Water Resour Manage (2007) 21:515–532
DOI 10.1007/s11269-006-9028-0

Water quality management of a stretch of river Yamuna: An interactive fuzzy multi-objective approach

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Received: 23 February 2005 / Accepted: 27 February 2006

Abstract This paper presents an Interactive Fuzzy Multi-Objective Linear Programming (IFMOLP) model for water quality management in a river basin. The IFMOLP model formulated will first evaluate dissolved oxygen (DO) concentrations or DO deficits at a point in different reaches depending on the overall Biochemical Oxygen Demand (BOD) concentration present in the respective drain. Subsequently, the model incorporates the aspirations and conflicting objectives of the decision maker (DM) by taking into consideration the aspects relevant for pollution control boards as well as dischargers responsible for generating wastewater. The uncertainty associated with specifying the water quality criteria (based on DO concentration or DO deficit) and treatment cost to remove pollution level is incorporated by interacting the decision maker. In this process DM is asked to specify the reference aspiration levels of achievement for the values of all membership functions generated with respect to each objective. This provides flexibility for the pollution control authorities and dischargers to specify their aspirations. IFMOLP model developed herein is then used in a case study for the evaluation of optimal BOD removal in different drains located across the river Yamuna at New Delhi, India. The presented model will simulate the allocation of waste load efficiencies with satisfactory results which will indicate usefulness of the model in managing more complex river basins along with better flexible policies of water management.

Keywords Water quality management · Yamuna river basin · Fuzzy multi-objective programming · Optimization · Dissolved oxygen (DO) · Biochemical oxygen demand (BOD) · Pollution

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1. Introduction

The growing urbanization and industrialization has increased problems associated with the disposal of wastewaters containing organic matter. The rivers and streams have often been treated as a convenient disposal site for various industrial and municipal wastes which causes the greatest detriment to a river's health. If the river water is overloaded with these organic wastes, the supply of dissolved oxygen in that water may be exhausted. With further addition of organic wastes, the river may not be able to recover unless other unpolluted river or stream meets it and provides sufficient dilution. For example increasing pollution in India's many rivers that once symbolized its ancient civilization are dying a slow death because millions of tons of industrial effluents and domestic wastes flow into them daily.

The present concern for river water quality has made it necessary for engineers and planners to study the impact of different kind of pollutants discharged into the rivers and then, adopt appropriate methodology to manage the water quality so that it does not degrade below a prescribed standard. Many researchers have presented alternative approaches to control water quality standards in surface waters. Most of them relate the effect of waste input to the water quality and the cost of treatment before its disposal into the river. It has been acknowledged that different kinds of uncertainty are involved in water quality management problems at different stages of decision-making process which makes the formulation of the problem quite complex. There are mainly two types of uncertainty that receive much attention as far as water quality management of a river basin is concerned. The first type of uncertainty is due to randomness associated with different components of a river basin system, such as quality and quantity of water available in the system, kind of waste input, its quality and kinetics involved within the system. This type of uncertainty is based on probability theory and can be expressed by probability distribution functions. Another kind of uncertainty is due to fuzziness or vagueness associated with describing the water quality goals and regional character of the pollution problem. This type of uncertainty is difficult to quantify and thus usually expressed in qualitative term, which can not be described by traditional probability distribution functions. In a majority of the cases, establishing the qualitative term is not precise but it also contains an element of vagueness.

Loucks and Lynn (1966) have presented one of the earlier works comprising the probabilistic considerations in stream quality estimation. Several other researchers have addressed water quality management problems as multiple objective optimization problems which describes mainly noncommensurate and conflicting objectives (Loucks, 1983; Tung and Hathhorn, 1989; Tung, 1992). General methods of solutions of such problems include the weighting method and the constraint method. Although these methods provide acceptable solutions, they are characterized by the difficulty of assigning unknown relative weights and setting upper bounds in the problem formulation. This results in an improper accounting of the aspirations of the various groups such as the pollution control agency and the dischargers. High-resolution numerical models are also being used by environmental engineers and river biologists to explore the complexity of river dynamics and to predict contaminant impact on water quality (Koussis et al., 1990; Mulligan and Brown, 1998; and Rodrigucz et al. 2004). It has been observed that analytical solutions derived from these numerical models perform well under pollutant loadings in which the error of the classical Streeter-Phelps model is sizeable.

It is apparent that significant theoretical advances in surface water quality management modeling have been made over the past 40 years. Some field studies have also been reported wherein simulation-optimization techniques under stochastic environment are applied to design surface water quality management strategies (Ecker, 1975; Fugiwara et al., 1988;

Lohani and Thanh, 1978; Singh and Ghosh, 2003b; Wen, 1989). Certain decision support systems have also been developed to evaluate river basin strategies and their impact on water quality in different river basins located mainly in European countries (Gils and Argiropoulos, 1991; Griensven, 2002; Kotti et al., 2005; Somlyody, 1997).

A thorough review of literature shows that very few attempts have been made to apply stochastic simulation-optimization water quality management models in India in practice. Most of the major cities and towns in India are situated by the side of river and almost all of them face severe water problems in terms of quantity and quality. They are producing enormous amount of domestic wastes and industrial wastes due to rapid growth in population, and inappropriate and unplanned urbanization, industrialization and irrigation projects. Therefore, there is an imperative need to identify proper water pollution reduction strategies in Indian context by developing a multiple-objective optimization model for water quality management of river basins under stochastic and fuzzy environment. The model should contain objectives of maximizing the likelihood of good management solutions, i.e., maximizing reliability, given water quality goals and a fixed budget level under the given constraints. Moreover, model frameworks need to integrate both types of uncertainties arising due to randomness associated with input parameters and vagueness associated with describing the goals related to water quality and pollutant abatement. It should be capable of incorporating the aspirations and conflicting objectives of policy makers by considering vagueness in their objectives apart from avoiding the difficulty of assigning unknown relative weights required in the solution of a multi-objective optimization problem.

This paper attempts to integrate both types of uncertainty by developing an interactive fuzzy multi-objective linear programming (IFMOLP) model for water quality management in a river basin. It not only includes different competing objectives but also incorporates the uncertainty due to vagueness involved with expressing the water quality criteria, pollution removal and treatment cost. Finally, to illustrate the practical application of the model, a case study of river Yamuna across New Delhi in India has been presented.

