J Sign Process Syst (2016) 83:83–92
DOI 10.1007/s11265-015-1046-9

Analysis of BICM-ID Receivers Exploiting Transformations of Extrinsic Information

M. Adrat[1] **· J. Lewandowsky**[2] **· M. Tschauner**[1] **· M. Antweiler**[1]

Received: 28 January 2015 / Revised: 12 June 2015 / Accepted: 14 September 2015 / Published online: 23 September 2015

Abstract In this article we analyze a novel idea to increase the applicability of *Bit Interleaved Coded Modulation with Iterative Decoding* (BICM-ID) to legacy waveforms. One essential design parameter of BICM-ID receivers with respect to the error correcting capabilities is the symbol mapping of the digital modulation scheme. A so-called *Semi-Set Partitioning* (SSP) symbol mapping is well known to provide higher stepwise gains in robustness in every iteration than a *Gray encoded* symbol mapping. The novel approach is based on the idea to make in a first step of BICM-ID the deliberately false assumption that a well performing symbol mapping has been used at the transmitter, even though in reality a less powerful symbol mapping was applied. In a second step, the mismatch in both symbol mappings is compensated by an innovative *Transformation of Extrinsic Information* (TEI). After having reviewed the innovative TEI idea in more detail, we will discuss the fundamentals of the required signal processing. In addition, as a novelty of this article we will analyze three different ways to implement the TEI approach. Several simulation results will be shown which demonstrate the theoretically achievable performance gains.

M. Adrat
marc.adrat@fkie.fraunhofer.de

J. Lewandowsky
j.lewandowsky@unibw.de

M. Tschauner
matthias.tschauner@fkie.fraunhofer.de

M. Antweiler
markus.antweiler@fkie.fraunhofer.de

1 Fraunhofer FKIE, Wachtberg, Germany

2 University of Federal Armed Forces Munich, Neubiberg, Germany

Keywords Software defined radio · Legacy waveform · Portability · Interoperability · Extrinsic Information · Box-Plus and Box-Minus Operations

1 Introduction

The key motivation for this research project is the assumption that in the future established legacy radios as well as modern *Software Defined Radios* (SDRs) will operate together in the same mission. Our objective is to provide the operator of an SDR an added value if compared to the operator of the legacy radio even if both are using the same waveform. This added value can be expressed in, e.g., an increased robustness of the waveform and thus, in an increased communication range. In order not to impair the interoperability on the air interface with the established legacy radios in mixed mode, the actions to be taken should be applied primarily on the receiver side.

It has already been analyzed in a preliminary study [1], whether a so-called *Bit Interleaved Coded Modulation with Iterative Decoding* (BICM-ID) [2] receiver structure alone can provide the desired profits. Such a receiver is characterized in that the decoding result of the error protection mechanism is fed back to the demodulator in the form of reliability information (so-called *extrinsic information*). The latter one can exploit this extra knowledge to improve its initial detection result. By multiple, iterative exchanges of reliability information between demodulation as well as error protection significant gains can be achieved in case of an appropriate parameterization of the error protection as well as the modulation schemes.

The preliminary studies [1] have demonstrated that it is not possible to achieve any gain by means of BICM-ID [2] alone when a *Gray encoded* symbol mapping is used for modulation. Around the turn of the millennium X. Li et al. [3] have already shown that when using a so-called *Semi Set Partitioning* (SSP) symbol mapping significant gains in *Bit Error Rate* (BER) performance can be realized. But, this assumes that the *SSP* symbol mapping is used at both ends of the communication scheme, the transmitting and the receiving end. However, the established legacy radios, to which interoperability is to be maintained, usually use a *Gray encoded* symbol mapping which is optimal for non-iterative receiver structures.

As part of the research project it was now investigated whether it is possible to realize gains in robustness by an innovative BICM-ID receiver structure even for still applying a *Gray encoded* symbol mapping at the transmitter. The basic idea is to make the deliberately false assumption in the demodulation that at the transmitting end a *(modified)*[1] *SSP* symbol mapping was used. This deliberately false assumption is corrected again in a subsequent novel *Transformation of Extrinsic Information* (TEI). Some first fundamental insights into the new approach have already introduced by us in [4].

In this article, we will briefly review the result of [4] that a round-trip transformation between a *Gray encoded* symbol mapping as well as a *(modified) SSP* symbol mapping exists. A particular challenge is that these transformations are not at the *bit-level* (i.e., in GF (2), *Galois Field*), but work with reliability information (i.e., so-called *Log-Likelihood Ratios* (LLRs or L-values) in the set of real numbers $\mathbb{R}$). This matter of fact offers several ways for implementations which will be analyzed for the first time in detail in this article. In addition, it will then be demonstrated in the context of a boundary experiment that in case of *Error-Free Feedback* (EFF) from the error protection mechanism to demodulation significant gains can theoretically be realized. However, so far these gains could not be obtained by simulation with any of the above mentioned practical implementations. Further research in this field is still required.

2 Transmission System with BICM-ID

Figure 1 shows the block diagram of a transmission system employing *Bit Interleaved Coded Modulation with Iterative Decoding* (BICM-ID).

Let us assume that a binary random source generates a sequence $\underline{x}$ of N bits $x \in \{0,1\}$. A channel encoder (*Forward Error Correction*, FEC) of rate r adds redundancy which can be exploited at the receiving end of the communication scheme for error correction. The channel encoded sequence $\underline{y}$ is then bit-interleaved. Digital modulation of order M maps $log_2(M)$ consecutive bits of the bit-interleaved sequence into a sequence $\underline{s}$ of symbols s.

After transmission of the individual symbols s over an *Additive White Gaussian Noise* (AWGN) channel with known channel quality E_S/N_0, a sequence $\underline{z}$ of noisy elements $z \in \mathbb{C}$ is received. E_S is the mean energy per symbol s and N_0 the noise power spectral density of the AWGN.

