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Dynamic Computable General Equilibrium Model and Sensitivity Analysis for Shadow Price of Water Resource in China

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Abstract A novel dynamic computable general equilibrium model based on National Water Resource Input Holding Output Table is proposed to calculate the Shadow Price of water resource in China. Unlike previous approaches, the dynamic Shadow Price of water resource is largely based on the scarcity extent and can reflect the marginal long-term value in the balanced growth path of China. Firstly, the basic concepts of dynamic Input Output analysis and Turnpike Theory are reviewed. Then, Dynamic Computable General Equilibrium (DCGE) is elaborated to calculate the Shadow Price, including the definition and computer-based algorithm. Furthermore, Shadow Price of water resource in China from 1949 to 2050 is calculated based on the DCGE. Also the sensitivity analysis of the DCGE for Shadow Price of water resource in China is presented. Dynamic Shadow Price of water resource has two meanings for China government: (1) Project evaluation. Every large-scale project in China must have national economic evaluation and the dynamic Shadow Price is prerequisite for national economic evaluation. (2) Market price of water resource. A lesson from this paper is that Shadow Price of water resource in domestic market of China should be rewritten according to the dynamic Shadow Price. In addition, the parallel computations approach could also be used to solve these problems in different countries or for different natural resources.

Key words water resource · shadow price · dynamic · computable general equilibrium · input output analysis · sensitivity analysis

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1 Introduction

Due to water resource shortage and economic development within last two decades, China faces serious problems of water supply shortage and water pollution. Per capita annual water resource of China in 1999 was 2,230 m^3, which is less than one-third of the global average of 7,800 m^3. Regional distribution of water resource in China is unbalanced (Chen et al., 2003). Water scarcity is the foremost problem for which solutions are urgently needed. Water prices can be considered as an instrument for satisfactory water allocation and must be justified in a satisfactory way. Dynamic Shadow Price of water resource has two meanings for China government: (1) Project Evaluation. Every large-scale project in China must have national economic evaluation and the dynamic Shadow Price is prerequisite for national economic evaluation. (2) Market price of water resource. How to access public projects and policy reforms when market price misleads us? Revenues and payouts at market price give distorted measures of social gains and losses. Social opportunity cost or 'Shadow Price' may be elegant coefficients to evaluate them. Assigning dynamic shadow price of water resource in China remains a challenging task. Public agencies and researchers in China routinely apply several techniques when assigning economic values to activities is not handled through normal market processes.

There are four main approaches to calculate Shadow Price of water resource in China: (1) Static computable general equilibrium (CGE) model. The Shadow Price of water resource is regarded as both the output price of the water resource sector and the input price of the non-water resource sector because these prices between the supply and the demand are equal (Shen et al., 1999). (2) Equilibrium price in Input Output Table (Zhong, 1984). The supplying sector of water resource is put into Input Output Table as a sector or a commodity, then theoretical price of the sector is regarded as the Shadow Price. The difficulty of this approach is how to construct the Input Output Table of water resource, because Input Output Table is the basic work in economics of water resource in China. (3) Marginal price (Chen et al., 2003). This approach is the most popular one in which the Shadow Price is equal to the derivative price of the production function. (4) Linear programming (LP) model (Liu and Chen, 2003). Optimal solution of the resource allocation can be used to get Shadow Price of the resource from the optimal solution of dual problem according to the water resource constraint line in the linear programming based on input output analysis. In the non-linear programming, Shadow Price is equal to the Lagrange Multipliers. In the dynamic programming, Shadow Price is equal to the vector in the Hamilton Matrices (Zhao, 1995). The above four models show the different processes of resource allocation based on the different economics theories. Each approach has its limitations. It is essential that the choice of method be guided by the nature of the good or service being valued and perhaps more crucially, by the type of information available to the researcher. This study develops and implements a dynamic method for assigning shadow prices to dual solution when data on the inputs and outputs of these activities are available.

Shadow Price of water resource is very important for the economic evaluation of the large-scale project. The model put forward by Shah et al. (1995) was developed to handle these problems of water allocation under these investments of conveyance and these investments in firm-specific conservation technology. The model provides Shadow Price for water authorities to evaluate the total effectiveness of the project.

Shadow Price contributes a self-consistent system of price based on the efficiency of the total project. Also the evaluation approaches by using the Shadow Price are more open and fair than its counterparts. This could help to solve an important problem of persuading users to accept the market water price even when the social psychology is suggested to be taken into account (Syme and Nancarrow, 1997).

Different economic explanation of the Shadow Price has led to different formal Shadow Price calculation. Kantorovich (1939) introduced the concept of Shadow Price and demonstrated it is optimal solution of the dual linear programming. The definition of Shadow Price can be applied to many different resources. The Shadow Price suffers from the fact that the gradient of the marginal function may not always exist. Gauvin (1980) introduced a definition of Shadow Price, which had practical implications from the marginal point of view. Horsley and Wrobel (2006) applied duality methods of linear programming to the problems of operation and rental valuation of a hydro plant and its river. Both of the two problems are approached by using time-dependent shadow pricing of water, and if the given market price for electricity (p) is a continuous function of time, then the shadow price function for water (ψ) is shown to be unique.

While the real-time data are applied in the above models, collecting the required interdependent data to calculate Shadow Price is also another hard work. So this problem can be solved by constructing National Input Output Table of water resource for China.