2. Interactive fuzzy multi-objective linear programming (IFMOLP)

Fuzzy multi-objective linear programming is the application of fuzzy set theory in which the aspiration levels concerning the multiple objective functions and constraints are not ordinary numbers but fuzzy numbers. The details of the topic can be found in the works of Klir and Yuan (1995), and Sakawa (1993). However, certain features of an interactive fuzzy multi-objective linear programming problem to represent a system behavior are discussed below.

A classical linear programming problem can be expressed in a vector matrix form as follows:

$$\left.\begin{array}{ll}\text{Optimize} & z = cx \\ \text{subject to} & Ax \leq b \\ & x \geq 0\end{array}\right\} \tag{1}$$

where $c = (c_1, c_2, \ldots, c_n)$ is an n dimensional row vector, $x = (x_1, x_2, \ldots, x_n)^T$ is an n dimensional column vector, $b = (b_1, b_2, \ldots, b_m)^T$ is an m dimensional column vector, and $A = [a_{ij}]$ is an $m \; x \; n$ coefficient matrix.

As water resources systems are usually characterized by multiple objectives which may refer to multiple economic, social, environmental and other objectives of water development, a fundamental characteristic of multiobjective water-resources problems is that the various objectives are often non-commensurate and can not be combined into a single objective. Moreover, the objectives may usually conflict with each other and any improvement in one objective can be achieved only at the expense of the other (Jain and Singh, 2003; Singh and Singh, 2000). Consequently, the aim in solving multi-objective optimization problem is to derive a compromise solution for a decision maker, which is also Pareto optimal based on subjective value judgments. For each of the objective functions $z_i(x) = c_i x$, $i = 1, 2,\ldots, k$, if it is assumed that the decision maker has a fuzzy goal such as a minimization problem (fuzzy min), then this type of statement can be quantified by eliciting a corresponding linear membership function, $\mu_i\ (z_i(x))$, and can be expressed as:

$$\mu_i(z_i(x)) = \begin{cases} 0; & z_i(x) \geq z_i^0 \\ \dfrac{z_i(x) - z_i^0}{z_i^1 - z_i^0}; & z_i^0 \geq z_i(x) \geq z_i^1 \\ 1; & z_i(x) \leq z_i^1 \end{cases} \tag{2}$$

where z_i^0 or z_i^1 denotes the value of z_i(x) such that the degree of membership function is 0 (i.e. when the ith goal or constraint is violated beyond its limit) or 1 if the ith goal or constraint is well satisfied respectively.

The solution to the multi-objective linear programming problem corresponding to the maximum value of the membership function of the resulting decision (z) can be obtained as per the fuzzy decision of Bellman and Zadeh (1970) and Zimmerman (1978). Though for obtaining the solution by this way, it has been implicitly assumed that the fuzzy decision or minimum operator is the proper representation of the fuzzy preferences of the decision maker. But in real life situations, the decision-maker does not always use the minimum operator when combining the fuzzy goals and/or constraints. Thus, it becomes evident that an interaction with the decision maker is necessary to specify the aspiration levels of achievement for the membership values of all membership functions, called the reference membership levels. For the decision maker's reference membership levels, $\bar{\mu}_i$, the corresponding optimal solution which is nearest to the requirements in the mini-max sense or better than that if the reference membership levels are attainable, is obtained by solving the following problem:

$$\underset{x \in X}{\text{minimize}} \quad \max_{i=1,2,\ldots,k} (\bar{\mu}_i - \mu_i(z_i(x)))$$

or equivalently,

$$\left.\begin{array}{ll} \text{minimize} & v \\ \text{subject to} & \bar{\mu}_i - \mu_i\,(z_i\,(x)) \leq v, i = 1, 2, \ldots, k \\ & x \in X. \end{array}\right\} \tag{3}$$

The above mini-max problem becomes a linear programming problem if all the membership functions $\mu_i(z_i(x))$, $i = 1, 2,\ldots,k$ are linear, and hence an (M−) Pareto optimal solution is obtained by directly applying the simplex method of linear programming. The decision maker must either be satisfied with the current (M−) Pareto optimal solution or act on this solution by updating the reference membership levels. In order to help the decision maker a degree of preference and trade-off information between a standing membership function $\mu_1(z_1(x))$ and each of the other membership functions can be expressed. Such trade-off information between $z_1(x)$ and $z_i(x)$ for each $i = 2, 3, \ldots, k$ is easily obtainable and may be referred elsewhere (Sakawa, 1993).

In this paper an interactive algorithm is used below in order to derive the compromise solution for the DM from the (M−) Pareto optimal solution set.

Step 1: Solve the problem as a linear programming problem by taking only one objective at a time.

Step 2: From the results of step 1, determine the corresponding values of each objective function, $z_i(x)$ at each solution derived.

Step 3: Calculate the individual minimum (worst value in the case of maximization problem and best value in the case of minimization problem) and individual maximum (worst value in the case of minimization problem and best value in the case of maximization problem) of each objective function under the given constraints.

Step 4: Derive a membership function from the decision maker for each of the objective functions. These membership functions may be linear, piecewise linear, hyperbolic etc.

Step 5: Set the initial reference membership levels to 1 for incorporating the initial opinion of the decision maker.

Step 6: For the current reference membership values assumed in the previous step, formulate the equivalent linear programming model of the IFMOLP problem using Equation (3). By solving this mini-max problem, the optimal solution is determined and the membership function value together with the trade-off rate information between the membership functions are estimated.

Step 7: If the decision maker is satisfied with the current level of the optimal solution, the process stops. Then the current optimal solution is the compromise solution of the decision maker. Otherwise, it asks the decision maker to update the current reference membership levels by considering the current values of the membership functions together with the trade-off rates between the membership functions and return to step 3.

3. IFMOLP model for water quality management

River basins have witnessed the rise and fall of many civilizations that have left indelible imprints on human history. It is the river waters that have continued to sustain man in many parts of the world throughout the history, as water is essential for human survival. It is also important to realize that management of water resources does not only mean the quantity of water available for different purposes but also its quality. With accelerated and uncontrolled developments, more and more waste products are being discharged to river-water courses which lead negative impact on water quality. The quality aspects become even more important in view of unpredictable and depleted natural flows. Consequently some design goals have to be established that would protect the water quality adequately as well as economically.

