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# Event-based Sediment Yield Modeling using Artificial Neural Network

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**Abstract** In the present study, a back propagation feedforward artificial neural network (ANN) model was developed for the computation of event-based temporal variation of sediment yield from the watersheds. The training of the network was performed by using the gradient descent algorithm with automated Bayesian regularization, and different ANN structures were tried with different input patterns. The model was developed from the storm event data (i.e. rainfall intensity, runoff and sediment flow) registered over the two small watersheds and the responses were computed in terms of runoff hydrographs and sedimentographs. Selection of input variables was made by using the autocorrelation and cross-correlation analysis of the data as well as by using the concept of travel time of the watershed. Finally, the best fit ANN model with suitable combination of input variables was selected using the statistical criteria such as root mean square error (RMSE), correlation coefficient (CC) and Nash efficiency (CE), and used for the computation of runoff hydrographs and sedimentographs. Further, the relative performance of the ANN model was also evaluated by comparing the results obtained from the linear transfer function model. The error criteria viz. Nash efficiency (CE), error in peak sediment flow rate (EPS), error in time to peak (ETP) and error in total sediment yield (ESY) for the storm events were estimated for the performance evaluation of the models. Based on these criteria, ANN based model results better agreement than the linear transfer function model for the computation of runoff hydrographs and sedimentographs for both the watersheds.

**Keywords** Automated Bayesian Regularization · ANN · Event-based · Runoff · Sediment yield · Sedimentograph · Small watersheds

**List of notations and abbreviation**

| | |
|---|---|
| $t_i$ | Target output at node $i$ |
| $a_i$ | Network output at node $i$ |
| N | Number of observation |

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| | |
|---|---|
| $\overline{X}_{k+1}$ | Weight factor at iteration ($k$+1) |
| $\overline{g}$ | $= \nabla f\left(\overline{X}_k\right)$ = error gradient vector |
| $Y_{\text{norm}}$ | Normalized dimensionless variable |
| $Y_i$ | Observed value of variable |
| $Y_{\min}$ | Minimum value of variable |
| $Y_{\max}$ | Maximum value of variable |
| $O_{(i)}$ | Output at $i$th hidden node |
| $O_n$ | Net output at $i$th hidden node |
| $Q_t$ | Direct runoff at time $t$ |
| $Q(t-r)$ | Direct runoff at lag-$r$ |
| $S_t$ | Sediment flow at time $t$ |
| $S_{\text{O}}$ | Observed sediment flow |
| $S_{\text{C}}$ | Computed sediment flow |
| $\overline{S}_{\text{O}}$ | Mean of observed sediment flow |
| $E_D$ | Sum of square error |
| $E_W$ | Sum of square network weights |
| $F$ | Objective function |
| $\lambda$ | Parameter of objective function |
| $\eta$ | Parameter of objective function |
| $S(t-p)$ | Sediment flow at lag-$p$ |
| $R_t$ | Rainfall intensity at time $t$ |
| $R(t-q)$ | Rainfall intensity at lag-$q$ |
| p, q, r | integer |
| $n$ | Chosen step size |
| $k$ | Lag |
| CE | Nash efficiency |
| EPS | Error in peak sediment flow rate |
| ETP | Error in time to peak |
| ESY | Error in sediment yield |
| RMSE | Root mean square error |
| CC | Correlation coefficient |

## 1 Introduction

The process of rainfall–runoff-sediment yield from watersheds is very complex, highly non-linear having temporal and spatial variability. The event-based modeling of this process has a vital role in hydrologic design and watershed management. Many models such as black-box, conceptual, and physically-based models have been developed especially for rainfall-runoff process. On the other hand, very few models are available for accurate estimation of sedimentograph from the storm events. In many situations, simple tools such as linear system theoretical models or black-box models have been used with advantage. However, these models normally fail to represent the non-linear process of rainfall–runoff-sediment yield transformation. The innovation of the artificial neural network (ANN) technique has added a new dimension to model such systems and has been applied in recent years, as a successful tool to solve various problems concerned with hydrology and water resources engineering (ASCE 2000a,b).

An ANN is a flexible mathematical structure having an inter-connected assembly of simple processing elements or nodes which emulates the functioning of neurons in the human brain. It has many distinct advantages and possesses the capability of representing the arbitrary complex non-linear relationship between the input and the output of any system. Mathematically, an ANN can be treated as a universal approximator having an ability to learn from examples without explicit physics (Vemuri 1992; ASCE 2000a, b). Such a model is easy to develop, yields satisfactory results when applied to complex systems which is poorly defined or implicitly understood. These are more tolerant to variable, incomplete or ambiguous input data. Hydrologic applications of ANN include the modeling of daily rainfall–runoff-sediment yield process, water-supply-system optimization, assessment of stream's ecological and hydrological response to climate change, rainfall–runoff forecasting, river flow forecasting, evapotranspiration process, drought forecasting, reservoir inflow modeling and operation, ground water quality prediction and ground water remediation (Smith and Eli 1995; Hsu et al. 1995; Fernando and Jayawardena 1998; Dawson and Wilby 1998; Campolo et al. 1999; Sajikumar and Thandaveswara 1999; Tokar and Markus 2000; Thirumalaiah and Deo 2000; Zhang and Govindaraju 2000; Birikundavyi et al. 2002; Rajurkar et al. 2002, 2004; Sudheer and Jain 2003; Kim and Valdes 2003; Moradkhani et al. 2004; Agarwal and Singh 2004; Olsson et al. 2004; Agarwal et al. 2005; Keskin and Terzi 2006; etc.). Minns and Hall (1996) applied ANN models for an event-based rainfall runoff modeling. The ANN was also applied in the unit hydrograph derivation (Lange 1998). Jain and Indurthy (2003) applied ANN models and compared with other conceptual and linear system models for event-based discharge predictions. Although, a few studies have been reported that focused on ANN-event-based sediment yield modeling and sediment concentration (Tayfur 2002; Nagy et al. 2002; Cigizoglu 2004; Agarwal et al. 2005; Raghuwanshi et al. 2006) but determination of sedimentograph during the storm event of short duration from the watersheds is scares. Looking into the facts, the present study is carried out to develop an event-based sediment yield model employing Artificial Neural Network technique for small watersheds.