So far it is elegant to use the Input Output analysis to solve the question on the water resource, such as the research on Colorado River developed by California and Arizona State in U.S.A (Carter and Ireri, 1970). Another paper listed out the water resource sector alone in Input Output Table (Bouhia, 1998). There are some applications of Input Output Table for water resource in China, such as Water Resource Input Output Table in Beijing of China (Xie et al., 1991), Water Resource Input Output Table of Shanxi Province of China (Chen et al., 2003), Water Resource Input Output Table in Huabei and Xinjiang region of China (Chen et al., 2003). All the above water resource Input Output Tables are Regional Water Resource Input Output Table. There are no national Input Output Table in China before. In our model we will construct and apply National Water Resource Input Holding Output Table for China.

In 2001, Liu and Chen (2003) developed a model to calculate Shadow Price of water. In this paper, an more direct and more practical improvement of that model is presented. Water scarcity is the foremost problem in some countries including China. Market price of water can be considered as an instrument to satisfy water allocation and must be justified in a reasonable way. Although Shadow Price has been widely appreciated, discussions of Shadow Price have been confined to static settings. For improving this point dynamic Input Output model and Turnpike model are improved and applied. The important properties of the water Shadow Price from 1949 to 2050 in China are obtained and analyzed. This paper develops a dynamic Shadow Price approach based on multi-periods Input Output optimizing model. Unlike previous approaches, it is based on the dynamic computable general equilibrium (DCGE) model to solve the problem on marginal long-term price of water resource.

In this paper, Section 2 will review the basic concepts of dynamic Input Output analysis and Turnpike Theory. Section 3 will elaborate the definitions and algorithms of the proposed dynamic calculation model for Shadow Price calculation in China.

Both the empirical specification and the data used to implement the model are described in Section 3. Section 3 will bring forward the result of Shadow Price from 1949 to 2050 in China as well as the sensitivity analysis by using National Water Resource Input Holding Output Table of China for 1999 in 51 sectors. Section 4 discusses policy implications and provides a summary.

2 Materials and Methods

2.1 Extended Dynamic Input Output Model and Turnpike Theory

The extended dynamic Input Output model and Turnpike model are introduced in this section. These two models provide the basis of the dynamic computable general equilibrium model. And they are also developed based on the separation of amount and structure for the calculation.

2.1.1 Dynamic Input Output Model

The linear static and dynamic Input Output models are well known in economic theory and economic practice (Leontief, 1951). The dynamic extended Input Holding Output model (Liu, 1995) is shown as follows :

$$\sum_{j=1}^{n} X_{ij}(t) + \sum_{j=1}^{n} Z_{ij}(t) + Yc_i(t) = X_i(t),$$
$$K_{ij}(t+1) = K_{ij}(t) + Z_{ij}(t) - D_{ij}(t+1),$$
$$i,\ j = 1, 2, ..., n \tag{1}$$

where $X_{ij}(t)$ is output in sector ith from sector jth in tth year, $\boldsymbol{Y}_{ci}(t)$ is final demand vector in tth year in sector ith, $K_{ij}(t)$ is the available capital goods in sector ith from sector jth in tth year, $Z_{ij}(t)$ is capital investment in sector ith from sector jth in tth year, $D_{ij}(t)$ is depreciation in sector ith from sector jth, $K_{ij}(t)$ is the available capital goods in sector ith from sector jth in tth year.

Direct input coefficient in tth year is defined as follows:

$$a_{ij}(t) = \frac{X_{ij}(t)}{X_j(t)} \tag{2}$$

where $a_{ij}(t)$ is direct input coefficient in sector ith from sector jth in tth year, $X_j(t)$ is the output vector in sector jth in tth year.

Added investment coefficient in tth year is defined as follows:

$$b_{ij}(t) = \frac{K_{ij}(t) - K_{ij}(t-1)}{X_j(t) - X_j(t-1)} \tag{3}$$

where $b_{ij}(t)$ is depreciation coefficient in sector ith from sector jth in tth year, K_{ij} $(t-1)$ is available capital goods in sector ith from sector jth in $t-1$th year, $X_j(t-1)$ is output vector in sector jth in $t-1$th year.

Depreciation coefficient in tth year is defined as follows:

$$\beta_{ij}(t) = \frac{D_{ij}(t)}{X_j(t)} \tag{4}$$

where $\beta_{ij}(t)$ is depreciation coefficient in sector ith from sector jth in tth year. Thus,

$$\sum_{j=1}^{n} a_{ij}(t)X_j(t) + \sum_{j=1}^{n}[b_{ij}(t+1)(X_j(t+1) - X_j(t)) + \beta_{ij}(t+1)X_j(t+1)] + Yc_i(t) = X_i(t), \quad i, j = 1, 2, ..., n \tag{5}$$

where $Yc_i(t)$ is the final demand vector in tth year in sector ith.

Equation (5) can be rewritten in matrix form:

$$[\boldsymbol{I} - \boldsymbol{A}(t) + \boldsymbol{B}(t+1)]\boldsymbol{X}(t) - [\boldsymbol{B}(t+1) + \boldsymbol{\beta}(t)] \boldsymbol{X}(t+1) = \boldsymbol{Y}c(t) \tag{6}$$

where $\boldsymbol{I}$ is unity matrix, $\boldsymbol{A}(t)$ is direct input coefficient matrix in tth year, $\boldsymbol{B}(t+1)$ is added investment coefficient matrix in $t+1$th year, $\boldsymbol{X}(t+1)$ is output matrix in tth year, $\boldsymbol{\beta}(t)$ is depreciation coefficient matrix in tth year, $\boldsymbol{Y}_c(t)$ is final demand matrix in tth year, $\boldsymbol{X}(t+1)$ is output matrix in $t+1$th year.