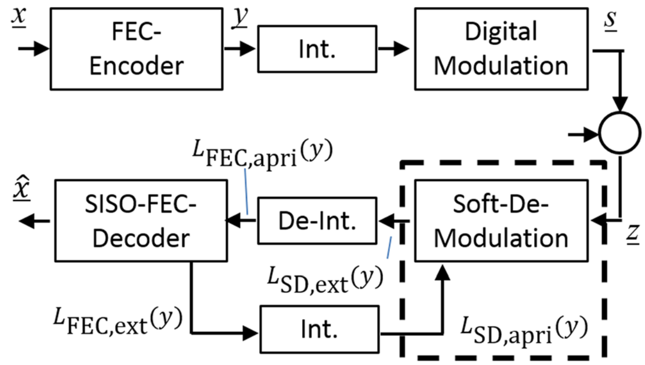

Figure 1 Block diagram of a transmission system with bit interleaved coded modulation with iterative decoding (BICM-ID).

The aim of the BICM-ID receiver is to recover the originally sent bits x as good as possible from the received sequence $\underline{z}$. For this purpose, the inner component of the iterative process, i.e., *Soft-Demodulation* (SD), determines so-called *extrinsic information* in terms of L-values $L_{SD,ext}(y)$ individually for each coded bit y. Please notice, that the sign of these L-values indicates the binary hard-decision in bi-polar format (i.e., a logical 0 becomes a bi-polar $+1$ and a logical 1 becomes a bi-polar -1) while the magnitude represents the reliability. Thus, the L-values can take any real value $L(y) \in \mathbb{R}$.

After de-interleaving the L-values $L_{SD,ext}(y)$ of soft-demodulation become a priori input knowledge $L_{FEC,apri}(y)$ for the *Soft-Input/Soft-Output* (SISO) FEC-Decoder. On the one hand, the SISO-FEC-Decoder can provide the hard decoded estimate $\hat{x}$ for the possibly sent data bit x. On the other hand, the SISO-FEC-Decoder can provide its decoding gain in terms of $L_{FEC,ext}(y)$, or the interleaved counterpart $L_{SD,apri}(y)$, as a priori knowledge to soft-demodulation. This new extra information helps soft-demodulation to refine the original L-values $L_{SD,ext}(y)$. In case of a proper configuration of all the system parameters, several iterations can provide reliability gains such that the number of residual bit errors in $\hat{x}$ decreases steadily.

The comparison of the originally sent sequence $\underline{x}$ and its estimated reconstruction $\hat{\underline{x}}$ allows to determine the *Bit Error Rate* (BER) as a function of the channel quality E_S/N_0.

2.1 Simulation Examples

Figure 2 shows the simulation examples for two different configurations of system parameters. In both examples, a terminated convolution code of rate $r=3/4$ with generator polynomial $G(133,171)_8$ and puncturing pattern *(1,1,0; 1,0,1)* is used to encode a sequence $\underline{x}$ of $N=894$ bits (plus 6 bits for termination). In addition, in both examples 8-PSK (Phase Shift Keying) is used for digital modulation. The key difference between both examples is the symbol mapping which is used in digital modulation.

[1] The SSP symbol mapping used here is not identical to the one proposed in [3], but it offers the same *Harmonic Mean* $d_h^2=2.877$ (see also Section 2.1).

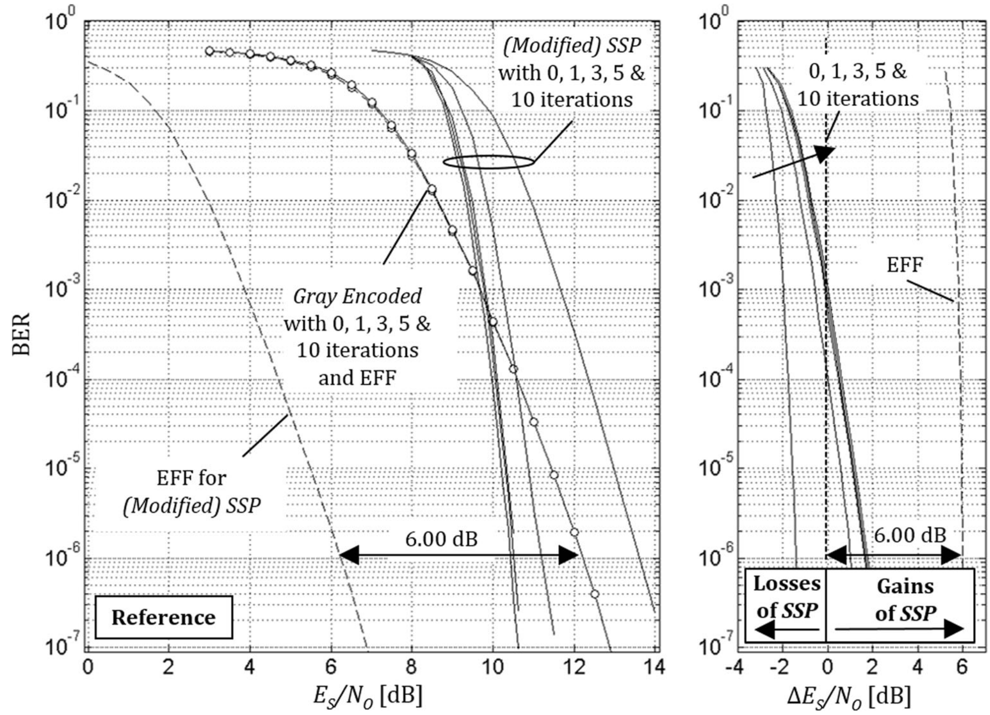

Figure 2 Bit error rate (BER) curves for BICM-ID receivers with different symbol mappings.

On the one hand, a *Gray encoded* symbol mapping is used (see Fig. 3a). On the other hand, a *(modified) SSP* symbol mapping is applied (see Fig. 3b).