In water resource management, the objective function is generally chosen to minimize the environmental impacts and treatment cost along with maximization of economic development and social welfare. The main objectives involved in water quality management for a river basin are: (i) maintaining the concentration level of water quality parameters such as dissolved oxygen (DO) within the permissible limit so that the water can be used efficiently for different purposes as desired by the pollution control boards and (ii) minimizing the wastewater treatment costs so that economic pressure on the concerned dischargers (like municipalities, industries) responsible for disposal of wastewater into the river can be reduced. The magnitude of interest in water quality management is the concentration levels of the water quality parameters that show the status of water quality. These water quality parameters are affected by various kinds of pollutants received from the dischargers or drains. If more wastewater is disposed without any treatment to river, water quality degradation is more. Therefore, there is a need to provide sufficient degree of treatment of wastewater before its disposal into a river so that necessary water quality goals could be achieved for its best-designated use. This is achieved by expressing pollutant level and water quality status in terms of certain parameters such as biological oxygen demand (BOD) and dissolved oxygen (DO) concentrations. By predicting BOD and DO concentrations or DO deficits at a point in space resulting from the discharge of biodegradable organic wastes, the degree of treatment can be estimated.

Therefore, the interactive fuzzy multi-objective linear programming (IFMOLP) model formulated here will first evaluate dissolved oxygen (DO) concentrations or DO deficits at a point in different reaches along the river depending on the overall BOD present in the respective drain. The overall BOD removal due to both decomposition and sedimentation (settling), which normally takes place after partially treated or untreated sewage outfalls drain into the stream is considered. Subsequently, the model incorporates the aspirations and conflicting objectives of the decision maker by taking into consideration the aspects relevant for pollution control boards as well as dischargers. The uncertainty associated with specifying the water quality criteria (based on DO concentration or DO deficit) and treatment cost to remove pollution level (expressed in terms of BOD concentration) is incorporated by interacting the decision maker. In the process DM is asked to specify the reference aspiration levels of achievement for the values of all membership functions with respect to the corresponding objective. The main objectives related to the pollution control boards and dischargers have been expressed as fuzzy sets and described in detail under following subtopics.

3.1. Objectives

The objectives of the problem are formulated by considering the aspirations of the decision maker corresponding to both pollution control board and the dischargers. These objectives are transformed to fuzzy goals by using fuzzy sets. The goals are then used in formulating the fuzzy decision for the water quality management problem.

The main goals involved in water quality management of a river basin are:

Goal 1: Maximizing the concentration level of water quality parameters (e.g. DO), which are producing positive impact on water quality of river and its environment. In this case the concentration of parameters should be as close as possible to the desirable level as perceived by the pollution control board. However, it should not be less than the minimum permissible level prescribed by the pollution control board (Sasikumar and Mujumdar, 1998).

Goal 2: Minimizing the concentration level of water quality parameters (e.g. toxic pollutants, DO deficit), which are producing negative impact on water quality of river and its environment. In this case the magnitude of parameters should be as close as possible to the level desired by pollution control board that is expected to be as low as possible.

Goal 3: Minimizing the treatment cost for percentage removal of pollutants by keeping it as close as possible to the aspiration level of the dischargers like municipalities and industries. However, it should satisfy the minimum permissible fraction removal level and should not be allowed to exceed the maximum permissible level of pollutants prescribed by the pollution control board.

3.2. Membership functions for fuzzy objectives

The membership functions of fuzzy sets are considered to represent the variation of the satisfaction level of each goal of the decision maker. These membership functions correspond to the above goals are formulated below:

Goal 1: The desirable level z_1^1 for the goal 1 to maximize the quantity of beneficial water quality parameter is assigned a membership value of 1. The minimum permissible level z_1^0 of this goal of the decision maker with regards to pollution control board is assigned a membership value of 0. The membership function for the fuzzy goal 1 is expressed as:

$$\mu_1(z_1) = \begin{cases} 1 & z_1 \geq z_1^1 \\ \dfrac{z_1 - z_1^0}{z_1^1 - z_1^0} & z_1^0 \leq z_1 \leq z_1^1 \\ 0 & z_1 \leq z_1^0 \end{cases} \tag{4}$$

Goal 2: The desirable level z_2^1 for the goal 2 to minimize the quantity of harmful water quality parameter is assigned a membership value of 1. The maximum permissible level z_2^0 of this goal of the decision maker with regards to pollution control board is assigned a membership value of 0. The membership function for the fuzzy goal 2 is expressed as:

$$\mu_2(z_2) = \begin{cases} 1 & z_2 \leq z_2^1 \\ \dfrac{z_2^0 - z_2}{z_2^0 - z_2^1} & z_2^1 \leq z_2 \leq z_2^0 \\ 0 & z_2 \geq z_2^0 \end{cases} \tag{5}$$

Goal 3: The desirable level z_3^1 for the goal 3 to minimize the treatment cost for the removal of pollutants is assigned a membership value of 1. The maximum acceptable level z_3^0 of this goal of the decision maker with respect to dischargers is assigned a membership value of 0. The membership function for the fuzzy goal 3 can be expressed same as Equation (5) by substituting subscript 2 with 3.

The objectives of fulfilling the aspiration levels of the decision maker with respect to pollution control board and restricting to the special case of only one parameter of significance for water quality i.e. minimizing the weighted sum of the DO deficit at selected mesh points along the river to fulfill its best designated use, the first objective of the decision maker corresponding to pollution control board reduces to goal 2 in which the desirable level of the DO deficit should not exceed the maximum permissible level. The minimization of the weighted sum of DO deficit can be formulated similarly as discussed elsewhere (Fuziwara et al., 1988) and can be expressed as given by Equation (6) mentioned below.