2.1.2 Turnpike Model

Turnpike model (Dorfman et al., 1958) is based on the linear programming in which the objective function is the maximum of the capital accumulation at the end of the objective term. When we calculate the balanced growth solution of Turnpike model, the dynamic Input Output model is used to extend the original structure of Turnpike model in order to avoid the constraint of Occluding Hypothesis in the Neumann model of Turnpike model (Neumann, 1945). The modified model without occluding constraints is in accord with the real status in China. Recall the dynamic Input Output model in Equation (6) as follows:

$$[\boldsymbol{I} - \boldsymbol{A}(t) + \boldsymbol{B}(t+1)]\boldsymbol{X}(t) - [\boldsymbol{B}(t+1) + \boldsymbol{\beta}(t)]\boldsymbol{X}(t+1) = \boldsymbol{Y}c(t)$$

Final demand coefficient in tth year is defined as follows:

$$\boldsymbol{C}(t) = \frac{\boldsymbol{Y}c(t)}{\boldsymbol{X}(t)} \tag{7}$$

where $\boldsymbol{C}(t)$ is final demand coefficient matrix.

Then, Equation (6) can be rewritten as follows:

$$\boldsymbol{X}(t) = \boldsymbol{A}(t)\boldsymbol{X}(t) + \boldsymbol{B}(t+1)(\boldsymbol{X}(t+1) - \boldsymbol{X}(t)) + \boldsymbol{\beta}(t)\boldsymbol{X}(t) + \boldsymbol{C}(t)\boldsymbol{X}(t) \tag{8}$$

where $\boldsymbol{C}(t)$ is final demand coefficient matrix. Suppose the equilibrium growth rate is the same in different sectors:

$$\boldsymbol{X}(t+1) = (1+\alpha)\boldsymbol{X}(t) \tag{9}$$

where α is the balanced growth development rate. Then,

$$\frac{1}{\alpha}\boldsymbol{X}(t) = \left[\boldsymbol{I} - \boldsymbol{A}(t)\boldsymbol{\beta}(t) - \boldsymbol{C}(t)\right]^{-1}\left[\boldsymbol{B}(t+1) + \boldsymbol{\beta}(t)\right]\boldsymbol{X}(t). \tag{10}$$

Thus,

$$\frac{1}{\alpha}\boldsymbol{X}(t) = \left[\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{\beta} - \boldsymbol{C}\right]^{-1}\left[\boldsymbol{B}(t+1) + \boldsymbol{\beta}\right]\boldsymbol{X}. \tag{11}$$

The existence of balanced growth solutions is discussed by using the extended model in Equation (11). We need to recall some basic issues related to non-negative systems. If a square matrix is non-negative (all of whose elements are non-negative), its Spectral Radius is an eigenvalue. And in correspondence of this eigenvalue there exists a non-negative eigenvector (Dorfman et al., 1958). Moreover, an irreducible matrix is characterized by one exact non-negative eigenvector (up to scalar multiplication) and this eigenvector is positive. Based on the hypothesis in Input Output model such as Hawkins Simon condition and Solow condition in Input Output analysis (Leontief, 1951), we can safely draw that the modulus of eigenvalue of the matrix $[\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{\beta} - \boldsymbol{C}]$ is below 1 and $[\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{\beta} - \boldsymbol{C}]^{-1}$ is non-negative matrix, then $[\boldsymbol{I} - \boldsymbol{A} - \boldsymbol{\beta} - \boldsymbol{C}]^{-1}[\boldsymbol{B}(t+1) + \boldsymbol{\beta}] = \boldsymbol{H}$ is non-negative matrix according to the Perron-Frobenius Theorem (Vanlint and Wilson, 1975). So, the balanced growth rate is equal to the reciprocal of the Perron-Frobenius solution. And the eigenvector is equal to the output structure on the balanced growth path (Leontief, 1951). This is an application of Perron-Frobenius Theorem to economic model.

2.1.3 Separating Amount and Structure of Output

The basic style of Input Output analysis is as follows:

$$\boldsymbol{X} = \boldsymbol{A}\boldsymbol{X} + \boldsymbol{Y} \tag{12}$$

where $\boldsymbol{X}$ and $\boldsymbol{Y}$ are output matrix and final demand matrix, respectively.

In order to impress the structure of the economic system, Equation (12) is changed as follows:

$$\dot{\boldsymbol{X}}\tilde{\boldsymbol{X}} = (\boldsymbol{I} - \boldsymbol{A})^{-1}\dot{\boldsymbol{Y}}\tilde{\boldsymbol{Y}} \tag{13}$$

where $\dot{\boldsymbol{X}}$, $\dot{\boldsymbol{Y}}$ are output amount matrix and final demand amount matrix, respectively. $\tilde{\boldsymbol{X}}$, $\tilde{\boldsymbol{Y}}$ are output structure matrix and final demand structure matrix, respectively. The element in $\tilde{\boldsymbol{X}}$ are output final ratios of every sector in the total gross output. The element in $\tilde{\boldsymbol{Y}}$ are final demand ratios of every sector in the total final demands.

2.2 Dynamic Computable General Equilibrium Model for Calculation of Shadow Price

The calculation of dynamic change in Shadow Price of water resource in China is consequently a project with a long time span, distinctive characters and complex structures. The conception of Dynamic Computable General Equilibrium model (DCGE) assumes that the level of production and resource control the structure of economic system on the balanced growth path. The balanced growth path is instrumental to the maintenance and improvement of Shadow Price. The balanced growth path conditions also give opportunity to calculate Shadow Price according to the effectiveness of the water resource used in the whole country. It also assumes that the increased input coefficient and the decreased gross amount of the water resource are synonymous with the increased Shadow Price.

