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Application of the Shuffled Frog Leaping Algorithm for the Optimization of a General Large-Scale Water Supply System

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Abstract A water supply system is a complex network of pipes, canals and storage and treatment facilities that collects, treats, stores, and distributes water to consumers. Increasing population and its associated demands requires systems to be expanded and adapted over time to provide a sustainable water supply. Comprehensive design tools are needed to assist managers determine how to plan for future growth. In this study, a general large-scale water supply system model was developed to minimize the total system cost by integrating a mathematical supply system representation and applying an improved shuffled frog leaping algorithm optimization scheme (SFLA). The developed model was applied to two hypothetical water communities. The operational strategies and the capacities for the system components including water transport and treatment facilities are model decision variables. An explicit representation of energy consumption cost for the transporting water in the model assists in determining the efficacy of satellite wastewater treatment facilities. Although the water supply systems studied contained highly nonlinear terms in the formulation as well as several hundred decisions variables, the stochastic search algorithm, SFLA, successfully found solutions that satisfied all the constraints for the studied networks.

Keywords Water supply system · Shuffled frog leaping algorithm · Decision support system · Decentralized treatment plants

Indices and Sets

N a set of nodes in a network (sources, users, and treatment plants)
A a set of arcs (i, j) from a node i to a node j in a network, $\forall i, j \in \mathbf{N}$

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$\mathbf{W}$	a set of pollutants
$\mathbf{T}$	a set of design period t, $\forall t \in \mathbf{T}$
$\mathbf{O}$	a set of operation period t, $\forall o \in \mathbf{O}$

Subsets,

$\mathbf{A_C}$	a set of canal connections, $\mathbf{A_C} \subseteq \mathbf{A}$
$\mathbf{A_P}$	a set of pipe connections, $\mathbf{A_P} \subseteq \mathbf{A}$
$\mathbf{A_U}$	a set of pump connections, $\mathbf{A_U} \subseteq \mathbf{A}$
$\mathbf{A_B}$	a set of connections through a recharge basin to an aquifer, $\mathbf{A_B} \subseteq \mathbf{A}$
$\mathbf{A_I}$	a set of seepage from users to an aquifer and riverbed infiltration, $\mathbf{A_I} \subseteq \mathbf{A}$
$\mathbf{N_U}$	a set of users, $\mathbf{N_U} \subseteq \mathbf{N}$
$\mathbf{N_S}$	a set of sources, $\mathbf{N_S} \subseteq \mathbf{N}$
$\mathbf{N_{SS}}$	a set of storage sources, $\mathbf{N_{SS}} \subseteq \mathbf{N_S} \subseteq \mathbf{N}$
$\mathbf{N_{NS}}$	a set of non-storage sources, $\mathbf{N_{NS}} \subseteq \mathbf{N_S}, \mathbf{N}$
$\mathbf{N_{IW}}$	imported water, $\mathbf{N_{IW}} \subseteq \mathbf{N_{NS}} \subseteq \mathbf{N_S} \subseteq \mathbf{N}$
$\mathbf{N_{RU}}$	river upstream node, $\mathbf{N_{RU}} \subseteq \mathbf{N_{NS}} \subseteq \mathbf{N_S} \subseteq \mathbf{N}$
$\mathbf{N_{RD}}$	river downstream node, $\mathbf{N_{RD}} \subseteq \mathbf{N_{NS}} \subseteq \mathbf{N_S} \subseteq \mathbf{N}$
$\mathbf{N_{WT}}$	a set of water treatment plants, $\mathbf{N_{WT}} \subseteq \mathbf{N}$
$\mathbf{N_{WWT}}$	a set of wastewater treatment plants, $\mathbf{N_{WWT}} \subseteq \mathbf{N}$

1 Introduction and Background

A water supply system is a collection of water transport structures, pumping stations, and water treatment and storage facilities that are managed to supply the desired amount of water with the desired quality to consumers. With increasing domestic and industrial water demands, developing a long term plan for sustainable water supply is challenging because of complexity of the system and future uncertainties.

In the southwestern Unites States and many arid and semi-arid regions, groundwater is a major water source but, oftentimes, it has been mined to meet increased water demands. As a result, groundwater table levels have fallen thus identifying alternative water sources is necessary. Since surface supplies are limited or already allocated, reclaimed and imported water are potential alternatives that may replace groundwater for agricultural and other purposes. In the future as new supplies become more limited, potable demands may be met with water reclaimed after a high level of treatment.

Water supply planning requires considering current demand, future growth, and available supplies. Supply costs include capital for construction, operation and maintenance. As reclaimed water becomes a more integral part of the water supply system, the cost of transporting water from a treatment facility to users becomes more critical in decision making. This introduces another complexity to the planning process in which the cost of distribution may exceed the gains in economies of scale that are obtained if wastewater treatment is centralized. Hence, smaller distributed wastewater treatment facilities may be a cost effective alternative over a single central plant.

Little research has been conducted on water supply system planning optimization. Ocanas and Mays (1981a, b) formulated and solved a water reuse planning optimization model using non-linear programming under steady and dynamic conditions. The steady state model consisted of a nonlinear objective function, linear and nonlinear constraints for a single period. A large-scale generalized reduced gradient technique was used to solve this optimization problem (Ocanas and Mays 1981a). In the follow-up paper, the problem structure was formulated in a dynamic water reuse planning model with single and multiple periods and solved by successive linear programming (Ocanas and Mays 1981b). Water quality was considered in both papers. In the dynamic model, the capacity expansion of treatment facilities was considered at the beginning of the period and operation costs were included in the objective function. This model provided a basis of the optimization structure for a water supply management system. Conveyance systems were considered as lumped units without detailed representations of energy loss and capacity. Later, Ejeta et al. (2004) applied a general approach to the studies in Rio Grande in New Mexico and Texas including a total suspended solids constraint. The objective of this study was to maximize total net benefit.

Recently, as the need for the reclaimed or reuse system increase, several decision support systems (DSSs) have been developed to evaluate the suitability of wastewater treatment plants and reuse systems. For example, a DSS for siting a wastewater treatment plant in a new urban development was developed by Makropoulos et al. (2007), Joksimovic et al. (2007), and Hochstrat et al. (2007) constructed (DSSs) for the studying water reuse systems. None of these models optimized the system of concern.

In this paper, the water reuse planning system formulated by Ocanas and Mays (1981a, b) is extended. The water reuse system in Ocanas and Mays (1981b) used only mass balance equation to formulate the system. A hydraulic energy relationship with a more realistic cost function is included and various transmission systems (pipes, canals, and pump connections) are considered. To make water supply system more general, multiple usage of a type of component was also allowed depending on user's need in a system. Decision variables include the capacities of conveyance and treatment facilities and the flow allocation within of the system for each time step. The resulting problem is highly nonlinear and can contain several hundred decisions. A stochastic search algorithm, the Shuffled Frog Leaping Algorithm (SFLA) (Eusuff et al. 2006), is improved and successfully applied to determine the water supply plans for two hypothetical communities.

2 Problem Description

In developing a water supply plan, the overall goal is to minimize the total costs of construction, operation and maintenance while meeting user demands for waters with different qualities. The supply system may have multiple sources and users and contain one or more water and wastewater treatment facilities. The problem considered in this paper extends earlier work by considering operational costs as a function of flow rates and selected component sizes. Unlike previous models, conveyance system hydraulics is directly embedded in the model to more realistically estimate operation expenses. One or more planning periods can be represented to allow delaying expansion investments or to take advantage of economies of scale by constructing excess capacity in early decision periods.

Figures 1 and 2 show the system schematics for the two communities. Potential flow paths are shown between sources, sinks and users that are denoted as nodes. Water uses are agricultural, domestic, industrial and large outdoor irrigation (golf courses, school, and parks). Available water supply sources can include surface reservoirs or groundwater aquifers that have storage capabilities and rivers that cannot store water over time. Any flow provided to domestic and industrial users from surface water sources must be treated at a water treatment facility (WT) while surface water provided to agricultural and outdoor uses does not need to be treated. Wastewater return flows from domestic and industrial uses must be treated at a wastewater treatment plant (WW). After wastewater treatment, reclaimed water may be supplied to agricultural or large irrigation areas, discharged to the river, or recharged to the aquifer through infiltration basins. Groundwater banking can also be achieved by recharging imported waters or surface supplies.

The primary system constraints are to satisfy conservation of mass at all locations and components in the system. In addition, each user has defined water quality and demand requirements, while all supplies are limited by flow capacity or storage volume. Conveyance system conditions related to canal capacity and pipeline/pump sizes are formulated to ensure proper sizing and energy consumption. The mathematical form of the problem is given in the next section.

