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Multi-Reservoir Operation Planning Using Hybrid Genetic Algorithm and Linear Programming (GA-LP): An Alternative Stochastic Approach

L. F. R. REIS[1,*], G. A. WALTERS[2], D. SAVIC[2] and F. H. CHAUDHRY[1]
[1]*São Carlos School of Engineering, University of São Paulo, Avenue Trabalhador São Carlense, 400, São Carlos, São Paulo, Brazil;* [2]*Department of Engineering, School of Engineering and Computer Science, University of Exeter, North Park Road, Exeter, EX4 4QF, U.K.*
*(*author for correspondence, e-mail: fernanda@sc.usp.br)*

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Abstract. Many models have been suggested to deal with the multi-reservoir operation planning stochastic optimization problem involving decisions on water releases from various reservoirs in different time periods of the year. A new approach using genetic algorithm (GA) and linear programming (LP) is proposed here to determine operational decisions for reservoirs of a hydro system throughout a planning period, with the possibility of considering a variety of equally likely hydrologic sequences representing inflows. This approach permits the evaluation of a reduced number of parameters by GA and operational variables by LP. The proposed algorithm is a stochastic approximation to the hydro system operation problem, with advantages such as simple implementation and the possibility of extracting useful parameters for future operational decisions. Implementation of the method is demonstrated through a small hypothetical hydrothermal system used in literature as an example for stochastic dual dynamic programming (SDDP) method of Pereira and Pinto (Pereira, M. V. F. and Pinto, L. M. V. G.: 1985, *Water Res. Res.* **21**(6), 779–792). The proposed GA-LP approach performed equally well as compared to the SDDP method.

Key words: genetic algorithm, hydrothermal system, linear programming, multi-reservoir systems, optimal operation, power generation

1. Introduction

Long-term multi-reservoir operation planning (MROP) aims to determine the operational decision schedule for each reservoir in a system in order to minimize the expected operational cost over the planning period within the context of uncertainty in natural inflows. MROP is complex as it involves consideration of the treatment of inflow uncertainty (problem I), the influence of current decisions on future system performance (problem II) and the form of operating rules (problem III). According to Seifi and Hipel (2001) "...The variability in the demand further complicates the problem. However, demands are somewhat more predictable or can be set by contracts and thus may sometimes be assumed to be deterministic."

There is certain interplay between the three fundamental problems involved in MROP. If operational policies encourage depletion of the reservoirs and low inflows occur, it may be necessary to import water in the case of water supply systems or energy in hydroelectric systems in order to meet the contracted demands. On the other hand, if these policies keep reservoir levels high, the system may be forced to spill even when the inflows are normal, thus wasting water and, for hydroelectric systems, energy as well.

Literature surveys by Yeh (1985), Wurbs (1993) and, more recently, Labadie (2004) reviewed the intensive research on the optimization of reservoir system operation. Many models have been suggested to deal with the stochasticity involved in problem I. These models are classified as simulation and explicit or implicit stochastic optimization models. Simulation models deal with problems II and III adopting trial seasonal operating policies which are used to simulate system operation with synthetic inflow sequences. System performance in response to the proposed policies is examined and the policies that produce the most desirable responses are adopted for future operation. Explicit stochastic models also deal with the three basic methodological problems mentioned above introducing probability distributions of inflows together with simplifying considerations like linear operating rules, reliability constraints, aggregation, decomposition, etc. to deal with systems with a large number of reservoirs. Stochastic dynamic programming (SDP) is a classic example of the explicit method. Implicit stochastic methods employ synthetic sequences and deterministic optimization to evaluate operation with minimum system simplification. There still remain some questions as regards the operating policies produced by deterministic optimization in the solution of a stochastic problem . . . "even if the deterministic problem were solved for a thousand different sequences" (Saad and Turgeon, 1988). It must be recognized, however, that the implicit methods explore the simplicity of deterministic optimization in the solution of a series of small size problems instead of the real size one. According to Labadie (2004), regression analysis applied to optimization solution to solve problem III result in poor correlations that invalidate the operating rules thus obtained. Further, attempts to infer rules by other methods may require extensive trial and error procedures with little general applicability.

The implicit stochastic method presented by Pereira and Pinto (1985, 1988) called Stochastic Dual Dynamic Programming (SDDP) deals with problems I and II accommodating, at least partially, the variety of future scenarios through a tree-like structure of synthetically generated inflows. Reis and Chaudhry (1994) applied SDDP to optimal operation of a cascade of six hydroelectric plants to characterize the variability of optimal responses considering different numbers of inflow scenarios and planning periods. Other researchers have tried to incorporate the inflow variety implicitly into the model. One example is the recent work by Seifi and Hipel (2001), which employed an interior-point method (IPM) for solving the equivalent deterministic problem for a finite number of scenarios.