The simulation results in the left part of Fig. 2 show that both system configurations provide different *BER-over-*E_S/N_0 behavior. A system design with a *Gray encoded* symbol mapping works best if no iterations are carried out, i.e., if soft-demodulation and SISO-FEC-decoding are realized only once. However, with a *Gray encoded* symbol mapping no noteworthy gains in BER can be realized by higher numbers of iteration. The BER performance remains the same for different numbers of iteration.

In contrast, significant gains in BER can be realized by several iterations if a *(modified) SSP* symbol mapping is applied. The dashed-curve illustrates the best possible performance in case of *Error-Free Feedback* (EFF) of reliability information from the SISO-FEC-decoder to the soft-demodulator. For a BER of 10^{-6} gains in E_S/N_0 of up to 6.00 dB are theoretically achievable. The gains (and losses for small numbers of iterations and BER values greater than ~10^{-3}) for all BER values are shown in the right part of Fig. 2.

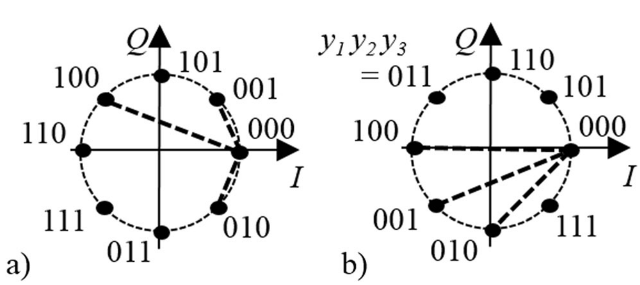

Figure 3 8-PSK signal constellation sets with (**a**) *Gray encoded* symbol mapping, (**b**) *(modified) SSP* symbol mapping.

3 Novel Idea with Transformation of Extrinsic Information

Obviously, in the BER regions which are relevant for a practical application, i.e., typically $BER < 10^{-3}$ for voice and $BER < 10^{-6}$ for data, a BICM-ID receiver with a *(modified) SSP* symbol mapping provides a much better performance than a receiver with a *Gray encoded* symbol mapping. One of the reasons is a higher so-called *Harmonic Mean* d_h^2 [3]. The *Harmonic Mean* is a measure which is directly related to the EFF case. It determines a measure for the average *Euclidean Distance* between these signal constellation points which differ in exactly one bit position. The dashed lines in Fig. 3 illustrate an example for the symbol mapping *000*. It can be seen that on average the distances are higher for the *(modified) SSP* symbol mapping if compared to the *Gray encoded* symbol mapping. Thus, the question arises if we can exploit the higher *Harmonic Mean* of a BICM-ID receiver with a *(modified) SSP* symbol mapping even if a *Gray encoded* symbol mapping is used at the transmitter ($d_h^{2,SSP}$=2.877 is greater than $d_h^{2,\,GRAY}$=0.809).

Our novel innovative idea tries to exploit the higher *Harmonic Mean* d_h^2 of a *(modified) SSP* symbol mapping on communication links where actually *Gray encoded* symbol mappings are used. For that purpose, we make the deliberately false assumption in the soft-demodulation that at the transmitting end a *(modified) SSP* symbol mapping was used. This deliberately false assumption is corrected again in a subsequent novel *Transformation of Extrinsic Information* from $L_{SD,ext}^{SSP}(y)$ to $L_{SD,ext}^{GRAY}(y)$. The respective inverse transformation needs to be applied to the reliability information $L_{SD,apri}^{GRAY}(y)$ (becomes $L_{SD,apri}^{SSP}(y)$), which is fed back from the SISO-FEC-decoder.

3.1 Transformations in case of Hard-Decision Decoding

In order to simplify matters and to improve comprehensibility of the new scheme with *Transformations of Extrinsic Information*, let us start with the extreme case of *Hard-Decision* decoding. In that case we can focus on the coded bits $y \in \{0,1\}$ instead of considering the reliability values $L(y) \in \mathbb{R}$. The transfer of our considerations to these L-values will follow in Section 3.2.

If we want to use a *(modified) SSP* symbol mapping in *Soft-Demodulation* even though a *Gray encoded* symbol mapping was used at the transmitter, the *Transformation* block in Fig. 4 needs to realize the mapping between both domains. In case of *Hard-Decision* decoding, this can simply be done by matrix operations in *GF(2)*, e.g., for the two 8-PSK signal constellation sets shown in Fig. 3 we have

$$y_{GRAY} = y_{SSP} \cdot \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{1}$$

For instance, the symbol mapping y_{SSP}= *(101)* (see Fig. 3b) becomes y_{GRAY}= *(001)* (see Fig. 3a). Thus, the matrix on the right hand side of Eq. (1) can be used to transform symbol labels from the *(modified) SSP* domain to the *Gray encoded* domain. It is easy to prove that the inverse for the specific example shown in Fig. 3 is given by

$$y_{SSP} = y_{GRAY} \cdot \begin{pmatrix} 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}. \tag{2}$$

Thus, as a <u>first important intermediate result</u> we can conclude that a transformation between both domains exists in case of *Hard-Decision* decoding.

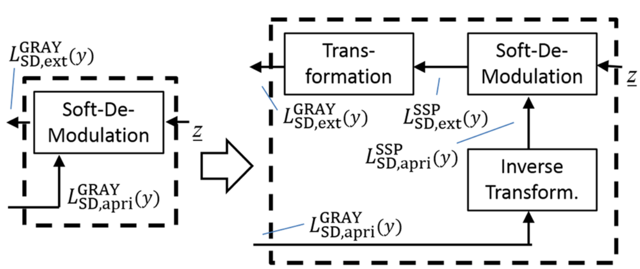

Figure 4 Soft-Demodulation with additional transformations at input and output.