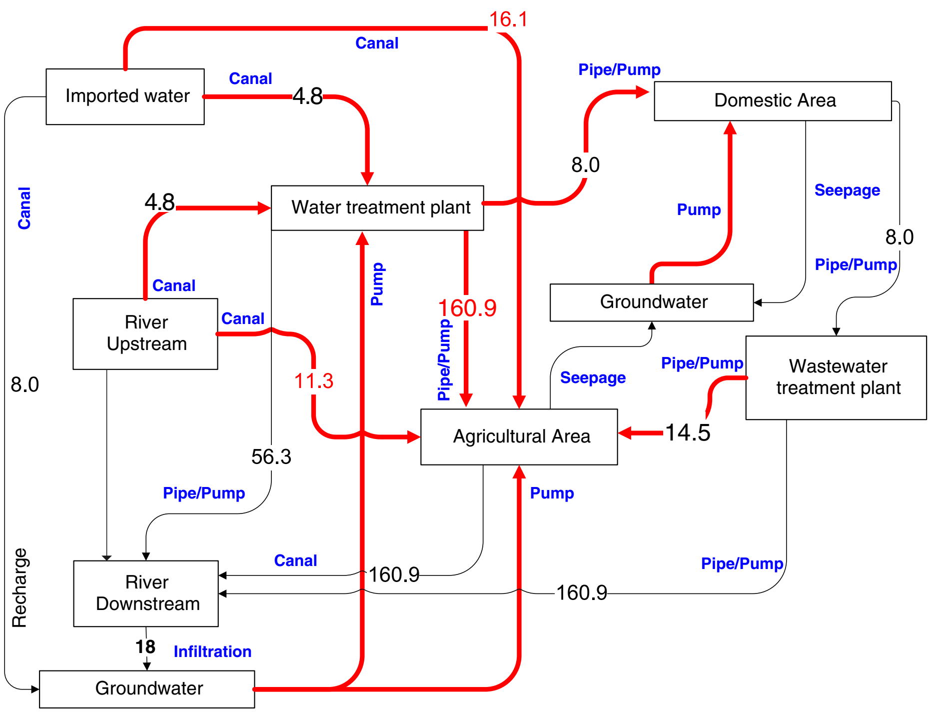

Fig. 1 Single wastewater treatment plant supply system schematic. *Bold arcs* represent the 10 decision variables and *thin arcs* are dependent flows that are computed from mass balance constraints (Note the *numbers in arcs* are pipe's or canal's length in km)

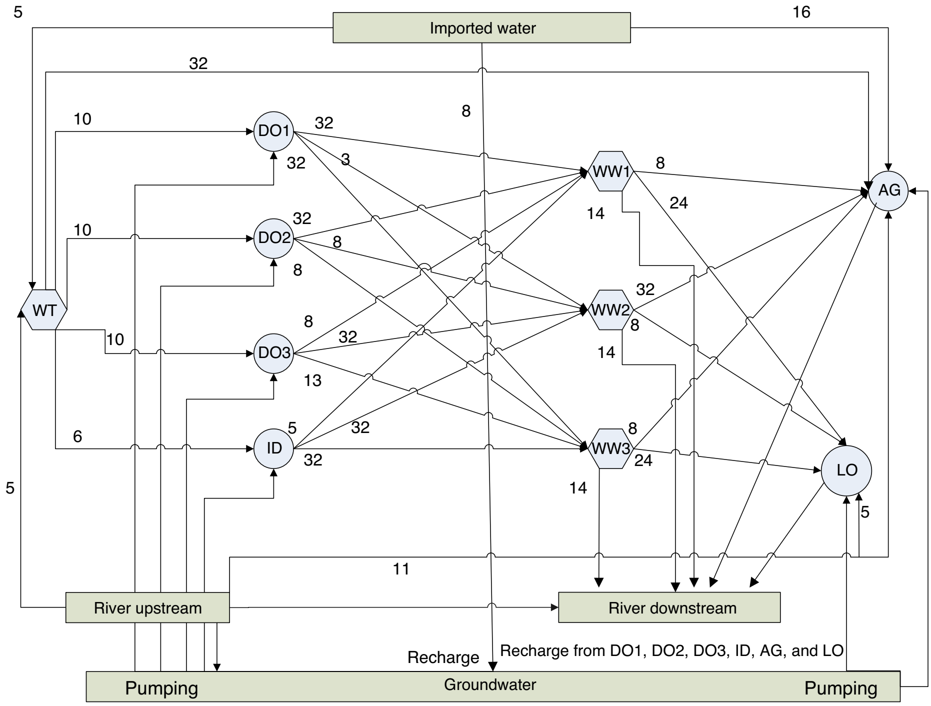

Fig. 2 Multi-wastewater plant system schematic (WT – water treatment plant, DO1, DO2, DO3 – the first, second and third domestic areas, respectively, ID – industrial area, WW1, WW2, WW3 – the first, second and third potential wastewater treatment plants, respectively, *AG* – agricultural area, LO – large outdoor area). (Note the *number in arcs* are pipe's or canal's length in km.)

3 Problem Formulation

A general system can be represented by a set of **N** nodes and **A** arcs. Arcs denote water transmission systems while nodes are locations where water is collected from or split between a set of arcs. The set of nodes is comprised of several subsets representing sources ($\mathbf{N_S}$) and users ($\mathbf{N_U}$). Sources are further divided into storage sources ($\mathbf{N_{SS}}$) that can retain water over in time (groundwater aquifers and surface reservoirs) and non-storage sources ($\mathbf{N_{NS}}$) that cannot store water over time (rivers and imported waters ($\mathbf{N_{IW}}$)). Note that a river is represented as upstream ($\mathbf{N_{RU}}$) and downstream nodes ($\mathbf{N_{RD}}$) that are connected by a river arc. Water is withdrawn by users from upstream nodes and returned from treatment plants to a downstream river node. Water user nodes represent domestic, industrial, and agricultural applications and water and wastewater treatment plants ($\mathbf{N_{WT}}$, and $\mathbf{N_{WWT}}$, respectively).

An arc transmits flow from a node i to a node j (arc ij). Arcs represent the sets of canals ($\mathbf{A_C}$), pipe connections ($\mathbf{A_P}$), pump connections ($\mathbf{A_U}$). Pipe connections may have pump stations at their upstream source depending on the elevation difference and energy losses between the two connected nodes. Recharge basin arcs ($\mathbf{A_B}$) can be used to transmit imported or treated water to the aquifer to bank or recharge water in the aquifer. In addition, rivers and users may recharge the aquifer through seepage or infiltration that is represented by infiltration arcs ($\mathbf{A_I}$).

The capacities of the water transmission structures such as pipes, pumps, and canals and treatment plants are described as structural variables. Flow allocations over the water supply network are operational variables. Lastly, the set of construction time periods (**T**) represents decision epochs at which facility construction and expansion is allowed. The set of operational time periods (**O**) corresponds to the time period over which the operational cost of the system is calculated.

The objective function consists construction and expansion, and operations and maintenance (O&M) costs for all components (pipes, canals, pumps, and treatment facilities) and a penalty term or:

$$\text{Minimize } Z^* = f_1\left(\kappa_{ij}^t\right) + f_2\left(\kappa_{ij}^t\right) + f_3\left(\chi_{ij}^t, H_{ij}^t\right) + f_4\left(w_i^t\right) + f_5\left(q_{ij}^t, w_i^t\right) + f_6 \tag{1}$$

Each term in Eq. 1 is a non-linear relationship with respect to the decision variables in parenthesis. Pipe construction and expansion costs in terms of pipe diameter $\left(\kappa_{ij}^t\right)$ over arc ij at construction time t are given by Clark et al. (2002):

$$\begin{aligned} f_1\left(\kappa_{ij}^t\right) = \sum_{t\in\mathbf{T}} \sum_{ij\in\mathbf{A_p}} x_{ij}^t \Big\{ & 57.198 + 0.35\kappa_{ij}^t + 0.62\kappa_{ij}^{t^{1.54}} + 0.0018\kappa_{ij}^{t^{1.9}} + 0.0062\kappa_{ij}^{t^{1.83}} \\ & -0.062\kappa_{ij}^{t^{0.73}} + 0.02\kappa_{ij}^{t^{1.8}} + 0.23\kappa_{ij}^{t^{0.93}} + 0.0022\kappa_{ij}^{t^{0.71}} \Big\} L_{ij} \end{aligned} \tag{1a}$$

Canal construction and expansion costs in terms of canal depth $\left(\kappa_{ij}^t\right)$ over arc ij at construction time t are (US Army Corps of Engineers 1980):

$$f_2\left(\kappa_{ij}^t\right) = \sum_{t\in\mathbf{T}} \sum_{ij\in\mathbf{A_C}} \left\{ 0.39\kappa_{ij}^{t^2} \frac{\text{ENR}^t\text{CITY}}{2877} + 55.30 L_{ij}\kappa_{ij}^t \frac{\text{ENR}^t\text{CITY}}{2877} \right\} \tag{1b}$$

Pump construction and expansion costs in terms of pump design discharge $\left(\chi_{ij}^t\right)$ and head $\left(H_{ij}^t\right)$ over arc ij at construction time t are given by Walski et al. (1987):

$$f_3\left(\chi_{ij}^t, H_{ij}^t\right) = \sum_{t\in\mathbf{T}} \sum_{ij\in\mathbf{A_P}\cup\mathbf{A_U}} \left(500\chi_{ij}^{t^{0.7}} H_{ij}^{t^{0.4}}\right) \tag{1c}$$

Water and wastewater treatment facility construction and expansion in terms of capacity of treatment facility $\left(w_i^t\right)$ in a node i at construction time t are approximated by Tang et al. (1987):

$$\begin{aligned} f_4\left(q_{ij}^t, w_i^t\right) = & \sum_{t\in\mathbf{T}} \sum_{i\in\mathbf{N_{WT}}} y_i^t \left(2897.13\, w_i^t + 35987\right) \\ & + \sum_{t\in\mathbf{T}} \sum_{i\in\mathbf{N_{WWT}}} y_i^t \left(10811.92\, w_i^t + 5454228\right) \end{aligned} \tag{1d}$$

