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An Improved I_a-S Relation Incorporating Antecedent Moisture in SCS-CN Methodology

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Abstract. Employing a large dataset of 84 small watersheds (area = 0.17 to 71.99 ha) of U.S.A., this paper investigates a number of initial abstraction (I_a)-potential maximum retention (S) relations incorporating antecedent moisture (M) as a function of antecedent precipitation (P_5), and finally suggests an improved relation for use in the popular Soil Conservation Service Curve Number (SCS-CN) methodology for determination of direct runoff from given rainfall. The improved performance of the incorporated $M = \alpha\sqrt{P_5 S}$ and $I_a = \lambda S^2/(S + M)$ relations, where λ is the initial abstraction coefficient, in the SCS-CN methodology exhibits the dependence of I_a on M, which is close to reality; the larger the M, the lesser will be I_a, and vice versa. Such incorporation obviates sudden jumps in the curve number variation with antecedent moisture condition, an unreasonable and undesirable feature of the existing SCS-CN model.

Key words: antecedent moisture, curve number, initial abstraction, NEH-4, soil conservation service

Introduction

The SCS-CN method was developed in 1954 and it is documented in Section 4 of the National Engineering Handbook (NEH-4) published by Soil Conservation Service (now called the Natural Resources Conservation Service), U.S. Department of Agriculture in 1956. The document has since been revised several times. It is one of the most popular methods for computing the volume of surface runoff for a given rainfall event from small agricultural, forest, and urban watersheds. The method is simple, easy to understand, and useful for ungaged watersheds. The method accounts for major runoff producing watershed characteristics, viz., soil type, land use/treatment, surface condition and antecedent moisture condition (Ponce and Hawkins, 1996; Mishra and Singh, 2003; Mishra *et al.*, 2004, 2005).

Due to spatial and temporal variability of rainfall, quality of measured rainfall-runoff data, and the variability of antecedent rainfall and the associated soil moisture amount, the SCS-CN method has sufficient room for variability (Ponce and Hawkins, 1996). The last source of variability is generally recognized as the antecedent moisture condition (AMC). Though the term antecedent is taken to vary from previous 5 days to 30 days (SCS, 1971), there exists no explicit guideline for varying the soil moisture with the antecedent rainfall of certain duration. NEH-4 (SCS, 1971) uses the antecedent 5-d rainfall for AMC, and it is usually practiced. AMC is categorized into three levels, AMC I (dry), AMC II (normal), and AMC III (wet), which statistically correspond, respectively, to 90, 10, and 50% cumulative probability of exceedance of runoff depth for a given rainfall (Hjelmfelt *et al.*, 1982).

Besides the above three AMC levels permitting unreasonable sudden jumps in curve numbers (CN), the constant initial abstraction coefficient (λ) in the SCS-CN methodology, which largely depends on climatic conditions (Ponce and Hawkins, 1996), is the most ambiguous assumption and requires considerable refinement. It is perhaps the reason that the past research endeavors suggested a need for further improvement, overhauling, or replacement of the method (Ponce and Hawkins, 1996; Mishra and Singh, 2002). To this end, Mishra and Singh (MS) (2002) among others (Mishra *et al.*, 2003, 2004, 2005) suggested SCS-CN-based relations incorporating antecedent moisture and 5-d antecedent precipitation amount. The MS and other models allow variation in λ, but treat I_a to be independent of M. However, in reality, the initial abstraction, which represents losses due to interception, surface storage, evaporation, and infiltration, varies inversely with the antecedent moisture. The higher the antecedent moisture, the lower will be the initial abstraction, and vice-versa. Thus, the objective of present study is to investigate a number of I_a-S relations for their performance on a large set of data from 84 small watersheds of US and finally suggest a modification to the I_a-S relationship for inclusion in the existing SCS-CN methodology.

SCS-CN-Based Models

EXISTING SCS-CN METHOD

The SCS-CN method consists of the water balance equation and two fundamental hypotheses (Mishra and Singh, 2003) which can be expressed, respectively, as:

$$P = I_a + F + Q, \tag{1}$$

$$\frac{Q}{P - I_a} = \frac{F}{S}, \tag{2}$$

$$I_a = \lambda S \tag{3}$$

where P = total precipitation, I_a = initial abstraction, F = cumulative infiltration, Q = direct runoff, S = potential maximum retention, and λ = initial abstraction coefficient. Notably, except for the initial abstractions, S includes all other losses. Here, all the variables, except λ which is non-dimensional, are dimensional [L] quantities. A combination of Equations (1) and (2) leads to the popular form of the existing SCS-CN method:

$$Q = \frac{(P - I_a)^2}{P - I_a + S} \tag{4}$$

In application to both gauged and ungauged watersheds, $\lambda = 0.2$, and parameter S is expressed as

$$S = \frac{25400}{\mathrm{CN}} - 254 \tag{5}$$

where S is in mm and CN is the curve number which depends on land use, hydrologic soil group, hydrologic condition, and antecedent moisture condition (SCS, 1971).

Various forms of the above model are represented in Table I by Models 1–3, in which Model 2 considers $\lambda = 0.2$ (the standard existing SCS-CN model), Model 1 allows variation in λ, and $\lambda = 0$ in Model 3.