3.2 Transformations in case of Soft-Decision Decoding

However, BICM-ID receivers are based on *Soft-Decision* Decoding. That means reliability information in terms of L-values is exchanged between the soft-demodulation and SISO-FEC-decoding components. In conclusion, simple matrix computations like in Eqs. (1) and (2) cannot be used to transform from the *(modified) SSP* domain into the *Gray encoded* domain and vice versa. However, techniques which are well known from *Turbo-decoding of binary block and convolutional codes* (e.g., [5]) and/or *Low Density Parity Check* (LDPC, e.g., [6]) decoding can be exploited.

3.2.1 The Box-Plus Operator ⊞

In [5] J. Hagenauer et al. have introduced the so-called *Box-Plus operation* ⊞ for Turbo-decoding of binary block and convolutional codes. The *Box-Plus operation* ⊞ is defined as

$$\begin{aligned} L(x_A \otimes x_B) &= L(x_A) \boxplus L(x_B) \\ &= \log \frac{1 + \exp(L(x_A) + L(x_B))}{\exp(L(x_A)) + \exp(L(x_B))}. \end{aligned} \tag{3}$$

Equation (3) allows determining the reliability value for the combination of two L-values $L(x_A)$, $L(x_B) \in \mathbb{R}$. Simply speaking, applying the *Box-Plus operation* ⊞ to L-values $L(x_A) \boxplus L(x_B)$ corresponds to the XOR-combination ⊗ of the represented binary values $x_A \otimes x_B$.

3.2.2 The Box-Minus Operator ⊟

The inverse operation to the *Box-Plus operation* ⊞ was at first introduced by T. Clevorn et al. [6] as *Box-Minus operator* ⊟ in the context of efficient belief propagation decoding of LDPC codes. This *Box-Minus operator* ⊟ is defined as

$$L(x_A) \boxminus L(x_B) = \log \frac{1 - \exp(L(x_A) + L(x_B))}{\exp(L(x_A)) - \exp(L(x_B))}. \tag{4}$$

Please note, with the *Box-Minus operator* ⊟ we can ensure that

$$(L(x_A) \boxplus L(x_B)) \boxminus L(x_B) = L(x_A). \tag{5}$$

It is also important to note that the *Box-Minus* operation, as it is defined in Eq. (4), provides a real numbered output value $L(x_A) \boxminus L(x_B) \in \mathbb{R}$ if and only if $|L(x_A)| < |L(x_B)|$. This side constraint is fulfilled per definition in the original context of decoding LDPC codes [6]. However, it remains to be the most challenging issue in the context of *Transformation of Extrinsic Information* which is discussed in the present article.

3.2.3 Different Options for Transformations in case of Soft-Decision Decoding

In this section we will discuss different options for the *Transformations of Extrinsic Information* in case of *Soft-Decision* decoding. For this purpose, we have to apply the *Box-Plus* and *Box-Minus* operations (see Sections 3.2.1 and 3.2.2) to our findings for *Hard-Decision* decoding (see Section 3.1).

Option A – Implementation with Box-Plus Operations only In a first option, we consider the two transformation matrices given in Eqs. (1) and (2) to be independent rate *r= 1* non-systematic linear block codes. With the assumption of mutual independence of both matrices we can directly apply the *Box-Plus* operation individually to both. Thus, Eq. (1) becomes (the lower index "*SD,ext*" is skipped to enhance readability)

$$\begin{aligned} L^{GRAY}(y_1) &= L^{SSP}(y_1) \boxplus L^{SSP}(y_3) \\ L^{GRAY}(y_2) &= L^{SSP}(y_1) \boxplus L^{SSP}(y_2) \boxplus L^{SSP}(y_3) \\ L^{GRAY}(y_3) &= L^{SSP}(y_2) \boxplus L^{SSP}(y_3) \; . \end{aligned} \tag{6}$$

in case of *Soft-Decision* decoding. Equation (6) determines the set of equations which need to be applied to the *L*-values in order to perform the *Transformation* (see Fig. 4) from the *(modified) SSP* domain into the *Gray encoded* domain.

Analogously, the *Inverse Transformation* (see also Fig. 4) from the *Gray encoded* domain into the *(modified) SSP* domain according to Eq. (2) becomes (the lower index "*SD,apri- is skipped to enhance readability)*

$$\begin{aligned} L^{SSP}(y_1) &= L^{GRAY}(y_2) \boxplus L^{GRAY}(y_3) \\ L^{SSP}(y_2) &= L^{GRAY}(y_1) \boxplus L^{GRAY}(y_2) \\ L^{SSP}(y_3) &= L^{GRAY}(y_1) \boxplus L^{GRAY}(y_2) \boxplus L^{GRAY}(y_3) \; . \end{aligned} \tag{7}$$

Please notice, while Eq. (2) is the inverse of Eq. (1) in *Hard-Decision* decoding, the set of Eq. (7) does not describe the exact mathematical inverse for the set of Eq. (6) in *Soft-Decision* decoding anymore. It is easy to prove that

$$(x_A \otimes x_B) \otimes x_B = x_A \tag{8}$$

but usually,

$$(L(x_A) \boxplus L(x_B)) \boxplus L(x_B) \neq L(x_A) \; . \tag{9}$$

Consequently, the assumption of mutual independence of both matrices causes a loss of mathematical correctness. Thus, with respect to Eq. (5) it is important to use the *Box-Minus* operation as well.