$$\underbrace{\sum_{i=k}^{m}\sum_{p=1}^{m_i} w_{ip}\left\{\sum_{k=l}^{i}\left[-\left(\frac{a_{ip}^k}{\sum_{l=0}^{i} q_l}\right) l_k x_k\right]\right\}}_{\text{First term which is dependent of decesion variables}} + \underbrace{\sum_{i=k}^{m}\sum_{p=1}^{m_i} w_{ip}\left\{\sum_{k=l}^{i}\left[\left(\frac{a_{ip}^k}{\sum_{l=0}^{i} q_l}\right) l_k\right] + \frac{I_{ip}}{\sum_{l=0}^{i} q_l}\right\}}_{\text{Second term which is independent of decesion variables}} \tag{6}$$

where a_{ip}^k is the multiplier at pth mesh point in ith reach due to BOD remaining in kth drain after the treatment and I_{ip} is the numerical value of the rest part after separating the $(1 - x_k)$ terms which is evaluated after combining all independent terms such as DO deficit due to stream flow and existing DO deficit in corresponding drains for a particular point. Both these terms are functions of flow, reaction kinetics, river geometry, deoxygenation constants, reaeration constants, settling rate constants, flow travel times and other physical parameters; all symbols have the usual meaning as described in Appendix I. The values of a_{ip}^k and I_{ip} can be evaluated by adopting the recursive rules. By ignoring all the terms that are independent of the decision variables i.e. BOD fraction removal (x_k) and by rearranging, Equation (6) is reduced to the maximization of

$$\sum_{k=l}^{m} a_k l_k x_k \tag{7}$$

where

$$a_k = \sum_{i=k}^{m} \sum_{p=l}^{p_i} \left(\frac{w_{ip} a_{ip}^k}{\sum_{l=0}^{i} q_l} \right).$$

The formulation of Equation (7) is associated with constraints of mass balance for pollution loading that guarantee continuity requirements in the river basin and water quality control constraints for minimum attainment level of water quality.

The second objective is to minimize the wastewater treatment cost of dischargers as mentioned in goal 3. It is assumed to be approximated by a linear function to the degree of removal of pollutant (in this case BOD fraction removal) as mentioned in the second of the Equation (8) under the same constraints as given for the first objective. Adopting these two equations, the multi-objective problem can be formulated as expressed below:

$$\left.\begin{aligned} &\text{Maximize} \sum_{k=l}^{m} a_k l_k x_k \\ &\text{Minimize} \sum_{k=l}^{m} c_k x_k \end{aligned}\right\} \tag{8}$$

Subject to

$$\left.\begin{aligned} &q_o L_{ip} + \sum_{k=1}^{i} q_k l_{ipk} = q_i LL_{ip} && \text{(Mass balance constraints for BOD loading)} \\ &q_o + \sum_{k=1}^{i} q_k = q_i && \text{(Mass balance constraints for stream flow)} \\ &DD_{ip}^D \le DD_{ip} \le DD_{ip}^U && \\ &LL_{ip}^D \le LL_{ip} \le LL_{ip}^U && \\ &xi^L \le xi \le xi^U && i = 1, 2, \ldots, m \end{aligned}\right\} \tag{9}$$

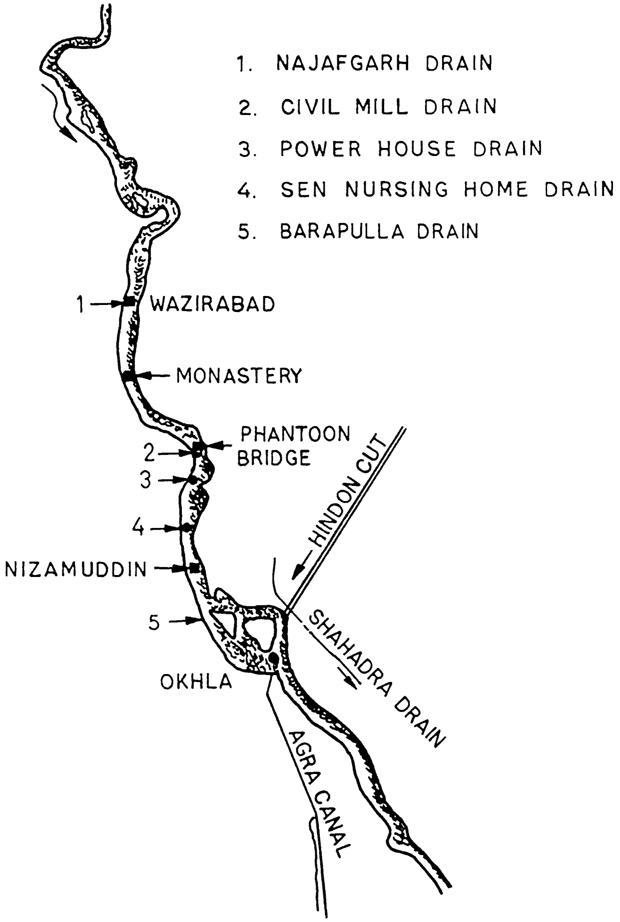


Fig. 1 Some selected drains along river Yamuna at Delhi

where all symbols have the usual meaning as described in appendix I. By using interactive algorithm and Equations (4), and (5), above multi-objective formulation can be transformed into an IFMOLP formulation and can be solved accordingly.

3.3. Application of the model

The Interactive Fuzzy Multi Objective Linear Programming (IFMOLP) model is applied on the river Yamuna across New Delhi, India, the layout of which is shown in Figure 1. The river Yamuna is a major tributary of river Ganga and forms the major source of drinking water for New Delhi besides serving many other towns and villages in Uttar Pradesh and Haryana states. The river enters New Delhi near Palla village, which is 23 km upstream from Wazirabad barrage of New Delhi. Through barrage at Wazirabad river water is trapped for drinking water supply to New Delhi. Generally a meager amount of water is allowed to flow beyond Wazirabad barrage in dry seasons. In this study, emphasis is restricted only on Wazirabad-Okhla stretch of 22 Kilometers, where major portion of water is the untreated or partially treated domestic and industrial wastewater coming through different drains. This river stretch ends into Agra Canal, which is used to augment its flow for irrigation in two neighboring states, Haryana and Uttar Pradesh in the downstream. The Yamuna river is polluted with domestic waste, silt, and industrial waste and there has been evidence of degradation and critical water quality conditions within this segment of the river (Singh and Ghosh, 2003a). In fact, the 22-km stretch between Wazirabad and Okhla barrage in New Delhi is only 2% of its catchment area, but it contributes about 80% of the river's total pollution load. This leads to significant degradation of water quality of river rendering the 500 km stretch downstream from New Delhi to Chambal confluence and water quality of river improves to certain extent only after this confluence. The major causes of water quality degradation are: (1) unabated increase in population of New Delhi, resulting in increased domestic population loads (2) rapid industrialization in the river basin area; and (3) decrease in flow of the river due to intensive abstractive use of surface and ground water in the basin area.