MISHRA-SINGH (MS) MODEL

Using the $C = S_r$ concept, where C is the runoff coefficient ($= Q/(P - I_a)$) and $S_r =$ degree of saturation, Mishra and Singh (2002) modified Equation (2) for antecedent moisture M as:

$$\frac{Q}{P - I_a} = \frac{F + M}{S + M} \tag{6}$$

which, upon substitution into Equation (1), leads to

$$Q = \frac{(P - I_a)\,(P - I_a + M)}{P - I_a + M + S} \tag{7}$$

Here, M is computed as:

$$M = \frac{(P_5 - 0.2S_I)S_I}{P_5 + 0.8S_I} \tag{8}$$

where P_5 is the antecedent 5-d precipitation amount, and S_I is the potential maximum retention corresponding to AMC I. Equation (8) assumes the watershed to be dry 5 days before the onset of the considered rain storm. Since $S_I = S + M$, it follows:

$$M = 0.5[-1.2\,S + \sqrt{0.64S^2 + 4P_5S}] \tag{9}$$

Here, $+$ sign before the square root is retained for $M \geq 0$. Equation (9) can be generalized by replacing 0.2 by λ, and the resulting M expressed as:

$$M = 0.5[-(1 + \lambda)S + \sqrt{(1 - \lambda)^2S^2 + 4P_5S}] \tag{10}$$

Table I. SCS-CN-Based rainfall-runoff models incorporating antecedent moisture

Model	Method or Model Types	Mathematical Eqs.	λ	Accounting of antecedent moisture (M)
1	SCS-CN Method	$Q = \frac{(P-I_a)^2}{P-I_a+S}$	Varying	Usual NEH-4 procedure
2		$\mathrm{I}_a = \lambda \mathrm{S}$	0.2	
3			0	
4	Mishra-Singh Models	$Q = \frac{(P-I_a)(P-I_a+M)}{P-I_a+M+S}$	Varying	$M = 0.5[-(1+\lambda)S + \sqrt{(1-\lambda)^2 S^2 + 4P_5 S}]$
5		$\mathrm{I}_a = \lambda S$	0.2	$M = 0.5[-1.2S + \sqrt{0.64S^2 + 4P_5 S}]$
6			0	$M = 0.5[-S + \sqrt{S^2 + 4P_5 S}]$
7	Proposed Modified Mishra-Singh (MMS) model	$Q = \frac{(P-I_a)(P-I_a+M)}{P-I_a+M+S}$	Varying	$M = 0.5[-(1+\lambda)S + \sqrt{(1-\lambda)^2 S^2 + 4P_5 S}]$
8		$I_a = \frac{\lambda S^2}{S+M}$	0.2	$M = 0.5[-1.2S + \sqrt{0.64S^2 + 4P_5 S}]$
9			Varying	$M = \alpha P_5$
10			0.2	
11			0	
12			Varying	$M = \alpha \sqrt{P_5 S}$
13			0.2	
14			0	
15			Varying	$M = 0.72\sqrt{P_5 S}$
16			0.2	
17			0.1	
18			0.08	
19			0	

Note All above formulations are dimensionally balanced.

Models 4–6 (Table I) are, as above, the various forms of the MS model.

PROPOSED MODIFIED MISHRA-SINGH (MMS) MODELS

In the MS model, I_a is given by Equation (3), which does not incorporate M. Since I_a, as above, greatly depends on M, Equation (3) can be modified for M as:

$$I_a = \frac{\lambda S^2}{S + M} \tag{11}$$

which is the proposed or modified non-linear $I_a - S$ relation. Here, for $M = 0$ or a completely dry condition (Mishra and Singh, 2003), $I_a = \lambda S$, which is the same as Equation (3). Thus, Equation (3) is a specialized form of Equation (11). A substitution of Equation (11) into Equation (7) yields

$$Q = \frac{\left(P - \frac{\lambda S^2}{S+M}\right)\left(P - \frac{\lambda S^2}{S+M} + M\right)}{P - \frac{\lambda S^2}{S+M} + M + S} \tag{12}$$

For determination of M in the above expressions, it is possible to use the relations (Equations (10) and (9)) given by Mishra and Singh (2002). Figure 1 shows M versus S plot for Equation (9) used in Model 5, a specific version of the MS model. It shows that for a given P_5, M increases first with increasing S, reaches a maximum value, and then decreases. The increasing trend is consistent with the fact that, for a given P_5, a watershed with larger retention capacity would retain greater amount of moisture. This increasing trend was incorporated in Models 12–19 through the proposed M-S relationships. These M-expressions coupled with Equation (12) form various versions (Models 7–19) of the Modified Mishra-Singh (MMS) model (Table I). In this table, α is a non-dimensional coefficient; α equal to 0.72 is the mean of the optimized α-values in Model 12, and this value is used in Models 15–19; and λ equal to 0.1 and 0.08 in Models 17 and 18, respectively, are the mean and median values of the optimized λ-values in Model 15.

Application

DATA

For evaluating the model performance, rainfall-runoff events were derived from the U. S. Department of Agriculture-Agricultural Research Service (USDA-ARS) Water Database, which is a collection of rainfall and stream flow data from small agricultural watersheds of the United States. In the present study, data for 22392 storm events from 84 watersheds varying from 0.17 to 71.99 ha were used.

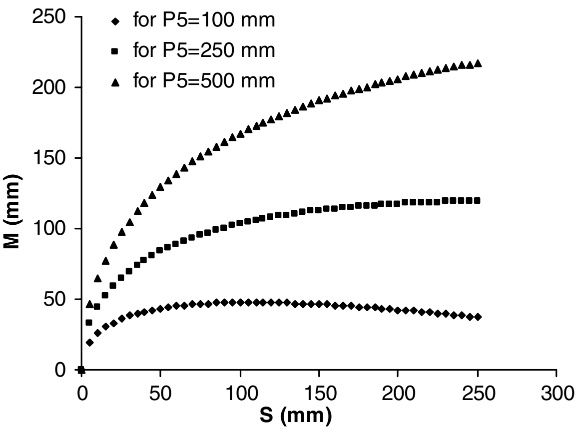


Figure 1. M versus S plot for Model 5.

PERFORMANCE CRITERION

For comparative evaluation of model performance, the root mean square error (RMSE) was taken as an index of the agreement between computed and observed values of runoff. It is expressed as:

$$\mathrm{RMSE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(Q_{\mathrm{obs}} - Q_{\mathrm{comp}})_i^2} \tag{13}$$

where Q_{obs} is the observed storm runoff (mm), Q_{comp} is the computed runoff (mm), N is the total number of rainfall-runoff events, and i is an integer varying from 1 to N. The higher the RMSE, the poorer is the performance of the model, and vice versa. RMSE $= 0$ indicates a perfect fit. The works of Madsen *et al.* (2002), Mishra *et al.* (2003), Itenfisu *et al.* (2003) are but a few examples among many others to cite the wide usage of RMSE.

