1}^{\pm}}{2} \right) + A_1^0 e_{1t}^{\pm} \right] \\ & - R_{tk_1}^{\pm}, \ \forall t; \ k_1 = 1, 2, \cdots, K_1^t \end{aligned} \tag{9b}$$

$$\begin{aligned} S_{(t+1)k_1k_2}^{\pm} = {} & S_{tk_1k_2}^{\pm} + \left(\widetilde{Q_{tk_2}^{\pm}} + R_{tk_1}^{\pm} \right) - \left[A_2^a e_{2t}^{\pm} \left(\frac{S_{tk_1k_2}^{\pm} + S_{(t+1)k_1k_2}^{\pm}}{2} \right) + A_2^0 e_{2t}^{\pm} \right] - R_{tk_1k_2}^{\pm}, \\ & \forall t; \ k_1 = 1, 2, \cdots, K_1^t; \ k_2 = 1, 2, \cdots, K_2^t \end{aligned} \tag{9c}$$

$$X_t^{\pm} - Y_{tk_1k_2}^{\pm} \le R_{tk_1k_2}^{\pm} - W_{tk_1k_2}^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t;\ k_2 = 1, 2, \cdots, K_2^t \tag{9d}$$

$$X_t^{\pm} + W_{tk_1k_2}^{\pm} \ge R_{tk_1k_2}^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t;\ k_2 = 1, 2, \cdots, K_2^t \tag{9e}$$

$$\Pr\left\{\begin{array}{l} S_{tk_1}^{\pm} \le \mathrm{RSC}_1^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t \\ S_{tk_1k_2}^{\pm} \le \mathrm{RSC}_2^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t; k_2 = 1, 2, \cdots, K_2^t \end{array}\right\} \ge 1 - q \tag{9f}$$

$$S_{tk_1}^{\pm} \ge \mathrm{RSV}_{1t}^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t \tag{9g}$$

$$S_{tk_1k_2}^{\pm} \ge \mathrm{RSV}_{2t}^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t;\ k_2 = 1, 2, \cdots, K_2^t \tag{9h}$$

$$\mathrm{De}_t^{\min} \le X_t^{\pm} \le \mathrm{De}_t^{\max}, \quad \forall t \tag{9i}$$

$$Z_{tk_1k_2}^{\pm}\begin{cases} = 1, & \text{if surplus water diversion is undertaken} \\ = 0, & \text{if otherwise} \end{cases}, \quad \forall t;\ k_1 = 1, 2, \cdots, K_1^t;\ k_2 = 1, 2, \cdots, K_2^t \tag{9j}$$

$$0 \le W_{tk_1k_2}^{\pm} \le M_{k_1k_2} Z_{tk_1k_2}^{\pm},\ \forall t;\ k_1 = 1, 2, \cdots, K_1^t;\ k_2 = 1, 2, \cdots, K_2^t \tag{9k}$$

$$X_t^{\pm} \ge Y_{tk_1k_2}^{\pm} \ge 0, \quad \forall t; k_1 = 1, 2, \cdots, K_1^t;\ k_2 = 1, 2, \cdots, K_2^t \tag{9l}$$

The detailed nomenclatures for the variables and parameters are provided in Appendix. In model (9), the decision variables can be sorted into two categories: continuous and binary. The continuous variables represent water-allocation targets, probabilistic shortage and surplus-flow-diversion levels, while the binary ones indicate whether individual surplus-flow-diversion actions need be undertaken. The objective is to maximize the expected net system benefit through allocating the water resources to the municipality from multi-reservoir over a multistage context. The objective value involves the benefit from suitably allocating water resources to users, the penalty for violating the promised targets, and the cost for diverting surplus flows. The constraints will help define the interrelationships among the decision variables and the water-resources management conditions. In detail, constraints (9b) and (9c) present the mass balance for water resources in each time period (i.e., the change in storage equals inflows minus releases and evaporation losses), where the evaporation loss is assumed to be a linear function of the average storage of reservoir. constraints (9d) and (9e) mean that the sum of optimal water allocated to the users and surplus water diverted (when flow level is high) will not exceed the amount of water released from the reservoir; constraint (9f) specify that the storage amount must not exceed each reservoir capacity under all scenarios, where the storage capacities are fixed with a probability level that represents the admissible risk of violating the uncertain capacity constraints; constraints (9g) and (9h) require that the storage in each reservoir will not lower a reserve level under all scenarios; constraint (9i) indicates that the allocated water must satisfy the users' minimum necessities but not exceed their maximum requirements; constraints (9j) and (9k)

Table 1 Stream inflows and economic data

	Planning period		
	$t = 1$	$t = 2$	$t = 3$
Low flow of stream 1 (probability = 0.2) (10^6 m^3)	[59.2, 65.5]	[65.1, 72.0]	[54.6, 60.5]
Medium flow of stream 1 (probability = 0.6) (10^6 m^3)	[95.4, 109.4]	[99.3, 113.8]	[90.2, 102.8]
High flow of stream 1 (probability = 0.2) (10^6 m^3)	[148.4, 170.6]	[155.6, 178.7]	[143.5, 163.6]
Low flow of stream 2 (probability = 0.4) (10^6 m^3)	[31.7, 35.3]	[33.4, 37.2]	[29.1, 32.4]
High flow of stream 2 (probability = 0.6) (10^6 m^3)	[67.2, 75.5]	[72.7, 81.6]	[64.5, 72.4]
Water allocation demand (10^6 m^3)	[132.5, 181.3]	[145.3, 194.2]	[137.4, 183.5]
Net benefit when water demand is satisfied ($/m^3)	[27.6, 33.2]	[32.3, 38.9]	[37.6, 45.3]
Penalty when water is not delivered ($/m^3)	[110.0, 130.0]	[120.0, 150.0]	[130.0, 160.0]
Fixed cost for surplus-flow diversion ($$10^6$)	[20.0, 25.0]	[23.4, 29.2]	[27.2, 34.0]
Variable cost for surplus-flow diversion ($/m^3)	[28.5, 34.0]	[32.8, 39.1]	[37.7, 45.2]