2.2.1 Basic Structure of DCGE

Two advantages of our model are given as follows: (1) Shadow Price is accord with the dynamic global optimal solution reflecting the dynamic order of the resources optimal allocation, and (2) DCGE model can be modified to calculate the Shadow Price of the certain year easily. Those solution sets of Shadow Price are useful to balance the development of an economic system. The shadow price in Tth year is calculated and the parameters and constraints in DCGE are shown as follows:

$$\text{Max} Z = (\boldsymbol{I} - \boldsymbol{A}(T))\boldsymbol{X}(T)$$

$$\text{s.t.} \begin{cases} \boldsymbol{A}(t)\boldsymbol{X}(t) + \boldsymbol{B}(t+1)(\boldsymbol{X}(t+1) - \boldsymbol{X}(t)) + \\ \boldsymbol{\beta}(t)\boldsymbol{X}(t) + \boldsymbol{C}(t)\boldsymbol{X}(t) \le \boldsymbol{X}(t) \\ t = 1, 2, ..., T-1 \\ \boldsymbol{A}(T)\boldsymbol{X}(T) + \boldsymbol{B}(T+1)(\tilde{\boldsymbol{X}}(T+1)\dot{\boldsymbol{X}}(T+1) - \\ \boldsymbol{X}(T)) + \boldsymbol{\beta}(T)\boldsymbol{X}(T) + \boldsymbol{C}(T)\boldsymbol{X}(T) \le \boldsymbol{X}(T) \\ \boldsymbol{A}_w(T)\boldsymbol{X}(T) \le \boldsymbol{W}(T) \\ \boldsymbol{H}(t) \le \boldsymbol{X}(t) \le \boldsymbol{h}(t) \\ \boldsymbol{L}(t) \le \boldsymbol{X}(t) \le \boldsymbol{l}(t) \\ \boldsymbol{X}(t) \ge 0, \dot{\boldsymbol{X}}(T+1) \ge 0 \\ t = 1, 2, ..., T \end{cases} \tag{14}$$

where $\boldsymbol{X}(t)$ is output matrix in tth year, $\boldsymbol{A}(t)$ is direct input coefficients matrix in tth year, $\dot{\boldsymbol{X}}(t)$ is output amount matrix in tth year, $\tilde{\boldsymbol{X}}(t)$ is output structure matrix in tth year, $\boldsymbol{C}(t)$ is final demand coefficient vector in tth year, $\boldsymbol{A}_w(t)$ is direct water input coefficient matrix in tth year, $W(t)$ is total water input in tth year, $\boldsymbol{B}(t)$ is investment coefficient matrix in tth year, $\boldsymbol{\beta}(t)$ is depreciation coefficient matrix in tth year. $\boldsymbol{H}(t)$, $\boldsymbol{h}(t)$, $\boldsymbol{L}(t)$, $\boldsymbol{l}(t)$ are other constraints and parameters. $\boldsymbol{A}(T)$ is direct input coefficients matrix in the specific Tth year, $\dot{\boldsymbol{X}}(T+1)$ is output amount matrix in the specific $T+1$th year, $\tilde{\boldsymbol{X}}(T+1)$ is output structure matrix in the specific $T+1$th year, $\boldsymbol{C}(T)$ is final demand coefficient vector in the specific Tth year, $\boldsymbol{A}_w(T)$ is direct water input coefficient matrix in the specific Tth year, $\boldsymbol{W}(T)$ is total water input in the specific Tth year, $\boldsymbol{B}(T+1)$ is investment coefficient matrix in the specific $T+1$th year, $\boldsymbol{\beta}(T)$ is depreciation coefficient matrix in the specific Tth year.

Some details of $\boldsymbol{H}(t)$, $\boldsymbol{h}(t)$, $\boldsymbol{L}(t)$, $\boldsymbol{l}(t)$ are given in the following section named ‘Other constraint’.

2.2.2 Notes for DCGE

A. Basic Constraint

DCGE is a new method to overcome the difficulties in calculation of dynamic Shadow Price in water resource project evaluation. The model proposed in this paper is different from the traditional analysis, because it is based on a large linear programming in the discrete time. The Shadow Price is calculated as well as a balance of economic system results.

During the whole process of calculation period, the time factor has been put into the equation to reflect relationship between the different time spots. Time Lag is set as one year. The constraint combines the dynamic Input Output model, Turnpike Theory, and theory on the separating of the amount and the structure. The objective function is the maximum of the Gross Demotic Product (GDP) in the objective year of the plan.

Data of National Input Output Table used in this calculation demonstrates a elegant approach to balance the economic developing. Multiplying the gross amount matrix with the structure matrix is the output matrix. The separation can improve the practical work. The Shadow Price of the water resource in balanced growth path is the dual solution according to the water resource constraint line in the objective year such as Line 7 in Equation (14). The primal and dual optima are characterized by the Kuhn–Tucker Conditions, which for LPs reduce to feasibility and complementary slackness.

B. Other Constraint

These above constraints are the basis of DCGE model. Some other constraints should be added to establish the general equilibrium model for avoiding the limit of the assumption of the Input Output analysis (Leontief, 1951; Zhong, 1984). The following are the other equilibrium and resources constraint which are shorten as $\boldsymbol{H}(t) \leq \boldsymbol{X}(t) \leq \boldsymbol{h}(t)$ and $\boldsymbol{L}(t) \leq \boldsymbol{X}(t) \leq \boldsymbol{l}(t)$ which are listed as follows:

(1) Constraint of production capacity is shown as follows:

$$X_i \leq \varphi_i \tag{15}$$

where φ_i is the maximum production capacity vector in sector ith.

(2) Constraint of labor capacity is shown as follows:

$$\frac{\sum_{i=1}^{n} X_i}{T_i} \leq \boldsymbol{L} \tag{16}$$

where T_i is the labor-working rate vector in sector ith, and L is the available labor amount.