PARAMETER ESTIMATION

Employing Equation (13), the model parameters were computed using the Marquardt (1963) algorithm of constrained least squares. Marquardt (1963) provided an elegant and improved version of the non-linear optimization method originally proposed by Levenberg (1944). The method primarily provides a smooth variation between the two extremes of the inverse-Hessian method and the steepest descent method. The latter method is used when the trial solution is far from the minimum and it tends continuously towards the former as the minimum is approached. This Levenberg-Marquardt method is also called as Marquardt method, which works well in practice and has become standard of non-linear least squares routines.

Taking the median CN corresponding to AMC II and varying it according to the NEH-4 AMC criterion (SCS, 1971; McCuen, 1982), Model 1 allowed variation of parameter λ in optimization. In Models 2 and 3, λ was taken equal to 0.2 and 0, respectively. Here, $\lambda = 0.2$ corresponds to the existing SCS-CN method, and $\lambda = 0$ was attempted for recent studies, for example, the study of Hawkins *et al.* (2001) recommends $\lambda = 0.05$ for field use. For the same reason, these values of λ were attempted in evaluating other models incorporating antecedent moisture, M. It is noted that the parameters CN, λ and α in all model formulations, where applicable, were allowed to vary within the prescribed range. In all the applications, the initial estimate of parameter CN was taken equal to 50, and variation was allowed from 1–100. The initial estimate of λ was taken equal to 0.2 and it was assumed to vary in the range (0, 0.5), for it can vary from 0 to ∞. Similarly, α was allowed to vary in the range (0.01, 2.0) with its initial value as 0.1. The computed values of parameters of some models for all the watersheds are shown in Table II, and the statistics for all the models in Table III.

PERFORMANCE EVALUATION

The RMSE-values resulting from application of each of the above models to each dataset of 84 watersheds was taken for comparative evaluation of all above models. Here, it is worth emphasizing that the lesser the RMSE, the better the model performance, and vice versa. Similarly, the mean of 84 RMSE-values resulting from a model application was taken for its overall comparative evaluation. The statistics of these RMSEs, viz., minimum, maximum, mean, median, coefficient of variation (CV) and confidence interval (C.I.) are shown in Table IV. Evidently, Model 12 performed the best, for it yielded the lowest value (=4.721 mm) of mean RMSE, which ranges from 0.249 to 10.630 mm. Also, at 95% confidence level, C.I. of RMSE by this model is 4.247–5.196 mm, lowest bounds among all. It explains the rationale of incorporating the antecedent moisture (a) in computation of the initial abstraction in the modified Mishra-Singh (MMS) model and (b) as a function of the antecedent precipitation and potential maximum retention (or CN). On the other hand, Model 3 performed the poorest of all, implying that the assumption of $\lambda = 0$ in Model 1 is not appropriate.

Model 9 which ranked second performed poorer than Model 12 for greater RMSE reasons. It implies that the antecedent moisture M depends not only on the antecedent rainfall amount but also on the curve number CN (or S), which describes the runoff-producing watershed characteristic. The better performance of both these models than others support the incorporation of λ and α in their mathematical formulations (Table I). Among Models 1, 4, 7, and 15 allowing variation in λ but with some constant (including zero) value of α, Model 15 performed better than Model 7, which is followed by Models 4 and 1. It implies that the M-expression of Model 15 yielded better results than those of Models 7 and 4, supporting the

Table II. Applications of models in 84 watersheds (RMSE in mm)