consider the issue of flooding management, when the flow levels are continuously high over multiple periods, more surplus would be generated, while desired water diversion plans can thus be obtained through using binary variables to indicate whether a particular surplus-flow-diversion action needs to be undertaken; constraint (9l) stipulates that the water shortage must not exceed the target and is non-negative. Besides, there are two assumptions for the above modeling formulation. Firstly, the random inflows of the two streams are assumed to take on discrete distributions, such that the IMJP model can be solved through linear programming method; secondly, the two random variables are assumed to be mutually independent, such that the probabilistic shortages correspond to joint probabilities.

Table 1 provides the inflow levels of the two streams and economic data over the planning horizon. Obviously, the water availabilities will fluctuate dynamically due to the varying river inflows. Shortage in water supply may be generated if the targeted water is not delivered; on the other hand, high stream inflows may lead to a raised surplus and thus mandate a decision of surplus-flow diversion for reservoirs. Besides, the storage capacities of reservoirs 1 and 2 are [27.0, 37.0] $\times$ 10^6 and [50.0, 63.0] $\times$ 10^6 m^3, respectively; the initial storages in reservoirs 1 and 2 are [19.5, 21.9] $\times$ 10^6 and [27.3, 30.1] $\times$ 10^6 m^3, respectively; in each period, the reserved storage levels for reservoirs 1 and 2 are [20.0, 24.0] $\times$ 10^6 and [32.5, 39.0] $\times$ 10^6 m^3, respectively.

4 Results and Discussion

In this case, random inflows can be conceptualized into a multilayer scenario tree, with a one-to-one correspondence between the previous random variable and one of the nodes (states of the system) in each stage. Figure 2 shows the structure of the scenario tree. For example, for stream 1, a three-period (four-stage) scenario tree can be generated with a branching structure of 1-3-3-3. In detail, there are one initial node at time 0 (the present) and three succeeding ones in period 1; each node in period 1 has three succeeding nodes in period 2, and so on for each node in period 3. These result in 27 nodes (scenarios) in period 3. Correspondingly, 258 scenarios will be generated for the two streams associated with different joint probabilities over the planning horizon. In addition, a set of chance constraints for the storage capacities of the two reservoirs are considered, which can help investigate the risk of violating

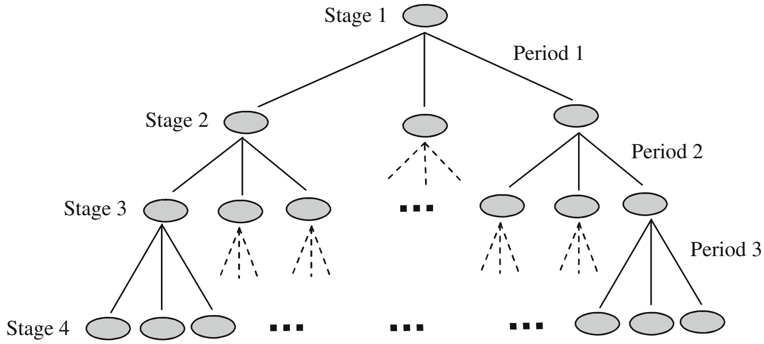

Fig. 2 Structure of scenario tree for stream 1

the capacity constraints and generate desired water-allocation and surplus-flow-diversion schemes. Nine conditions (i.e. nine IMJIP models) are examined based on multiple joint probabilities and individual probabilities. An increased probability level means a raised risk of violating the constraints of reservoir capacities. Each IMJIP model can be transformed into two submodels that correspond to the lower and upper bounds of the objective function values. In each model, there are 1074 variables and 2148 constraints; among them, the number of binary variables is 258. The results through the IMJIP model will help answer the following questions: (1) how to identify the desired water-allocation plan with a minimized risk of penalty, (2) how to generate an optimized surplus-flow-diversion scheme with sound timing and sizing considerations, and (3) how to achieve a maximized system benefit with the least risk of system disruption over a multi-period planning horizon.

Table 2 presents the solutions for system benefit, penalty level, and diversion cost; they would vary with the joint probability (q) level. For example, under conditions 2, 5 and 8 (i.e., when $q = 0.01$, 0.05 and 0.10, respectively), the system benefits ($f^{\pm}$) would be \$[6499.8, 15550.6] $\times$ 10^6, \$[6728.7, 15730.5] $\times$ 10^6 and \$[6811.7, 15808.3] $\times$ 10^6, respectively; the penalties would be \$[3145.7, 6558.8] $\times$ 10^6, \$[3257.9, 6690.4] $\times$ 10^6 and \$[3254.8, 6781.1] $\times$ 10^6, respectively; the diversion costs would be \$[829.6, 1198.4] $\times$ 10^6, \$[705.7, 1008.2] $\times$ 10^6 and \$[654.4, 923.2] $\times$ 10^6, respectively. Moreover, the solutions for the system benefit, penalty level, and diversion cost

Table 2 Solutions of system benefits, penalty levels, and diversion costs (unit: \$$10^6$)