## 2 Proposed Artificial Neural Network Configuration

A commonly applied three-layered feedforward type ANN network (Fig. 1) is considered for the development of event-based sediment yield model. As shown in Fig. 1, in a feedforward network, the input quantities are fed to input nodes, which in turn pass them on

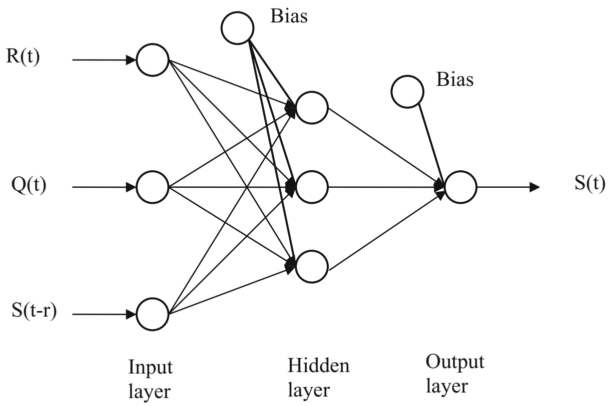


**Fig. 1** Typical three layer feed-forward artificial neural network

to the hidden layer nodes after multiplying by a weight. The weighted sum of the received input from the each input nodes is calculated and passed to the subsequent layer. It associates it with a bias, and then passes the result on through a transfer function. The output nodes do the same operation as that of a hidden node. Before its application to any problem, the network is first trained, whereby the target output at each output node is compared with the network output, and the difference or error is minimized by adjusting the weights and biases through some training algorithm. In this study, a back-propagation training technique is used to make sure that the training is meanfully applied.

## 2.1 The Back-propagation Algorithm

In the back-propagation scheme, network weights and biases are adjusted by moving them along the negative gradient of error function during each iteration until the desired convergence is achieved, i.e.,

$$\overline{X}_{k+1} = \overline{X}_k - n\overline{g} \tag{1}$$

where, $\overline{X}_{k+1}$ = weight vector at iteration ($k$+1), $\overline{X}_k$ = weight vector at iteration $k$, $n$ = chosen step size, $\overline{g} = $ error gradient vector $= \nabla f\left(\overline{X}_k\right)$, and $f\left(\overline{X}_k\right)$ = error function for weight vector $\overline{X}_k$.

## 2.2 Network Training

Training is done by gradient descent algorithm. There are two different ways in which this gradient descent algorithm can be implemented e.g. incremental mode and batch mode. In the incremental mode, the gradient is computed and the weights are updated after each iteration whereas in the batch mode, all the pattern is applied to the network before the weights are updated. The weights and biases of the network are updated only after the entire training. In the present study, the gradient descent algorithm is implemented in batch mode and is done through Automated Regularization. In this framework, the weights and biases of the network are assumed to be random variables with specified distributions. The regularization parameters are related to the unknown variances associated with these distributions and carried out using the Bayesian regularization. Bayesian regularization is the modification of the Levenberg–Marquardt training algorithm to produce networks that generalizes well to reduce the difficulty of determining the optimum network architecture. In this training the algorithm run until the effective number of parameters has converged. The detailed procedure of training using Automated Bayesian Regularization is explained as follows.

The training of the network is said to be satisfactory when the target output at each output node is closed to the network output. The objective of the training algorithm is to minimize the sum of square error $E_D$, which is defined as follows.

$$E_D = \sum_{i=1}^{N} (t_i - a_i)^2 \tag{2}$$

where, $N$ = total number of output nodes; $t_i$ = target output at node $i$; and $a_i$ = network output at node $i$. When the network satisfies this condition, the generalization is said to be satisfactory. The process function is constrained to conform to a high degree of smoothness by assigning the appropriate weights to the network configuration. A further step in the regularization adds one more additional term to objective function $F=F_D$, so that:

$$F = \lambda E_D + \eta E_W \tag{3}$$

where $E_W$ the sum of is squared network weights, $\lambda$ and $\eta$ are the objective function parameters. The relative size of $\lambda$ and $\eta$ provide the emphasis for the training. If $\eta \ll \lambda$, training algorithm drive the errors smaller, whereas $\eta \gg \lambda$ emphasize weight size reduction in the training at the expense of network errors, thus producing smoother network response. Hence, the main problem of implementing appropriate regularization is based on assigning the correct values to objective function parameters. It involves optimization of objective function parameters $\eta$ and $\lambda$ in a Bayesian framework (Foresee and Hagan 1997; Haykin 1999; Pradhan and Ramu 2004).

### 2.3 Data Normalization

The data set was normalized in the range of [0, 1] using the following equation.

$$Y_{\text{norm}} = \frac{Y_i - Y_{\min}}{Y_{\max} - Y_{\min}} \tag{4}$$

where, $Y_{\text{norm}}$ = normalized dimensionless variable; $Y_i$ = observed value of variable; $Y_{\min}$ = minimum value of the variable; and $Y_{\max}$ = maximum value of the variable. The tan-sigmoid transfer function can accept $Y_{\text{norm}}$ values in the range [0, 1]. The tan-sigmoid transfer can be defined as follows.

$$O_{(i)} = \frac{2}{\left\{1 + \exp\left(-O_{n(i)}\right)\right\}^{-1}} \tag{5}$$

where, $O_{(i)}$ is the output of the $i$th hidden node, $O_{n(i)}$ is the net output at $i$th hidden node.

## 3 Linear Transfer Function Model

Linear transfer function model for two variable, viz., rainfall and runoff, can be define as follows.

$$Q_t = \sum_{j=1}^{p} a_j R_{t-j+1} + \sum_{j=1}^{q} b_j Q_{t-j} \tag{6}$$

where, $Q_t$ is the dependent variable, $R_t$ is the independent variable; $p$ and $q$ are the time response; and $a_j$, and $b_j$ are the time invariant parameters of linear transfer function. The parameters of the linear transfer function can be computed using the ordinary least-squares method. When the sediment yield is considered as a dependent variable over the rainfall and runoff then the linear transfer function for rainfall–runoff-sediment yield process can be written as follows.

$$S_t = \sum_{j=1}^{p} a_j S_{t-j} + \sum_{j=1}^{q} b_j R_{t-j+1} + \sum_{j=1}^{r} c_j Q_{t-j+1} \tag{7}$$

where, $S_t$ is the dependent observation; $R_t$ and $Q_t$ are the independent variable observation; $p$, $q$ and $r$ are the time response; and $a_j$, $b_j$, and $c_j$ are the parameters of linear transfer function. The time response, $p$ can be approximately estimated through the auto-correlation function whereas $q$ and $r$ can be approximated through lag-$k$ cross-correlation between the two variables (Salas et al. 1980).