The river network is divided into 5 reaches and each reach is assumed to receive a point source of BOD waste load from a drain located at beginning of the reach. For the purpose of the analysis BOD inputs from the five major drains, namely Najafgarh drain, Civil Mill drain, Power House drain, Sen Nursing Home drain, and Barapulla drain which contribute about 80% of BOD load to river Yamuna upstream of Okhla barrage are considered. The model was calibrated using historical 1995–96 water quality data and verified for another set of the field data collected by the Central Pollution Control Board (CPCB). The data available were arranged into the format needed for the model. The calibrated model has been later applied to evaluate the likely impact on the river of various wastewater treatments the objective of which is to show the application and utility of the proposed model (Singh and Ghosh, 2003a).

Mean values of all the analysis parameters have been given in Table 1. The DO deficit and BOD concentration just upstream of Najafgarh drain are considered as zero and 2.82 mg/L respectively whereas the saturated DO concentration in all the reaches of the river is taken as 8.38 mg/L at 25 °C. The average discharge of the river just upstream of Najafgarh drain is taken as 18.66 m^3/sec. Site specific parameters such as the cost for treatment facilities and the deoxygenating rate coefficient, reaeration rate coefficient are independently investigated and determined as discussed by Singh and Ghosh (2003a), and in ADSORBS/32/1999–2000.

The five reaches considered in the river network contain length of 7.1 km, 1.8 km, 1.4 km, 3.6 km and 11.8 km which are further divided into 10, 10, 10, 10 and 16 number of mesh points respectively with a total of 56 mesh points from outfall of Najafgarh drain to the end of fifth reach. The width, slope, Manning's coefficient, molecular diffusion coefficient at 25 ^{o}C and transition time for every reach of the river stretch are taken as 200 m, 0.0001, 0.05 $m^{-1/3}$s, 0.00021106 m^2/day and 0.0303 days respectively whereas mean of deoxygenating rate coefficient (k_i) for all reaches are taken as 1.3 day^{-1}. However, the reaeration rate constant, k_{ri}, in any ith reach is evaluated by the equation proposed by O'Connor and Dobbins (1956).

The objective functions to be optimized consider both the economic and environmental factors. The preceding formulation results in a model with two objective functions one to be maximized and the other to be minimized along with the constraint set consisting of minimum attainment levels for water quality, the interaction between BOD and DO, and mass balance relations for both water quality and quantity along the river basin. The proposed model is used to determine the optimal BOD removal efficiencies for five dischargers to maintain water quality (in terms of DO concentrations) in every reach of the river.

The operation and maintenance (O & M) costs of treatment plants for BOD removal have been determined from Rowan et al. (1961). Lack of data inhibited to make an extensive study

Table 1 Data related to main stream and dischargers

Description	Variables	Mean
Water characteristics of river Yamuna before outfall of first drain	Stream flow ($q_\circ$ in m^3/sec)	18.66
	Initial BOD in stream ($L_\circ$ in mg/L)	2.82
	Initial DO Deficit ($d_\circ$ in mg/L)	0.0
Wastewater characteristics and cost of treatment for Najafgarh drain	Drain flow (q_1 in m^3/sec)	19.59
	BOD in the drain (l_1 in mg/L)	52.67
	DO Deficit in the drain (d_1 in mg/L)	8.38
	O&M cost of treatment per unit degree of removal (US $)	2292.60
Wastewater characteristics cost and treatment for civil mill drain	Drain flow (q_2 in m^3/sec)	0.94
	BOD in the drain (l_2 in mg/L)	176.08
	DO Deficit in the drain (d_2 in mg/L)	8.38
	O&M cost of treatment per unit degree of removal (US $)	312.90
Wastewater characteristics and cost of treatment for power house drain	Drain flow (q_3 in m^3/sec)	0.63
	BOD in the drain (l_3 in mg/L)	137.83
	DO deficit in the drain (d_3 in mg/L)	8.38
	O&M cost of treatment per unit degree of removal (US $)	226.02
Wastewater characteristics and cost of treatment for Sen nurshing home drain	Drain flow (q_4 in m^3/sec)	0.92
	BOD in the drain (l_4 in mg/L)	186.50
	DO Deficit in the drain (d_4 in mg/L)	8.38
	O&M cost of treatment per unit degree of removal (US $)	307.25
Wastewater characteristics and cost of treatment for Barapulla Drain	Drain flow (q_5 in m^3/sec)	4.98
	BOD in the drain (l_5 in mg/L)	54.50
	DO deficit in the drain (d_5 in mg/L)	8.38
	O&M cost of treatment per unit degree of removal (US $)	1268.16

Source: ADSORBS/32/1999-2000 and Singh and Ghosh (2003a)

and hence the cost figures obtained from above reference are directly evaluated. The cost figures are then converted to recent 1995 figures by assuming suitable interest rate. It has been decided to use 1995-dollar value as most of the data given in Table 1 belong to 1995–96. However, using the optimal cost obtained in 1995 along with suitable interest rate, O & M cost for future can also be computed. All the cost figures are converted to monthly cost per unit removal and the cost curves have been developed to demonstrate the variation of the cost function and the application of the model. It may be noted that all the cost figures are given in US dollars because no reliable literature on cost estimate of treatment plants is available as far as Indian currency is concerned. It is also not logical to convert directly the treatment cost expressed in US dollars into Indian rupees because exchange rate of rupee with respect to dollars is not stable. Thus, the cost estimates presented here are not the absolute one.

Rather, they are used for comparative estimating purposes and general guide for determining optimal BOD removal and allocating optimal treatment efficiencies. Another point of interest is that though the cost in US dollars and rupees are not same, overall objectives of decision maker for minimizing the treatment cost and allocating optimal treatment efficiencies would be fulfilled because if the cost given in US dollars is minimized, the cost in Indian currency would also be minimized in the same proportion and in both the cases the optimal BOD removal of wastewater received from various drains would be the same.