Condition	Joint probability	Individual probability	System benefit	Penalty	Diversion cost
1	$q = 0.01$	$q_1 = 0.001, q_2 = 0.009$	[6,518.9, 15,577.3]	[3,194.4, 6,663.8]	[800.8, 1,153.3]
2		$q_1 = 0.005, q_2 = 0.005$	[6,499.8, 15,550.6]	[3,145.7, 6,558.8]	[829.6, 1,198.4]
3		$q_1 = 0.009, q_2 = 0.001$	[6,436.9, 15,517.2]	[3,091.9, 6,474.2]	[863.5, 1,274.5]
4	$q = 0.05$	$q_1 = 0.01, q_2 = 0.04$	[6,688.0, 15,748.4]	[3,277.6, 6,758.7]	[691.4, 990.4]
5		$q_1 = 0.025, q_2 = 0.025$	[6,728.7, 15,730.5]	[3,257.9, 6,690.4]	[705.7, 1,008.2]
6		$q_1 = 0.04, q_2 = 0.01$	[6,671.4, 15,668.5]	[3,187.5, 6,666.7]	[760.3, 1,069.7]
7	$q = 0.10$	$q_1 = 0.01, q_2 = 0.09$	[6,747.2, 15,823.4]	[3,259.4, 6,854.4]	[646.4, 912.9]
8		$q_1 = 0.05, q_2 = 0.05$	[6,811.7, 15,808.3]	[3,254.8, 6,781.1]	[654.4, 923.2]
9		$q_1 = 0.09, q_2 = 0.01$	[6,736.1, 15,692.6]	[3,180.9, 6,631.2]	[742.7, 1,050.1]

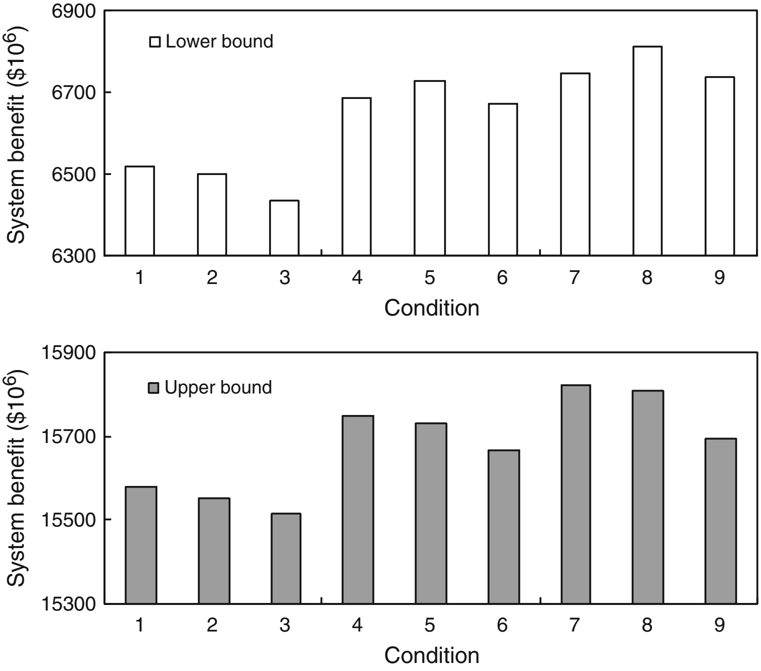

Fig. 3 System benefits under different probability levels

would vary with individual probability (q_i) level of each reservoir-capacity constraint. For example, under conditions 4, 5 and 6 (i.e. with the same joint probability of 0.05), the system benefits would be \$[6688.0, 15748.4] $\times$ 10^6 ($q_1 = 0.01$ and $q_2 = 0.04$), \$[6728.7, 15730.5] $\times$ 10^6 ($q_1 = 0.025$ and $q_2 = 0.025$), and \$[6671.4, 15668.5] $\times$ 10^6 ($q_1 = 0.04$ and $q_2 = 0.01$), respectively. Figures 3 and 4 show the variations of system-benefit and diversion-cost with the joint probability level. Variations in the q level correspond to the decision makers' preferences regarding the tradeoff among

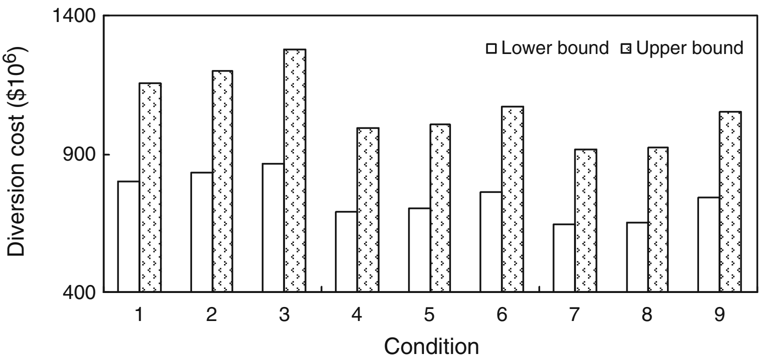

Fig. 4 Diversion costs under different probability levels

system benefit, diversion cost, and constraint-violation risk. A lower joint probability level would result in a lower system benefit and a lower constraint-violation risk; conversely, a higher joint probability would sacrifice the system safety in order to reduce the surplus-flow-diversion cost.

Moreover, under each joint probability level, the solution of $f_{\text{opt}}^{\pm}$ is expressed as an interval. Given different water-availability and storage-capacity conditions as well as their underlying probability levels, the expected system benefit would change correspondingly between f_{opt}^{-} and f_{opt}^{+}. Planning for a lower system benefit would be associated with a lower risk of violating the water-allocation constraints; conversely, a desire for a higher benefit would correspond to a higher possibility of violating the constraints. A tradeoff thus exists between the system benefit and the constraint-violation risk.