(3) Other constraint of resources is shown as follows:

$$\sum_{i=1}^{n} g_{ki} X_i \leq h_k \tag{17}$$

where g_{ki} is the input coefficient vector of the kth resource in sector ith, and h_k is the available resource amount vector of the kth resource in sector ith.

(4) Constraint of the equilibrium between the import and export is shown as follows:

$$\sum_{i=1}^{n} e_i X_i = \sum_{i=1}^{n} F_i \tag{18}$$

where e_i is input coefficient of the import commodity vector in sector ith, and F_i is export of the in sector ith.

(5) Constraint of the equilibrium between the reward of the income and the consumable is shown as follows:

$$\sum_{j=1}^{n} a_{vj} X_j + \boldsymbol{V}^* - \boldsymbol{U} \leq \boldsymbol{Y_w} + \boldsymbol{W} \tag{19}$$

where $\sum_{j=1}^{n} a_{vj} X_j$ is income of dwellers in the property sector, V^* is income of non-property sector, U is non-commercial payout of dwellers, Y_w is supply of consumable, and W is import consumable.

(6) Equilibrium between accumulation and consumption is shown as follows:

$$\sum_{i=1}^{n} Y_{iw}^* + \boldsymbol{\mu} \geq Y_w \tag{20}$$

where Y_{iw}^* is consumption amount in sector ith in the previous term, Y_w is consumption amount in sector ith in the present term, μ is other factors in the previous term.

(7) Equilibrium between forming and occupying is shown as follows:

$$\sum_{i=1}^{n} Y_k \geq \sum_{i=1}^{n} \tilde{k}_i (\hat{L}_i - L_i) \tag{21}$$

where Y_k is capital forming amount vector in the previous term, $\tilde{k}_i$ is average fund occupying amount per person in sector ith, $\hat{L}_i$ is the number of the employee in the previous term, L_i is number of the employee in the present term.

2.2.3 Notes for Long-term Marginal Cost in the DCGE

Most commercial LP solvers return the objective function coefficients for the slack and surplus variables at the optimum, called the shadow prices or dual prices. This paper develops and implements a unique formulation of a linear programming model in that the dynamic shadow price of water resource is characterized as a dual solution. The approach requires Input Output analysis to bear a shadow price. Kantorovich (1939) provided an economic interpretation of the dual variables as guides for the coordination of allocate decision. This provides the first insight into the symmetric relationships between activity levels of processes and Shadow Price of resource. The Lagrange Multipliers are given the names 'Shadow Price' and 'dual activity' in linear programming where these changes can be analyzed by sensitivity analysis. Why Shadow Price of the water resource in balanced growth path are the dual solution according to constraint line of the water resource? The following prove can explain this question. Equation (14) can be shortened as:

$$\text{Max}\boldsymbol{Z} = \boldsymbol{CX}$$
$$\text{s.t.} \begin{cases} \boldsymbol{AX} \le \boldsymbol{b} \\ \boldsymbol{A}_w\boldsymbol{X} \le \boldsymbol{b}_w \\ \boldsymbol{X} \ge 0 \end{cases} \tag{22}$$

where $\boldsymbol{A}_w\boldsymbol{X} \le \boldsymbol{b}_w$ is the water resource constraint in the original linear programming in Equation (14). The optimal basis $\boldsymbol{B}$ is given, then the optimal solution of the dual problem is $\boldsymbol{Y}^* = (y_1^*, ..., y_m^*) = \boldsymbol{C}_B\boldsymbol{B}^{-1}$. The input water resource of b_i is equal to the optimal solution of the dual problem y_i^*, then $\boldsymbol{Z}^* = \boldsymbol{y}^*\boldsymbol{b}$, and $\boldsymbol{Z}^* = \sum_{i=1}^{m} y_i^* b_i$. This comes from the basic dual programming theory in Linear Programming.

Thus,

$$y_i^* = \frac{\partial \boldsymbol{Z}^*}{\partial b_i}, \quad i = 1, 2, ..., m, \quad \boldsymbol{y}_m^* = \frac{\partial \boldsymbol{Z}^*}{\partial \boldsymbol{b}_w} \tag{23}$$

In a word, the Shadow Price of water resource $\boldsymbol{y}_m^*$ are the measure of the objective function when the resource $\boldsymbol{b}_w$ are changed, as well as the marginal long-term value. If the gross of water resource increase 1 m^3, the objective function (the amount of GDP) will increase one Yuan. So the unit of Shadow Price for water resource is Yuan/m^3. The dual solution in the sixth line constraint in Equation (14) have an clear economical meaning. The meaning is the Shadow Price of water resource. The Shadow Price of water resource is equivalent to the reductions in the optimal (or nearly optimal) value of the objective function in Equation (14) as GDP are reduced by 1 Yuan. In the following real-time results in Table I if the gross quantities of water resource in China increase 1 m^3 in 1999, GDP of China will increase 3.85 Yuan.

2.2.4 Sensitivity Analysis for the DCGE

The Objective in the sensitivity analysis is changing the left-hand side values of constraints—water consumption coefficient. That is a study of how the optimal solution of a linear program would change if some of the numbers used in the formulation of the problem were to change.