As the rainfall–runoff-sediment yield is a complex non-linear process, then the linear transfer function of the form given in Eq. 7 can be written as:

$$S(t) = f\left\{ \begin{matrix} S(t-1), \ldots..S(t-p), R(t), R(t-1), \ldots.. \\ R(t-q), Q(t), Q(t-1), \ldots..Q(t-r) \end{matrix} \right\} \tag{8}$$

Based on Eq. 8 the input pattern of the ANN-based sediment yield model is chosen followed be confirming the lag response of the system using the autocorrelation function and cross-correlation coefficients.

## 4 Model Performance

The performance of ANN model is assessed by the satisfying the defined objective function of the model. Also, to test the applicability of the model for hydrologic problem following statistical criteria are also applied.

### 4.1 Root Mean Square Error (RMSE)

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(S_{Oi} - S_{Ci})^2} \tag{9}$$

where, RMSE is the root mean square error, $N$ is number of observations; $S_{Oi}$ and $S_{Ci}$ are the ordinates of observed and computed sedimentographs, respectively.

### 4.2 Correlation coefficient (CC)

$$CC = \frac{\sum_{i=1}^{N}\left(S_{Oi} - \overline{S}_O\right) \times \left(S_{Ci} - \overline{S}_C\right)}{\sqrt{\sum_{i=1}^{N}\left(S_{Oi} - \overline{S}_O\right)^2 \times \sum_{i=1}^{N}\left(S_{Ci} - \overline{S}_C\right)^2}} \tag{10}$$

where CC is the correlation coefficient; $S_{Oi}$ and $S_{Ci}$ are the ordinates of observed and computed sedimentographs, respectively; $\overline{S}_O$ and $\overline{S}_C$ are the mean value of observed and computed values of sedimentograph for the storm event.

### 4.3 Nash's Efficiency (CE)

$$CE = 1 - \frac{\sum_{i=1}^{N}(S_{Oi} - S_{Ci})^2}{\sum_{i=1}^{N}\left(S_{Oi} - \overline{S}\right)^2} \tag{11}$$

where CE is the Nash's efficiency (Nash and Sutcliffe 1970); $S_{Oi}$ and $S_{Ci}$ are the ordinates of observed and computed sedimentographs, respectively; and $\overline{S}$ is the mean value of observed sedimentograph of the storm event.

Along with the above statistical criteria following event based error criteria are also used to test the results.

4.4 Error in Peak Sediment Flow (EPS)

$$EPS = \frac{(S_{OP} - S_{CP})}{S_{OP}} \times 100\% \tag{12}$$

where $EPS$ = error in peak sediment flow rate; $S_{OP}$=observed peak sediment flow rate; and $S_{CP}$=computed peak sediment flow rate.

4.5 Error in Time to Peak (ETP)

$$ETP = \frac{(T_{OP} - T_{CP})}{T_{OP}} \times 100\% \tag{13}$$

where $ETP$ = is the error in time to peak of sediment flow rate; $T_{OP}$ = observed time to peak of sediment flow rate; and $T_{CP}$ = computed time to peak of sediment flow rate.

4.6 Error in sediment yield (ESY)

$$ESY = \frac{(SY_O - SY_C)}{SY_O} \times 100\% \tag{14}$$

where $ESY$ = error in total sediment yield during the storm; $SY_O$ = observed sediment yield; and $SY_C$ = computed sediment yield.

## 5 Study Area and Data Preparation

The ANN based sediment yield model is developed for the storm-event data of rainfall, runoff and sediment flow from two small watersheds of different climatic and physiographic characteristics. The two watersheds selected are W-2 watershed of Treynor catchment (USA; Fig. 2) and W7 of Goodwin Creek experimental watershed (Mississippi; Fig. 3). Both the watersheds are very much susceptible to soil erosion due to variable range of surface and highly erodible nature of soil. The W-2 watershed is an experimental watershed (Latitude: 40°10′10″ N and Longitude: 95°39′ W), which is located in Pottawattamie Country in South-Western Iowa, USA, about 26 km east of Missouri river near a small town of Treynor. W-2 is a micro-watershed having drainage area 0.335 km$^2$ with average land slope of 8.0%. Length and width of the watershed is nearly 670.0 and