				Model 2		Model 6		Model 12				Model 18	
Sl. No.	Watershed	Area (ha)	Events	CN_{med}	RMSE	CN	RMSE	λ	CN	α	RMSE	CN	RMSE
1	9004	23.96	94	77.8	4.59	32.7	2.13	0.04	32.50	0.87	1.83	42.80	1.92
2	16010	40.47	325	88.7	3.4	50.7	2.55	0.04	51.50	0.59	2.41	57.84	2.43
3	17001	11.02	586	91	5.86	73.8	5.89	0.15	77.80	0.96	5.78	75.89	5.80
4	17002	20.21	546	90.9	5.91	72	5.99	0.13	76.90	0.84	5.89	74.99	5.90
5	17003	5.08	137	89.6	5.57	65.2	5.59	0.17	68.40	1.79	5.06	68.36	5.37
6	26010	0.55	879	91.5	4.22	69.4	4.14	0.02	69.30	0.42	4.10	71.82	4.14
7	26013	0.68	572	84.9	4.25	40.6	3.77	0.00	46.00	0.02	3.68	49.85	3.90
8	26014	0.26	695	90.7	3.87	69.3	4.34	0.06	69.90	0.61	4.28	70.72	4.29
9	26016	0.59	358	90.5	3.83	58.7	3.34	0.01	62.10	0.22	3.36	63.47	3.46
10	26018	0.48	106	93.9	4.92	70.4	4.01	0.00	66.00	0.64	4.10	74.51	4.30
11	26031	49.37	77	84.7	3.89	43.7	3.2	0.00	55.90	0.06	2.99	59.82	3.48
12	26791	32.05	1475	92.7	4.16	78.4	4.1	0.11	79.20	0.88	3.99	78.57	4.00
13	26863	0.17	197	95.6	3.54	84.3	2.97	0.00	83.60	0.53	2.94	86.60	3.00
14	34001	0.9	258	89.6	5.76	69.8	5.64	0.03	69.20	0.55	5.60	71.60	5.62
15	34002	1.95	247	91.1	6.02	72.5	6.21	0.03	70.00	0.77	6.21	74.69	6.24
16	34006	0.71	275	90.3	5.53	67.5	6.14	0.02	65.00	0.66	6.13	70.22	6.17
17	34007	0.81	262	90.1	5.48	69.6	6.06	0.06	70.40	0.81	6.03	73.24	6.04
18	34008	1.91	231	88.8	5.67	63.1	6.23	0.04	60.00	0.65	6.14	64.71	6.17
19	35001	13.52	158	90.6	7.11	76.7	6.44	0.15	81.90	0.58	6.29	78.40	6.34
20	35002	1.3	151	89.2	5.09	65.2	4.53	0.00	66.30	0.20	4.26	68.19	4.51
21	35003	1.27	107	88.6	7.71	75.9	6.75	0.26	82.00	1.02	6.29	77.67	6.50
22	35005	2.14	128	85.2	5.98	56.1	4.8	0.03	56.50	0.37	4.59	58.68	4.69
23	35008	3.68	129	86.7	3.85	62.2	4.09	0.05	55.40	1.08	3.85	63.60	3.90
24	35009	5.42	120	87.1	5.19	68.2	4.36	0.07	66.40	0.72	3.96	67.31	3.96
25	35010	6.35	113	85.4	3.99	61.4	4.28	0.06	54.80	1.14	3.88	62.52	3.95
26	35011	38.36	99	81	4.32	28	3.19	0.00	22.40	0.40	3.21	41.59	3.78
27	37001	6.76	195	89.5	9.87	74.7	10.48	0.07	78.60	0.37	10.51	77.15	10.57
28	37002	37.23	388	87.3	8.58	67.6	8.86	0.13	69.80	1.21	8.70	70.12	8.75
29	42006	70.42	819	88	7.47	69.6	7.73	0.14	70.70	1.29	7.37	70.43	7.49
30	42007	52.61	148	87.9	11.4	69.1	10.21	0.03	68.60	0.78	10.04	72.62	10.07
31	42008	17.12	162	84.6	11.51	69.1	11.07	0.17	72.20	1.08	10.63	69.27	10.70
32	42010	7.97	224	90.2	9.7	73.9	8.54	0.00	73.60	0.52	8.48	77.57	8.54
33	42012	53.42	277	88	8.3	69.4	7.75	0.11	75.00	0.56	7.56	72.07	7.59
34	42013	32.33	36	88.3	7.23	78.3	5.15	0.13	81.70	0.81	4.92	80.02	4.94
35	42014	6.6	273	86.3	7.23	64.1	6.65	0.07	63.50	0.68	6.32	64.58	6.32
36	42015	16.19	128	88.7	9.09	74.8	8.04	0.21	78.90	1.29	7.55	76.68	7.69
37	42016	8.42	293	87.5	6.58	63.2	6.31	0.10	64.10	1.04	5.89	64.75	5.94
38	42017	7.53	237	87.5	9.26	71.2	8.28	0.22	75.70	1.61	7.72	74.05	7.97
39	42037	4.57	181	87.7	6.8	69.2	7.34	0.24	79.40	0.57	7.05	71.50	7.17
40	42038	2.27	158	87.4	9.8	69.3	10.13	0.20	75.60	1.24	9.79	72.36	9.93
41	42039	4.01	237	85.2	8.15	68.6	8.54	0.24	75.00	0.86	7.90	67.26	7.99

(*Continued on next page.*)

Table II. (*Continued*)

				Model 2		Model 6		Model 12				Model 18	
Sl. No.	Watershed	Area (ha)	Events	CN_{med}	RMSE	CN	RMSE	λ	CN	α	RMSE	CN	RMSE
42	42040	4.57	226	83.2	9.53	69.7	9.76	0.37	79.60	0.68	8.91	66.87	9.06
43	44005	1.46	135	77.7	4.3	51.2	4.78	0.18	65.10	0.40	3.97	50.57	4.10
44	44006	1.38	149	79.6	5.64	61.4	6.86	0.25	69.40	0.99	5.67	57.48	5.89
45	44007	1.53	524	93.8	4.5	81.3	4.81	0.24	84.70	1.03	4.57	80.81	4.63
46	44008	1.47	515	93.3	4.25	80.1	4.96	0.13	82.50	0.60	4.79	79.60	4.81
47	44009	1.63	537	92.1	3.76	69.3	4.01	0.19	78.30	0.84	3.81	73.18	3.86
48	44013	1.53	248	91	3.9	67.9	3.75	0.03	66.90	0.52	3.63	70.06	3.67
49	44014	1.61	262	90.7	3.71	61.8	4.37	0.01	60.20	0.54	4.36	68.33	4.48
50	44015	1.56	295	91.6	3.96	72.5	4.35	0.08	76.80	0.66	4.34	76.25	4.34
51	44016	1.48	309	93.6	4.61	73.9	4.95	0.03	72.40	0.81	4.92	77.71	4.97
52	44017	1.38	276	92	3.41	71.7	3.75	0.07	75.00	0.69	3.68	75.33	3.68
53	44018	1.36	274	92.6	3.78	74.4	4.07	0.09	75.80	0.98	3.94	76.93	3.97
54	44019	1.46	303	93.5	3.67	75.3	4.08	0.02	74.40	0.68	4.06	78.57	4.10
55	44020	1.44	281	92	4.05	76.3	4.15	0.16	81.60	0.74	4.02	78.17	4.04
56	44021	1.6	321	93.2	4.06	76.1	4.11	0.12	79.60	0.82	4.00	78.15	4.01
57	44022	1.51	320	85.1	4.9	67.6	4.51	0.11	73.50	0.27	3.95	65.82	4.08
58	44023	1.66	258	92.3	3.41	75.4	4.24	0.15	81.40	0.63	4.11	77.37	4.15
59	44024	1.64	264	92.5	4.34	64.1	4.5	0.00	55.70	1.00	4.37	71.39	4.65
60	44025	1.59	238	91.5	4.34	70	4.86	0.06	72.90	0.71	4.85	74.13	4.86
61	44026	1.55	241	91.4	3.74	71.8	4.49	0.17	80.50	0.61	4.31	75.31	4.37
62	44027	1.64	277	93.1	3.91	76.7	4.42	0.07	74.90	1.17	4.29	78.52	4.35
63	44028	1.7	269	92	3.85	73.3	4.59	0.23	81.90	0.84	4.37	76.40	4.44
64	56001	59.41	229	96.2	1.88	35.6	0.72	0.00	43.90	0.09	0.70	55.39	0.82
65	56002	71.99	123	95.3	1.82	45.7	0.72	0.00	45.50	0.15	0.71	63.79	0.80
66	56003	21.49	192	97.1	1.01	11	0.25	0.00	19.60	0.15	0.25	55.44	0.31
67	61002	18.41	386	83.1	4.12	51.4	4.45	0.07	58.80	0.28	4.24	53.78	4.31
68	61004	25.5	342	84.6	4.08	43	3.38	0.00	39.30	0.43	3.42	54.20	3.64
69	62014	0.59	134	93.3	6.89	82.5	6.87	0.21	83.80	1.55	6.60	83.86	6.80
70	63102	1.46	132	94.1	2.58	84.8	2.48	0.12	87.70	0.34	2.20	83.27	2.25
71	63103	3.68	94	90.7	3.14	74.9	2.07	0.12	85.50	0.12	1.57	78.18	1.86
72	63104	4.53	83	93	2.79	78	2.39	0.23	89.80	0.22	2.11	80.68	2.25
73	66004	2.56	389	89.7	2.49	37.6	1.35	0.03	36.80	0.82	1.28	52.30	1.35
74	66005	3.86	244	90.3	2.52	61.1	2.27	0.06	46.80	2.00	2.00	63.40	2.04
75	68013	40.47	201	95	1.91	33.1	0.74	0.00	31.10	0.16	0.73	56.82	0.83
76	68014	13.35	40	92	1.99	25	1.5	0.00	8.70	0.93	1.46	49.22	1.69
77	69032	17.91	198	85.8	4.09	65.8	4.46	0.20	77.90	0.59	4.23	69.35	4.32
78	69033	12.11	156	84.3	4.45	68.9	5.52	0.22	77.20	0.58	4.92	67.20	5.03
79	69034	5.16	94	83.8	4.53	53.7	5.35	0.03	53.50	0.49	5.39	58.72	5.43
80	69036	10.73	113	78.2	5.52	59.3	5.18	0.13	66.80	0.50	4.66	59.13	4.72
81	69037	11.04	123	79	3.97	48.7	3.88	0.10	60.00	0.36	3.50	51.89	3.59
82	69044	7.77	225	90.5	3.99	69.7	3.63	0.04	69.00	0.69	3.60	72.73	3.64
83	69045	11.15	250	87	2.94	62.7	3.31	0.08	64.40	0.66	3.12	64.29	3.12
84	70011	2.91	41	77.5	5.05	65.3	6.62	0.16	53.20	2.00	3.71	59.23	4.56