Figure 5 provides the solution of water-allocation plan under $q = 0.01$ (i.e. condition 2 listed in Table 2). There are 258 scenarios for water allocation associated with different probabilities over the planning horizon. Each allocated flow is the difference between the promised target and the probabilistic shortage under a given stream condition with an associated probability level (i.e. $A_{tk_1k_2\,\text{opt}}^{\pm} = X_{t\,\text{opt}}^{\pm} - Y_{tk_1k_2\,\text{opt}}^{\pm}$). The results indicate that, under this condition, the optimized water-allocation targets would be 132.5×10^6 m^3 in period 1, $[151.7, 179.6] \times 10^6$ m^3 in period 2, and $[151.6, 179.7] \times 10^6$ m^3 in period 3. Deficits would occur if the available water

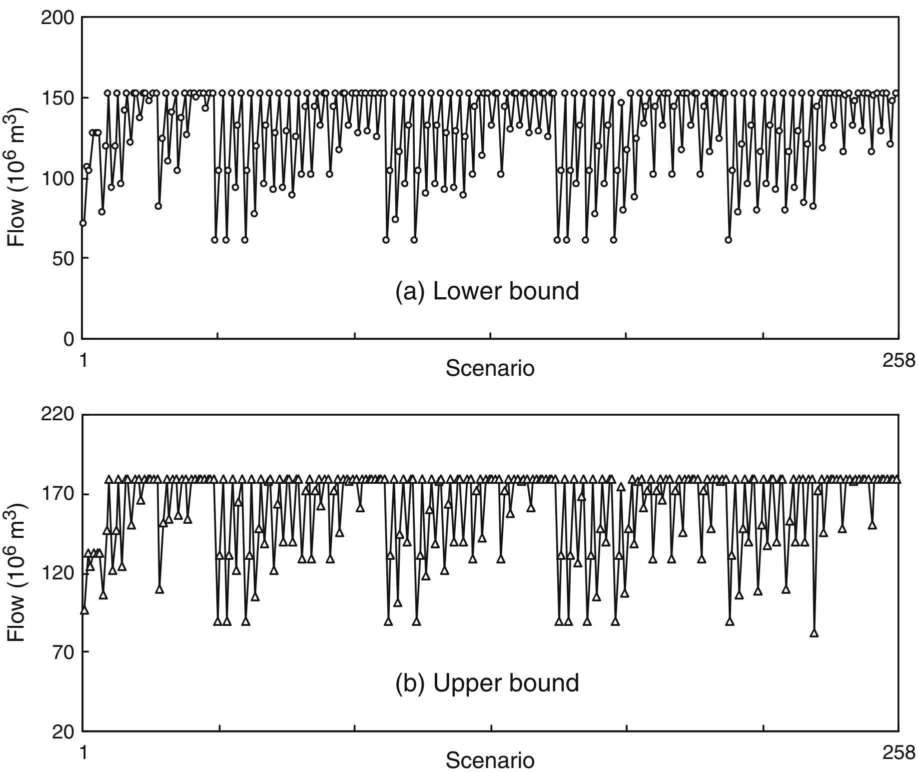

Fig. 5 Optimized water-allocation plan under $q = 0.01$ (condition 2)

amounts are less than the promised targets. For example, under the worst-shortage scenario (i.e. when inflows of the two streams are both low during the entire planning horizon), the shortages would be [35.9, 61.6] $\times 10^6$ m^3 in period 1, 73.9 $\times 10^6$ m^3 in period 2, and 90.3 $\times 10^6$ m^3 in period 3. Correspondingly, the actual water allocations would be [70.9, 96.6] $\times 10^6$ m^3 in period 1, [77.8, 105.7] $\times 10^6$ m^3 in period 2, and [61.3, 89.4] $\times 10^6$ m^3 in period 3. The total of allocated water would be 210.0 to 291.7 $\times 10^6$ m^3 in the three periods; however, the total water demand over the planning horizon would be [415.2, 559.0] $\times 10^6$ m^3, indicating a serious shortage in water supply. Thus, the municipality would have to obtain water from other sources to satisfy its essential demands.

The solutions for most of the non-zero water shortage values ($Y^{\pm}_{tk_1k_2}$) are interval numbers. These imply that (a) under advantageous conditions (e.g., when the available water amounts approach their upper bounds), the shortage levels may be low, and (b) under demanding conditions, the shortage levels may be raised. Figure 6 presents the optimized water-shortage pattern when $q = 0.01$ (i.e. condition 2). Under advantageous conditions, the number of scenarios subjecting to water-shortage risks would be 112; however, under demanding conditions, such a number would be increased to 147 (occupying approximately 57.0% of the total water-allocation scenarios; in this case, the number of total water-allocation scenarios would be 258 over the planning horizon).

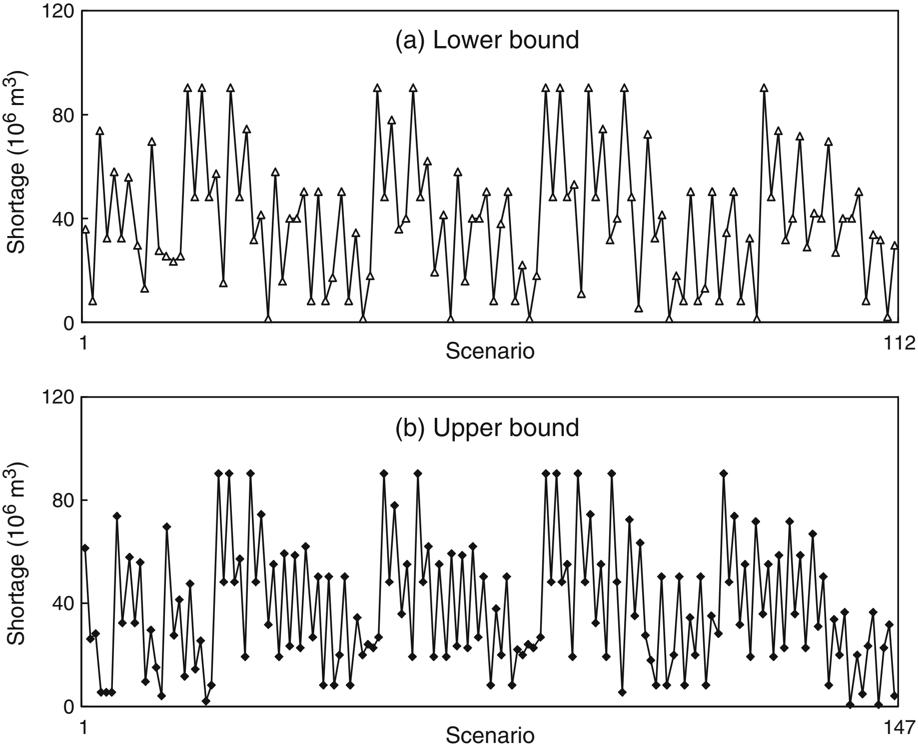

Fig. 6 Optimized water-shortage pattern under $q = 0.01$ (condition 2)