Oliveira and Loucks (1997) addressed all the three basic problems using genetic algorithms (GAs) to explicitly derive multi-reservoir operating policies in terms of stored volumes or target releases as piecewise linear functions for two within-year seasons in terms of stored volumes or target releases for which the inflection points are unknown. They set the optimization problem as that of finding the coordinates of such inflection points to define the optimal operating policy for inflow series obtained from a stochastic flow generator.

There have been a number of studies using GAs to solve MROP problems. Wardlaw and Sharif (1999) demonstrate applicability of GA to reservoir systems considering optimization of benefits for an example four-reservoir system over 2-h operating periods. Sharif and Wardlaw (2000) applied GA approach to optimize a three-reservoir multipurpose system over a 36 ten-day periods in the second. When the objective function is expressed in terms of optimal reservoir storage and releases to obtain rule curves for the basic problem III, these authors observed that the number of decision variables (chromosome length) increases with the number of reservoirs and planning periods, making it increasingly difficult to satisfy the problem constraints. Cai *et al.* (2001) presented a combined GA and linear programming (LP) strategy for solving large nonlinear problems that are difficult, if not impossible, using currently available NLP solver. They used GA to optimize reservoir surface levels, called “complicating variables”, for linearizing the operation problem in each time period to be later solved sequentially for different time periods. According to these authors if careful choices of the complicating variables are made, a fairly standard GA is capable of finding high quality solutions to reservoir problems in reasonable computing times. Labadie (2004) in his comprehensive in-depth review considers also directions for future research, recognizing the ability of GA to be linked with trusted simulation models as a great advantage.

In view of the computational advantages of combined GA-LP strategies to deal with large non-linear problems, this paper proposes a simple alternative scheme to solve MORP problems as posed for SDDP that uses a hybrid process involving GA and LP. It retains the SDDP advantages in dealing with problems I and II while it generates useful parameters to produce answers to problem III. This paper formulates the problem of optimal operation of reservoir systems for hydrothermal electric generation, presents the hybrid GA-LP procedure and demonstrates the method through the application example involving a simple hydrothermal system used previously by Pereira and Pinto (1985). The paper is organized in seven sections. In Section 2, the optimal operation problem is posed within the classical SDDP framework and, in Section 3, the proposed GA-LP method is presented. Section 4 describes the optimization algorithms and Section 5 presents a solution example for a simple hydrothermal system, presenting explicitly the procedures involved in combined GA-LP optimization of the system operation problem. The results for the example problem are presented and discussed in Section 6 and the conclusions are summarized in Section 7.

2. Optimal Reservoir Operation Problem

The optimal operation problem for a hydrothermal system can be expressed as that of minimizing the expected cost of energy deficit resulting from variability of the incremental inflows $\mathbf{A}_t (t = 1, 2, \ldots, T)$, over the planning period, which is here assumed to be divided into T time periods or stages (years, months, weeks, etc.). If α_t is the expected value of the optimal operating cost of the system from any stage t to the last stage T, and $\mathbf{X}_t$ is the state of the system at the beginning of stage t, the recursive equation for stochastic dynamic programming (SDP) can be set as (Pereira and Pinto, 1985):

$$\alpha_t(\mathbf{X}_t) = \underset{A_t/X_t}{E} \left\{ \min \left[C_t(\mathbf{U}_t) + \frac{1}{\beta} \alpha_{t+1}(\mathbf{X}_{t+1}) \right] \right\} \tag{1}$$

Where E_{A_t/X_t} represents the expected value, over all possible inflow vectors $\mathbf{A}_t$, conditional on the state vector $\mathbf{X}_t$. $C_t(\mathbf{U}_t)$ represents the cost of operation corresponding to decision vector $\mathbf{U}_t$ for stage t and β is the discount factor.

The problem in (1) is subject to the following constraints:

$$\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{A}_t, \mathbf{U}_t) \tag{2}$$

$$g_{t+1}(\mathbf{X}_{t+1}) \geq 0 \tag{3}$$

$$h_t(\mathbf{U}_t) \geq 0 \tag{4}$$

The set of equations (2) represents the state transition relationships, (3) represents reservoir volume constraints and (4) incorporates bounds on the outflows from a hydroelectric plant.

$\mathbf{X}_t$ includes all variables that may influence future operational performance, defined here as the current storage volumes in the reservoirs, $\mathbf{V}_t$, and inflows during the previous stage, $\mathbf{A}_{t-1}$. Note that $\mathbf{X}_0$ is assumed known. Therefore

$$X_t = \begin{bmatrix} \mathbf{V}_t \\ \mathbf{A}_{t-1} \end{bmatrix} \tag{5}$$

The decision vector $\mathbf{U}_t$ includes the outflow through the turbines, $\mathbf{Q}_t$, and the outflow over the spillway, $\mathbf{S}_t$. Therefore

$$\mathbf{U}_t = \begin{bmatrix} \mathbf{Q}_t \\ \mathbf{S}_t \end{bmatrix} \tag{6}$$

The transition Equations (2) correspond to the water balance equation:

$$V_{t+1}(i) = V_t(i) + A_t(i) - Q_t(i) - S_t(i) + \sum_{j \in M_i} [Q_t(j) + S_t(j)] \tag{7}$$

M_i is the set of hydroelectric plants immediately upstream of plant i. This assumes that there is no time lag between release from an upland reservoir and the water entering the downstream reservoir which is valid when the chosen time period for a stage is much larger than the actual time lags. Further, water losses due to evaporation from reservoir water surface are neglected in this equation obtaining linear constraints.