Equation (22):

$$\text{Max}\boldsymbol{Z} = \boldsymbol{CX}$$
$$\text{s.t.} \begin{cases} \boldsymbol{AX} \le \boldsymbol{b} \\ \boldsymbol{A}_w\boldsymbol{X} \le \boldsymbol{b}_w \\ \boldsymbol{X} \ge 0 \end{cases} \tag{24}$$

Table I Shadow price of water resource from 1949 to 2050 in China

Year	Shadow prices of water resources (Yuan/m^3)	Shadow prices of water resources (US$/m^3)
1949	0.48	0.06
1959	0.11	0.01
1965	2.01	0.24
1980	2.89	0.35
1993	3.46	0.42
1994	3.57	0.43
1995	3.69	0.45
1996	3.80	0.46
1997	3.92	0.47
1998	3.79	0.46
1999	3.85	0.47
2000	3.86	0.47
2005	3.99	0.48
2008	4.10	0.50
2010	4.29	0.52
2015	4.47	0.54
2020	4.58	0.55
2025	4.77	0.58
2030	4.93	0.60
2040	5.11	0.62
2050	5.39	0.65

can be shorten as follows:

$$\mathrm{Max}\boldsymbol{Z} = \boldsymbol{CX}$$
$$\text{s.t.} \begin{cases} \boldsymbol{AX} \leq \boldsymbol{b} \\ \boldsymbol{X} \geq 0 \end{cases} \tag{25}$$

Let $\boldsymbol{A} = (\boldsymbol{B}, \boldsymbol{N})$, $\boldsymbol{X} = \begin{Bmatrix} \boldsymbol{X}_B \\ \boldsymbol{X}_N \end{Bmatrix}$, $\boldsymbol{C} = (\boldsymbol{C}_B, \boldsymbol{C}_N)$, $\boldsymbol{B}$ is basic set, $\boldsymbol{N}$ is non-basic set. Then:

$$\begin{Bmatrix} \boldsymbol{B} & \boldsymbol{N} \\ \boldsymbol{C}_B & \boldsymbol{C}_N \end{Bmatrix} \begin{Bmatrix} \boldsymbol{X}_B \\ \boldsymbol{X}_N \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{b} \\ \boldsymbol{Z} \end{Bmatrix} \tag{26}$$

Then, simplex method is as follows:

$$\begin{Bmatrix} \boldsymbol{B} & \boldsymbol{N} & \boldsymbol{b} \\ \boldsymbol{C}_B & \boldsymbol{C}_N & \boldsymbol{Z} \end{Bmatrix} \to \begin{Bmatrix} \boldsymbol{I} & \boldsymbol{B}^{-1}\boldsymbol{N} & \boldsymbol{B}^{-1}\boldsymbol{b} \\ \boldsymbol{C}_B & \boldsymbol{C}_N & \boldsymbol{Z} \end{Bmatrix} \to \begin{Bmatrix} \boldsymbol{I} & \boldsymbol{B}^{-1}\boldsymbol{N} & \boldsymbol{B}^{-1}\boldsymbol{b} \\ 0 & \boldsymbol{C}_N - \boldsymbol{C}_B\boldsymbol{B}^{-1}\boldsymbol{N} & \boldsymbol{Z} - \boldsymbol{C}_B\boldsymbol{B}^{-1}\boldsymbol{b} \end{Bmatrix}$$

Then, $\boldsymbol{\sigma}_N = \boldsymbol{C}_N - \boldsymbol{C}\boldsymbol{B}^{-1}\boldsymbol{N}$, $\boldsymbol{Z}_0 = \boldsymbol{C}_B\boldsymbol{B}^{-1}\boldsymbol{b} \to \boldsymbol{X}_B + \boldsymbol{B}^{-1}\boldsymbol{N}\boldsymbol{X}_N = \boldsymbol{B}^{-1}b \to \boldsymbol{\sigma}_N\boldsymbol{X}_N = \boldsymbol{Z} - \boldsymbol{Z}_0 \to \boldsymbol{Z} = \boldsymbol{Z}_0 + \boldsymbol{\sigma}_N\boldsymbol{X}_N$. If $\boldsymbol{\sigma}_N \leq 0$, when $\boldsymbol{X}_N = 0$, $\boldsymbol{Z}$ has the maximum value Z_0, then $\boldsymbol{X}_B = \boldsymbol{B}^{-1}\boldsymbol{b}$, then the optimal rule is $\boldsymbol{\sigma}_j \leq 0$, $j \in \boldsymbol{J}_N$. If coefficient $\boldsymbol{A}$ is changed, in column $\boldsymbol{K}$th of $\boldsymbol{P}_K$ in $\boldsymbol{A}$, $\boldsymbol{P}_k^{\cdot} = \boldsymbol{P}_k + \triangle\boldsymbol{P}_K$. (1) If $\boldsymbol{P}_k \notin \boldsymbol{B}$, $\boldsymbol{X}_k$ is the non-basic set, check digit corresponding to $\boldsymbol{X}_k$ is $\boldsymbol{\sigma}_k^{\cdot} = \boldsymbol{C}_k - \boldsymbol{C}_B\boldsymbol{B}^{-1}(\boldsymbol{P}_k + \triangle\boldsymbol{P}_k)$. (2) If $\boldsymbol{\sigma}_k \leq 0$, then basic set is not changed, optimal solution is not changed. (3) If $\boldsymbol{\sigma}_k^{\cdot} > 0$, then $\boldsymbol{B}^{-1}\boldsymbol{P}_k$ is changed to $\boldsymbol{B}^{-1}(\boldsymbol{P}_k + \triangle\boldsymbol{P}_k)$, $\boldsymbol{\sigma}_k$ is changed to $\boldsymbol{\sigma}_k^{\cdot}$. The simplex method is used for iterative process sequentially.

In this paper, we want to get the 'Lower Limit' and 'Upper Limit' of water consumption coefficient $\boldsymbol{A}_w$. In the ranging information, the 'Lower Limit' and 'Upper Limit' refer to the maximum changes from the 'current water consumption coefficient' which will keep the optimum solution at the same basis. Remember that the basis is the division of the variable into the basic and non-basic sets.