Option B – Implementation with Box-Minus Operations in the Inverse Transformation In a second option, we use the same set of Eq. (6) to perform the *Transformation* (see Fig. 4) from the *(modified) SSP* domain into the *Gray encoded* domain. Hence, in order to ensure that the *Inverse Transformation* is described by a set of equations which is mathematically inverse to (6), it is easy to prove that Eqs. (2) and (7) respectively, must be re-written as

$$\begin{aligned} L^{SSP}(y_1) &= L^{GRAY}(y_2) \boxminus L^{GRAY}(y_3) \\ L^{SSP}(y_2) &= L^{GRAY}(y_2) \boxminus L^{GRAY}(y_1) \\ L^{SSP}(y_3) &= L^{GRAY}(y_1) \boxminus \left(L^{GRAY}(y_2) \boxminus L^{GRAY}(y_3)\right) \; . \end{aligned} \tag{10}$$

Thus, as a second important intermediate result we can conclude that also in case of *Soft-Decision* decoding transformations between both domains, the *(modified) SSP* domain as well as the *Gray encoded* domain, exist. However, in order to make sure that both transformations are exactly inverse to each other the *Box-Minus* operation $\boxminus$ becomes necessary.

However, introducing the *Box-Minus* operation $\boxminus$ reveals some new challenges for a practical implementation because it provides real-valued outputs only for specific relations of the inputs (i.e., $|L(x_A)| < |L(x_B)|$). Unfortunately, this cannot be guaranteed in the BICM-ID receiver because of the *Soft-Demodulation* block which is located between both transformations (see Fig. 4).

Option C – Implementation with Box-Plus Operations in the Inverse Transformation The third option is complementary to Option B. That means, if the set of Eq. (7) is used to perform the *Inverse Transformation* (see Fig. 4) from the *Gray encoded* domain into the *(modified) SSP* domain, then *Box-Minus* operation needs to be applied in the *Transformation.* From that follows

$$\begin{aligned} L^{GRAY}(y_1) &= L^{SSP}(y_3) \boxminus L^{SSP}(y_1) \\ L^{GRAY}(y_2) &= L^{SSP}(y_1) \boxminus \left(L^{SSP}(y_3) \boxminus L^{SSP}(y_2)\right) \\ L^{GRAY}(y_3) &= L^{SSP}(y_3) \boxminus L^{SSP}(y_2) \; . \end{aligned} \tag{11}$$

The key issues and challenges with Option C are the same as for Option B. They are just shifted from the *Inverse Transformation* in Fig. 4 to the *Transformation*.

Summary of Options Table 1 shows a summary of the sets of equations which are used by the different options under consideration in this article.

Option A has the benefit that is utilizes the *Box-Plus* operation only which allows avoiding the issues and challenges with the *Box-Minus* operation. However, Option A suffers

Table 1 Summary of options under consideration.

	Transformation (see Fig. 4)	Inverse Transformation (see Fig. 4)
Option A	Eq. (6)	Eq. (7)
Option B	Eq. (6)	Eq. (10)
Option C	Eq. (11)	Eq. (7)

from the fact that the sets of Eqs. (6) and (7) are not exactly mathematically inverse to each other.

In contrast, the set of Eqs. (6) and (10) of Option B as well as (7) and (11) of Option C have proven to be mathematically inverse, but in both options we have to face the issue that the *Box-Minus* operation provides real-valued outputs only under side constraints which are not fulfilled by nature.

Before analyzing the BER performances of all these three options in Section 4, we will discuss a first attempt to solve the *Box-Minus* operation issue next.

3.2.4 The Box-Minus Operation Issue in the EFF Case

For that purpose, let us consider the extreme case of *Error-Free Feedback* (EFF). On one hand, the EFF case will give us an idea about the best possible theoretically attainable performance of the BICM-ID receiver. On the other hand, it allows us avoiding the *Box-Minus* operation issue. In the EFF case the L-values $L_{SD,apri}^{GRAY}(y)$ take the values $\pm\infty$ (i.e., $+\infty$ for $y=0$ and $-\infty$ for $y=1$). Table 2 summarizes the results of the *Box-Minus* operation according to Eq. (4) for all combinations of $L(x_A)$ and $L(x_B)$ in the EFF case.

Thus, in the EFF case we can use a lookup table (see Table 2) instead of implementing the *Box-Minus* operation as defined in Eq. (4).

3.2.5 First Attempt to Solve the Box-Minus Operation Issue

The EFF case is an extreme case that gives insights in the best possible theoretically attainable performance of the BICM-ID receiver. For a practical implementation under regular conditions it remains to be a challenging task to provide solutions for situations in which $|L(x_A)| \nless |L(x_B)|$. As a first attempt, we propose to replace all these *Box-Minus* operations of Eqs. (10) or (11) by *Box-Plus* operations whenever $|L(x_A)| \nless |L(x_B)|$. For instance, the set of Eq. (10) becomes

$$\begin{array}{lcl} L^{SSP}(y_1) & = & L^{GRAY}(y_2) \boxplus L^{GRAY}(y_3) \\ L^{SSP}(y_2) & = & L^{GRAY}(y_2) \boxminus L^{GRAY}(y_1) \\ \multicolumn{3}{l}{L^{SSP}(y_3) = L^{GRAY}(y_1) \boxminus \left(L^{GRAY}(y_2) \boxplus L^{GRAY}(y_3)\right)} \end{array} \tag{12}$$

if $L^{\text{GRAY}}(y_2) > L^{\text{GRAY}}(y_3)$. Note, on a case-by-case decision we only replace the operation and not the entire line in which $|L(x_A)| \nless |L(x_B)|$ (see, e.g., last line of Eq. (12)). Thus, in certain situations we propose to use the *Box-Plus* operation instead of using the *Box-Minus* operation. Of course, this means again a loss of mathematical correctness as already explained in Section 3.2.3 (see e.g., Eq. (9)).

Anyways, the main purpose of this article is to review the basic idea of an innovative BICM-ID receiver with a novel *Transformation of Extrinsic Information* [4] and to perform an analysis of various implementation options. For this we have introduced the fundamental math in Sections 3.1, 3.2.1 and 3.2.2 as well as an approach to determine the best possible theoretically attainable performance in Section 3.2.4. The solution for the *Box-Minus* operation issue proposed in Section 3.2.5 may just be considered as a first attempt allowing a first implementation for simulation purposes. Better approaches might exist. This is a matter of ongoing research work at our institutes.