The total load, L_t, has to be met through hydro generation, by thermally generated energy, TG_t, or by energy imported from other systems, IMP_t, the last two usually having higher unit cost.

$$\sum_{i \in N} [\rho_i Q_t(i)] + TG_t + IMP_t = L_t \tag{8}$$

N is the number of hydro plants in the system and ρ_i is the generation characteristic of the plant i considered constant to keep the problem linear.

The constraints on the system state (3) can be represented by upper and lower bound on the volumes ($\underline{\mathbf{V}}_{t+1}$, $\bar{\mathbf{V}}_{t+1}$) as

$$\underline{\mathbf{V}}_{t+1} \leq \mathbf{V}_{t+1} \leq \bar{\mathbf{V}}_{t+1} \tag{9}$$

The constraints on the decision variables define the upper bounds on the flows through the turbines ($\bar{\mathbf{Q}}_t$), lower bound on the total outflow ($\underline{QS}_t$) and upper bound on the thermal generation ($\overline{TG}$) as

$$\mathbf{Q}_t \leq \bar{\mathbf{Q}}_t \tag{10}$$

$$\mathbf{Q}_t + \mathbf{S}_t \geq \underline{\mathbf{QS}}_t \tag{11}$$

$$TG_t \leq \overline{TG} \tag{12}$$

Although the generation characteristics ρ_i depend on the reservoir water levels, these are assumed constant corresponding to average working levels making the problem constraints linear.

In view of the computational difficulties in the solution of the recursion in (1), Pereira and Pinto (1985, 1988) presented an explicit method for the solution of optimal operation problem by Stochastic Dual Dynamic Programming (SDDP) which employs a bifurcated structure of future synthetically generated inflows. They adopted a subdivision of a multistage problem into a sequence of various two-stage sub-problems which were solved by a stochastic extension of Benders' decomposition. This SDDP approach avoids the difficulties of dimensionality of classical dynamic programming whilst retaining detailed representation of the hydro systems. However, the computational effort can become prohibitively large as the procedure for solution by Benders' decomposition requires additional linear constraints during successive iterations covering all stages in the planning period.

This paper reformulates the above MROP problem in terms of GA and LP while keeping the tree-like structure proposed by these authors to deal with the basic problem I. The optimization problem formulated in terms of LP represents an approximation in that it linearizes the effects of variable head and reservoir water surface area. Although this part of the proposed procedure can also be solved by GA or NLP, these nonlinearities can be dealt with by an iterative procedure incurring comparably smaller computational effort.

3. GA-LP Hybrid Approach

3.1. FORMULATION OF PROBLEM I

As indicated in Section 2, future inflow variety is accommodated here through a treelike structure of synthetically generated inflows. Instead of one possible sequence representing future inflows, various equally probable inflow vectors $\mathbf{A}_t^1, \mathbf{A}_t^2, \ldots, \mathbf{A}_t^{SCEN(t)}$ are considered at stage t. $SCEN(t)$ represents the number of possible inflow vectors at stage t which depends on the number of alternative inflows NB (one, two, or more) arranged in a branched structure (a simple sequence, binary tree, etc.). It is given by

$$SCEN(t) = NB^{(t-1)} \tag{13}$$

The resulting evolution of the reservoir system state would have a treelike structure such as that in Figure 1, where each bifurcation corresponds to alternative future inflow vectors of a five-stage binary tree.

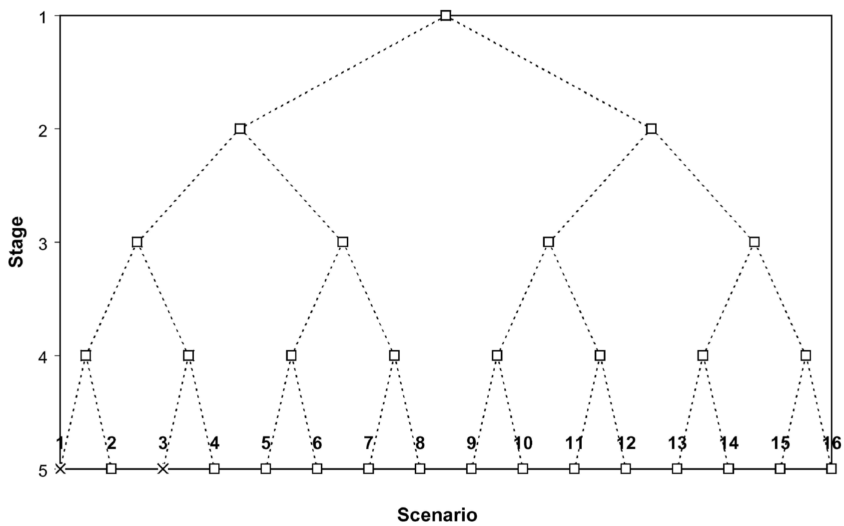

Figure 1. Treelike structure represented by a five-stage binary tree.