The results indicate that any change in q would yield varied reservoir-storage capacities and thus result in varied water allocation patterns. Moreover, the temporal and spatial variations of water demand and availability may also result in varied water-allocation plans. Figure 7 provides the optimized water-allocation plan under condition 8 (when $q = 0.10$). Under this condition, the optimized water-allocation targets would be 132.5×10^6 m^3 in period 1, $[152.5, 180.1] \times 10^6$ m^3 in period 2, and $[157.8, 183.5] \times 10^6$ m^3 in period 3, which are different from those under condition 2 (when $q = 0.01$). Moreover, the shortage would vary under different q levels. For example, when inflows of streams 1 and 2 are respectively medium and low over the planning horizon, the shortages would be $[3.0, 25.4] \times 10^6$ m^3 (in period 1), 37.2×10^6 m^3 (in period 2), and 36.2×10^6 m^3 (in period 3) under condition 8. In comparison, under condition 2, the shortages would be $[8.1, 28.2] \times 10^6$, 32.1×10^6, and 31.9×10^6 m^3 in periods 1, 2 and 3, respectively.

Moreover, the water-allocation plans would vary with individual probability (q_i) of each reservoir-capacity constraint (even under the same joint probability level). Figure 8 provides the optimized water-allocation plan under condition 7 [i.e. joint probability level is 0.10, and individual probabilities (of reservoirs 1 and 2) are 0.01 and 0.09, respectively]. In fact, different q_i levels correspond to different available storage capacities, leading to varied water allocation patterns over a multistage

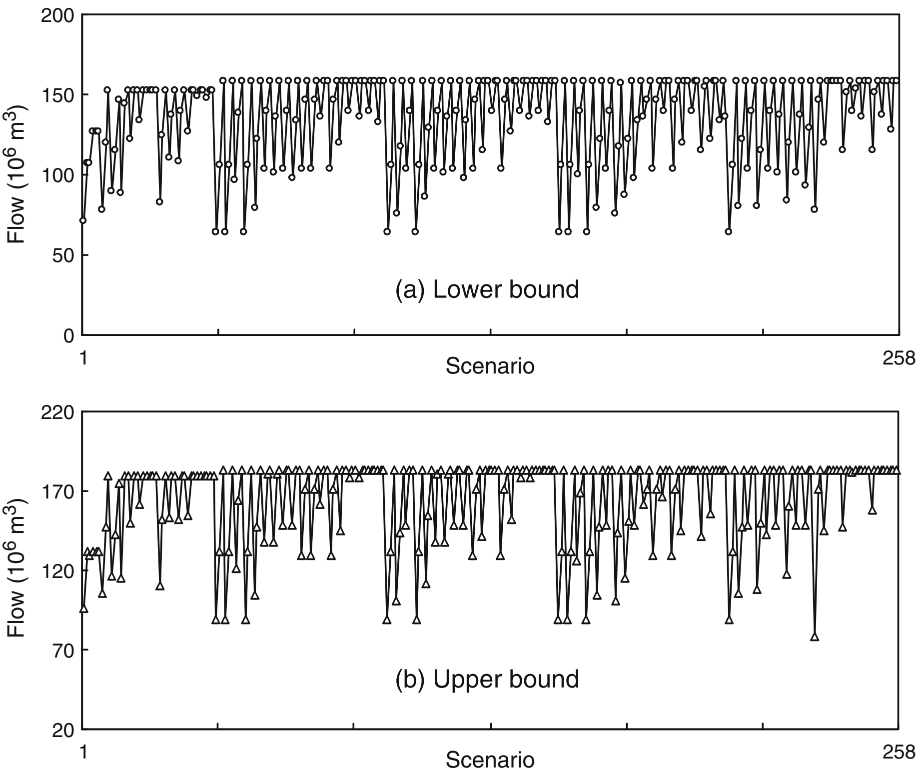

Fig. 7 Optimized water-allocation plan under $q = 0.10$ (condition 8)