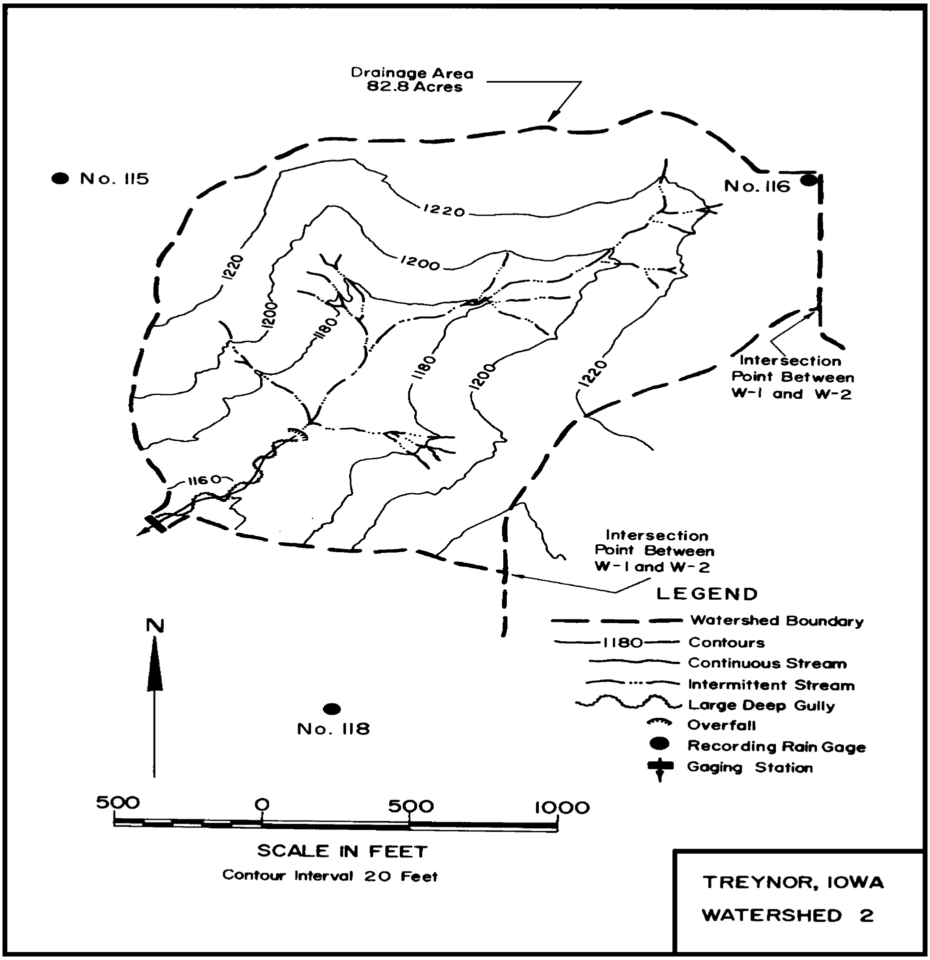


**Fig. 2** Topographic map of W-2 watershed of Treynor, Iowa

500.0 m, respectively. Climatic condition of the watershed is sub-tropical with average annual rainfall of 814.0 mm. Silty soil is the major soil group experienced in the watershed. Major land uses of the watershed are agriculture (= 95%) and pasture (= 5%). The watershed is looked under the Soil Conservation Service Land Resource Area M-107, Iowa and Missouri Hills. The W-7, a sub-watershed of Goodwin Creek experimental watershed (Latitude: 34°15′10.342″ N and Longitude: 89°51′34.479″ W) having drainage area of 1.60 km$^2$ with average slope of nearly 6.5% is located in the southeast quarter of Panola Country, in northern Mississippi. Length and width of the watershed is approximately 2,050.0 and 800.0 m. Major soil texture experienced in the watershed is silty in nature. A combination of timber (= 26%), agriculture (= 14%) and pasture (= 60%) are the common land use of the watershed. Climate of the watershed is humid with average annual rainfall of 1,292 mm. Watershed is operated under USDA-ARS, National Sedimentation Laboratory (NSL), Oxford, Mississippi.

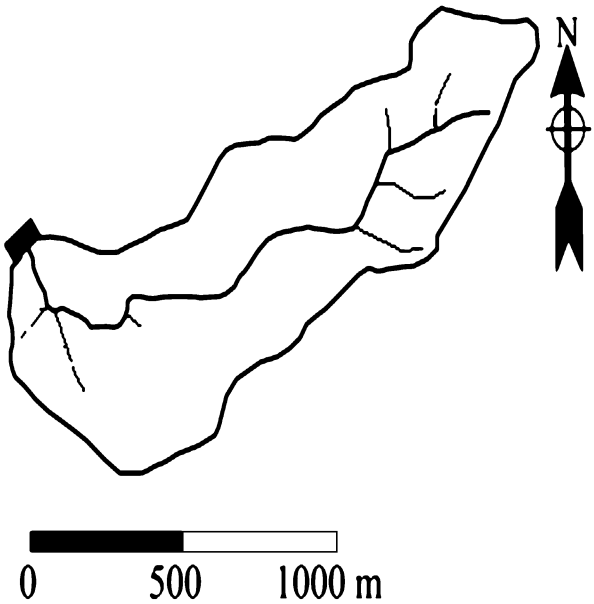


**Fig. 3** Topographic map of W-7 watershed of Goodwin Creek Watershed

In totality, twenty two storm events of unequal base time were analyzed in the application of proposed model. Detailed description of the storm events viz. date of storm event, duration of storm, total depth of rainfall, runoff volume and sediment yield for both the watersheds are given in Table 1. The observations of rainfall (mm), runoff ($m^3/s$) and sediment concentration (ppm) for the watersheds are available in few minute intervals (1–4 min interval) therefore, the data were first made in equal intervals of one minute through linear interpolation and sediment concentration values were converted into sediment flow rate (N/s) by multiplying the sediment concentration to the corresponding discharge. For training of the network, the data sets were divided into two sets viz., training (70%) and validation (30%; Table 1). ANN application requires normalized input and output data set and is performed using Eq. 4 in the range of [0, 1] to avoid any saturation effect that may arise from the use of sigmoid activation function.

## 6 Application of ANN Model and Results

Since the sediment flow is dependent on the discharge as well as on the grass rainfall intensity, therefore the computation of runoff hydrograph was taken up first and discussed in the following section.

### 6.1 Computation of Runoff Hydrograph

Determination of sedimentograph requires the runoff depth produced by rainfall from the watershed. The event data of rainfall intensity and runoff with their previously lagged data

**Table 1** Details of storm events used in the analysis

| Watershed | Storm-event date | Storm duration (min) | Rainfall depth (mm) | Runoff volume (mm) | Peak rate of runoff ($m^3/s$) | Time to peak of runoff (min) | Sediment yield (*N*) | Peak sediment flow rate (N/s) | Time to peak of sediment flow (min) |
|---|---|---|---|---|---|---|---|---|---|
| W-2: Treynor catchment of IOWA, USA | 15.8.77 (C) | 202.0 | 47.07 | 10.06 | 2.66 | 59.0 | 601786.4 | 776.8 | 57.0 |
| | 12.6.80 (C) | 240.0 | 29.35 | 36.0 | 3.70 | 38.0 | 3507955.0 | 3363.9 | 40.0 |
| | 5.9.80 (C) | 162.0 | 11.43 | 1.128 | 0.292 | 49.0 | 64725.8 | 81.1 | 50.0 |
| | 8.7.81 (C) | 148.0 | 40.35 | 7.65 | 2.38 | 42.0 | 1457606.0 | 1534.7 | 42.0 |
| | 1.8.81 (C) | 121.0 | 41.56 | 8.71 | 2.94 | 28.0 | 705759.0 | 807.2 | 25.0 |
| | 14.6.82 (C) | 131.0 | 38.14 | 12.88 | 4.27 | 40.0 | 5145998.0 | 6546.4 | 41.0 |
| | 17.9.82 (C) | 200.0 | 29.84 | 1.69 | 0.30 | 47.0 | 76827.5 | 69.43 | 47.0 |
| | 29.8.75 (V) | 390.0 | 41.85 | 7.44 | 1.50 | 82.0 | 170249.6.2 | 130.8 | 82.0 |
| | 18.6.80 (V) | 198.0 | 16.44 | 4.75 | 2.40 | 84.0 | 1201140.0 | 2261.6 | 84.0 |
| | 26.8.81 (V) | 226.0 | 19.30 | 0.90 | 0.202 | 71.0 | 33303.6 | 47.18 | 72.0 |
| | 30.5.82 (V) | 178.0 | 16.62 | 2.92 | 0.40 | 102.0 | 471122.0 | 269.23 | 111.0 |
| | 30.6.82 (V) | 122.0 | 26.21 | 5.83 | 2.15 | 58.0 | 1267358.0 | 1648.6 | 58.0 |
| W-7: Goodwin Creek watershed, Mississippi | 17.10.81 (C) | 517.0 | 68.46 | 22.21 | 5.21 | 149.0 | 1511802.0 | 318.36 | 149.0 |
| | 25.5.81 (C) | 437.0 | 37.88 | 8.38 | 3.19 | 62.0 | 438607.3 | 147.7 | 89.0 |
| | 3.6.82 (C) | 864.0 | 46.95 | 2.68 | 0.92 | 112.0 | 165407.8 | 79.56 | 94.0 |
| | 12.9.82 (C) | 570.0 | 33.29 | 3.73 | 1.01 | 330.0 | 100885.0 | 31.75 | 322.0 |
| | 30.11.82 (C) | 518.0 | 19.40 | 2.11 | 0.49 | 278.0 | 66844.0 | 19.56 | 278.0 |
| | 30.6.82 (V) | 454.0 | 26.59 | 7.24 | 2.49 | 61.0 | 417573.7 | 135.53 | 76.0 |
| | 11.8.82 (V) | 658.0 | 42.48 | 8.7 | 2.226 | 210.0 | 239114.0 | 92.94 | 206.0 |
| | 27.8.82 (V) | 702.0 | 83.17 | 46.7 | 10.37 | 107.0 | 2683771.0 | 711.3 | 103.0 |