Due to computational difficulties of the SDDP formulation for MROP problem, an alternative approach is proposed in this paper for the solution of the same problem retaining, however, the treelike inflow structure in order to account for inflow uncertainty. This new approach considers independent one-period minimization sub-problems for various inflow sequences governed by their own state transition equations.

3.2. FORMULATION OF PROBLEM II

To encourage more efficient utilization of the stored water to supply the demand in future stages, cost reduction factors (*CRFs*) are introduced into the objective function of the LP problems in the form of negative penalties on the volumes remaining in the reservoirs at the end of each stage. These *CRFs* are parameterized through weights which vary according to reservoir and yearly season, the weights being determined from an overall optimization problem defined in terms of the minimization of the total expected operational cost over the planning period. This overall problem for the determination of weights is solved through using a Genetic Algorithm. This two-step optimization is necessary for obtaining the said weights. However, once determined, the weights may be used to update operational decisions in the light of new inflow forecasts using LP optimization directly. These weights summarize the system operating rule in an indirect manner and have the ability to allow formulation of a linear optimization problem which can give system releases. This possibility represents an important advantage since computational burden is significantly reduced.

The LP problem for individual inflow scenario at a given stage t adopting a linearized operational cost instead of $C_t(\mathbf{U}_t)$ in the original problem in Equation (1) is posed as

$$\min_{U_t} \left[\mathbf{C}_t^T \cdot \mathbf{U}_t - G \cdot \mathbf{CRF}_t^T \cdot \mathbf{V}_{t+1}\right] \tag{14}$$

subject to constraints (7)–(12). Note that the optimization problem thus formulated considers constraints (7) and (8) in linear form neglecting evaporation and variable head in power generation. In case these effects are highly relevant, the resulting nonlinearities can be managed by iterative procedures within the linear programming framework as suggested by Loucks *et al.* (1981) with reference to hydroelectric power production and by Pinheiro (2003) to account for variable reservoir water surface area.

$\boldsymbol{C}_t$ is the vector of unit costs of the different elements of decision vector $\mathbf{U}_t$ and T denotes the operation of transposition. $\mathbf{CRF}_t$ is the vector of cost reduction factors of size equal to the number of reservoirs in the system. As mentioned above, the cost reduction factors are applied to the reservoir volumes $\mathbf{V}_{t+1}$ remaining at the end of the present stage t to discourage depletion of the reservoirs at this stage. G is a constant multiplier to adjust the relative magnitude of the incentives towards conserving water in the reservoirs. A sensitivity analysis can be performed to determine adequate value of G in order to obtain quick convergence of GA algorithm.

An individual element of the vector of cost reduction factors in the LP objective function for stage t, $\mathbf{CRF}_t$, corresponding to reservoir k is constructed as a product of seasonal weight w_t and reservoir weight w_{T+k}. Thus the vector $\mathbf{CRF}_t$ is:

$$\mathbf{CRF}_t = \begin{bmatrix} w_t \cdot w_{T+1} \\ w_t \cdot w_{T+2} \\ \cdots \\ w_t \cdot w_{T+K} \end{bmatrix} \tag{15}$$

K is the total number of reservoirs in the system.

It should be noted that we have replaced the two-period linear optimization sub-problems of SDDP, which involved progressive augmentation of constraint set, by one-period optimization problems solved through LP. Variable elements of $\mathbf{CRF}_t$ are parameterized and an overall optimization is formulated to estimate the parameters for the whole planning horizon. Further, the weights are surrogates for the reservoir operating rules subject of the basic problem III.

3.3. FORMULATION OF PROBLEM III

The overall unconstrained optimization problem aims at identifying optimal weights that minimize the expected value of operational cost of the system defined as the total sum of the minimized costs of operation, each corresponding to individual inflow scenarios determined by LP.