As a part of the study, it was assumed that the Najafgarh drain, the largest polluter, discharges only 50% of its total discharge into the river and remaining 50% of discharge was diverted elsewhere. The above assumption is considered to show the complete utility of the model as it reduces high proportion of the flow of the largest drain, however, model can equally be applied to any magnitude of discharge of any drain. It is considered here that each plant could be operated at any given level of treatment within its upper and lower limits for a given month. It is also considered that the pollution control authorities impose a minimal BOD fraction removal of 35% (at least primary treatment) for all the drains whereas maximum fraction removal should not exceed 95%. The aspiration level of allowable DO deficits within every reach of the stream have been taken as 3.2 mg/L, which allow the desired dissolved oxygen concentration levels in all the reaches.

3.4. Results and discussions

The computer programme developed in C++ was run to determine the weighted sum of DO deficit in terms of both the coefficients $a_k l_k$ and the BOD fraction removal (x_k) for each drain. The value of $a_k l_k$ was initially evaluated by assigning equal weights of unity to all 56-mesh points. The values of $a_k l_k$ for all five reaches were obtained as 83.44, 9.74, 3.14, 8.20 and 15.17 respectively. The O and M cost for the proposed treatment plants are taken as given in Table 1. Thus, both the objectives of Equation (8) were formulated. Solving the Equations (8) and (9) using IFMOLP algorithm, the optimal BOD removal rates x_1, x_2, x_3, x_4, and x_5 for treatment plants of Najfgarh drain, Civil Mill drain, Power house drain, Sen Nurshing Home drain, and Barapulla drain respectively are estimated. The model first determines the individual minimum and maximum of each of the objective function under given constraints as given in Table 2.

Table 2 shows that a hypothetical decision maker chooses his aspiration level to generate the membership function of each objective. He establishes the type of membership functions and corresponding assessment values. For objective 1 of maximization of $a_k l_k x_k$, let the decision maker assume any $z_1^1 = 113$ for totally desirable level and $z_1^0 = 57$ for an unacceptable level of the objective along with linear membership function though he can take any value within the specified limit. This gives the weighted sum of DO deficit of all mesh points as 110.79 mg/L for z_1^1 and 166.79 for z_1^0 against the permissible weighted sum of DO deficit of 179.2 mg/L (i.e. 3.2 × 56).

Similarly model allows choosing the aspiration level for objective 2 of minimization of cost. Let the decision maker selects z_2^0 = \$240,000 for totally unacceptable level as he may

Table 2 Individual minimum and maximum of objective functions

Objective function	Minimum	Maximum
z_1	55.44	113.71
z_2	206990	418658.94

Table 3 Interactive process for compromising solution when the DM is biased towards a Goal

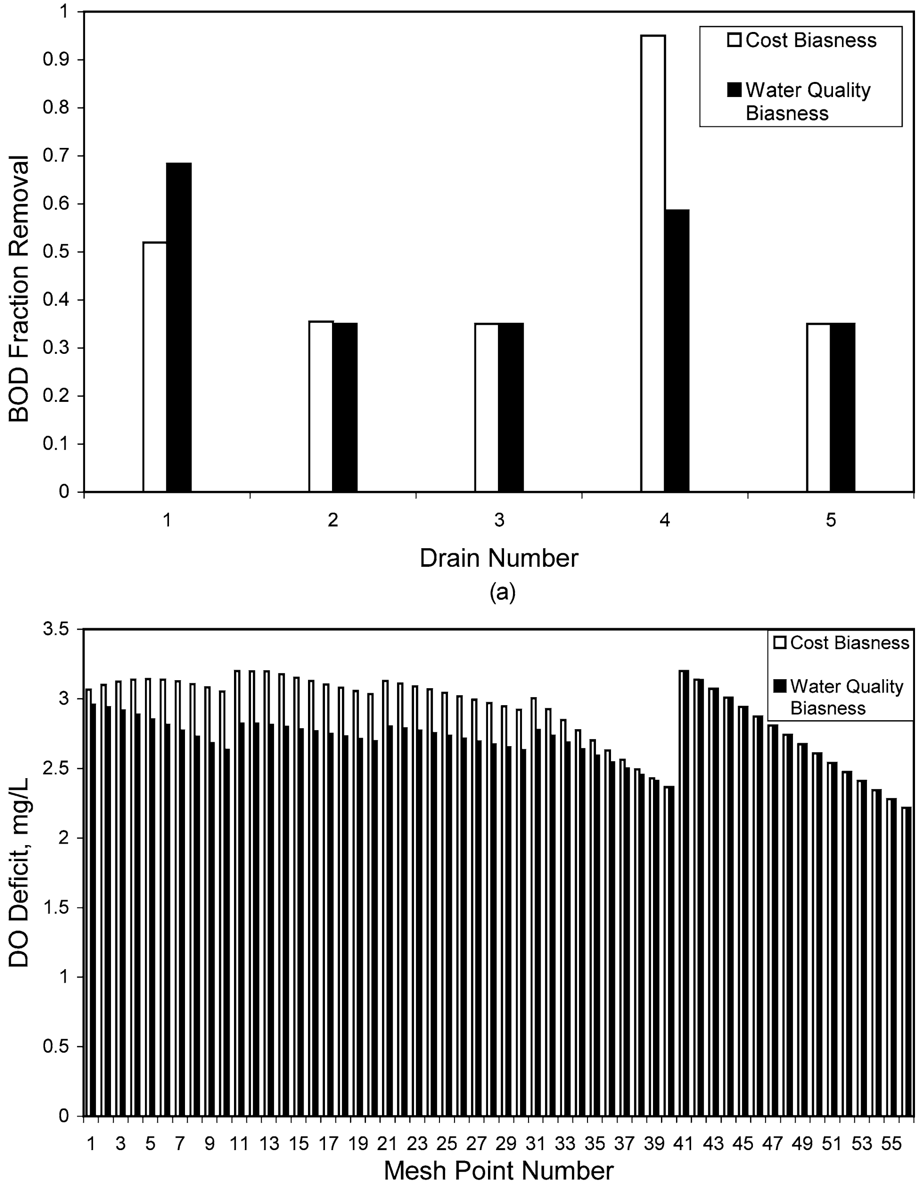
	Iterations for Biasness towards reduction of cost			Iterations for Biasness towards enhancement of water quality		
Parameters	1	2	3	1	2	3
Rm_1	1.0	0.15	0.10	1.0	0.5	0.7
Rm_2	1.0	0.80	0.90	1.0	0.4	0.5
Cm_1	0.2238	0.10	0.0717	0.2238	0.2429	0.2619
Cm_2	0.2238	0.75	0.8714	0.2238	0.1429	0.0619
z_1	69.53	62.60	61	69.53	70.6	71.67
z_2	232700	215600	211700	232700	235400	238000
x_1	0.6509	0.5446	0.5195	0.6509	0.6672	0.6836
x_2	0.35	0.35	0.3544	0.35	0.35	0.35
x_3	0.35	0.35	0.35	0.35	0.35	0.35
x_4	0.6594	0.8959	0.95	0.6594	0.623	0.5866
x_5	0.35	0.35	0.35	0.35	0.35	0.35
− dm (2)/dm(1)	294.11	294.11	224.34	294.11	294.11	294.11