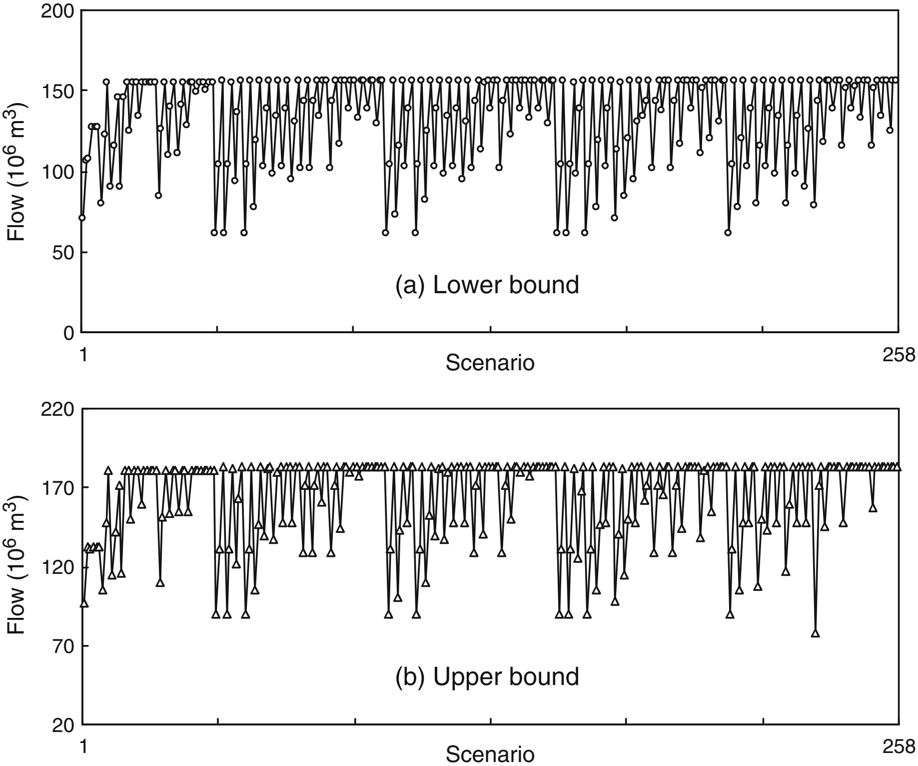

Fig. 8 Optimized water-allocation plan under $q_1 = 0.01$ and $q_2 = 0.09$ (condition 7)

context. The results (in Table 2) also indicate that condition 8 would correspond to the highest lower-bound system benefit (f^{-} = \$6811.7 million), and condition 7 would be linked to the highest upper-bound system benefit (f^{+} = \$15823.4 million).

Surplus would occur if the flows are continuously high, and a surplus-flow-diversion project would be undertaken to avoid spill from reservoirs. The results indicate that varied stream inflows would lead to changed surplus-flow-diversion schemes. Figure 9 presents the solutions of optimized surplus-flow-diversion scheme under condition 2 (when $q = 0.01$). For example, when the inflows of two streams are continuously high during the entire planning horizon, the total available flows would be $[698.7, 794.4] \times 10^6$ m^3; however, the total targets would be $[435.8, 491.0] \times 10^6$ m^3 while the available capacities (of the two reservoirs) would be $[79.5, 102.8] \times 10^6$ m^3. Consequently, surplus flows have to be diverted to minimize flooding risk. The results indicate that, under this scenario, the diverted flows would be 59.1×10^6 m^3 in period 1, 77.2×10^6 m^3 in period 2, and 52.8×10^6 m^3 in period 3. The number of scenarios for diverting the surplus flows would be 78 under advantageous conditions, and 107 under demanding conditions (Fig. 9).

The results also indicate that the surplus-flow-diversion schemes would be vary with q and q_i levels. Figure 10 provides the solution of surplus-flow-diversion scheme under condition 8 ($q = 0.10$), which is different from that under $q = 0.01$ (Fig. 9). For example, when inflows of the two streams are high during the entire planning

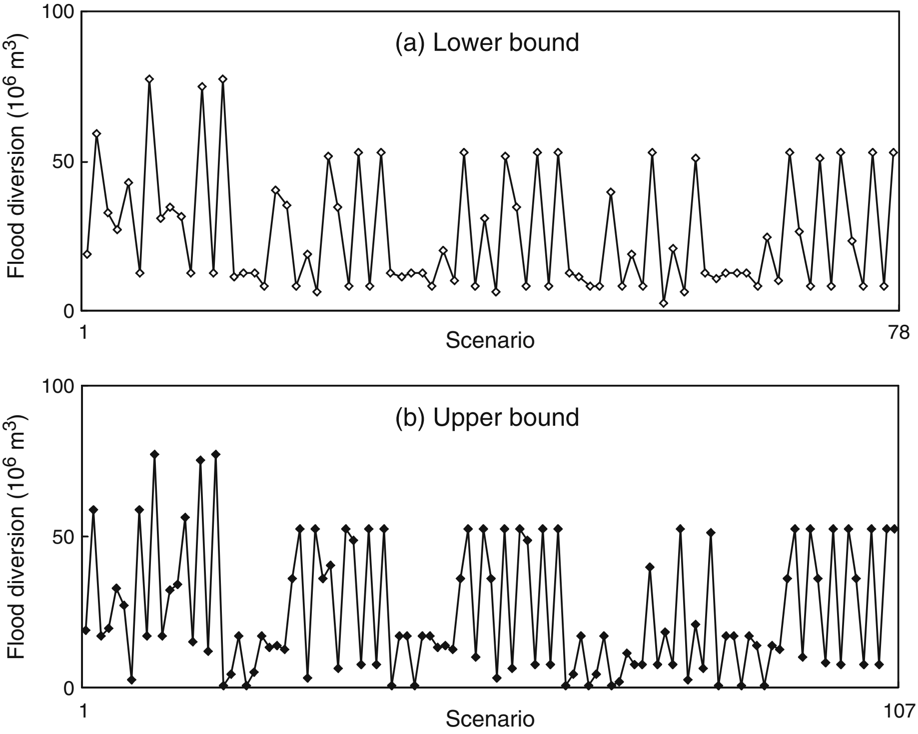

Fig. 9 Optimized surplus-flow-diversion scheme under $q = 0.01$ (condition 2)

horizon, the diverted surplus flows would respectively be 50.6×10^6, 76.7×10^6 and 49.0×10^6 m^3 in periods 1, 2 and 3; the total diverted flow would be 176.3×10^6 m^3 in the three periods, which is less than that under condition 2 (i.e. 189.1×10^6 m^3). Moreover, the number of scenarios with surplus flow diversion would be decreased to 74 and 86 under condition 8 ($q = 0.10$). Generally, an increased q level