The total number of LP problems to be dealt with for evaluation of each solution obtained through GA in the hybrid procedure is

$$TSCEN = \sum_{t=1}^{T} SCEN(t) \tag{16}$$

The solution of the overall unconstrained optimization problem is the focus of the GA minimization for which the fitness function is evaluated by summing the costs related to energy imports and thermal generation or water deficits determined for various stages by LP. Thus the fitness function for GA is formulated simply as:

$$\text{Fitness} = \sum_{t=1}^{T} \bar{C}_t(\mathbf{U}_t) \tag{17}$$

$\bar{C}_t(\mathbf{U}_t)$ represents the expected value of cost for stage t, calculated as

$$\bar{C}_t(\mathbf{U}_t) = \frac{1}{SCEN(t)} \cdot \sum_{j=1}^{SCEN(t)} C_t^j \left(\mathbf{U}_t^j\right) \tag{18}$$

$C_t^j(\mathbf{U}_t^j)$ is the cost corresponding to the individual inflow vector j at stage t.

This formulation provides **CRF** parameter values which summarize system operating rules in an indirect manner. Keeping **CRF** parameters in memory can prove useful for solving one-period linear optimization sub-problems for inflows other than those used in the full overall optimization problem. The absence of recourse to overall optimization can save computing time and resources.

4. Optimization Algorithms

The proposed method for the solution of MROP problems involves two optimization steps. The one-period optimization sub-problems are expressed in terms of LP and the overall non-linear optimization problem is proposed for solution by GA. Although the overall optimization problem may be computationally time consuming, GAs are well-suited to the solution of large scale nonlinear water resources management problems as noted in Section 1.

There are numerous efficient codes available for linear programming and genetic algorithms. For the demonstration of the GA-LP model, an example problem is posed and solved in the following section. The linear programming problems involved were solved via the revised simplex algorithm DLPRS available in Microsoft Fortran PowerStation. As the example problem has few variables, a simple steady state GA was employed using real-value representation, roulette wheel selection, one-point arithmetic crossover and uniform random gene by gene mutation. A population of 30 alternative solutions was used and the model was permitted to run a maximum of 1500 generations, using uniform crossover probability of 0.7 and uniform mutation probability of [1/(string length)]. The value of G indicated by sensitivity analysis produced convergence of overall optimization by GA in much smaller number of generations. Application of GA-LP algorithms to the example problem presented below required only a few seconds to converge on a Pentium III-700 MHz-256 Mb computer.

5. Application Example: Four Reservoir Hydrothermal System

Implementation of the proposed GA-LP method is demonstrated through a small hypothetical hydrothermal system (Figure 2) used in literature as an example for stochastic dual dynamic programming (SDDP) method of Pereira and Pinto (1985). This four-reservoir hydrothermal system is studied to calculate optimal system

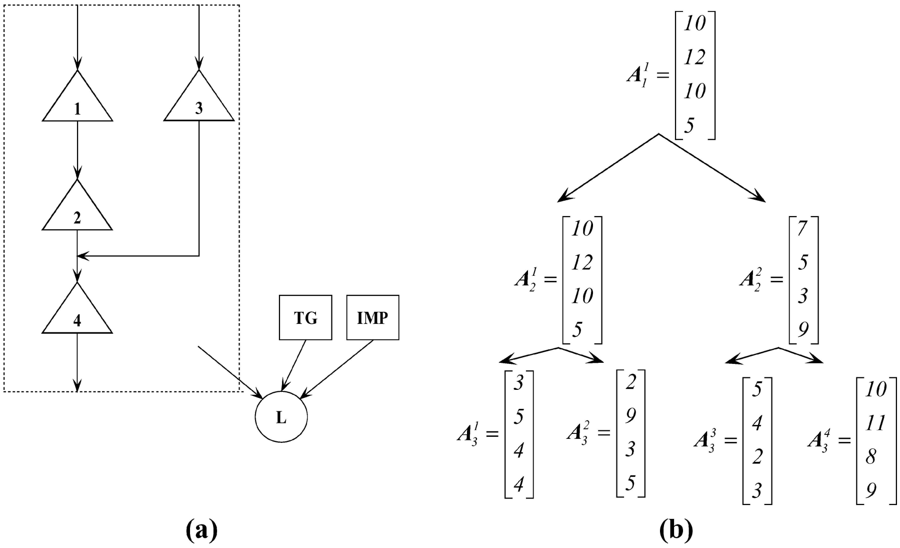

Figure 2. (a) Four-reservoir hydrothermal system and (b) binary tree of reservoir inflow vectors A_t^j at time period t, $j = 1, 2, \ldots, SCEN(t)$ (Pereira and Pinto, 1985).

trajectories, for which the initial state $\mathbf{X}_0$ is assumed known as well as the variety of future inflow vectors as a binary tree in Figure 2(b). This simple problem considers minimization of the operational costs of the hydrothermal system over three stages only without reference to seasons.[1] The generation characteristics of the system are in Table I reproduced from Pereira and Pinto (1985). In this example, two equally likely inflow vectors are considered at each stage and the initial reservoir state vector is $\mathbf{V}_0 = [50\ 40\ 50\ 50]^T$.