2.2.5 Notes for Existence and Uniqueness in DCGE

The existence and the uniqueness have not been achieved until nonlinear programming are guaranteed for this Shadow Price from mathematics programming on dual solution. In the practical estimation of Shadow Price, the assumption that the economy is on a balanced growth path is invoked.

2.3 Computer-based Algorithm of DCGE

A heuristic DCGE algorithm can be outlined as follows:

Step 1: Use the Turnpike model in Equation (11) to calculate the structure of the output in T+1th year $\tilde{\boldsymbol{X}}(T+1)$.

Step 2: Solve the LP beginning with $\tilde{\boldsymbol{X}}(T+1)$. During the process of calculation, perhaps there is no feasible solution in the model in some time. Then the upper and the lower limit can been adjusted. The constraint and balance conditions can be also added and reduced to get the right results.

Step 3: Get the Shadow Price along with the balanced growth path for each year. The process of calculation can be divided into short term. For example if the result of Shadow Price in 2005 is calculated by using the 1999–2005 model, then the Shadow Price denotes the equilibrium Shadow Price of 2005 based on economics equilibrium from 1999 to 2005. Based on National Water Resource Input Holding Output Table for 1999, 1949–1999 Shadow Price backwards and the 1999–2050 Shadow Price forward can be calculated as a whole system. For the long-time span, Time Lag can be prolonged to 2 years or more to reduce the complexity of the algorithm.

Step 4: Sensitivity analysis for the DCGE.

2.3.1 Notes for Data and the Parameter

A. Input Holding Output Table

Many approaches presented in Section 1 to calculate the Shadow Price are tentative and have suffered from data inadequacies. To solve this problem, the design and implementation of a National Water Resource Input Holding Output Table are described. The research group leaded by Xikang Chen, consisted of about 22 researchers and professors, who spent more than one and half year to construct National Water Resource Input Holding Output Table of China for 1999 (Chen et al., 2003). The table shorten to 19 sectors is the main data resource for our calculation. The style of the table can be shown in Table II.

It is a great system engineering work to construct the table. There are four steps to construct National Water Resource Input Holding Output Table for China (Chen et al., 2003).

Table II Water resource input holding output table in China

<table>
<tr><td colspan="3" rowspan="3">Input
Output
Holding</td><td colspan="2">Intermediate Demands</td><td rowspan="3">Final Demands</td><td rowspan="3">Total Output

Total Water Resource</td></tr>
<tr><td>Non-water Conservancy Sector</td><td>Water Conservancy Sector</td></tr>
<tr><td>1,2 …,s</td><td>s+1,s+2,…,n</td></tr>
<tr><td rowspan="2">Input</td><td>Non-water Conservancy Sector</td><td>1, 2, …, s</td><td colspan="2" rowspan="2">X_{ij}</td><td rowspan="2">Y_{ij}</td><td rowspan="2">X_j</td></tr>
<tr><td>Water Conservancy Sectors</td><td>s+1, s+2, …, n</td></tr>
<tr><td colspan="3">Primary Input</td><td colspan="2">V_j</td><td></td><td></td></tr>
<tr><td colspan="3">Total Input</td><td colspan="2">X_j</td><td></td><td></td></tr>
<tr><td rowspan="3">Water</td><td colspan="2">Fresh Water</td><td colspan="2" rowspan="2">F_{ij}</td><td rowspan="2">Z_{ij}</td><td rowspan="2">W_j</td></tr>
<tr><td colspan="2">Recycle Water</td></tr>
<tr><td colspan="2">Waste Water Emission</td><td colspan="2">P_{ij}</td><td>R_{ij}</td><td>WA_j</td></tr>
<tr><td rowspan="3">Holding</td><td colspan="2">Fixed Assets</td><td colspan="2">D_{ij}</td><td></td><td></td></tr>
<tr><td colspan="2">Circulating Capital</td><td colspan="2">C_{ij}</td><td></td><td></td></tr>
<tr><td colspan="2">Labor Force</td><td colspan="2">L_{ij}</td><td></td><td></td></tr>
</table>

Step 1: Constructing 1999 Input Output Table with 40 sectors for 31 provinces, autonomous regions and municipalities of China.
Step 2: Collect water use data and estimate the amounts of water, used by each sector. The water amount is in physical units.
Step 3: Constructing holding part of the basic table.
Step 4: Revision work.

B. Other Parameters

Some parameters of DCGE should be estimated and emended for each year, such as input coefficient, value added coefficient, final demand, water input coefficient, added capital coefficient and depreciation capital coefficient by using the nonlinear and the key emendation way in Statistics and Economics. A study in the computational efficiency and complexity through a series of empirical tests is also important in DCGE.

3 Results and Discussion

3.1 Computer-based Result of DCGE Model for Shadow Price

3.1.1 Result and Analysis for Shadow Price of Water Resource from 1949 to 2050 in China

To calculate of DCGE, the basic variables of output are $19 \times 19 \times 100 = 36{,}100$, not including the slack variables and others. The scale of calculation is so enormous that the common software can't work it out, such as Matlab, Microsoft Excel and so on. We use the 'Lingo' software (see http://www.lindo.com) to simulate and get the fitful result. The following Shadow Price for China which are based on the unchanged price index for 1999 in 51 sectors. The Shadow Price in Table I doesn't reflect the inflation of average market price, the exchange rate etc. So if the effect of the inflation of average market price and the exchange rate is included in the Shadow Price, the Shadow Price will be revised.