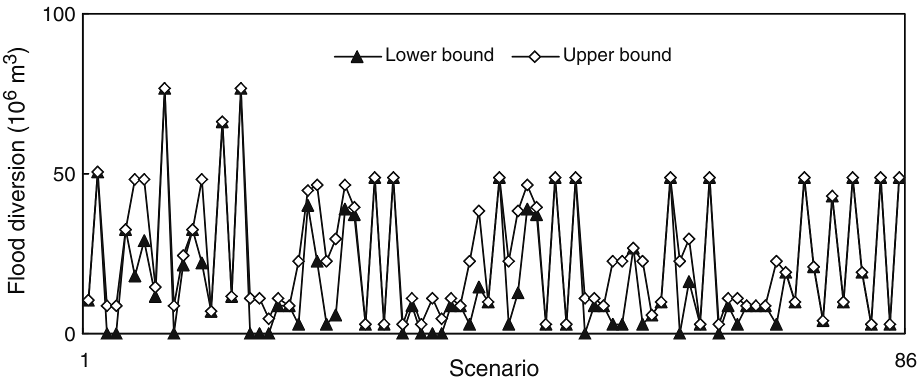

Fig. 10 Optimized surplus-flow-diversion scheme under $q = 0.10$ (condition 8)

means a raised risk of violating joint-probabilistic constraints and, at the same time, a decreased strictness for the reservoir-capacity constraints and thus a reduced surplus-flow-diversion cost, and vice versa.

Considering the constraints of reservoir capacities as a set of deterministic values (i.e. $q = 0$), the resulting system benefit would be \$[6410.0, 1545.2] $\times$ 10^6, which is lower than those from IMJIP under a range of joint probabilities. Meanwhile, the diversion cost would be \$[877.3, 1310.1] $\times$ 10^6 when $q = 0$, which is higher than that from the IMJIP model. These are attributed to the fact that no violation (or relaxation) on the reservoir capacity constraints is allowed when $q = 0$, leading to reduced system capacities, and thus raised needs for surplus-flow diversion. Figures 11 and 12 present the solutions of water-allocation and surplus-flow-diversion plans when $q = 0$. When inflows of the two streams are continuously high during the planning horizon, the total diverted flows would be 193.3 $\times$ 10^6 m^3 when $q = 0$, which is higher than those from IMJIP (under a range of q levels). Moreover, the number of scenarios with surplus flow diversion would be increased to 81 and 110 under $q = 0$, more than those from the IMJIP model. Therefore, when $q = 0$, the results can only provide decision support for planning water allocation and surplus diversion under one extreme condition. In comparison, the IMJIP model can incorporate more uncertain information within its modeling framework, and

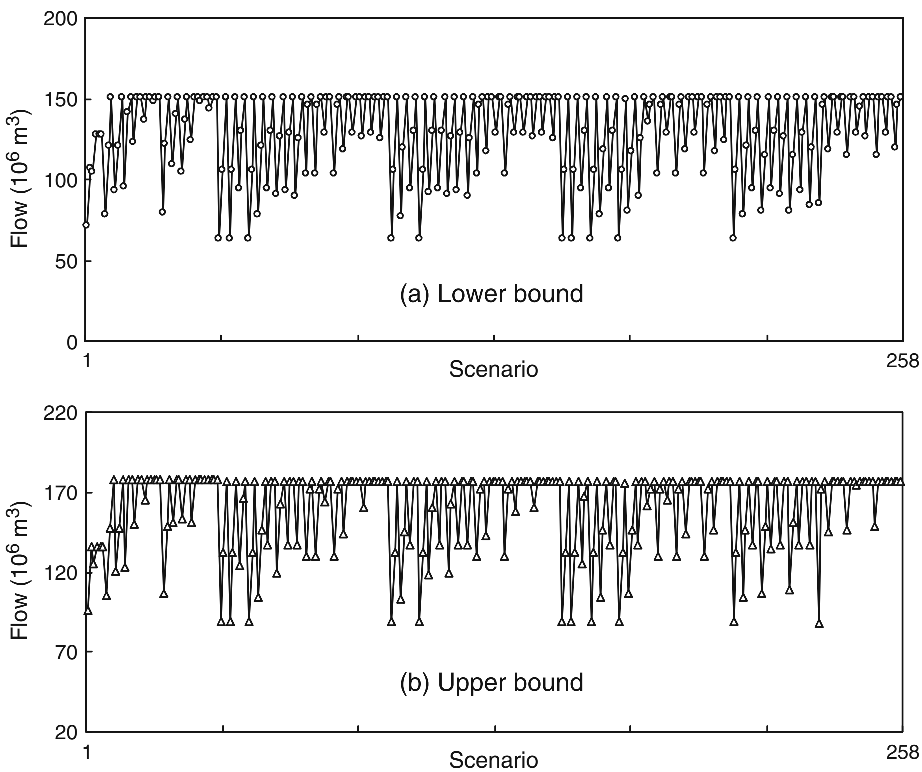

Fig. 11 Optimized water-allocation plan under $q = 0$

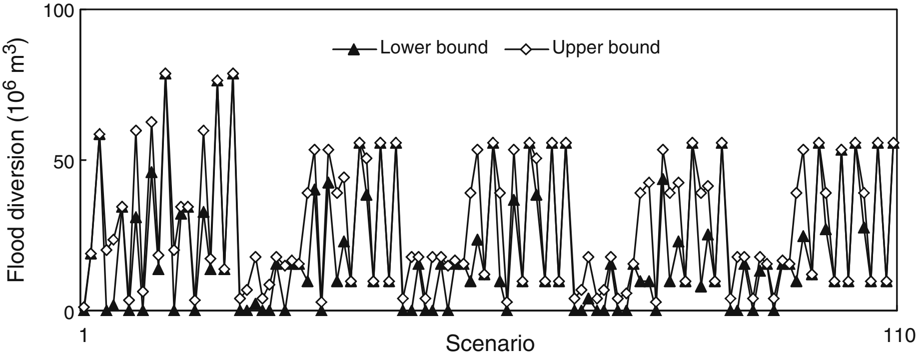

Fig. 12 Optimized surplus-flow-diversion scheme under $q = 0$

its results can effectively support in-depth analyses of the interrelationship between system benefit and constraint-violation risk.