Table I. Characteristics of the Generating System

Hydro plant	Maximum storage $\bar{V}$	Maximum outflow $\bar{Q}$	Generation characteristic, ρ
1	500	100	0.8
2	400	140	
3	500	100	
4	500	240	
Thermal plant	Maximum capacity	Unit cost	
TG	50	1	
IMP	∞	10	

Note. Load (L) to be met is 200 and all the data are in consistent volume units.

Table II. Constraints set for LP problem at a given stage for scenario *SCEN*(t)

Thermal generation TG	Imported energy IMP	Decision variables: Final volumes **V** V_1	V_2	V_3	V_4	Outflows **Q** Q_1	Q_2	Q_3	Q_4	Spillages **S** S_1	S_2	S_3	S_4	Known $(V_0 + A_{\mathrm{SCEN}})_i$
0	0	1	0	0	0	1	0	0	0	1	0	0	0	$=(V_0 + A_{\mathrm{SCEN}})_1$
0	0	0	1	0	0	−1	1	0	0	−1	1	0	0	$=(V_0 + A_{\mathrm{SCEN}})_2$
0	0	0	0	1	0	0	0	1	0	0	0	1	0	$=(V_0 + A_{\mathrm{SCEN}})_3$
0	0	0	0	0	1	0	−1	−1	1	0	−1	−1	1	$=(V_0 + A_{\mathrm{SCEN}})_4$
1	1	0	0	0	0	0.8	0.8	0.8	0.8	0	0	0	0	$\geq L$

Note. i denotes reservoir number; $\rho_i = 0.8$.

The vector of cost reduction factors for stage t for this example is written as $\mathbf{CRF}_t = [w_t w_{T+i}]$, $t = 1, \ldots, T$; $i = 1, 2, \ldots, K$. Note that planning horizon T and total number of reservoirs K in this example are respectively 3 and 4. The number of variables in the string formed by weights in the range from 0 to 1 is thus $7 (= 3 + 4)$.

Once the decision variable vector $\mathbf{W}^l = (w_1^l, w_2^l, w_3^l, w_4^l, w_5^l, w_6^l, w_7^l)^T$ representing a possible solution l is determined by GA, a chain of linear programming problems is solved for 14 decision variables each, whose constraints are given in Table II. In this table, the first four equations correspond to the mass balance in reservoirs 1, 2, 3 and 4 of the system, where $A_{SCEN\,k}$ represents inflow to reservoir k for scenario *SCEN*(t). The last inequality represents the load to be met by hydro generation, thermal generation and imported energy. Table III presents the lower and upper bounds assumed for the decision variables as well as the respective cost coefficients in the objective function. The sensitivity analysis on constant G was performed for 10 different random initial populations of solutions covering values of 1, 10, 100 and 1000 in the objective function of one-period LP problems whose solutions produce information for the GA fitness evaluation. Setting $G = 1$ did not produce convergence even in the maximum number of 1500 generations allowed for. Also $G = 1000$ was inadequate to have convergence of the optimization model to the optimal solution although convergence was obtained to near optimal solutions. Only constant G value of 10 always guaranteed convergence for all random populations and produced various optimal solutions in terms of decision variables with the same value of the objective function. On the other hand, $G = 100$ produced convergence to the optimal objective function value only for 8 of the 10 initial populations. Results of this analysis for these two G values are presented in the next section.

Figure 3 presents the block diagrams for GA-LP hybrid procedure involving solution of individual LP problems, GA implementation and fitness evaluation. The two stopping criteria for termination of GA employ the coefficient of variation (CV) of fitness values of the solutions in the population at each generation and the maximum number of GA generations (*nger*).

Table III. Bounds and cost coefficients for decision variables in each LP problem

Variable	TG	IMP	V_1	V_2	V_3	V_4	Q_1	Q_2	Q_3	Q_4	S_1	S_2	S_3	S_4
Lower bound	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Upper bound	50	200	500	400	500	500	100	140	100	240	500	400	500	500
Cost coefficient	1	10	$-G \cdot w_t \cdot w_4$	$-G \cdot w_t \cdot w_5$	$-G \cdot w_t \cdot w_6$	$-G \cdot w_t \cdot w_7$	0	0	0	0	0	0	0	0

Note. t denotes the stage.