Table III. Range of parameters

Model	CN				λ				α (not same for all)			
No.	Min.	Max.	Mean	Median	Min.	Max.	Mean	Median	Min.	Max.	Mean	Median
1	36.30	94.70	82.18	86.10	0.00	0.33	0.13	0.13	NA	NA	NA	NA
2	77.50	97.10	88.91	89.90	0.20	0.20	0.20	0.20	NA	NA	NA	NA
3	10.90	90.90	59.69	64.95	0.00	0.00	0.00	0.00	NA	NA	NA	NA
4	19.00	89.70	66.98	70.25	0.00	0.21	0.04	0.02	NA	NA	NA	NA
5	61.83	91.45	81.37	82.21	0.20	0.20	0.20	0.20	NA	NA	NA	NA
6	16.48	84.62	62.18	66.17	0.00	0.00	0.00	0.00	NA	NA	NA	NA
7	11.80	88.00	67.42	72.10	0.00	0.21	0.05	0.03	NA	NA	NA	NA
8	61.21	91.06	80.65	81.50	0.20	0.20	0.20	0.20	NA	NA	NA	NA
9	1.00	89.60	68.19	71.80	0.00	0.31	0.08	0.07	0.01	2.00	0.87	0.80
10	57.30	90.22	78.54	79.32	0.20	0.20	0.20	0.20	0.10	2.00	0.82	0.80
11	9.90	85.30	62.83	66.50	0.00	0.00	0.00	0.00	0.00	2.00	0.78	0.65
12	8.70	89.80	66.72	69.95	0.00	0.37	0.10	0.07	0.02	2.00	0.72	0.67
13	56.01	89.43	76.74	78.04	0.20	0.20	0.20	0.20	0.10	1.80	0.77	0.80
14	1.00	84.58	61.00	65.16	0.00	0.00	0.00	0.00	0.00	2.00	0.48	0.40
15	1.00	87.70	65.75	70.90	0.00	0.33	0.10	0.08	0.72	0.72	0.72	0.72
16	53.94	89.62	76.70	77.53	0.20	0.20	0.20	0.20	0.72	0.72	0.72	0.72
17	44.75	87.24	70.42	72.12	0.10	0.10	0.10	0.10	0.72	0.72	0.72	0.72
18	41.59	86.60	68.66	70.57	0.08	0.08	0.08	0.08	0.72	0.72	0.72	0.72
19	1.98	83.03	55.76	61.09	0.00	0.00	0.00	0.00	0.72	0.72	0.72	0.72

NA = Not Applicable.

incorporation of M in I_a description.

Among the models with constant or zero values of λ and α, Model 18 ranking seven performed the best (Table IV). Since the values of λ (=0.08) and α (=0.72) in the M-expression of this model are fixed, their field estimation is obviated. Furthermore, Model 18 has two and one parameters less than those in Models 12 & 9 and Models 15, 7, 4, 1, respectively. Among the other one-parameter models, Model 6 (ranked as 12) performed much better than Model 2 (existing SCS-CN method) (ranked as 17).