5 Conclusions

An inexact multistage joint-probabilistic programming (IMJP) method has been developed through incorporating techniques of multistage stochastic programming with recourse (MSP), joint-probabilistic constraint programming (JPC), and interval-parameter programming (IPP) within a general optimization framework. The developed IMJP can deal with uncertainties expressed as joint probabilities and interval values; moreover, dynamics of water allocation and surplus-flow diversion can be taken into account based on multilayer scenario trees. Furthermore, it can help examine the risk of violating joint-probabilistic constraints, and can facilitate analyses of multiple policy scenarios that are associated with economic penalties and/or possibilistic losses when the pre-regulated targets are violated.

The developed method has been applied to a case study of water-resources management within a multi-stream, multi-reservoir and multi-period context, where joint probabilities exist in terms of water availabilities and storage capacities. The mixed integer linear programming (MILP) technique has been introduced into the IMJP framework to facilitate dynamic analysis for decisions of surplus-flow diversion. The results indicate that reasonable solutions for continuous and binary variables have been generated. They can be used to help decision makers to identify desired system designs against water shortage and for flood control, as well as to determine which of these designs will lead to optimized system objective. Decisions at a lower risk level would lead to an increased reliability in fulfilling system requirements but with a lower system benefit; conversely, a desire for increasing the benefit level could result in a raised risk of violating the system constraints.

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Appendix

Nomenclatures

Variables:

$f^{\pm}$	objective function value, expected net system benefit over the planning horizon ($);
$R_{tk_1}^{\pm}$	auxiliary variable, released flow from reservoir 1 in period t under scenario k_1 (m^3);
$R_{tk_1k_2}^{\pm}$	auxiliary variable, released flow from reservoir 2 in period t under scenarios k_1 and k_2 associated with joint probabilities of (m^3);
$S_{tk_1}^{\pm}$	auxiliary variable, storage level in reservoir 1 in period t under scenarios k_1 (m^3);
$S_{tk_1k_2}^{\pm}$	auxiliary variable, storage level in reservoir 2 in period t under scenarios k_1 and k_2 (m^3);
$X_t^{\pm}$	first-stage decision variable, water allocation target that is promised to the municipality in period t (m^3);
$Y_{tk_1k_2}^{\pm}$	recourse decision variable, shortage level by which the water-allocation target is not met under scenarios k_1 and k_2, which is associated with joint probabilities of (m^3);
$W_{tk_1k_2}^{\pm}$	recourse decision variable, the amount of surplus flow to be diverted in period t under scenarios k_1 and k_2 (m^3);
$Z_{tk_1k_2}^{\pm}$	binary variable, which is used for identifying whether a surplus-flow-diversion action needs to be undertaken in period t under scenarios k_1 and k_2.

Parameters:

A_1^0	storage-area coefficient for reservoir 1;
A_2^0	storage-area coefficient for reservoir 2;
A_1^a	area (per unit of active storage volume) above ;
A_1^a	area (per unit of active storage volume) above ;
$\mathrm{De}_t^{\min}$	minimum water demand of the municipality in period t (m^3);
$\mathrm{De}_t^{\max}$	maximum water demand of the municipality in period t (m^3);
$e_{1t}^{\pm}$	average evaporation rate for reservoir 1 in period t;
$e_{2t}^{\pm}$	average evaporation rate for reservoir 2 in period t;
$\mathrm{FC}_t^{\pm}$	fixed-charge cost for surplus-flow diversion in period t ($10^6);
K_1^t	number of possible scenarios for stream 1 in period t;
K_2^t	number of possible scenarios for stream 2 in period t;
$M_{tk_1k_2}$	variable upper bounds for surplus-flow diversion in period t under scenarios k_1 and k_2, which is assumed to be sufficiently large (m^3);
$\mathrm{NB}_t^{\pm}$	net benefit per unit of water allocated in period t ($/m^3);

P_{tk_1} probability of occurrence of scenario k_1 (for stream 1) in period t, with and $\sum_{k_1=1}^{K_1^t} P_{tk_1} = 1$;
P_{tk_2} probability of occurrence of scenario k_2 (for stream 2) in period t, with and $\sum_{k_2=1}^{K_2^t} P_{tk_2} = 1$;
$\mathrm{PE}_t^{\pm}$ penalty per unit of water not delivered in period t (\$/m3), and $PE_t > NB_t$;
q joint probability of violating constraints of the reservoir-storage capacities;
$\widetilde{Q}_{tk_1}^{\pm}$ random inflow into stream 1 in period t under scenarios k_1 (m^3);
$\widetilde{Q}_{tk_2}^{\pm}$ random inflow into stream 2 in period t under scenario k_2 (m^3);
$\mathrm{RSC}_1^{\pm}$ storage capacity of reservoir 1 (m^3);
$\mathrm{RSC}_2^{\pm}$ storage capacity of reservoir 2 (m^3);
$\mathrm{RSV}_{1t}^{\pm}$ reserved storage level for reservoir 1 (m^3);
$\mathrm{RSV}_{2t}^{\pm}$ reserved storage level for reservoir 2 (m^3);
t time period, $t = 1, 2, \ldots, T$;
$\mathrm{VC}_t^{\pm}$ variable cost for surplus-flow diversion in period t (\$/m^3).

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