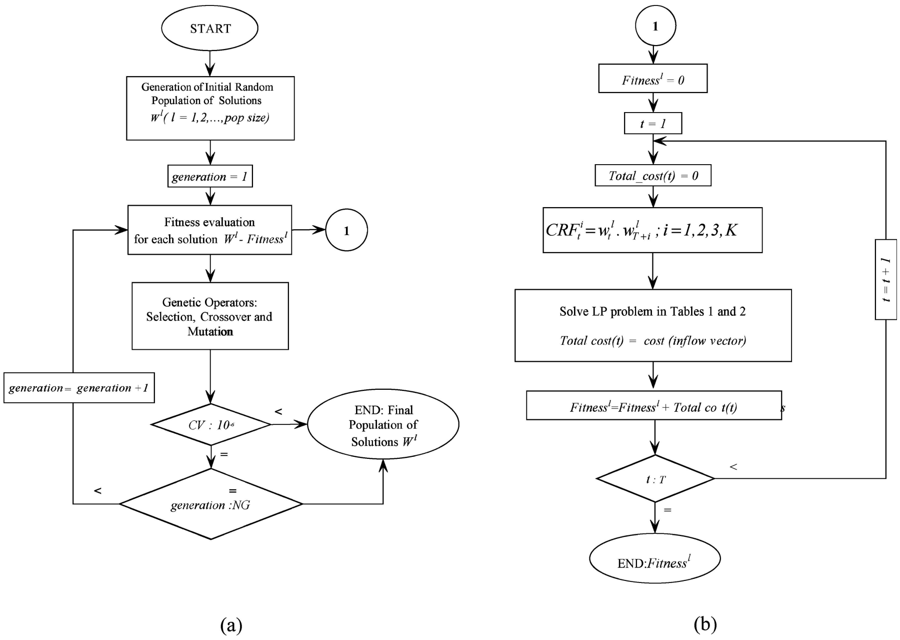

Figure 3. Block diagrams: (a) GA implementation; (b) Fitness evaluation for each solution W^l vector.

6. Results and Discussion

The sensitivity analysis showed that, even for a reduced number of decision variables, the order of magnitude of G is important in the identification of good solutions. Large G values (1000) seem to strongly penalize bad solutions, skipping their neighborhood regions on the response surface and, as a result, produce poor solutions. GA simulations suggest that G has to be only large enough to make elements of the vector $G \cdot \mathbf{CRF}_t^T$ comparable to the unit costs of thermal energy and imports.

Once the weights have been determined, they can simply be substituted into the objective function (Table III, last row) and the corresponding linear problem solved subject to the constraints of Table II, to obtain the operational decision variables such as imports, storages and releases. If any inflow forecasts are available, one can run the linear programming problem to determine future decisions. Thus, the weights are surrogates for the reservoir operating rules. They have the advantage that no *a priori* relationship between water availability and decisions such as linear operating rules need to be specified.

Tables IV and V present best alternative sets of optimal weights (w_1, w_2, w_3, w_4, w_5, w_6, w_7) for G values of 10 and 100, respectively, where the first three weights refer to the stages of the planning period and the last four refer

Table IV. Best combinations of weights for $G = 10$ in various best solutions

Solution	w_1	w_2	w_3	w_4	w_5	w_6	w_7
1	0.47	0.84	0.00	0.82	0.00	0.00	0.45
2	0.90	0.93	0.16	0.66	0.00	0.00	0.22
3	0.95	0.89	0.00	0.46	0.00	0.17	0.00
4	0.64	0.73	0.15	0.83	0.51	0.03	0.14
5	0.65	0.48	0.07	0.82	0.22	0.04	0.42
6	0.77	0.89	0.23	0.71	0.43	0.00	0.00
7	0.52	0.94	0.14	0.63	0.38	0.15	0.24

Table V. Best combinations of weights for $G = 100$ in various best solutions

Solution	w_1	w_2	w_3	w_4	w_5	w_6	w_7
1	0.78	0.34	0.00	0.31	0.09	0.00	0.00
2	0.54	0.26	0.00	0.84	0.29	0.00	0.00
3	0.54	0.11	0.00	0.84	0.29	0.00	0.00
4	0.54	0.12	0.00	0.84	0.29	0.00	0.00
5	0.72	0.57	0.00	0.31	0.07	0.00	0.00
6	0.17	0.13	0.00	0.31	0.15	0.00	0.00

to the reservoirs. In spite of several possible optimal combinations of weights in Tables IV and V for the same optimal total expected operational cost, a reduced number of corresponding operational plans were identified. For $G = 10$, three operational plans were found whose details are presented in Table VI. For $G = 100$ only one operational plan was obtained, which is the same as the third one shown in Table VI, for $G = 10$. The results in Table VI permit the evaluation of the lowest expected cost. Since a discount factor is not considered and each scenario at any stage is equally likely, the expected cost corresponding to the solutions in Tables IV or V is evaluated as $[1 \times 50 + 10 \times 0] + [1 \times 50 + 10 \times 0] + [1 \times (50 + 50 + 50 + 34.4)/4 + 10 \times (21.2 + 18 + 22 + 0)/4] = 299.1$ cost units.

Similar calculation for the results in Table VI revealed that the best solution obtained by GA are identical for all the three plans in terms of minimum expected total cost (299.1 cost units), as well as the cost at each stage of the planning period. No spill was produced during the operation indicated by the optimal solution. Despite the distinct differences in the evolution of volumes in reservoirs and flows through the turbines in different plans, the final volumes (zero) are the same at the end of the planning period. One can observe that imports of energy from neighboring systems only occur during the third (last) stage of the planning period, when the thermal capacity is completely exploited ($TG = 50$).