As a better alternative to the above, the model performance was further evaluated quantitatively by ranking the above RMSE values. This approach is advantageous in a sense that it assigns relative weight in terms of marks based on the RMSE values indicating how better a model performed among all the above 19 models (Table I) on one rainfall-runoff data set of a watershed. The model with least RMSE was assigned the maximum marks (=19), implying that it ranked first; that with highest RMSE the minimum marks (=1), implying that it ranked last; and the others ranked in between these two. These marks were further converted into percent marks. Then, the percent marks obtained by each model in each of the above 84 applications were

Table IV. Model performance based on RMSE

	RMSE							
						C. I. at 95% confidence level		
Model No	Min. (mm)	Max . (mm)	Median (mm)	Mean (mm)	C.V. (%)	Lower bound (mm)	Upper bound (mm)	Performance position
1	0.31	11.47	4.065	4.815	50.022	4.299	5.330	6th
2	1.01	11.51	4.33	5.096	43.589	4.621	5.571	17th
3	0.31	12.47	5.24	5.827	48.056	5.228	6.426	19th
4	0.251	10.906	4.358	4.807	47.188	4.322	5.292	5th
5	0.697	11.165	4.55	5.103	43.434	4.629	5.577	18th
6	0.244	10.932	4.423	4.886	46.459	4.400	5.371	12th
7	0.251	10.897	4.302	4.792	47.246	4.308	5.276	4th
8	0.775	11.023	4.438	5.025	43.497	4.558	5.493	16th
9	0.215	10.78	4.289	4.734	47.715	4.251	5.217	2nd
10	0.598	10.795	4.364	4.891	44.721	4.423	5.359	13th
11	0.244	10.938	4.373	4.851	47.083	4.363	5.340	9th
12	0.249	10.63	4.287	4.721	47.005	4.247	5.196	1st
13	0.468	10.632	4.374	4.852	44.502	4.391	5.314	10th
14	0.237	10.866	4.369	4.877	46.49	4.392	5.362	11th
15	0.268	10.688	4.325	4.766	46.627	4.291	5.242	3rd
16	0.494	10.7	4.402	4.898	44.228	4.434	5.361	14th
17	0.329	10.694	4.363	4.828	45.604	4.358	5.299	8th
18	0.306	10.704	4.358	4.826	45.769	4.353	5.298	7th
19	0.237	10.866	4.548	4.958	45.274	4.478	5.438	15th

averaged. Thus, the higher the average marks (percent) obtained by a model, the better is its performance than others.

Table V shows the resulting average percent marks with other statistics for each model and also the corresponding rank. It is evident from this table that Model 12 secures the maximum average percent marks with least coefficient of variability (CV) and maximum lower and upper bounds of C.I. at 95% confidence level, indicating its best performance. Also, the ranks of Models 9, 15, 7, and 18 were the same as those based on mean RMSE. It is worth noting that both Models 18 and 13 obtained the same mean marks. However, for lesser CV reasons, the former was considered superior to the latter. Compared to the above mean RMSE-based evaluation, the ranks of Models 4 and 1 were reversed. Similarly, Models 6 and 2 ranked 13th and 14th, respectively, slightly different from that based on mean RMSE.

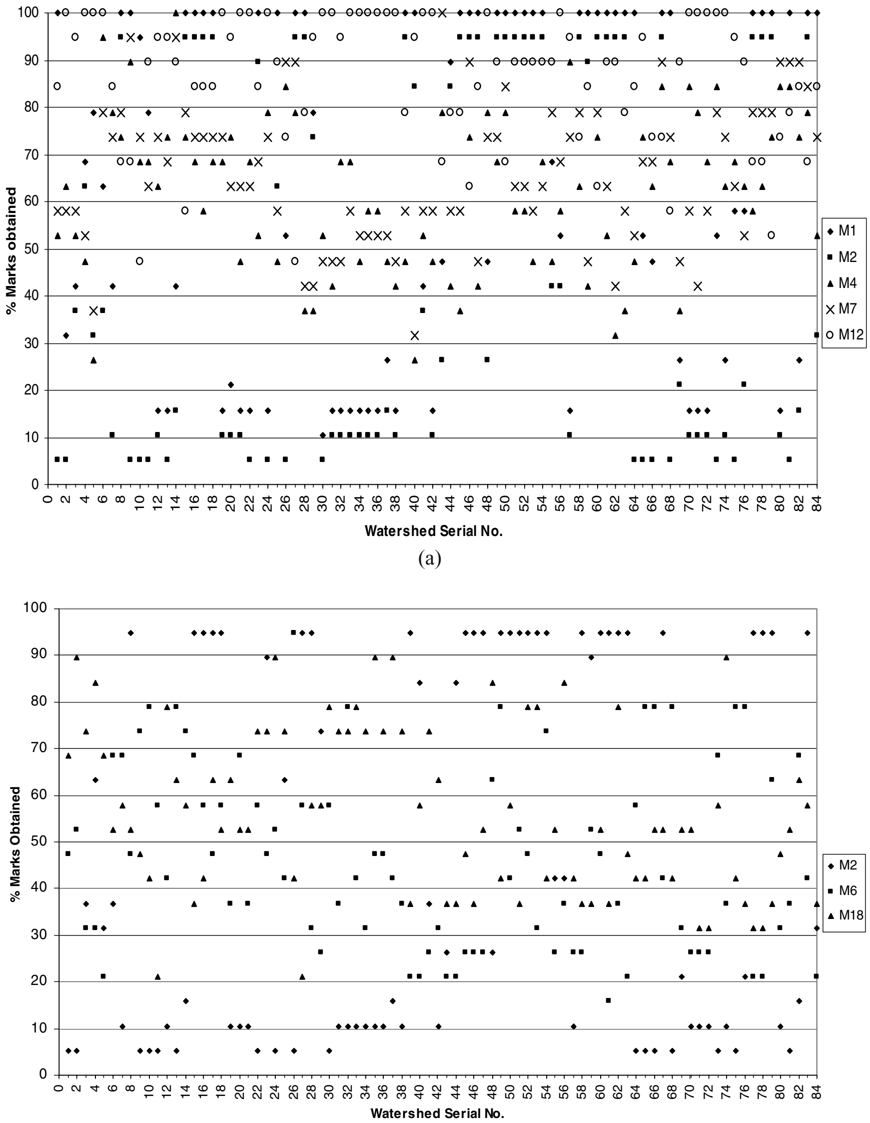
The too high value of CV (=87.51%) of marks secured by Model 2 indicates a large variation in its marks, showing inconsistency in model performance. M1, M2,