not have enough budget to run the project and $z_2^1 = \$207500$ for totally desirable level for objective 2. Pareto optimum value for current membership (Cm) and objective functions (z_i) are evaluated and accordingly BOD fraction removal efficiencies are obtained for initial reference membership value (Rm) of each objective. If the decision maker is not satisfied with the current membership values, the decision maker updates the reference membership levels. As both the objectives contradict each other, decision maker may have choice to become biased either towards the water quality or towards the cost and the compromise could be obtained by interaction with the decision maker.

Table 3 summarizes interactive processes for the compromise solution if the decision maker is biased about the reduction of cost of treatment. In the case of fuzzy minimization objective, when membership value increases, corresponding objective value reduces and therefore, the cost can be reduced by increasing the reference membership value of the corresponding objective as shown by different iterations. At the third iteration the compromise solution of the hypothetical decision maker is derived.

Similarly, if the decision maker is predetermined about the improvement of water quality, compromising solution can be obtained as shown in Table 3. In this case of fuzzy maximization objective when membership value increases, corresponding objective value also increases and therefore, the water quality can be enhanced by increasing the reference membership value of the corresponding objective as shown by different iterations. Thus, the model reduces the overall permissible DO deficit if the decision maker is more biased towards water quality improvement and reduces the cost if he is biased towards saving the cost of treatment. It should be noted here that any improvement of one objective function has been achieved only at the expense of other.

The bar charts shown in Figure 2 compare the BOD removals, the DO deficits, and the required treatment costs for both water quality enhancement and cost minimization criteria. The biasness of water quality results into higher BOD fraction removal of Najafgarh drain (drain no. 1) and lower BOD removal of Sen Nurshing home drain (drain no. 4) as shown in Figure 2(a). However, other drains have the same BOD fraction removal in both the cases. Since the Najafgarh drain being the largest drain, there is an overall higher BOD removal and as a consequence a lower DO Deficit values with higher DO concentration levels in the reaches. As shown in Figure 2(b), the maximum DO deficit is well below the maximum allowable DO deficit except first mesh point of fifth reach (i.e. mesh point no. 41) where it is equal to the permissible one. This may be due to the effects of all upstream drains, which are treated with lesser BOD removal to satisfy aspiration level of other objective. Further, the higher BOD removal and the huge discharge of Najafgarh drain requires highest treatment cost as shown in Figure 2(c).



(a)

(b)

Fig. 2 *(Continued on next page)*

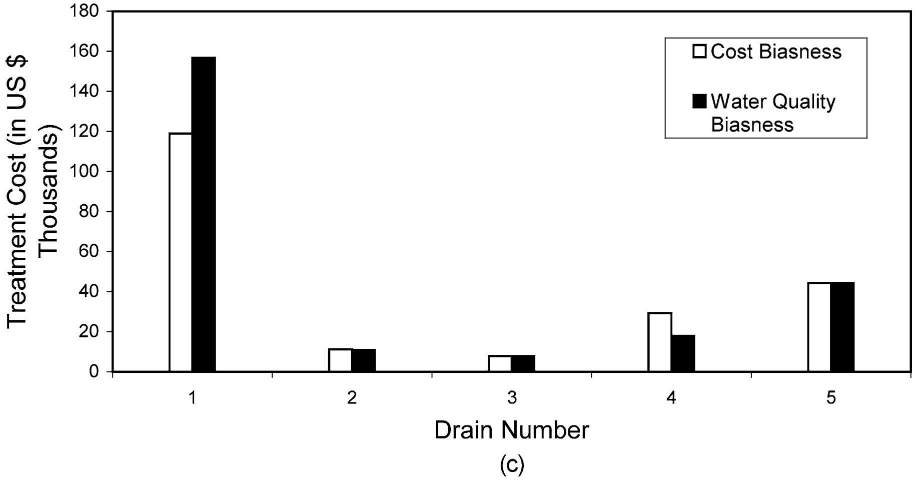


Fig. 2 (a) Bar chart of BOD fraction removal for individual drain (b) Bar chart of DO deficit at different mesh points and (c) Bar chart of treatment cost for individual drain

The biasness of cost minimization suggests lower BOD fraction removal of Najafgarh drain (drain no. 1) and DO deficit values in all the reaches approaching to the maximum permissible DO deficit. As shown in Table 3, the value of current membership function is 0.0717 for objective 1 and 0.8714 for objective 2 in the case of cost minimization criteria. This gives the optimal (minimum) satisfaction level equal to 0.0717. The upper and lower bound of satisfaction level reflect two extreme scenarios in the system. The upper bound equal to 1 indicates that the goals have been completely satisfied and therefore represents no conflict scenario. The lower bound equal to 0, indicates that at least one goal has a zero satisfaction level and therefore represents a conflict scenario. Any intermediate value represents the degree of conflict that exists in the system. It may be expected for a water quality management problem that the value of minimum satisfaction level will be closer to zero than to unity as shown. This indicates that conflict scenario can not be avoided which is mainly due to the compound effect of the conflicting objectives.