Operation and maintenance of pipes, canals, pumps, and treatment facilities are given by Clark et al. (2002), US Army Corps of Engineers (1980), Walski et al. (1987), Tang et al. (1987):

$$
\begin{aligned}
& f_6\left(q_{ij}^t, w_i^t\right) \\
& \sum\nolimits_{o\in\mathbf{O}}\left[\frac{1}{(1+I)^o}\left[\sum\nolimits_{ij\in\mathbf{A_P},\{t|t\leq o\}} x_{ij}^t\left(27.7+0.3q_{ij}^o\right)L_{ij}\right.\right. \\
& \quad +\sum\nolimits_{ij\in\mathbf{A_C},\{t|t\leq o\}}\left(0.0254L_{ij}q_{ij}^{o^{0.572}}+\left(0.078+0.0135q_{ij}^o\right)L_{ij}\frac{\mathrm{ENR}^{\mathrm{o}}}{1850}\right) \\
& \quad +\sum\nolimits_{ij\in\mathbf{A_P}\cup\mathbf{A_U},\{t|t\leq o\}}\mu_{ij}^t\left\{79.47\Delta_{ij}^o q_{ij}^o+4560q_{ij}^{o^{0.58}}+320q_{ij}^{o^{0.935}}\frac{\mathrm{ENR}^{\mathrm{o}}\mathrm{CITY}}{2877}\right\} \\
& \quad \left.\left.+\sum\nolimits_{i\in\mathbf{N_{WT}},\{t|\leq o\}} y_i^t\left(28.97w_i^t+360\right)+\sum\nolimits_{i\in\mathbf{N_{WWT}},\{t|t\leq o\}} y_i^t\left(108.12\,w_i^t+54542\right)\right]\right]
\end{aligned}
\tag{1e}
$$

where $\mathbf{A_P}$, $\mathbf{A_U}$, $\mathbf{A_C}$, $\mathbf{N_{WT}}$, and $\mathbf{N_{WWT}}$ are the set of pipe, pump, and canal arcs from node i to j and water and wastewater treatment plant nodes, respectively. The superscript t indicates the construction and expansion time while o denotes the operation period used for evaluating O&M costs.

The constant terms associated with the pipe construction costs require binary decision variables x to identify whether or not pipe i in the set of links, $\mathbf{A_P}$, will be installed with diameter κ over its length of L. Similarly, μ is a binary decision variable representing whether or not a pump will be installed with design pump discharge and head, χ and H, respectively. Note that pumps can be installed at the beginning of defined pipelines or stand alone as pump connections. The binary decision variable, y_i, indicates whether a water or wastewater treatment plant, will be built with design capacity, w. No discrete variables are needed for the other components as their decision variables are permitted to go to zero and do not have a fixed cost. Canal variables are represented by the continuous canal depth κ for the given canal length L.

Operation and maintenance costs (Eq. 1e) are calculated for each component and summed over the planning period, O. O&M costs are functions of the component capacity and flow, q, that is also a model decision variable.

The objective function also includes several system defined parameters. Δ in the pump terms corresponds to the elevation difference between two nodes, i and j. CITY is a construction cost factor that varies by location. The ENR cost factor at year t incorporates the inflation rate in all construction costs and is given by:

$$\mathrm{ENR}^t=-7.7\times10^9+15.7t-12.0\times10^3t^2+4.1t^3-0.5\times10^{-3}t^4 \tag{2}$$

3.1 Simple Bound Constraints on System Flows and Component Sizes

Bounds on the decision variable are:

$$\kappa_{ij}, \chi_{ij}, H_{ij} \geq 0 \qquad \forall \mathbf{A_C} \cup \mathbf{A_P} \cup \mathbf{A_U} \tag{3}$$

$$w_i \geq 0 \qquad \forall \mathbf{N_{WT}} \cup \mathbf{N_{WWT}} \tag{4}$$

$$x_{ij} \in \{0, 1\} \qquad \forall \mathbf{A_P} \tag{5}$$

$$\kappa_{ij} \leq M_{ij} x_{ij} \qquad \forall \mathbf{A_P} \tag{6}$$

$$\mu_{ij} = \{0, 1\} \qquad \forall \mathbf{A_P} \cup \mathbf{A_U} \tag{7}$$

$$\chi_{ij} \leq M_{ij} \mu_{ij} \qquad \forall \mathbf{A_P} \cup \mathbf{A_U} \tag{8}$$

$$H_{ij} \leq M_{ij} \mu_{ij} \qquad \forall \mathbf{A_P} \cup \mathbf{A_U} \tag{9}$$

$$y_i \in \{0, 1\} \qquad \forall \mathbf{N_{WT}} \cup \mathbf{N_{WWT}} \tag{10}$$

$$w_i \leq M_i y_i \qquad \forall \mathbf{N_{WT}} \cup \mathbf{N_{WWT}} \tag{11}$$

$$q_{ij} \geq 0 \qquad \forall \mathbf{A} \tag{12}$$

$$q_{ij} \leq M_{ij} x_{ij} \qquad \forall \mathbf{A_P} \tag{13}$$

$$q_{ij} \leq M_{ij} \mu_{ij} \qquad \forall \mathbf{A_U} \tag{14}$$

M_{ij} and M_i are assigned large values or the upper bound of the corresponding component size. From these constraints, if the binary variable corresponding to a design component is set to zero, the design and operation variables must be equal to zero. Otherwise, the component can be added to the model and water is allocated through the component.

Lower bounds are given for pipe diameters, canal depths and pump capacities in the set of construction variables, $\mathbf{A_P}$, $\mathbf{A_C}$, and $\mathbf{A_U}$ (3). Equation 4 defines lower bounds on water and wastewater treatment plant capacities in the sets $\mathbf{N_{WT}}$ and $\mathbf{N_{WWT}}$. As noted, binary variables, x, (5) are introduced to denote the non-existence/existence of a pipe connection. If the connection is included, i.e., $x = 1$, pipe diameters are bounded by the maximum diameter (6). The second set of binary variables, μ_{ij}, (7) is associated with pump installation and Eqs. 8 and 9 define the upper bounds of pump capacity and discharge, respectively. The last set of binary variable, y_i, represents water and wastewater treatment plants construction (10) and is bounded by the maximum construction capacity (11). Finally, the operational pipe and pump flows have a lower bound of zero and an upper bound equal to their capacities (12–14).

3.2 Node Constraints

In addition to simple bounds on system flows, q, mass balances at system nodes, demands and flow availability and requirements also limit the flow rate ranges. By conservation of mass, imported water and water and wastewater treatment plants, must balance for each operational period, o in $\mathbf{O}$, or:

$$\sum\nolimits_j q^o_{ji} - \sum\nolimits_j q^o_{ij} = 0 \qquad i \in (\mathbf{N_{NS}} \backslash \mathbf{N_{RD}}) \cup \mathbf{N_{WT}} \cup \mathbf{N_{WWT}} \forall \mathbf{O} \tag{15}$$

Similar mass balances at user nodes (the set $\mathbf{N_U}$) are expanded to include consumptive use $\left(\mathrm{CU}_j^o\right)$ and seepage losses to the aquifer (LOSS_j) terms or:

$$\sum_j q_{ji}^o - \sum_j q_{ij}^o = \mathrm{CU}_j^o + \mathrm{LOSS}_j \quad i \in \mathbf{N_U}, \forall \mathbf{O} \tag{16}$$

Mass balances for storage nodes (surface reservoirs and groundwater aquifers) are written over time in terms of water elevation, WEL by:

$$\mathrm{WEL}_i^o = \mathrm{WEL}_i^{o-1} + \frac{\sum_j q_{ji}^o - \sum_j q_{ij}^o}{\mathrm{AREA}_i} \tag{17}$$

where AREA_i is a surface area of storage nodes.

In addition to mass balance, other physical limits may be imposed at nodes. For non-storage nodes, the total flows supplied to a treatment facility must be less than the plant capacity, w_j, for each operational period, o in $\mathbf{O}$, or:

$$\sum_i q_{ij}^o \leq w_j^t \quad j \in \mathbf{N_{WT}} \cup \mathbf{N_{WWT}}, \quad t \in \mathbf{T}, \quad o \in \mathbf{O} \tag{18}$$

Similarly, the release from downstream river nodes must exceed the minimum flow requirement in the river, RQ or:

$$\sum_j q_{ji}^o - \sum_j q_{ij}^o \geq \mathrm{RQ_i} \quad i \in \mathbf{N_{RD}} \in \mathbf{N_{NS}}, \forall \mathbf{O} \tag{19}$$

To meet demands, the sum of the flows entering a user node j must equal or exceed the nodal demand, D_j^o or:

$$\sum_i q_{ij}^o \geq D_j^o \quad j \in \mathbf{N_U}, \quad \forall \mathbf{O} \tag{20}$$

where i is the set of nodes supplying demand node j. When minimizing costs, the net total inflow at user nodes should be strictly equal to the demand since, exceeding it, will incur additional costs.