Table V. Model performance based on marks obtained by models

Model No.	Marks Obtained (%) Min.	Max.	Median	Mean	C.V.	C. I. at 95% confidence level Lower bound	Upper bound	Ranking
1	10.53	100	73.68	64.6	56.14	56.84	72.35	5th
2	5.26	94.74	26.32	45.18	87.51	36.72	53.63	14th
3	5.26	100	5.26	21.99	135.59	15.62	28.37	18th
4	26.32	100	63.16	62.28	26.46	58.76	65.80	6th
5	5.26	73.68	15.79	21.12	75.28	17.72	24.51	19th
6	15.79	94.74	42.11	46.37	42.64	42.14	50.59	13th
7	31.58	100	63.16	66.23	22.75	63.01	69.45	4th
8	5.26	89.47	23.68	29.45	64.12	25.41	33.49	17th
9	36.84	100	84.21	79.89	20.1	76.45	83.32	2nd
10	15.79	94.74	36.84	49.75	51.29	44.29	55.21	12th
11	15.79	100	50	53.76	41.15	49.03	58.49	10th
12	47.37	100	89.47	86.28	15.99	83.33	89.23	1st
13	15.79	94.74	57.89	56.33	45.14	50.89	61.77	8th
14	15.79	100	47.37	51.25	53.87	45.35	57.16	11th
15	31.58	94.74	78.95	74.25	20.99	70.91	77.58	3rd
16	10.53	84.21	42.11	43.92	50.69	39.16	48.68	15th
17	15.79	89.47	52.63	55.89	32.39	52.02	59.76	9th
18	21.05	89.47	52.63	56.33	31.7	52.51	60.15	7th
19	10.53	94.74	28.95	35.15	63.83	30.35	39.95	16th

M4, M6, M7, M12, and M18 in Figure 2a, b stands for models 1, 2, 4, 6, 7, 12, and 18 respectively. It is evidenced from these figures exhibiting Model 2 obtaining less than 30% marks in about half (43) of the total 84 watersheds, and more than 90% marks in about one-third (27) of these watersheds. Thus, the model performed very well on some watersheds and very badly on several other watersheds. On the other hand, Model 12 secured more than 60% marks in 79 (more than 94%) watersheds, and more than 45% marks in remaining 5 watersheds (Figure 2a). It secured more than 90% marks in 35 (or 42%) watersheds. Among one parameter (i.e. CN) models, Model 18 obtained less than 30% marks in only 2 watersheds, while Models 6 and 2 in 21 and 43 of the watersheds, respectively, as shown in Figure 2b.

For illustration, Figures 3a–h depicts the runoff computed by Models 2, 6, 12, and 18 against the observed runoff for the watersheds 9004 and 42007 respectively. It is evident from these figures that most data points computed using Model 2- lie far away of ±20% error bands whereas most of them due to Model 12 lie within



(a)

(b)

Figure 2. (a) Percent marks obtained by Models 1, 2, 4, 7 & 12 in all the 84 watersheds. (b) Percent marks obtained by Models 2, 6, & 18 in all the 84 watersheds.

the bands, indicating the latter to be better than the former. The Models 6 and 18 perform within these two extremes. Thus, in general, Model 12 performed the best, and Model 2 the poorest, as above.

For further evaluation, a reliability analysis was carried out using Weibull method (Gumbel, 1954), which is simple and analyses complete data series to yield prob-

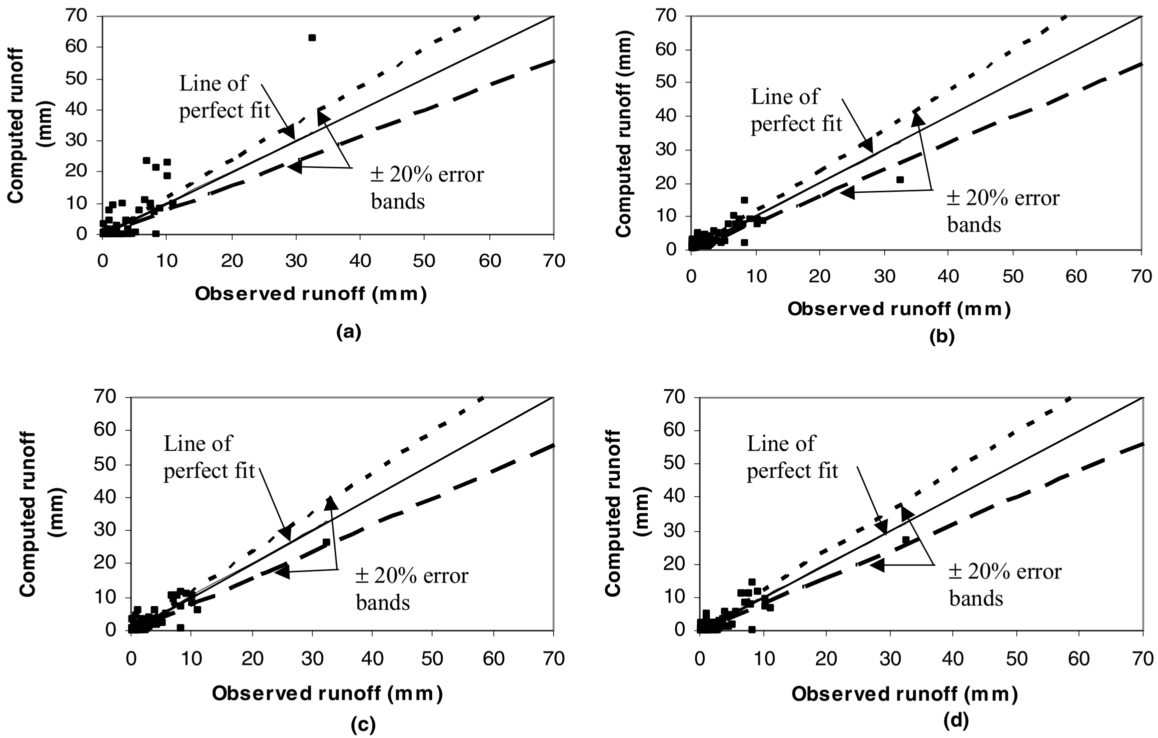
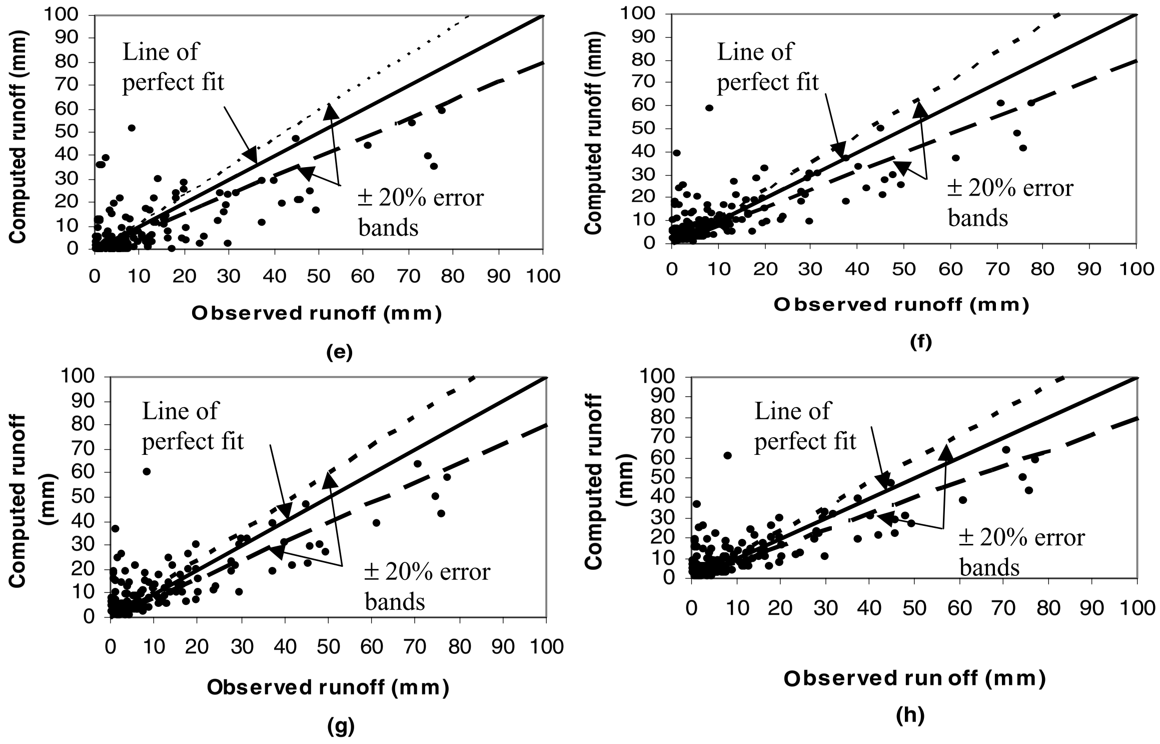