*C* Calibration event; *V* Validation event were considered as input to the model. Runoff hydrograph was computed utilizing the following combination of rainfall and runoff data of previous lags up to $q$ using the three-layer feedforward back-propagation artificial neural network:

$$Q(t) = f\{Q(t-1), .....Q(t-q), R(t), R(t-1), .....R(t-p)\} \tag{15}$$

The effective lag-response of the data is decided on the basis of auto correlation function (ACF) and Lag-$k$ cross-correlation of the watershed's rainfall–runoff data of the storm events. Also, lag of the model was decided on the basis of travel time using the Kirpich formula (Kirpich 1940) and is given as follows.

$$t_c = 0.0195 L^{0.77} S^{-0.385} \tag{16}$$

where $t_c$ is the travel time (min), $L$ is the length of the stream (m) and $S$ is the slope (m/m). Based on Eq. 16, estimated values of travel time for the W-2 and W-7 watersheds were 7.72 min (≈ 8.0 min) and 19.8 min (≈ 20 min), respectively. Based on the ACF and Lag-$k$ cross-correlation as well as from the travel time the following model structures were decided for W-2 watersheds.

$$\text{QM1}: \; Q(t) = f\{R(t-1), R(t), Q(t-1)\} \tag{17}$$

$$\text{QM2}: \; Q(t) = f\{R(t), R(t-8), Q(t-1)\} \tag{18}$$

$$\text{QM3}: \; Q(t) = f\{R(t), R(t-8), Q(t-8)\} \tag{19}$$

Similarly for the W-7 watershed following model were used in the analysis.

$$\text{QM1}: \; Q(t) = f\{R(t-1), R(t), Q(t-1)\} \tag{20}$$

$$\text{QM2}: \; Q(t) = f\{R(t), R(t-20), Q(t-1)\} \tag{21}$$

$$\text{QM2}: \; Q(t) = f(R(t), R(t-20), Q(t-20)) \tag{22}$$

For feedforward back-propagation ANN model, the number of input nodes in the input layer was taken equal to the number of input variables (i.e. $R(t-1)$, $R(t)$ and $Q(t-1)$). Since no clear-cut guidelines are available for the number of hidden nodes in the hidden layer(s) (Vemuri 1992) therefore, number of hidden nodes were initially taken equal to the number of input nodes and increased up to twice based on the minimization of error criteria. However, corresponding to one output, only one node was taken in the output layer. Training of the network was performed using the proposed ANN model (i.e. Section 2) at MATLAB platform. Different ANN ($p$, $q$, $r$) structures were tried for the analysis in which the variable $p$ is the number of input nodes, $q$ is the number of hidden nodes in the hidden layer and $r$ is the number of output nodes, and finally based on the performance criteria (i.e. RMSE, CC, CE) best representing ANN model was adopted for further analysis. The different ANN structures along with model used, and estimated values of statistical indices for the W-2 and the W-7 watersheds are presented in Table 2. Based on the performance criteria, model QM1 with ANN (3, 3, 1) structure was found to be best fitted with observed data for both the watersheds and therefore selected for further evaluation of the model

**Table 2** Performance criteria of ANN and linear transfer function model of rainfall–runoff processes for W-2 and W-7 watersheds

| Watershed | Model | ANN model | ANN-based model | | | | | Linear transfer function model | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | RMSE | CC (C) | CC (V) | CE (C) | CE (V) | RMSE | CC (C) | CC (V) |
| W-2 | QM1 | (3,3,1) | 0.054 | 0.99 | 0.99 | 0.99 | 0.98 | 0.07 | 0.99 | 0.75 |
| | | (3,6,1) | 0.028 | 0.99 | 0.79 | 0.99 | −0.93 | | | |
| | QM2 | (3,3,1) | 0.047 | 0.99 | 0.99 | 0.99 | 0.98 | | | |
| | QM3 | (3,3,1) | 0.244 | 0.89 | 0.87 | 0.79 | 0.75 | | | |
| W-7 | QM1 | (3,3,1) | 0.025 | 0.99 | 0.99 | 0.99 | 0.99 | 0.03 | 0.99 | 0.68 |
| | | (3,6,1) | 0.028 | 1.00 | 0.68 | 1.000 | −5.93 | | | |
| | QM2 | (3,3,1) | 0.024 | 0.99 | 0.99 | 0.99 | 0.99 | N.U. | | |
| | QM3 | (3,3,1) | 0.270 | 0.94 | 0.48 | 0.90 | 0.18 | | | |
| | | (3,6,1) | 0.265 | 0.95 | 0.95 | 0.90 | 0.90 | | | |

*N.U.* Not utilized

performance in the reproduction of runoff hydrographs for the watersheds. It is clear from the Table 2 that the order of model performance for both the watersheds is QM1 $\geq$ QM2 $>$ QM3. The model performance was also evaluated by comparing the observed and computed runoff hydrographs in calibration and validation for these watersheds. The comparison of observed and computed runoff hydrographs for the sample storm events are shown in Figs. 4 and 5, respectively for W-2 and W-7 watershed. It is clear from Figs. 4 and 5 that the results obtained from QM1 model with (3, 3, 1) structure gives better agreement with the observed data. The results of the runoff hydrographs obtained from the event-based rainfall–runoff model were used in the development ANN based rainfall–runoff-sediment yield model and is discussed in the following section.