For storage nodes, the water level must be maintained above the required water elevation, REL or:

$$\mathrm{WEL}_i^o \geq \mathrm{REL}_i \quad i \in \mathbf{N_{SS}}, \quad \forall \mathbf{O} \tag{21}$$

3.3 Arc Related Flow Constraints

A significant extension in this formulation is the direct consideration of energy and flow relationships for pipe/pump systems (the sets $\mathbf{A_P}$ and $\mathbf{A_U}$). Conservation of energy is applied to these arcs as:

$$\left[\left(\frac{4}{3}H_{ij}^t - \frac{H_{ij}^t q_{ij}^{o^2}}{3\chi_{ij}^{t^2}}\right) - \frac{8f\,L_{ij}}{g\pi^2\kappa_{ij}^{t^5}} q_{ij}^{o^2} - \Delta_{ij}^o\right]_i \geq H_{\min,j} - 10000\left(1-\mu_{ij}^t\right) \tag{22}$$
$$ij \in \mathbf{A_P}, \quad t \in \mathbf{T}, \quad o \in \mathbf{O}$$

where the terms correspond to the pump head added (in parenthesis), the pipe friction head loss (the Darcy–Weisbach equation with a constant friction factor), and the elevation difference, Δ, between the upstream and downstream nodes. The downstream head at node j (left side in Eq. 22) must satisfy a defined minimum requirement, $H_{min,j}$. When it is necessary to meet this requirement, a pump station may be constructed by setting μ equal to 1 that allows the χ and H to be non-zero in Eqs. 8 and 9. Note the pump relationship is developed assuming a quadratic form (USEPA 2000) in which the cutoff head (maximum head at zero flow) equals $4/3H$ and the maximum flow equals twice the design flow (2χ).

For flows to domestic, industrial, agricultural and large outdoor demand nodes that may be pumped directly to the user from the aquifer or a nearby location (the set $\mathbf{A_U}$). A similar energy equation is written for a pump station without the pipe as:

$$\left(\frac{4}{3}H_{ij}^{t} - \frac{H_{ij}^{t} q_{ij}^{o^2}}{3\chi_{ij}^{t^2}} - \Delta_{ij}^{o}\right) \geq H_{\min,j} - 10000\left(1-\mu_{ij}^{t}\right) \quad ij \in \mathbf{A_U}, \quad t \in \mathbf{T}, \quad o \in \mathbf{O} \tag{23}$$

To maintain hydraulic efficiency, pump connection flows are required to be between 50% and 150% of the pump flow capacity for all pumps (the sets $\mathbf{A_P}$ and $\mathbf{A_U}$):

$$q_{ij}^{o} \geq 0.5\chi_{ij}^{t} \quad ij \in \mathbf{A_P} \cup \mathbf{A_U}, \quad t \in \mathbf{T}, \quad o \in \mathbf{O} \tag{24}$$

$$q_{ij}^{o} \leq \chi_{ij}^{t} \quad ij \in \mathbf{A_P} \cup \mathbf{A_U}, \quad t \in \mathbf{T}, \quad o \in \mathbf{O} \tag{25}$$

Canal flows are limited by the canal capacity. Maximum canal flows are computed using Manning's open channel flow equation that relates the channel characteristics (slope (S), roughness (n), and geometry), the channel depth that is a decision variable (κ), and the channel flow rate $\left(q_{ij}^{o}\right)$ for all channels $\mathbf{A_C}$ as:

$$q_{ij}^{o} \leq \frac{1.49}{n_{ij}}\sqrt{2}\left(\sqrt{1+z_{ij}^{2}} - z_{ij}\right)\kappa_{ij}^{t\frac{8}{3}} S_{ij}^{\frac{1}{2}} \quad ij \in \mathbf{A_C}, \quad t \in \mathbf{T}, \quad o \in \mathbf{O} \tag{26}$$

Note the canal shape is assumed as the most efficient trapezoidal channel.

Infiltration (q_{ij}^{o}, i is river and j is groundwater) through a river bed to an aquifer is calculated by Darcy's equation:

$$q_{ij}^{o} = \left(\frac{\mathrm{WEL}_{\mathrm{RI}}^{o} - \mathrm{WEL}_{\mathrm{GW}}^{o}}{\mathrm{EL}_{\mathrm{RI}} - \mathrm{WEL}_{\mathrm{GW}}^{o}} \mathrm{COND}\right)(\mathrm{BOT})\, L_{\mathrm{RI}} \quad ij \in \mathbf{A_I} \tag{27}$$

and

$$\mathrm{WEL}_{\mathrm{RI}}^{o} = \frac{\sum_j \mathrm{q}_{ji}^{o} - \sum_j \mathrm{q}_{ij}^{o}}{(\mathrm{BOT})\, V_{\mathrm{RI}}} \tag{28}$$

if a node i is river where $\mathrm{WEL}_{\mathrm{RI}}^{o}$ is water elevation in the river in the year o, $\mathrm{WEL}_{\mathrm{GW}}^{o}$ is the groundwater elevation in the year o, $\mathrm{EL}_{\mathrm{RI}}$ is the riverbed elevation, and L_{RI}, V, BOT and COND are the river length, velocity, bottom width and channel hydraulic conductivity, respectively.

Groundwater flows from recharge basins is computed as the product of the rate of recharge depth times the basin area or:

$$q_{ij}^{o} = \mathrm{Area}_{\mathrm{basin}} V \tag{29}$$

where $\mathrm{Area}_{\mathrm{basin}}$ is groundwater basin area and V is the recharge rate.

3.4 Water Quality Constraints

Water quality constraints ensure that water supplied to users or returned to sources satisfy their water quality requirements. Flows entering a node are assumed to be completely mixed and the outflow water quality of a node is the flow weighted average of the incoming water concentrations, c. This value must be below a defined minimum value, WQR. When water is used or treated, it undergoes a change in quality. For a treatment facility or a river, the qualities leaving the nodes are assumed to be improved by removal efficiency (WQRE) or:

$$c_i^o = \frac{\sum_j c_{ji}^o q_{ji}^o (1 - \mathrm{WQRE}_i)}{\sum_j q_{ij}^o} \leq \mathrm{WQR}_i \quad i \in \mathbf{N_{NS}} \cup \mathbf{N_{WT}} \cup \mathbf{N_{WWT}}, \forall \mathbf{O} \tag{30}$$

where WQRE is the removal efficiency of the contaminant in the river or treatment facility.

Consumers detrimentally affect water quality that is represented by an incremental addition to the contaminant concentration, WQΔ. The water quality leaving the user node is then computed by:

$$c_i^o = \frac{\sum_j c_{ji}^o q_{ji}^o}{\sum_j q_{ij}^o} + \mathrm{WQ}\Delta_i \leq \mathrm{WQR}_i \quad i \in \mathbf{N_U}, \forall \mathbf{O} \tag{31}$$

Water quality is assumed unchanged during transport in canals and pipes.

If the node represents storage components, all stored water is assumed to be completely mixed with the inflow water and the average contaminant concentration is computed by:

$$\begin{cases} c_i^0 \leq \mathrm{WQR}_i & \text{at year} = 0 \\ c_i^o = \dfrac{c_i^{o-1}\left(\mathrm{WEL}_i^{o-1}\mathrm{AREA}_i\right) + \sum_j c_{ji}^o q_{ji}^o (1 - \mathrm{WQRE}_i)}{\left(\mathrm{WEL}_i^o \mathrm{AREA}_i\right)} \leq \mathrm{WQR}_i & \\ \text{otherwise} \quad i \in \mathbf{N_{SS}}, \forall \mathbf{O} & \end{cases} \tag{32}$$

3.5 Objective Function Penalty Term

Random search techniques can be useful in solving a large nonlinear problem. However, these techniques are only applicable to unconstrained optimization problems. To insure the constraints are satisfied, constraint violations are generally embedded as a penalty term in the objective function (Eq. 1). Here, a simple penalty term, f_6, that scales the absolute value of constraint violations by a large user defined number is applied for 12 of the above constraints (Eqs. 18–32 (excluding Eqs. 27, 28, and 29)). Constraints (3–14) are direct functions of a single independent decision variable and can be restricted during the random search.

Total penalty term, f_6, is the sum of penalties from each of the 12 constraints or:

$$f_6 = fn_{18} + fn_{19} + fn_{20} + fn_{21} + fn_{22} + fn_{23} + fn_{24} + fn_{25} + fn_{26} + fn_{30} + fn_{31} + fn_{32} \tag{33}$$

where fn_i is the penalty term for Eq. i.

Since the objective is to minimize cost, the positive penalty term will be minimized to satisfy the constraints and optimize the solution. The penalty term is zero when the constraint is satisfied. A scaled violation term by a user defined value M' is added if the constraint is not met. For example, the term for Eq. 26 is given by:

$$fn_{26} = \begin{cases} 0 & \text{, if } q_{ij}^{o} \leq \dfrac{1.49}{n_{ij}}\sqrt{2}\left(\sqrt{1+z_{ij}^{2}}-z_{ij}\right)\kappa_{ij}^{t\frac{8}{3}}S_{ij}^{\frac{1}{2}} \\ M'\left(q_{ij}^{o}-\dfrac{1.49}{n_{ij}}\sqrt{2}\left(\sqrt{1+z_{ij}^{2}}-z_{ij}\right)\kappa_{ij}^{t\frac{8}{3}}S_{ij}^{\frac{1}{2}}\right), & \text{otherwise} \end{cases}$$
$$ij \in \mathbf{A_C},\ t \in \mathbf{T},\ o \in \mathbf{O} \tag{34}$$

A single M' value is applied for all constraints because it is difficult to predict which constraint will have significant effect on the objective function. For simplicity, this large factor is not changed during the optimization process.

3.6 Summary of Formulation

The set of equations (1–32) forms the basis for the water supply optimization problem. The formulation is general and can consider multiple demand centers and supply sources, one or more treatment facilities and linkages between users. Distances and system topography can also be represented. One or more planning periods can be evaluated with multiple years within each planning period. The decision variables are a mixture of continuous and binary variables. The equations are highly nonlinear. A heuristic stochastic search algorithm known as the SFLA (Eusuff et al. 2006) is improved and applied to solve the problem.