Figure 3. (a) Observed runoff versus computed runoff by Model 2 in watershed-9004. (b) Observed runoff versus computed runoff by Model 6 in watershed-9004. (c) Observed runoff versus computed runoff by Model 12 in watershed-9004. (d) Observed runoff versus computed runoff by Model 18 in watershed-9004. (e) Observed runoff versus computed runoff by Model 2 in watershed-42007. (f) Observed runoff versus computed runoff by Model 6 in watershed-42007. (g) Observed runoff versuss computed runoff by Model 12 in watershed-42007. (h) Observed runoff versus computed runoff by Model 18 in watershed-42007.



(e) (f)

(g) (h)

Figure 3. (Continued...)

Table VI. Probability analysis of performances of important models under study

	Maximum RMSE (mm) at different percent probabilities		
Model no.	60%	75%	90%
1	4.58	5.91	8.80
2	4.92	5.98	8.85
4	4.69	6.13	8.19
6	4.86	6.16	8.26
12	4.66	6.03	7.81
18	4.72	6.04	7.98

ability distribution. In this study, the probability that the RMSE due to a model will be less than or equal to any given value is determined by this method. In other words, maximum RMSE is determined at a given probability level. Table VI presents maximum RMSE by these models at 60, 75, and 90% probability levels. Evidently, at 60 and 75% probability levels, the maximum RMSE due to Model 1 is the minimum of all. However, at 90% level, it is maximum of all. Model 12 however exhibits the minimum value, and it is followed by Model 18. It implies that in terms of reliability, Model 12 performs the best, followed by Model 18.

Here, it is worth emphasizing that the recommendation of Model 6 for field use by Mishra *et al.* (2003) was based on its containing only one parameter (CN) and allowing direct incorporation of antecedent moisture and, in turn, obviating sudden jumps in CN with AMC. In this study, out of 19 models considered, this model was, however, ranked as 12 based on mean RMSE, and as 13 based on mean marks obtained. On the other hand, Model 18 (with $\lambda = 0.08$) encompassing all advantageous features of Model 6 was ranked as 7, performing much better than both Models 6 and 2. Furthermore, since Model 18 also accounts for the dependence of initial abstraction on antecedent moisture, it can form to be a more viable alternative to Models 6 and 2 for field application.

Limitations of The Proposed Model

The following are the limitations of the proposed model:

(i) Similar to the existing method, the proposed model also does not consider the effect of rainfall intensity or duration on runoff.

(ii) Since the inferences drawn from this study are primarily based on the model application to small watersheds (0.17–71.99 ha), their applicability is limited to these watersheds only, for the proposed model does not account for the spatial scale effects.

Conclusions

The following conclusions can be drawn from the study:

1. Formulation of the modified Mishra-Singh (MMS) model and its variations is more rational than that of the existing SCS-CN model, for the latter is a specific form of the former.
2. Model 12 with varying λ performs the best of all nineteen SCS-CN-based models. The existing SCS-CN Model either performs very well (in some cases) or very poorly (in many cases), and therefore, is not consistent in its performance.
3. The expressions for antecedent moisture (M) in Models 12 and 15 yield better results than those in Models 7 and 4.
4. Among one-parameter models, Model 18 performs the best, and much better than Models 2 and 6. Model 18 accounting for the realistic dependence of initial abstraction on antecedent moisture (and others ignoring it) forms to be a more viable alternative to both these models for field application.

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