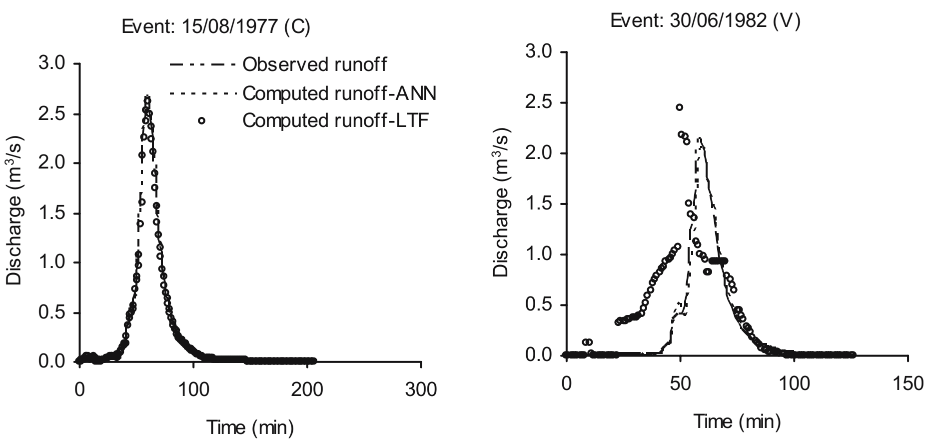


**Fig. 4** Comparison of observed and computed runoff hydrographs for W-2 watershed (*C*=Calibration, *V*= Validation)

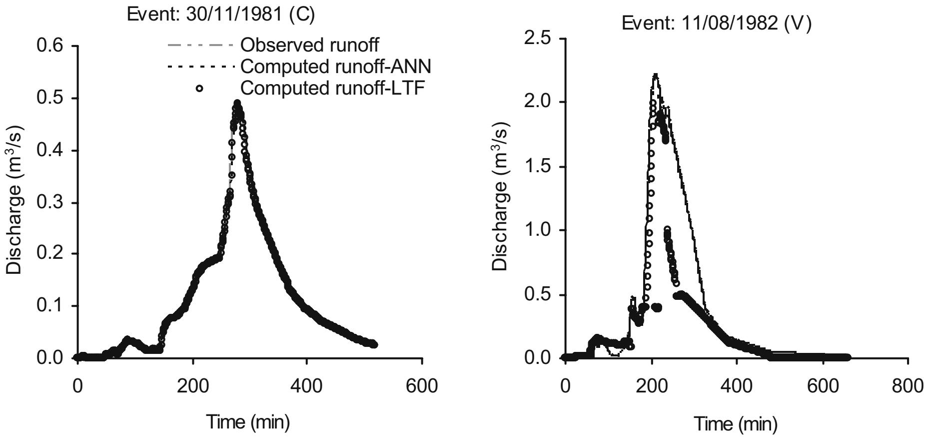


**Fig. 5** Comparison of observed and computed runoff hydrographs for W-7 watershed (*C*=Calibration, *V*= Validation)

## 6.2 Computation of Sedimentograph

The storm event data of rainfall intensity, runoff and sediment flow rate, along with their previously lagged data were considered as input to the model. The model input can be given as follows.

$$S(t) = f\left\{ \begin{matrix} S(t-1), \ldots..S(t-p), R(t), R(t-1), \ldots. \\ R(t-q), Q(t), Q(t-1), \ldots..Q(t-r) \end{matrix} \right\} \tag{23}$$

Again, the autocorrelation function and lag cross-correlation analysis were used to know the time response parameters, $p$, $q$, and $r$. Based on the autocorrelation and cross-correlation analysis and travel time following model structures were decided for W-2 watershed were used.

$$\text{SM1}: S(t) = f\{R(t-2), R(t-1), R(t), Q(t-2), Q(t-1), S(t-2), S(t-1)\} \tag{24}$$

$$\text{SM2}: S(t) = f\{R(t-1), R(t), Q(t-1), Q(t), S(t-2), S(t-1)\} \tag{25}$$

$$\text{SM3}: S(t) = f\{R(t-8), R(t), Q(t)\} \tag{26}$$

$$\text{SM4}: S(t) = f\{R(t-8), R(t), Q(t), Q(t-8), S(t-8)\} \tag{27}$$

Similarly, for W-7 watershed, following model were used for the computation of sediment flow rate.

$$\text{SM1}: S(t) = f\{R(t-1), R(t), Q(t-1), Q(t), S(t-2), S(t-1)\} \tag{28}$$

**Table 3** Performance criteria of ANN and linear transfer function model of rainfall–runoff-sediment flow processes for W-2 and W-7 watersheds

| Watershed | Model | ANN model | ANN-based model | | | | | Linear transfer function model | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | RMSE | CC (C) | CC (V) | CE (C) | CE (V) | RMSE | CC (C) | CC (V) |
| W-2 | SM1 | (7,7,1) | 25.03 | 0.99 | 0.96 | 0.99 | 0.90 | 0.07 | 0.99 | 0.75 |
| | | (7,14,1) | 7.97 | 0.99 | 0.67 | 0.99 | −0.82 | | | |
| | SM2 | (6,6,1) | 15.07 | 0.99 | 0.82 | 0.99 | 0.64 | N.U. | | |
| | | (6,12,1) | 5.18 | 0.99 | 0.83 | 0.99 | −3.97 | | | |
| | SM3 | (3,3,1) | 127.7 | 0.94 | 0.76 | 0.88 | 0.50 | | | |
| | | (3,4,1) | 133.0 | 0.93 | 0.85 | 0.87 | 0.71 | | | |
| | SM4 | (3,6,1) | 35.3 | 0.99 | 0.85 | 0.99 | 0.72 | | | |
| | | (3,9,1) | 30.13 | 0.99 | 0.89 | 0.99 | 0.78 | | | |
| W-7 | SM1 | (6,6,1) | 0.574 | 0.99 | 0.99 | 0.99 | 0.98 | 0.90 | 0.99 | 0.96 |
| | | (6,12,1) | 0.2427 | 1.000 | −0.522 | 1.0000 | −29.63 | | | |
| | SM2 | (3,3,1) | 34.86 | 0.98 | 0.89 | 0.96 | 0.79 | N.U. | | |
| | SM3 | (4,5,1) | 2.27 | 0.99 | 0.81 | 0.99 | 0.66 | | | |