4. Conclusions

The IFMOLP formulation presented in this paper involves conflicting objectives of improving the water quality and reducing treatment cost. The fuzziness associated with establishing the water quality criteria and the aspirations of the decision maker have been effectively determined using appropriate membership functions for the different fuzzy goals by continuously interacting with the decision maker. By interacting with the decision maker, IFMOLP model evaluates optimal treatment efficiencies for a number of drains located on the river. A salient feature of the IFMOLP model is allotting weights for DO deficit at each mesh point. As the decision maker is continuously interacting with the problem, problem itself generates weights for DO deficits at each mesh point and there is no need for generating any weights allotment procedure as done in simulation model (Fujiwara et al., 1988). The model provides flexibility for all objectives of the decision maker to specify their aspirations independently. By interacting with the decision maker, the different aspiration levels for improving water quality and minimizing treatment costs are specified and the compromise solutions of the treatment levels have been achieved. This application ensures that the various conflicting objectives are simultaneously satisfied with minimum satisfaction level, while optimizing the treatment level.

The case study shows that there is a dire need to allocate highest treatment efficiency for Najafgarh Drain to remove BOD of wastewater before its disposal into the river Yamuna. Action is immediately required to trap the drain through a trunk sewer line near its outfall and convey the wastewater to suitable wastewater treatment facilities, which would produce effluent conforming to an acceptable quality for discharge into the river Yamuna without causing undue deterioration. A number of extensions and applications of the model may be possible. One of the greatest difficulties in using the model lies in the non-availability of adequate data. This is even more so in developing countries around the world. Actual cost data are extremely difficult to obtain and therefore cost estimates adopted in this study are not absolute one. Rather, they have been used for comparative estimating purposes. In fact, there is a considerable need for applied research and strategy evaluation in the areas of extracting information from water quality monitoring and information utilization within water quality management. However, a more practical implementation of nonlinear multi-objective programming with fuzzy parameters would be a future improvement.

Appendix I: List of notation

The notations used in this paper will have the following meaning:

a_{k}	Function of river parameters such as a_{ip}^{k}, w_{ip}, q_k etc.
a_{ip}^{k}	Multiplier at pth mesh point in ith reach due to BOD remaining in kth drain after the treatment and is functions of river flow ($q_\circ$), river geometry and characteristics such as H_i, n_i, S_i and W_i, reaction kinetics parameters like D_i,k_i, k_{ri}, k_{si}, t_{ip} and other physical parameters.
c_i	Cost of ith treatment plant as function of fractional BOD removal rate (x_i)
d_i	The DO deficit in ith discharging drain
d_{ipk}	DO deficit occurring at the (i, p)th mesh point due to action of the kth drain alone, where $k = 1, 2, \ldots, \mathrm{m}$ and is function of d_i, k_i, k_{si}, k_{ri}, $L_\circ$, L_{is}, L_{in}, T_i and t_{ip}
D_i	Diffusion constant of oxygen at temperature (Temp °C) in m^2/day in ith reach $= 1.76 \times 10^{-4}\ (1.037)^{\mathrm{Temp}-20}$ m^2/day
D_{ip}	DO deficit occurring at the (i, p)th mesh point due to action of stream flow alone and is function of $D_\circ$, k_i, k_{ri}, $L_\circ$, and t_{ip}
$D_\circ$	DO deficit in river just above the outfall of first drain
DD_{ip}	Overall DO deficit at any pth mesh point in the ith reach and is function of D_{ip}, d_{ipk} $q_\circ$, and q_k
DD_{ip}^{D}	Desirable DO deficit at any pth mesh point in ith reach
DD_{ip}^{U}	The upper limit of DO deficit at any pth mesh point in ith reach
H_i	Average depth of flow in ith reach
I_{ip}	The numerical value of the rest part after separating the $(1 - x_k)$ terms which is evaluated after combining all independent terms such as DO deficit due to stream flow and existing DO deficit in corresponding drains for a particular point.
k_i	Deoxygenation constant in ith reach of the river
k_{ri}	Reaeration rate constant in any ith reach of the river
k_{si}	Deoxygenation rate constant for settleable organic matter in ith reach of river

l_{ipk}	BOD remaining at the (i, p) mesh point due to action of the kth drain alone and is function of L_{is}, L_{in}, k_i, T_i and t_{ip}
l_k	BOD concentration in kth wastewater drain before it enters to the treatment plant for $k = 1, 2, \ldots$, m that involves both settleable (L_{is}) and non-settleable(L_{in}) parts of BOD loadings whereas l_k becomes $L_o$whenBOD of main stream flow is considered
L_o	The BOD concentration in river just above the outfall of first drain
L_{in}	Non-settleable portion of the initial BOD concentration in ith reach just after the drain outfall
L_{ip}	BOD remaining at the (i, p) mesh point due to action of stream flow alone and is function of k_i, $L_\circ$ and t$_{ip}$
L_{is}	The settleable portion of the initial BOD concentration in ith reach just after the drain outfall
LL_{ip}	Overall BOD at any pth mesh point in the ith reach and is function of L_{ip}, l_{ipk}, $q_\circ$, and q_k etc.
LL_{ip}^D	Desirable BOD concentration at any pth mesh point in the ith reach
LL_{ip}^U	The upper limit of BOD Concentration at any pth mesh point in ith reach
m	Total number of reaches
n_i	Manning's constant in the ith reach in m$^{-1/3}$sec
o_k	e_kth quantile of a stochastic variable $a_k l_k$
m_i	Total number of mesh points considered in the ith reach
q_k	Flow rate of kth discharger for $k = 1, 2, \ldots, m$
$q_\circ$	Stream flow rate
S_i	Slope of the river in ith reach
t_{ip}	Travel time of flow from the top of ith reach to its pth mesh point
T_i	Transition period of flow in ith reach and expressed in m/day
W_i	Width of river in ith reach
w_{ip}	Weight on DO deficit at ith mesh point in ith reach
xi^L	Minimum degree of BOD removal at any ith plant
xi^U	Maximum degree of BOD removal at any ith plant
x_k	BOD fraction removal in kth drain
z	Fuzzy decision
$z_i(x)$	ith objective function of a multi objective linear programming problem for $i = 1, 2, \ldots, k$ linear objective function
z_i^0	Value of objective function $z_i(x)$ such that the degree of membership function is 0, i.e. an ith objective function is violated beyond its limit
z_i^1	Value of objective function $z_i(x)$ such that the degree of membership function is 1, i.e. the ith objective is well satisfied
λ	Minimum satisfaction level
$\bar{\mu}_i$	Reference membership level of ith objective function defined by DM
$\mu_i\ (z_i(x))$	Linear membership function of any objective function $z_i(x)$

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