Table 2 Results of the $\boxminus$ operation in the EFF case.

$L(x_A)$	$L(x_B)$	$L(x_A) \boxminus L(x_B)$
$+\infty$	$+\infty$	$+\infty$
$+\infty$	$-\infty$	$-\infty$
$-\infty$	$+\infty$	$-\infty$
$-\infty$	$-\infty$	$+\infty$

4 Simulation Results and Analysis of Different Options

Figures 5, 6, and 7 show the simulation results for the different options of novel BICM-ID receiver with a *Transformation of Extrinsic Information* (TEI, see right part of Fig. 4 and Table 1). The simulation settings are the same as in Section 2.1 (see Fig. 2) for the classic BICM-ID schemes with a standard *Soft-Demodulation* block (see left part of Fig. 4).

4.1 Simulation Results for Option A

The left part of Fig. 5 shows the BER curves for Option A, i.e., for the implementation of the *Transformation* and *Inverse Transformation* using *Box-Plus* operations only.

Obviously, the BER curve for the EFF case of the new BICM-ID receiver with *Transformation of Extrinsic Information* (TEI) outperforms the BICM-ID receiver with a classic *Soft-Demodulator* for a *Gray-encoded* symbol mapping considerably. For instance, for a BER of 10^{-6} gains in E_S/N_0 of up to 5.51 dB are theoretically achievable (see right part of Fig. 5).

However, we are not able to realize these theoretical gains with Option A. Like the simulation results for the *(modified) SSP* symbol mapping shown in Fig. 2 there is an expected loss in *BER-over-*E_S/N_0 performance if we realize *Soft-Demodulation* and *SISO-FEC* decoding only once. One reason for this loss can be found in the fact that both transformations are not exactly inverse to each other when we use *Box-Plus* operations only. Anyways, but unlike the simulation results for the *(modified) SSP* symbol mapping the performance does not improve for higher numbers of iteration. Unfortunately, the BER behavior remains the same for different numbers of iteration.

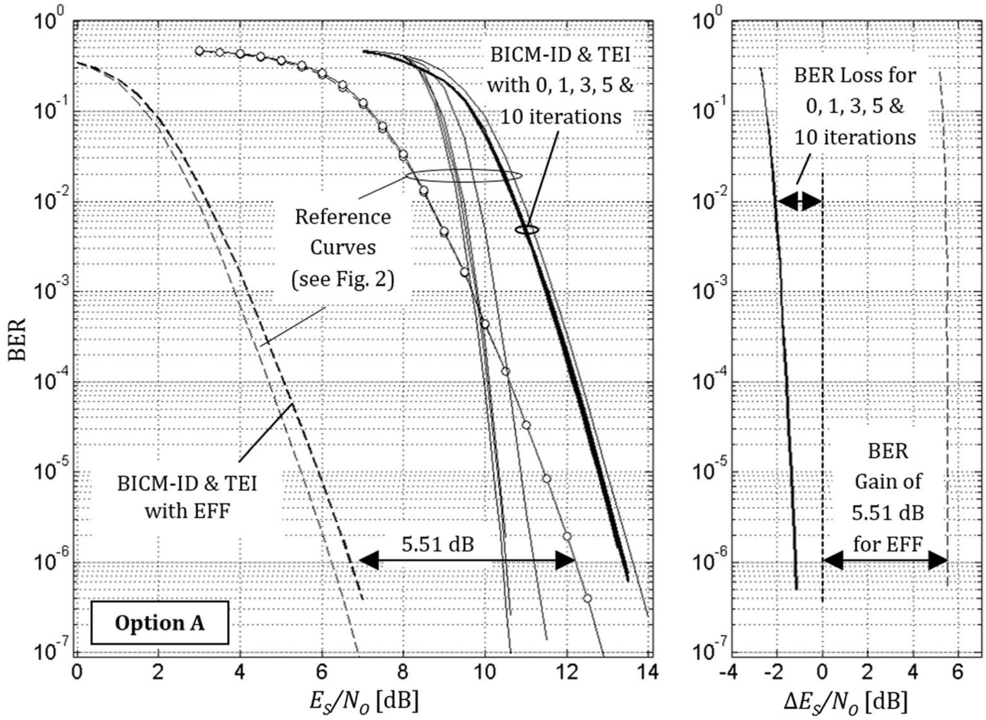

Figure 5 BER curves for a BICM-ID receiver with *transformation of extrinsic information* (TEI) – Option A.

4.2 Simulation Results for Option B

Figure 6 shows the BER curves for Option B, i.e., for the implementation of the *Inverse Transformation* (see Fig. 4) considering *Box-Minus* operations.

It can easily be seen that the performance is more or less the same as for Option A. The EFF curve promises a noteworthy theoretically achievable gain (e.g., for a BER of 10^{-6} gains in E_S/N_0 of up to 5.51 dB), however so far we were not able to realize any of these theoretical gains. One reason is that the first attempt to solve the *Box-Minus* operation issue (as introduced in Section 3.2.5) is not the most appropriate solution. No additional gains can be achieved by the iterations. Further research work (like an EXIT-chart analysis [7]) is necessary to