*N.U.* Not utilized

$$\text{SM2}: \ S(t) = f\{R(t-20), R(t), Q(t)\} \tag{29}$$

$$\text{SM3}: \ S(t) = f\{R(t-20), R(t), Q(t), Q(t-20)\} \tag{30}$$

The defined three layer feedforward back-propagation ANN model for rainfall–runoff-sediment yield were developed for the selected watersheds utilizing the data of runoff hydrograph computed from the ANN based rainfall–runoff model which has been already explained. The different ANN structures along with model identity, and computed values of statistical indices (i.e. RMSE, CC, CE) for W-2 and W-7 watersheds are given in Table 3. Based on these statistical criteria, model SM1 with ANN (7, 7, 1)

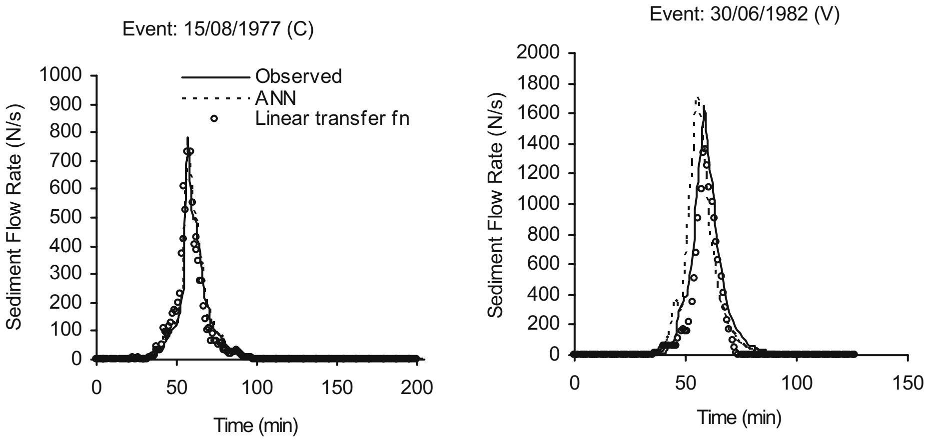


**Fig. 6** Comparison of observed and computed sedimentographs for W-2 watershed (*C*=Calibration, *V*= Validation)

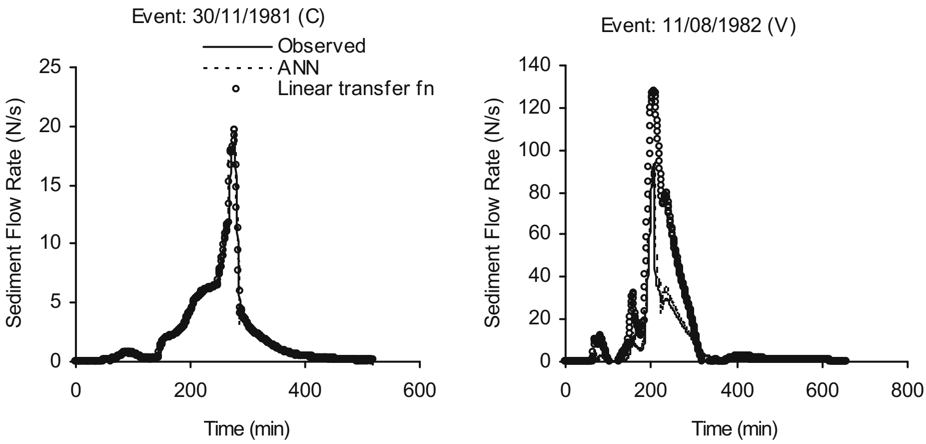


**Fig. 7** Comparison of observed and computed sedimentographs for W-7 watershed (*C*=Calibration, *V*=Validation)

structure was found to be best fitted with the observed data for W-2 watershed and therefore, selected for further evaluation of the model performance in the reproduction of sedimentographs. The comparative performance of the models for W-2 watershed are $SM1(7,7,1) > SM4(3,9,1) > SM3(3,4,1) > SM2(6,6,1)$. On the other hand, for W-7 watershed, SM1 (6, 6, 1) model gives better prediction of sedimentographs as compared to the other models. The performance order of all the models based on the statistical criteria is $SM1(6,6,1) > SM2 > SM3$. Using the best fitted models the sedimentographs were further analyzed. The model's performance was first evaluated through the comparison of observed and computed sedimentographs in calibration and validation for the watersheds. The comparison of observed and computed sedimentographs for the representative storm events are shown in Figs. 6 and 7 respectively for W-2 and W-7 watershed. The error criteria (*CE*, *EPS*, *ETP*, and *ESY*) were also computed for each storm events of calibration as well validation events for W-2 and W-7 watersheds and are given in Tables 4 and 5.

**Table 4** Performance evaluation of ANN and linear transfer function model for W-2 watershed

| Event date | ANN-based model | | | | Linear transfer function model | | | |
|---|---|---|---|---|---|---|---|---|
| | CE | EPS (%) | ETP (%) | ESY (%) | CE | EPS (%) | ETP (%) | ESY (%) |
| 29/08/75 (C) | 0.957 | −20.106 | 0.000 | −18.67 | 0.6320 | −76.364 | 3.659 | −11.95 |
| 15/08/77 (C) | 0.991 | 6.169 | −1.754 | −0.206 | 0.9168 | −20.002 | −1.754 | −2.67 |
| 12/06/80 (C) | 0.999 | −0.822 | 2.500 | 0.63 | 0.9970 | −0.327 | 2.500 | 1.98 |
| 18/06/80 (C) | 0.996 | 3.160 | 0.000 | 1.113 | 0.9780 | −9.415 | 0.000 | 1.735 |
| 05/09/80 (C) | 0.923 | 2.432 | 14.000 | −15.95 | 0.8874 | −7.455 | 12.000 | −2.58 |
| 08/07/81 (C) | 0.997 | −1.590 | 0.000 | 1.37 | 0.9942 | −5.515 | 0.000 | 1.32 |
| 01/08/81 (C) | 0.989 | −7.377 | 0.000 | −1.68 | 0.9263 | −43.244 | 0.000 | −1.15 |
| 26/08/81 (C) | 0.836 | 9.597 | 4.167 | −27.8 | 0.8217 | 4.597 | 1.389 | −4.72 |
| 30/05/82 (V) | 0.966 | 17.385 | −0.901 | 9.86 | 0.9768 | 0.710 | −0.901 | 8.953 |
| 14/06/82 (V) | 0.579 | 29.763 | 7.317 | 36.31 | 0.5900 | 57.954 | 2.439 | 60.70 |
| 15/06/82 (V) | 0.632 | 19.557 | 25.000 | 14.74 | 0.6670 | 39.368 | 0.000 | 47.66 |
| 30/06/82 (V) | 0.811 | −3.975 | 5.172 | 2.01 | 0.8488 | 17.567 | −1.724 | 27.92 |
| 17/09/82 (V) | 0.809 | −15.483 | 6.383 | −14.64 | −7.973 | 13.280 | −2.128 | 63.05 |