4 Shuffled Frog Leaping Algorithm

The posed water supply problem is non-linear and non-convex. The degree of nonlinearity causes difficulties in solving the problem using non-linear programming. A nonlinear programming model was constructed in MINOS, however, no improvement was achieved from the initial solutions. Therefore, a stochastic search technique, the SFLA (Eusuff et al. 2006), was applied. To improve the speed of convergence, the algorithm was slightly modified.

SFLA is applicable to problems with continuous and discrete decision variables. It is a descent based stochastic search method that begins with an initial population of frogs whose characteristics, known as memes, represent the decision variables.

Memes are units of knowledge that, like an idea, improve as more information is gained. In a physical domain, memes represent a frog's position that is improved by comparing its objective function value with other frogs and changes in a frog's characteristics (memes) are made by moving (leaping) in the direction of lower objective function value in case of minimum problem. In a frog's domain, he is moving to locations with higher food concentrations. This movement is similar to the evolution of an idea that changes as each individual learns something from others.

In SFLA, the total population is partitioned into groups (memeplexes) that search independently. After a number of iterations of local search algorithm, the groups are combined and reformed to pass information between individuals, similar to the cross-fertilization of ideas between different organizations. This goal of the overall process is to determine global optimal solutions. As such, a random component is also included to generate new points that maintain a diverse search. The process of local group searches and population shuffles continues until convergence to an optimum is reached or a user-defined number of frogs have been evaluated.

The global exploration and local search algorithms are presented in the following steps. This algorithm is slightly modified from Eusuff et al. (2006) in the local search that extends the potential step size to be beyond the current frog locations. In Eusuff et al., a new point was limited to fall between the existing frogs.

4.1 Global Exploration

Step 0. Initialize: Select m and N, where m = number of memeplexes and N = number of frogs in each memeplex. Therefore, the total population size in the swamp, $F = mN$.

Step 1. Generate a virtual population: Sample F virtual frogs $U(1), U(2), \ldots, U(F)$ in the feasible space $\Omega \subset \Re^d$ where d is the number of decision variables (i.e., number of menotype(s) in a meme carried by a frog). The 'ith' frog is represented as a vector of decision variable values $U(i) = \left(U_i^1, U_i^2, \ldots, U_i^d\right)$, $i = 1, \ldots, F$.

Compute the performance value, $f(i)$, for each frog $U(i)$.

Step 2. Rank frogs: Sort the F frogs in order of decreasing performance value in case of minimum problem. Store them in array $\boldsymbol{X} = \{U(i), f(i), I = 1 \ldots F\}$, so that $i = 1$ represents the frog with the best performance value. Record the best frog's position in the entire population (F frogs), $\boldsymbol{P_X}$, (where $\boldsymbol{P_X} = U(1)$).

Step 3. Partition frogs into memeplexes: Partition array $\boldsymbol{X}$ into m memeplexes $\boldsymbol{Y}^1, \boldsymbol{Y}^2, \ldots, \boldsymbol{Y^m}$, each containing n frogs, such that:

$$Y^k = \left[U(j)^k, f(j)^k \,\middle|\, U(j)^k = U(k+m(j-1)), f(j)^k = f(k+m(j-1)), \right. \\ \left. j = 1, \ldots, n\right], k = 1, \ldots m$$

For example, for $m = 3$, rank 1 goes to memeplex 1, rank 2 goes to memeplex 2, rank 3 goes to memeplex 3, rank 4 goes to memeplex 1 again, and so on.

Step 4. Memetic evolution within each memeplex: Evolve each memeplex $\boldsymbol{Y^k}$, $k = 1, \ldots, m$ according to the frog leaping algorithm (FLA) outlined below.

Step 5. Shuffle memeplexes: After a defined number of memetic evolutionary steps within each memeplex, replace $\boldsymbol{Y}^1, \ldots, \boldsymbol{Y^m}$ into $\boldsymbol{X}$, such that $\boldsymbol{X} = \left\{\boldsymbol{Y^k}, k = 1, \ldots, m\right\}$. Sort $\boldsymbol{X}$ in order of decreasing performance value. Update the population best frog's position, $\boldsymbol{P_X}$.

Step 6. Check convergence: If the convergence criteria are satisfied, stop. Otherwise, return to Step 3. Typically, the decision on when to stop is made by a pre-specified number of consecutive time loops when at least one frog carries the "best memetic pattern" without change. Alternatively, a maximum total number of function evaluations can be defined.

4.2 Local Exploration: Frog Leaping Algorithm

Step 0. Set *im* (iteration count) and *iN* (shuffle count) equal to zero. The number of iterations and shuffles are limited to user-defined values *ic* and *is*, respectively. Form an initial random set of frogs and evaluate each frogs objective function value.

Step 1. Set $im = im + 1$.

Step 2. Set $iN = iN + 1$.

Step 3. Construct a sub-memeplex: The frogs' goal is to move towards the optimal ideas by improving their memes. An individual frog is updated using the presently available information. The new frog is returned to the memeplex and another frog is updated. This strategy is consistent with evolution of ideas since the best information available is used unlike a genetic algorithm in which the entire population is updated prior to using any information gained. To complete this process, a subset of the memeplex, ($\boldsymbol{Y^k}$, $k = 1, \ldots, m$), called a sub-memeplex, ($\boldsymbol{Z^{iq}}, iq = 1, \ldots, q$) is considered. The sub-memeplex selection strategy from a memeplex ($\boldsymbol{Y^k}$) having n frogs is to give higher weights to frogs that have higher performance values and less weight to those with lower performance values. The weights are assigned with a triangular probability distribution, i.e.,

$$p_j = 2\,(n + 1 - j)\big/(n\,(n + 1)), \quad j = 1, \ldots, n$$

such that, within a memeplex, ($\boldsymbol{Y^k}$, $k = 1, \ldots, m$), the frog with best performance has the highest probability of being selected for the sub-memeplex, $p_1 = 2/(n + 1)$ and the frog with worst performance has the lowest probability, $p_n = 2/(n(n + 1))$.

Here, q distinct frogs are selected randomly from n frogs in each memeplex ($\boldsymbol{Y^k}, k = 1, \ldots, m$) to form the sub-memeplex array ($\boldsymbol{Z^{iq}}, iq = 1, \ldots, q$). The sub-memeplex is sorted so that frogs are arranged in order of decreasing performance. Record the best ($iq = 1$; $iq = 1, \ldots, q$) and worst ($iq = q$; $iq = 1,\ldots, q$) frog's position in the sub-memeplex as vectors P_B and P_W, respectively.

Step 4. Improve the worst frog's position: When improving a frog's position, it can adapt their ideas from the best frog within the memeplex (group), $\boldsymbol{P_B}$, or from the global (population) best, $\boldsymbol{P_X}$. The direction, step size and new position are first computed for the frog with worst performance in the sub-memeplex ($\boldsymbol{Z^{iq}}$, $iq = 1, \ldots, q$). The computation includes identifying the direction of improvement (gradient) and the magnitude of change (step length) in that direction. The direction of change (positive or negative) is defined by the movement toward the sub-memeplex best or ($\boldsymbol{P_B} - \boldsymbol{P_W}$) where $\boldsymbol{P}$ represents the location vector. This change involves both in magnitude as well as direction of the decisions.

The magnitude of the step size is randomly selected as a proportion of change in direction. It is limited by the maximum step size, $\boldsymbol{S}_{\max}$. The present version of the algorithm considers a pre-specified fraction of the bound of the variable value as $\boldsymbol{S}_{\max}$. Mathematically, the step size is defined as:

$$\text{Step size, } |\mathbf{s}| = \text{MIN}\left[2\,(\text{rand}\ (\mathbf{P}_\text{B} - \mathbf{P}_\text{W}))\,,\ \mathbf{S}_{\mathbf{max}}\right]$$

where *rand* is a random number in the range [0,1]. The new position is then computed by:

$$Z_{(iq=q)} = \boldsymbol{P}_W + \boldsymbol{s} \tag{35}$$

Note that the multiplier of 2 in the step size calculation allows the frog's new position to be between the two frogs or move beyond the better frog's location depending on *rand*. Then, if the new position is beyond feasible boundary for any decision variable, it is forced to be the boundary value. This modification improves upon convergence seen in Eusuff et al. (2006).

Compute the new performance value $f_{(iq=q)}$. If the new $f_{(iq=q)}$ is better than the old $f_{(iq=q)}$, i.e., if the move produces a benefit, then replace the old $\boldsymbol{Z}_{(iq=q)}$ with the new one and go to *Step* 7. Otherwise go to *Step* 5.

Step 5. If *Step* 4 cannot produce a better result then the step and new position are computed for that frog by:

$$\text{Step size, } |\boldsymbol{s}| = \text{MIN}\left[2\left(\text{rand}\left(\boldsymbol{P}_X - \boldsymbol{P}_W\right)\right),\ \boldsymbol{S}_{\mathbf{max}}\right]$$

and the new position is computed by Eq. 35.

Compute the new performance value $f_{(iq=q)}$ for point $\boldsymbol{Z}_{(iq=q)}$. If the new $f_{(iq=q)}$ is better than the old $f_{(iq=q)}$, i.e., if the evolution produces a benefit, then replace the old $\boldsymbol{Z}_{(iq=q)}$ with new one and go to *Step* 7. Otherwise go to *Step* 6.

Step 6. Censorship: If the new position is either infeasible or worse than the old position, the spread of the defective meme is stopped by randomly generating a new frog '*r*' at a feasible location to replace the frog whose new position was not favorable to progress. Compute $f(r)$ and set $\boldsymbol{Z}_{(iq=q)} = r$ and $f_{(iq=q)} = f(r)$.