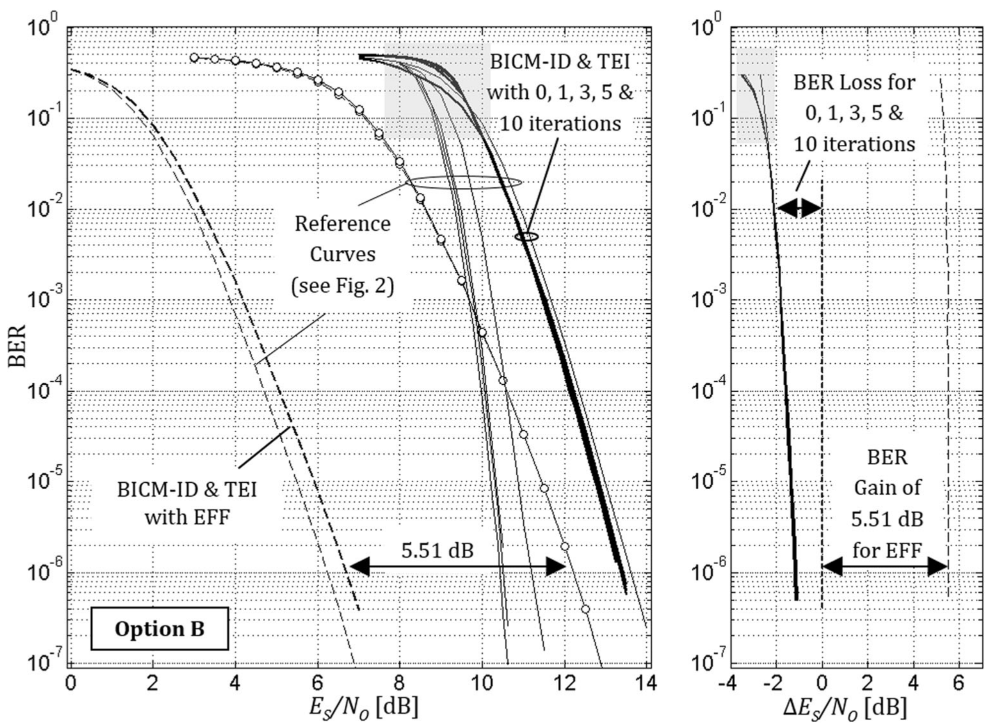

Figure 6 BER curves for a BICM-ID receiver with *transformation of extrinsic information* (TEI) – Option B.

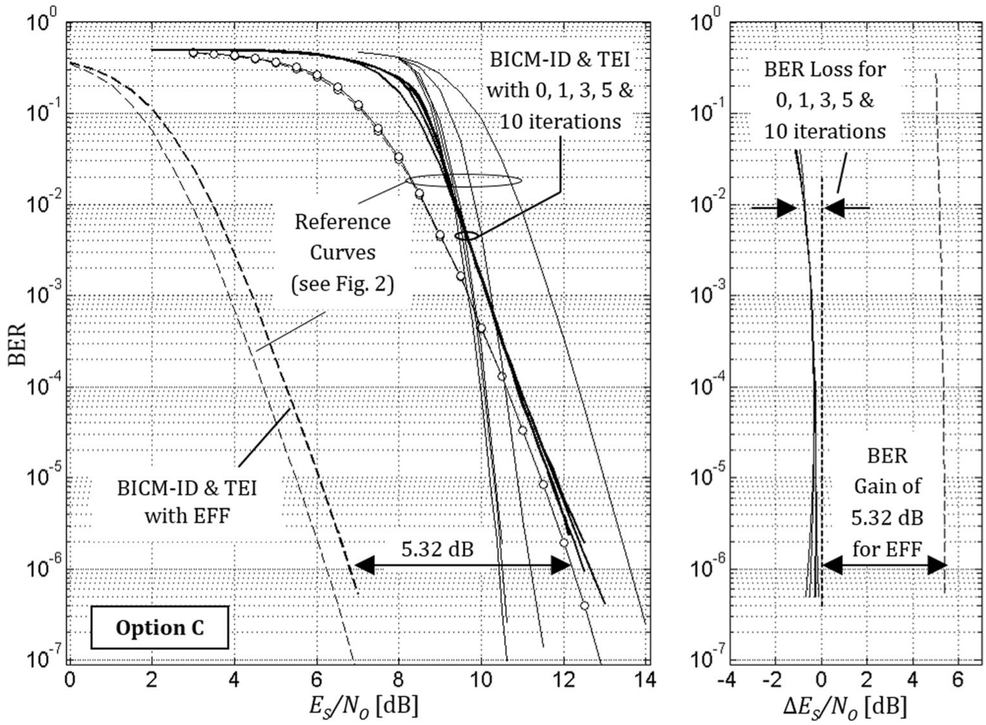

Figure 7 BER curves for a BICM-ID receiver with *transformation of extrinsic information* (TEI) – Option C.

find a better solution for the challenging *Box-Minus* operation issue.

Please notice, the only difference between the simulation results for Options A and B can be found the small grey-shaded area. The best result is achieved for the initial iteration (i.e., 0 iterations). For higher numbers of iterations the BER performance is slightly less. From this observation it can be concluded that the first attempt to solve the *Box-Minus* operation issue is not best suited because it introduces some additional errors.

4.3 Simulation Results for Option C

Finally, Fig. 7 shows the BER curves for Option C, i.e., for the complementary approach to Option B in which we consider *Box-Minus* operations in the implementation of the *Transformation* (see Fig. 4).

Obviously, the BER performance of Option C looks different from the results for Options A and B. For the first time we have found an implementation which allows to approach the reference curves of the classical *Gray encoded* system quite closely. However, still none of the BER curves outperforms this reference.

In addition, the remaining principle behavior is the same as for the other two options. The EFF curves promises significant gains (e.g., for a BER of 10^{-6} gains in E_S/N_0 of up to 5.32 dB), but neither the initial iteration nor any higher number of iteration can close the gap to the EFF curve. Again, like in Option B the initial iteration is slightly better than the other ones.

Thus, from Figs. 5, 6 and 7 it can be concluded as a third important intermediate result that Option C is the favorable implementation approach because it provides the best BER performance of all three options. Please note, only Option B was discussed in [4]. However, even though the BER curves of Option C come close to the reference and even though the EFF curve of Option C promises considerable gains, none of the curves outperforms the reference and none of the promised gains have been realized so far. The first attempt to solve the *Box-Minus* operation issue (see Section 3.2.5) was not powerful enough.

4.4 Outlook

In our still ongoing research, we have already done a similar analysis for 16-QAM modulation as well. The principle behavior shown by the respective simulation results is the same as described in this article for 8-PSK modulation.