Table VI. Results obtained for $G = 10$ and $G = 100$ (bold values)

		Operational Plan 1				Operational Plan 2				Operational Plan 3			
Stage 1	$\mathbf{V}_f$	22.25				60.00				60.00			
		0.00				0.00				45.75			
		0.00				45.75				0.00			
		204.75				0.00				0.00			
	Q	37.75				0.00				0.00			
		89.75				52.00				6.25			
		60.00				14.25				60.00			
		0.00				121.25				121.25			
	TG	50.00				50.00				50.00			
	IMP	0.00				0.00				0.00			
Stage 2	$\mathbf{V}_f$	0.00		0.00		43.33		43.33		43.33		43.33	
		0.00		0.00		0.00		0.00		0.00		0.00	
		0.00		0.00		0.00		0.00		0.00		0.00	
		130.00		130.00		0.00		0.00		0.00		0.00	
	Q	30.25		29.25		24.67		23.67		24.67		23.67	
		36.25		34.25		30.67		28.67		76.42		74.42	
		4.00		3.00		49.75		48.75		4.00		3.00	
		117.00		121.00		82.42		86.42		82.42		86.42	
	TG	50.00		50.00		50.00		50.00		50.00		50.00	
	IMP	0.00		0.00		0.00		0.00		0.00		0.00	
Stage 3	$\mathbf{V}_f$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Q	3.00	2.00	5.00	10.00	46.33	45.33	48.33	53.33	46.33	45.33	48.33	53.33
		8.00	11.00	9.00	21.00	51.33	54.33	52.33	64.33	51.33	54.33	52.33	64.33
		4.00	3.00	2.00	8.00	4.00	3.00	2.00	8.00	4.00	3.00	2.00	8.00
		146.00	149.00	144.00	168.00	59.33	62.33	57.33	81.33	59.33	62.33	57.33	81.33
	TG	50.00	50.00	50.00	34.40	50.00	50.00	50.00	34.40	50.00	50.00	50.00	34.40
	IMP	21.20	18.00	22.00	0.00	21.20	18.00	22.00	0.00	21.20	18.00	22.00	0.00

Note. $\mathbf{V}_f$ denotes volume vector at the end of a given stage.

Finally, the results show that the simplified scheme used to represent the problem in terms of weights was adequate for the small hypothetical system studied, producing total expected cost of 299.1 units against 298.45 units reported by Pereira and Pinto (1985) obtained by SDDP.

7. Conclusions

This paper proposed and evaluated a new stochastic approach to obtain optimized decisions on the operation of reservoirs systems employing genetic algorithms and linear programming. Future inflow variability at each stage is incorporated through a treelike structure of synthetically generated inflows as SDDP. It is shown that the weights, introduced into the formulation to discourage reservoir depletion in the initial stages of the planning period, are useful parameters that can be employed in the determination of optimal releases, imports, etc. in response to future inflow predictions, without fixing *a priori* linear or non-linear rules of operation. While these weights represent incentives towards storing water as cost reduction factors (CRFs) in the linear programming problem objective function, the total expected cost of operation constitutes the objective function of the overall minimization problem by genetic algorithm.

The applicability of the proposed formulation was demonstrated through an example considering a hypothetical hydrothermal system with four reservoirs inflow realizations represented by a binary tree with depth equal to three time period which is the planning horizon of the system. The solution for the minimum additional costs of imports and thermal generation during the planning period in order to meet energy deficits is found to be very close to the SDDP solution presented by Pereira and Pinto (1985).

The hybrid scheme proposed in this paper offers some advantages as compared to SDDP. First, it avoids iterations throughout the tree of inflows saving computational effort. Second, CRF parameter values (i.e., weights) summarize system operation rules in an indirect manner any time this is necessary. This advantage of the new method is even more important as one needs to solve only one or more one-period optimization subproblems. Further, it is possible to replace LP by nonlinear programming or keep LP with an iterative procedure to deal with nonlinear formulations which may result from considerations of head dependent generation characteristics or evaporation. The new method also avoids progressive increase in the number of linear constraints resulting from "Benders Cuts" added to the LP problems during the iterative solution process of SDDP. Although the hybrid approach is less demanding in terms of memory requirements, it is computationally more time consuming in view of parallel evaluations involved in GAs. However, GA remains a promising choice for the study of hydrosystems with large number of reservoirs. The proposed model requires a user-defined normalizing constant to scale penalty terms introduced into the one-period LP problems whose value needs to be identified by sensitivity analysis. More research is required regarding automatic choice of G instead of sensitivity analysis. The authors are preparing an application of the proposed approach to the operation of an existing water supply reservoir system which may bring to light more information with regard to the normalizing constant.

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Note

1. A more realistic example differentiating explicitly between the seasons and stages in the formulation of *CRF*s shall be presented in the follow-up paper which considers a water supply reservoir system.

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