Note that the Shadow Price in Table I is at Chinese constant price in 1999. The exchange rate between RMB and U.S. Dollar is 1 US$=8.2765 Yuan. The unusual result in the Shadow Price set is discussed as follows:

(1) 1949 is the year when P. R. China was built so the starting time spot is selected as 1949.
(2) The Shadow Price from 1949 to 1965 is low because the economic system in China had been adjusted during this time.
(3) The result is based on the Chinese constant price in 1999. The volatility of exchange rate, inflation rate and so on cannot be reflected.
(4) All the Shadow Price of water resource is positive and the average growth is 48.71% from 1949 to 2050.
(5) The water Shadow Price is the average Shadow Price of water resource in different sectors of China. As we know, the Shadow Price of water resource used in industry sector is much higher than agriculture sector.

3.1.2 Sensitivity Analysis Result for 1999 Shadow Price of Water Resource in China

A sensitivity analysis of finding upper and lower bounds to keep the Shadow Price fixed. A rigorous algorithm of obtaining these exact values is presented in this subsection. An advantage of using the software named 'Lingo' is that it has a special package to calculate the sensitivity analysis result. 'Lingo' also provides the simple ranging information that can be used for sensitivity analysis. The important lesson can be got from the sensitivity analysis of overall variable is that the total available water resource and the direct input water coefficient are the key factors. The above two factors can effect the Shadow Price greatly. In Table III, the lower limit and upper limit of water consumption coefficient are given when the Shadow Price for 1999 is equal to 3.85 Yuan/m^3.

It is possible that small changes in water consumption coefficients may have a dramatic impact on optimal solution. In Table III, if specific water consumption coefficients in specific sectors change between lower limit and upper limit, the Shadow Price of China in 1999 will be unchanged.

Table III Limit of input water coefficient of China for 1999

Sector	Output (10,000 Yuan)	Consumption of water resource (Mil.m^3)	Water consumption coefficient (m^3/10,000 Yuan)	Lower limit (m^3/10,000 Yuan)	Upper limit (m^3/10,000 Yuan)
Agriculture	245,671,720.000	381,795.988	1,554.090	1,552.950	1,557.939
Heavy industry	76,131,050.884	3,646.917	47.903	47.902	47.907
Food & tobacco	149,631,590.706	6,795.831	45.417	45.416	45.420
Clothes industry	70,275,851.786	3,590.282	51.088	51.087	51.093
Lumber & furniture	25,704,101.682	1,323.060	51.473	51.471	51.477
Petroleum industry	35,131,120.972	1,077.528	30.672	30.671	30.673
Non-ferrous metals industry	104,468,131.409	3,675.273	35.181	35.180	35.183
Ferrous metals industry	57,959,301.829	2,462.910	42.494	42.493	42.497
Machinery industry	103,494,521.044	6,527.708	63.073	63.071	63.079
Transportation industry	64,995,880.246	2,075.316	31.930	31.929	31.932
Electricity industry	72,589,754.050	3,165.297	43.605	43.604	43.608
Communication industry	67,051,215.051	1,715.172	25.580	25.580	25.581
Instrument & equipment	11,034,049.064	680.533	61.676	61.674	61.682
Repairing industry	7,390,588.030	177.069	23.959	23.958	23.960
Other industry	32,932,604.478	1,992.707	60.509	60.507	60.514
Manufacturing industry	54,123,929.578	15,375.141	284.073	284.035	284.202
Textile, chemistry, paper, industry	446,178,783.768	43,118.771	96.640	96.636	96.655
Electric power industry	42,049,766.707	34,066.965	810.158	809.848	811.204
Other sector	742,536,768.183	21,234.078	28.597	28.596	28.598

Table IV Comparison between shadow price and average market price

Year	Average market price (Yuan/m^3)	Shadow price (Yuan/m^3)	Average market price (US$/m^3)	Shadow price (US$/m^3)
1997	0.735	3.92	0.089	0.474
1998	0.750	3.79	0.090	0.458
1999	0.970	3.85	0.117	0.465

3.1.3 Comparison between Shadow Price and Average Market Price

The comparison between Shadow Price and average market price of water resource can be shown in Table IV.

The directed market price by China government are far lower than the SP which has been assumed in this paper. This average market price of water resource in China come from National Bureau of Statistics of China (www.stats.gov.cn). In Nov. 6th, 2000, the water price for Beijing residential use is 1.3 Yuan/m^3 (0.16 US$/m^3). The water average price includes the water price of industry, agriculture, services and so on. The water price for the residential use is higher than that for the most other sectors. So the government should increase the water market price according to the water shadow price.

4 Conclusions and Recommendation

The directed price by China government are far lower than the SP assumed in this paper, and that in several cases should not have been made if decisions on water resource project evaluation are to be based on directed price by China government criteria alone. Based on these empirical findings, there is an alternative perspective approach, which supports the hypothesis that the present economic efficiency alone to estimate the value of the water resource is incomplete. The main objective of this paper is to propose a new method as a sustainability indicator to evaluate the price. The important properties of the shadow prices were obtained and analyzed. The parallel computations approach could be used to solve the problems in these different courtiers or for different natural resources in China numerically.

China government still controls the price of water resource so the result of SP can give the useful decision support. In recent years, the price of water resource have been elevated many times by China government. In the author's point of view the reason why the marketing price of water resource is below the real price is due to the Marxism price theory which is applied by China government. The price theory of the Marxism can reflect the society need labor time and the input and output during the process of the production but it ignores the real society benefit of the labor and the material.

Generally speaking, it seems to be widely accepted by now that the SP should be viewed as reflecting at least three dimensions: national economy, social cultural system and ecological system. China government should improve the Market Price of water resource.

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