Step 7. Upgrade the memeplex: After the memetic change for the worst frog in the sub-memeplex, replace $\boldsymbol{Z}^{\boldsymbol{iq}}$ in their original locations in $\boldsymbol{Y}^{\boldsymbol{k}}$. Sort $\boldsymbol{Y}^{\boldsymbol{k}}$ in order of decreasing performance value.

Step 8. If $iN < is$, go to *Step* 2.

Step 9. If $im < ic$, go to *Step* 1. Otherwise return to global search to shuffle memeplexes.

Steps 4 and 5 of the FLA are similar in philosophy to particle swarm optimization. A descent direction is identified for a particular frog and the frog is moved in that direction. Here, however, since the global search is also introduced in the shuffle operation, only the local minimum is used rather than the complete population best (Step 4) unless no improvement is made (Step 5). Since a descent direction is implicitly applied, it may be fruitful to perform a line search rather than a random step but the simpler approach is taken here.

5 Applications

The optimization problem (1–32) has been formulated for the water supply systems shown in Figs. 1 and 2 for 20-year planning periods. New structural component construction is permitted at the outset (year 1) and new components or existing component expansion may be added after 10 years. Biochemical Oxygen Demand (BOD) is used as the representative water quality parameter.

5.1 Single Wastewater Treatment Plant System

The first system to be optimized (Fig. 1) consists of single water and wastewater plants, multiple sources (imported water, groundwater aquifer, and surface water) and two demands centers (domestic and agricultural). Three types of water transport structures are used depending upon the connection: canal, pipe and/or pump. All canal flows for the conveyance of imported and raw water sources are driven by gravity. Agricultural areas and the water treatment plant directly pump groundwater from aquifers available near their location so that they do not require a pipe link. Other flows are transported through pipes that may require a pump station to supply the energy necessary to pass flow through the pipeline and satisfy the minimum pressure head requirement at the outlet (14.0 m of water = 137.9 kPa = 20 psi). Groundwater replenishment through recharge basins is assumed to occur at a constant rate of 9.1 m/year (30 ft/year). Seepage losses from users to the aquifer are assumed as 0.1% of total user demand.

Input parameters for the single wastewater treatment plant system are shown in Tables 1 and 2. The system contains the total of 18 arcs with different lengths (Fig. 1). As seen in Fig. 1, the network consists of six canal depth construction decisions (6) and seven pump/pipeline arcs with their three design decisions (pipe diameter and pump flow capacity and head) for a total of 21 decisions. The network also includes two pump links for which the pump design flow and head (4 design decisions in total) must be selected and two treatment facilities with plant capacity as decision variables

Table 1 Input parameters for the single wastewater treatment plant system application

Parameter	Value	Unit
Darcy–Weisbach coefficient, f	0.02	
Manning's coefficient, n	0.014	
Canal side slope, S	2	
Hydraulic conductivity, COND	9.144	m/year
Imported water, IWA	0.062	km^3/year
Initial population, POP^0	300,000	
Population growth rate, POPGR	2.7	%/year
Interest rate, I	2.0	%/year
City multiplier, CITY	1	
Annual precipitation, PP	69.8	mm/year
Basin area, AREA	132,771	km^2
Required water elevation of groundwater, REL	397	m
Required discharge of downstream river, RQ	2.83	m^3/s
Initial water quality of groundwater, c_{ik}^0, i = GW	0.04	mg/l
Water quality of imported water, c_{ik}^0, i = IW	30.0	mg/l
Precipitation water quality, c_{ik}^0, i = PP	1.0	mg/l
Upstream river bottom width, BOT	3.05	m
Upstream river length, L_{RI}	4,023	m
Average velocity of upstream river, V_{RI}	1.52	m/s
Agricultural consumptive use, CU	0.26	km^3/year

GW groundwater, *IW* imported water, *PP* precipitation

Table 2 Nodal input parameters for the single wastewater treatment plant system application

Nodes	Area (km^2)	Elevation (m)	Water quality requirement (mg/l)	Quality degradation (mg/l)	Pollutant removal efficiency (%)
Imported water	0	671	30	0	0
Groundwater	13,277	518[a]	30	0	0.8
Upstream river	0	641	30	0	0.2
Downstream river	0	653	30	0	0.2
Water treatment plant	0	610	3	0	0.99
Domestic area	0	579	5	200	0
Agricultural area	133	610	30	100	0
Wastewater treatment plant	0	640	30	0	0.99

[a] Initial water surface elevation

(total of 2 decisions). Thus, 33 design decision variables are to be determined for each of the two planning periods or a total of 66 design decisions.

Ten of the 18 arc flows in each period are independent control decision variables that are also selected by the optimization model. The remaining 8 are dependent variables that are computed from the mass balance constraints defined in Eqs. 15 and 16. Therefore, the final optimization problem contains a total 86 of decision variables for the two design periods (66 design and 20 control decision variables).

The following SFLA parameters were selected from experience and preliminary testing: the total number of population (F = 3,000), memeplex (m = 10), frogs in each memeplex (N = 300), evolutionary steps (iN = 300), and frogs in a sub-memeplex (iq = 300) are established and applied in the single wastewater treatment plant system. The problem was run on a Dell Inspiron with a Centrino Duo T2300 1.6 GHz and 1 GB of RAM and was solved in 5.5 min after 131 thousand function evaluations. Figure 3 shows the progress of solution with respect to the number of function evaluations. The penalty term that accounts for constraint violations fell dramatically in early iterations after which the total system cost gradually decreased. Total construction and operation cost for the single treatment plant system for the 20-year period is \$771 million (present value for year 0) or an annual cost of \$47 million.

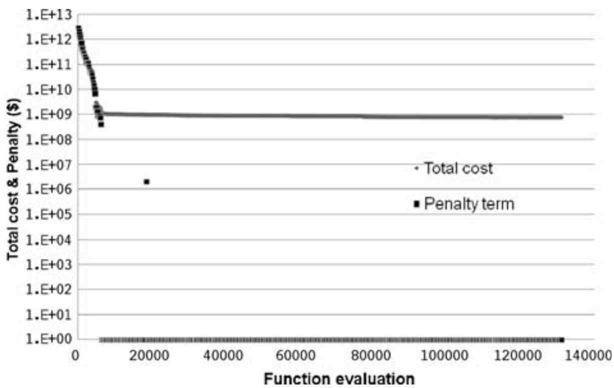

Fig. 3 Best solution and penalty term change with the number of function evaluations in the single WWT plant system

Table 3 Optimal solution for the single wastewater treatment plant system application

Leaving node	Entering node	Flow allocation (m^3/s)		Pipe diameter (mm)		Canal Depth (m)		Design pump discharge (m^3/s)		Design pump head (m)	
		First period	Second periods	First period	Second periods	First period	Second periods	First period	Second periods	First period	Second periods
IW	AG	0.00	0.00			0	0				
IW	WT	0.00	0.00			0	0				
RI_U	WT	0.34	2.07			1	1				
RI_U	AG	0.59	0.59			1	1				
WT	DO	0.70	2.88	590 (600)	1,417 (1,500)			0.00	0.00	0	0
WT	AG	0.00	0.00	0	0			0	0	0	0
GW	DO	1.03	1.03	520 (600)	737 (900)			0.68	0.69	61	74
WW	AG	0.02	0.02	144 (200)	144 (200)			0.00	0.00	0	0
GW	WT	0.36	0.80					0.24	0.55	145	145
GW	AG	7.67	7.67					5.13	8.87	87	87
IW	GW	0.39	0.39			1	1				
WT	RI_D	0.00	0.00	0	0			0	0	0	0
DO	GW	0.00	0.00								
DO	WW	1.11	1.42	832 (900)	832 (900)			0.74	1.03	107	107
WW	RI_D	1.08	1.40	1,172 (1,200)	1,172 (1,200)			0.74	0.93	152	183
AG	GW	0.14	0.14								
AG	RI_D	0.00	0.00			0	0				
RI_U	GW	0.00	0.00								

The values in () indicate commercial pipe diameter available.

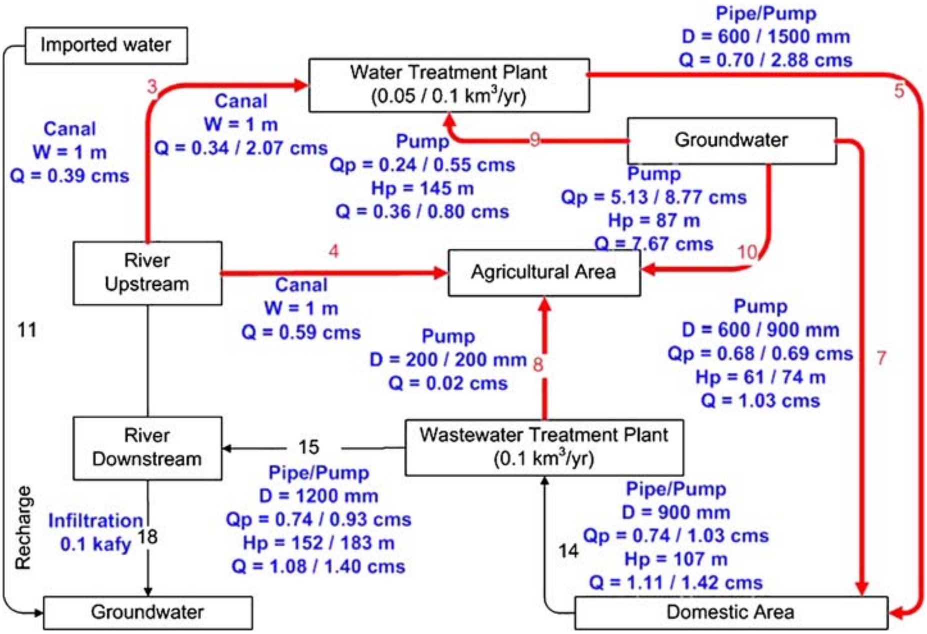

Fig. 4 Optimal solution of single wastewater plant system application showing the flow allocations and arc capacities/design variables—results from first design period/second design period, respectively. (Note the single value indicates no expansion in the capacity or flow allocations)