**Table 5** Performance evaluation of ANN and linear transfer function model for W-7 watershed

| Event Date | ANN-based model | | | | Linear transfer function model | | | |
|---|---|---|---|---|---|---|---|---|
| | CE | EPS (%) | ETP (%) | ESY (%) | CE | EPS (%) | ETP (%) | ESY (%) |
| 17/10/81 (C) | 0.9998 | −0.059 | 0.000 | 0.076 | 0.9995 | 0.968 | 0.000 | 0.279 |
| 30/11/81 (C) | 0.9899 | −0.312 | 0.000 | −0.312 | 0.9976 | −0.654 | 0.000 | −2.571 |
| 05/04/82 (C) | 0.9687 | −6.386 | 3.871 | −5.945 | 0.9321 | −12.697 | 2.581 | −18.047 |
| 25/05/82 (C) | 0.9504 | −0.210 | 2.247 | −0.060 | 0.9994 | −0.765 | 0.000 | −0.524 |
| 03/06/82 (C) | 0.9787 | −1.800 | 2.632 | 0.050 | 0.9989 | −0.893 | 0.000 | −0.659 |
| 30/06/82 (V) | 0.9872 | 2.164 | 2.128 | 4.985 | 0.8330 | 33.507 | −14.894 | −3.804 |
| 11/08/82 (V) | 0.9910 | −1.498 | −0.485 | −6.131 | −0.6630 | −37.616 | −0.485 | −125.15 |
| 27/08/82 (V) | 0.9718 | 14.941 | −1.942 | 5.423 | 0.9060 | 20.234 | −3.883 | −10.12 |
| 12/09/82 (V) | 0.9969 | −1.039 | −0.311 | −3.278 | −1.0400 | −55.339 | −0.311 | −149.53 |

## 7 Comparison of ANN Model with Linear Transfer Function Model

Relative performance of the ANN based rainfall–runoff and rainfall–runoff-sediment yield models were carried out by comparing the results obtained by fitting the linear transfer function (i.e. Eqs. 6 and 7) models using the same input variable as used in the best fitted ANN model. For the development of rainfall–runoff-sediment yield model, the rainfall–runoff process was fitted first using the observed rainfall–runoff data by ordinary least-squares method and then the rainfall–runoff process was coupled with the sediment yield process. For both the processes, fitted linear transfer function models for W-2 watershed are as follows.

$$Q(t) = -0.0001\,R(t-1) + 0.0011\,R(t) - 0.979\,Q(t-1) \tag{31}$$

$$\begin{aligned} S(t) = {} & -0.206\,R(t-1) + 0.098\,R(t) - 602.84\,Q(t-1) + 613.00\,Q(t) \\ & - 0.168\,S(t-2) + 1.1398\,S(t-1) \end{aligned} \tag{32}$$

Similarly, for W-7 watershed, the fitted linear transfer function (LTF) models for rainfall–runoff and rainfall–runoff-sediment flow process are given by Eqs. 33 and 34, respectively.

$$Q(t) = -0.00006\,R(t1) + 0.00015\,R(t) - 0.999\,Q(t-1) \tag{33}$$

$$\begin{aligned} S(t) = {} & -0.009\,R(t1) - 0.007\,R(t) - 44.24\,Q(t-1) + 45.11\,Q(t) \\ & - 0.409\,S(t-2) + 1.386\,S(t-1) \end{aligned} \tag{34}$$

The fitting criteria viz., RMSE and CC for the linear transfer function models were also computed and given in Tables 2 and 3 for W-2 and W-7 watershed respectively. Using the linear transfer function models of rainfall–runoff process and rainfall–runoff-sediment yield, the runoff hydrographs and sedimentographs were computed for the watersheds. Runoff hydrographs obtained from the linear transfer function model are shown in Figs. 4 and 5 for sample storm events of W-2 and W-7 watershed, respectively. These figures clearly show that the runoff hydrographs of validation events are not well simulated by LTF model with respect to hydrograph characteristics such as, peak, time to peak and time to peak and time based.

The performance evaluation of the sediment yield model was carried out on the basis of visual comparison of observed and computed sedimentographs as well as by computing the error criteria such as CE, EPS, ETP and ESY (refer Section 4). The comparisons between the observed and computed sedimentographs are shown in Figs. 6 and 7 for W-2 and W-7 watersheds, respectively. The event-wise estimated values of error criteria such as CE, EPS, ETP and ESY are given in Tables 4 and 5 for W-2 and W-7 watersheds, respectively. The Figs. 6 and 7 as well as the Tables 4 and 5 clearly show that the sedimentographs computed by the LTF model are quite satisfactory for the calibration events however; the prediction efficiency for the validation events was inferior than the proposed ANN models.

The graphical (Figs. 4 through 7) as well as the error criteria used (Tables 4 and 5) show that the ANN based model produce the sedimentographs closer to the observed one as compared to the LTF model. Also, the parameters of runoff hydrographs and sedimentographs (i.e. peak, time to peak and volume) along with rising and recession segments was reproduced well by ANN based model. Thus, the shape of the sedimentographs is very well preserved in case of ANN based model for calibration as well as in validation events.

## 8 Conclusions

The present study used the well accepted feedforward back-propagation artificial neural network with different structures. The training of the network was accomplished under batch mode by gradient descent algorithm with automated Bayesian regularization. The tan-sigmoid and pure linear transfer functions were used in the hidden layer and output layer, respectively. Different ANN structures along with the different combinations of input variables were used for the analysis. Based on the performance criteria viz., RMSE, CC, and CE, a best fit ANN model was selected and used for the computation of runoff hydrographs. Using the same methodology and output of the ANN based rainfall–runoff model (i.e., runoff hydrographs), sedimentographs were computed and compared with observed ones. The relative performance of ANN based model was also evaluated by comparing the results with other existing model, for which LTF model was developed using the similar inputs as used in the best fitted ANN based model. The fitting of the linear transfer function model was carried out using the least-squares method. The results obtained from ANN-based model when compared with the linear transfer function model using the error criteria viz. Nash's efficiency (CE), error in peak (EPS), error in time to peak (ETP) and error in volume (ESY), endorse its applicability for the computation of runoff hydrographs as well as the sedimentographs.

Thus, the proposed model based on the artificial neural network technique has been found to be satisfactory for the event-based computation of sedimentograph without any computational burdens.

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