Table 4 Storage water sources and water demands for the single wastewater treatment plant system application

Year	Groundwater elevation (m)	Downstream river discharge (m^3/s)	Domestic demand (km^3/year)
2001	518.2	59.3	0.044
2002	517.5	59.4	0.045
2003	516.9	59.4	0.046
2004	516.3	59.4	0.047
2005	515.6	59.4	0.048
2006	515.0	59.5	0.049
2007	514.4	59.5	0.051
2008	513.7	59.5	0.052
2009	513.1	59.5	0.053
2010	512.4	59.6	0.055
2011	511.9	59.1	0.056
2012	511.4	59.1	0.057
2013	510.8	59.2	0.059
2014	510.3	59.2	0.060
2015	509.8	59.2	0.062
2016	509.2	59.3	0.063
2017	508.7	59.3	0.065
2018	508.2	59.3	0.067
2019	507.6	59.4	0.068
2020	507.1	59.4	0.070

Table 5 Construction and operation cost for the single wastewater treatment plant system application

Cost	Pipes	Pumps	Canals	Treatment plants
Construction ($\times 10^6$ \$)	190.79	26.07	33.59	6.08
Operation at year 1 ($\times 10^6$ \$/year)	21.22	5.39	0.07	0.06
Expansion ($\times 10^6$ \$)	5.86	2.77	0.00	0.15
Operation at year 10 ($\times 10^6$ \$/year)	22.72	6.22	0.13	0.06

Table 3 lists the optimal component designs and the optimal network solution is depicted graphically in Fig. 4. Water and wastewater treatment plant capacities are 0.05 and 0.05 km^3/year during the first design period, respectively. The water and wastewater treatment plants are not expanded at year 10 suggesting that economies of scale made oversizing in the first period to be more desirable than future expansion.

Increased transport capacity was required to/from the domestic area in year 10 to convey the increased demands. Although the flow allocation from the aquifer to the farms remains constant over time, the groundwater pump capacity was expanded to overcome the required lift and the drop in the aquifer water level (Table 4).

The system has abundant water in the downstream river and aquifer (Table 4) to preserve sustainability. Therefore, the domestic area is mainly supplied from the river through the water treatment plant, and water pumped from the aquifer is the main source for agricultural area.

BOD concentrations in the aquifer and the downstream river remain steady and below their 30 mg/l water quality requirements (Table 4). Influents to domestic and agricultural area also have better quality than their required values of 5 and 30 mg/l, respectively.

Table 6 Nodal input parameters for the multiple wastewater treatment plant system application

Nodes	Area (km^2)	Elevation (m)	Water requirement (mg/l)	Quality degradation (mg/l)	Pollutant removal efficiency (%)
Imported water	0	671	30	0	0
Groundwater	13,277	518[a]	30	0	0.8
Upstream river	0	640	30	0	0.2
Downstream river	0	549	30	0	0.2
Water treatment plant	0	610	3	0	0.99
Domestic area 1	258.0	579	5	200	0
Domestic area 2	344.0	610	5	200	0
Domestic area 3	516.0	613	5	200	0
Industrial area	2,063.9	579	5	200	0
Agricultural area	244.9	610	30	100	0
Large outdoor area	173.4	610	30	100	0
Wastewater treatment plant 1	0	610	30	0	0.99
Wastewater treatment plant 2	0	579	30	0	0.99
Wastewater treatment plant 3	0	640	30	0	0.99

[a] Initial water surface elevation.

As shown in Table 5, pipe construction is the dominant cost for this system and economies of scale compel some pipe to be constructed in the initial period. Pipe and pump connections require significant operation costs as compared with the operation of canal and treatment plants.

5.2 Multiple Wastewater Treatment Plant System

As shown in Fig. 2, to investigate a more general system, the second configuration consists of six users – three domestic areas, one industrial, one agricultural, and one large outdoor area – and three wastewater treatment plants. In general, input parameters used for the multiple wastewater treatment plant system are the same as for the single wastewater treatment plant system except for the initial population at the domestic areas. Initial populations of domestic area 1, 2, and 3 are 300,000, 400,000, and 600,000, respectively. Table 6 summarizes the multiple wastewater treatment plant system nodal parameters.

This network has 44 arc connections with lengths given in Fig. 2 and includes 6 canals (6 parameters for canal depth), 29 pipes of which pump station could be built depending on energy relationship (29 parameters for each pipe diameter, pump design capacity, and pump head), 2 pumps (2 parameters for each pump design capacity and head), and 4 treatment plants (4 parameters for capacities). Structures can be built or expanded in year 0 and 10. Total structural design variables are 101 ($=6 + 3 \times 29 + 2 \times 2 + 4$) for each design period and a total 202 for whole operational period.

Flow allocations through 23 arcs out of 44 are defined as operation decision variables while the remaining 21 arcs are dependent variables that are computed by mass balance equations. In total, the final problem consists of 248 decision variables (2 design periods $\times$ 124 decision variables (101 and 23 for design and operation variables, respectively)).

The problem was solved using the computer system cited in the previous section in about 70 min and nearly 582 thousands function evaluations. The optimal cost for the system was \$837 million as the present value in the starting year of the planning period and the estimated annual cost was \$51 million. The final optimal solution is depicted in Fig. 5. Although the optimal solution found may not be the global optimal solution due to high discrete nonconvexity associated with the study system, the optimization process demonstrates the improvement in overall system cost and reduction in the penalty term (Fig. 6).

Since the system has sufficient local water to meet user demands, imported water is not purchased. Domestic areas 1 and 2 are supplied from the aquifer which has sufficient water quality for domestic demands, and domestic area 3 and industrial area are supplied from upstream river through water treatment plant.

Pipe, pump, and canal capacities are given in Table 7. The water treatment plant capacity is 0.25 km^3/year at the first design period and is not expanded. Wastewater treatment plant 1 and 3 are 0.13 km^3/year and 0.25 km^3/year, respectively. Wastewater treatment plant 2 is not built because of its high elevation. Canals carry water from the upstream river to agricultural and large outdoor areas. Large outdoor turf uses are partially supplied with treated wastewater. As for the single treatment plant

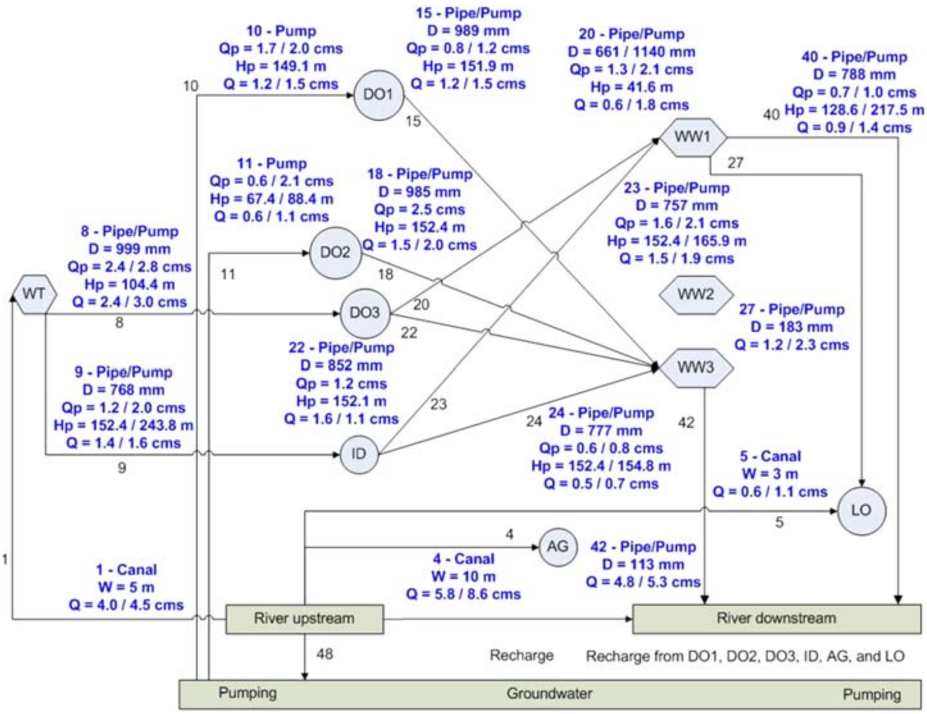

Fig. 5 Optimal solution of multiple wastewater plant system application showing the flow allocations and arc capacities/design variables—results from first design period/second design period, respectively. (Note the single value indicates no expansion in the capacity or flow allocations)

case, most pipe construction occurred at the outset due to economies of scale. Pipe construction cost dominates the total system cost (Table 8). Groundwater quality is close to zero due to the effect of soil purification. Water quality parameters for influent of users, water elevation, and source discharge are summarized in Table 9. Water demands increase over time (Table 10).

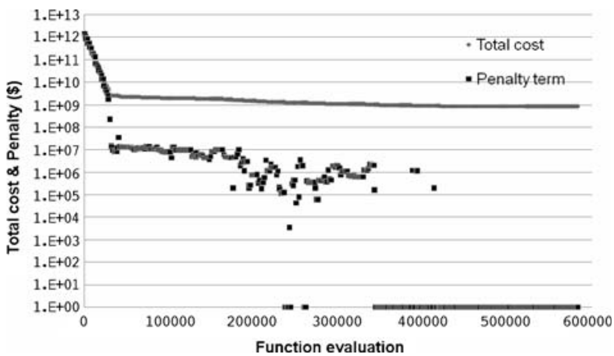

Fig. 6 Optimal system cost and penalty term changes with the number of function evaluations for the multiple WWT plant system

Table 7 Optimal solution for the multiple wastewater treatment plant system application

Leaving node	Entering node	Flow allocation (m^3/s)		Pipe diameter (mm)		Canal Depth (m)		Design pump discharge (m^3/s)		Design pump head (m)	
		First period	Second periods	First period	Second periods	First period	Second periods	First period	Second periods	First period	Second periods
RI_U	WT	4.0	4.5			5	5				
IW	WT	0.0	0.0			0	0				
IW	AG	0.0	0.0			0	0				
IW	GW	0.0	0.0			0	0				
RI_U	AG	5.8	8.6			10	10				
RI_U	LO	0.6	1.1			3	3				
WT	DO1	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WT	DO2	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WT	DO3	2.4	3.0	999	999			2.4	2.8	104.4	104.4
WT	ID	1.4	1.6	768	768			1.2	2.0	152.4	243.8
GW	DO1	1.2	1.5	0	0			1.7	2.0	149.1	149.1
GW	DO2	1.6	2.9	0	0			1.1	1.7	82.8	90.9
GW	DO3	0.0	0.0	0	0			0.0	0.0	0.0	0.0
GW	ID	0.6	1.1	0	0			0.6	2.1	67.4	88.4
DO1	WW1	0.0	0.0	0	0			0.0	0.0	0.0	0.0
DO1	WW2	0.0	0.0	0	0			0.0	0.0	0.0	0.0
DO1	WW3	1.2	1.5	989	989			0.8	2.0	151.9	151.9
DO2	WW1	0.0	0.0	0	0			0.0	0.0	0.0	0.0
DO2	WW2	0.0	0.0	0	0			0.0	0.0	0.0	0.0
DO2	WW3	1.5	2.0	985	985			2.5	2.5	152.4	152.4

Table 7 (continued)

Leaving node	Entering node	Flow allocation (m^3/s)		Pipe diameter (mm)		Canal Depth (m)		Design pump discharge (m^3/s)		Design pump head (m)	
		First period	Second periods	First period	Second periods	First period	Second periods	First period	Second periods	First period	Second periods
DO3	WW1	0.6	1.8	661	1,140			1.3	2.1	41.6	41.6
DO3	WW2	0.0	0.0	0	0			0.0	0.0	0.0	0.0
DO3	WW3	1.6	1.1	852	852			1.2	1.2	152.1	152.1
ID	WW1	1.5	1.9	757	757			1.6	2.1	152.4	165.9
ID	WW2	0.0	0.0	0	0			0.0	0.0	0.0	0.0
ID	WW3	0.5	0.7	777	777			0.6	0.8	152.4	154.8
WW1	AG	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WW1	RI_D	1.2	2.3	183	183			0.0	0.0	0.0	0.0
WW1	LO	0.9	1.4	788	788			0.7	1.0	128.6	217.5
WW2	AG	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WW2	RI_D	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WW2	LO	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WW3	AG	0.0	0.0	0	0			0.0	0.0	0.0	0.0
WW3	RI_D	4.8	5.3	113	113			0.0	0.0	0.0	0.0
WW3	LO	0.0	0.0	0	0			0.0	0.0	0.0	0.0
GW	AG	0.0	0.0					0.0	0.0	0.0	0.0
GW	LO	0.0	0.0					0.0	0.0	0.0	0.0
DO1	GW	0.0	0.0								
DO2	GW	0.0	0.0								
DO3	GW	0.0	0.0								
ID	GW	0.0	0.0								
RI_U	GW	0.0	0.0								
AG	GW	0.0	0.0								
LO	GW	0.0	0.0								

Table 8 Construction and operation cost for the multiple wastewater treatment plant system application

Cost	Pipes	Canals	Pumps	Treatment plants
Construction ($\times 10^6$ \$)	116.81	85.40	31.42	20.29
Operation in the 1st period ($\times 10^6$ \$/year)	22.45	0.40	2.67	0.20
Expansion ($\times 10^6$ \$)	3.42	0.17	3.97	0.04
Operation in the 2nd period ($\times 10^6$ \$/year)	26.66	0.64	1.92	0.20

6 Conclusions

In this study, a large-scale general water supply system optimization model was formulated as a integer-nonlinear problem. The formulation was applied to two moderate and large hypothetical water networks. Difficulties in solving the problem using gradient based NLP methods led to the application of a heuristic stochastic search algorithm, the SFLA. A minor improvement to SFLA was made over previous work by Eusuff et al. (2006).

Single (7 nodes) and multiple (13 nodes) wastewater treatment plant system have been optimized in terms of the total system cost. The total number of decision variables in the single and multiple wastewater treatment plant system applications are 86 and 248, respectively. The resulting annual minimum costs were \$47 million and \$51 million, respectively.

Table 9 Water quality change, groundwater elevation, and downstream river discharge for the multiple wastewater treatment plant system application

Year	Down-stream river BOD (mg/l)	DO1 influent BOD (mg/l)	DO2 influent BOD (mg/l)	DO3 influent BOD (mg/l)	ID influent BOD (mg/l)	AG influent BOD (mg/l)	LO influent BOD (mg/l)	Ground-water elevation (m)	Down-stream river discharge (m^3/s)
2001	1.703	0.040	0.040	0.008	0.011	0.800	1.548	520.68	11.88
2002	1.709	0.008	0.008	0.008	0.008	0.800	1.548	537.72	12.02
2003	1.716	0.002	0.002	0.008	0.007	0.800	1.548	554.04	12.17
2004	1.723	0.001	0.001	0.008	0.007	0.800	1.548	569.60	12.32
2005	1.729	0.000	0.000	0.008	0.007	0.800	1.548	584.40	12.47
2006	1.735	0.000	0.000	0.008	0.006	0.800	1.548	598.41	12.63
2007	1.741	0.000	0.000	0.008	0.006	0.800	1.548	611.60	12.79
2008	1.748	0.000	0.000	0.008	0.006	0.800	1.548	623.96	12.96
2009	1.753	0.000	0.000	0.008	0.006	0.800	1.548	635.46	13.13
2010	1.759	0.000	0.000	0.008	0.006	0.800	1.548	646.08	13.31
2011	2.846	0.000	0.008	0.000	0.006	0.800	1.467	650.78	11.95
2012	2.802	0.000	0.008	0.000	0.006	0.800	1.467	654.38	12.13
2013	2.761	0.000	0.008	0.000	0.006	0.800	1.467	656.87	12.32
2014	2.722	0.000	0.008	0.000	0.006	0.800	1.467	658.21	12.52
2015	2.687	0.000	0.008	0.000	0.005	0.800	1.467	658.37	12.72
2016	2.654	0.000	0.008	0.000	0.005	0.800	1.467	657.31	12.93
2017	2.624	0.000	0.008	0.000	0.005	0.800	1.467	655.02	13.14
2018	2.595	0.000	0.008	0.000	0.005	0.800	1.467	651.44	13.36
2019	2.568	0.000	0.008	0.000	0.005	0.800	1.467	646.56	13.58
2020	2.543	0.000	0.008	0.000	0.005	0.800	1.467	640.32	13.81

Table 10 Water demands in the multiple wastewater treatment plant system application (m^3/s)

Year	Domestic area 1	Domestic area 2	Domestic area 3	Industrial area	Large outdoor
2001	1.38	1.81	2.66	1.60	2.54
2002	1.42	1.85	2.73	1.64	2.61
2003	1.45	1.90	2.80	1.69	2.68
2004	1.49	1.95	2.87	1.73	2.75
2005	1.53	2.00	2.94	1.78	2.83
2006	1.56	2.05	3.02	1.83	2.90
2007	1.60	2.10	3.10	1.87	2.98
2008	1.64	2.16	3.18	1.93	3.06
2009	1.68	2.21	3.26	1.98	3.15
2010	1.73	2.27	3.35	2.03	3.23
2011	1.77	2.33	3.43	2.09	3.32
2012	1.82	2.39	3.52	2.14	3.41
2013	1.86	2.45	3.62	2.20	3.50
2014	1.91	2.51	3.71	2.26	3.59
2015	1.96	2.58	3.81	2.32	3.69
2016	2.01	2.64	3.91	2.38	3.79
2017	2.06	2.71	4.01	2.45	3.89
2018	2.11	2.78	4.12	2.51	4.00
2019	2.17	2.85	4.23	2.58	4.11
2020	2.22	2.93	4.34	2.65	4.22

Available water were sufficient to meet the user demands in both applications, so no external water was purchased. Therefore, the initial water treatment and transportation facility capacities were sufficient to cope with water demand increases in the later years. Pipe construction and pump operation were dominant cost terms.

Further research efforts are needed to develop a more detailed water supply system. For example, future formulations should include discrete pipe diameters or considering parameter uncertainties. Alternative random search techniques such as Shuffled Complex Evolution can be applied to compare their performances. Finally, although the authors attempted to identify the latest cost equations they are dated and updated cost relationships are needed to improve the applicability of these techniques.

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