Thus, in a next step we will perform an EXIT-chart analysis [7] (*Extrinsic Information Transfer,* EXIT) of all options. The EXIT-chart analysis tool will allow us visualizing the convergence behavior of the BICM-ID process. So-called *EXIT characteristics* for the *Soft-Demodulator* as well as the *SISO-FEC Decoder* will determine bounds for a so-called *Decoding Trajectory* which illustrates the stepwise increase of *extrinsic information* in every iteration. Based on this visualization, we hope to much better understand the reasons for the absence of gains by the iterations and to propose a more powerful solution for the *Box-Minus* operation issue.

Typically, there are two effects which can prohibit such gains. The first one is a flat *EXIT characteristic* of *Soft-Demodulation* (like for the *Gray encoded* symbol mapping) and the second one is an intersection of both *EXIT characteristics*. The first reason can be neglected here because the EFF performance already indicates theoretically achievable performance improvements. Thus, we can expect that the *EXIT characteristic* of the *Soft-Demodulation* in combination with the *Transformation* and the *Inverse Transformation* (see Fig. 4) is not flat anymore. If an intersection of both EXIT characteristics turns out to be the limiting factor, then a (slightly) different system configuration typically helps.

In addition, we will investigate in detail the reasons for the better BER performance of Option C if compared to the one of Option B.

5 Conclusions

In this article we have analyzed different approaches to implement a novel idea for BICM-ID receivers with innovative *Transformations of Extrinsic Information*. The key motivation for introducing the new signal processing was to provide an added value (higher robustness, longer communication ranges) to operators of modern SDRs while preserving interoperability to legacy equipment. For this purpose, in a first step, the deliberately false assumption is made that a well performing symbol mapping has been used at the transmitter, even though in reality a less powerful symbol mapping was applied. In a second step, the mismatch in the symbol mappings at the transmitter and receiver is compensated by the innovative *Transformation of Extrinsic Information*.

After having reviewed the fundamentals of the required novel signal processing, we have discussed several options for implementations. In addition, we have demonstrated by simulation that the theoretically achievable performance gains in the *Error-Free Feedback* case are considerable. However, for a practical implementation under regular conditions the *Box-Minus* operation turned out to be a critical element. Finding a proper solution for the *Box-Minus* operation issue is still a matter of ongoing research work.

References

1. Leduc, J., Adrat, M., Antweiler, M., & Elders-Boll, H. (2010). *Legacy waveforms on software defined radios: benefits of advanced digital signal processing* (Proc. of NATO RTO information systems technology panel symposium (IST - 092 / RSY - 022)). Poland: Breslau.
2. Li, X., & Ritcey, J. A. (1997). Bit interleaved coded modulation with iterative decoding. *IEEE Communication Letters, 1*(6), 169–171.
3. Li, X., Chindapol, A., & Ritcey, J. A. (2002). Bit-interleaved coded modulation with iterative decoding and 8-PSK signaling. *IEEE Transactions on Communications, 50*(8), 1250–1257.
4. Adrat, M., Osten, T., Tschauner, M., Antweiler, M., Lewandowsky, J. (2014). *Novel Transformations of Extrinsic Information applied to Innovative BICM-ID Receivers: Fundamentals and Limits*. Proc. of SDR15-WInnComm-Europe, Rome (Italy).
5. Hagenauer, J., Offer, E., & Papke, L. (1996). Iterative decoding of binary block and convolutional codes. *IEEE Transactions on Information Theory*, 429–445.
6. Clevorn, T., & Vary, P. (2005). The box-minus operator and its application to Low-complexity belief propagation decoding. *Proceedings of IEEE Vehicular Technology Conference (VTC Spring), 1*, 687–691.
7. ten Brink, S. (2001). Convergence behavior of iteratively decoded parallel concatenated codes. *IEEE Transaction on Communications*, 1727–1737.

Marc Adrat received his Dipl.-Ing. degree in electrical engineering and the Dr.-Ing. degree (PhD) from RWTH Aachen University, Germany, in 1997 and 2003, respectively. Since April 2005, he is with the Fraunhofer Institute for Communication, Information Processing and Ergonomics (FKIE). He is the head of the Software Defined Radio Research Group in the Communication System department. His current research interests include software defined radio, cognitive radio, (military) waveform design as well as concepts for waveform development environments. Matters of particular interest are wideband networking waveforms (like COALWNW) as well as waveform development environments for portable, interoperable and energy-efficient waveforms.

Jan Lewandowsky received his M.Sc. degree in Communication Technology from the University of the Federal Armed Forces in Munich in 2011. During his further work at this university, he worked on a collaborative research project with the Fraunhofer Institute for Communication, Information Processing and Ergonomics (FKIE) from 2012 until 2014. He joined the Institute of Communications at the Hamburg University of Technology in 2014, where his current research activities focus on practical applications of information theory in channel coding and in particular in the decoding of LDPC codes.

Matthias Tschauner received the Dipl.-Ing. degree in electrical engineering from the RWTH Aachen University in 2011. Since 2011, he is a research assistant within the Communication Systems department at the Fraunhofer Institute for Communication, Information Processing and Ergonomics (FKIE). His research interests include the areas of software defined radio technology, wireless communication systems, and waveform design. A matter of particular interest includes the area of digital modulation and coding theory.

Markus Antweiler received the Dipl.-Ing. and Dr.-Ing. (PhD) degrees from the RWTH Aachen University in 1986 and 1992, respectively. The focus in his industry career was on system design and verification of digital communication transceivers and implementation in ASICs and FPGA technology. Projects he has worked for were in the area of digital modulation/demodulation and coding/decoding for satellite communication, microwave links, cellular and wireless communication systems. In 2004, he joined the Fraunhofer Institute for Communication, Information Processing and Ergonomics (FKIE). Dr. Antweiler is heading the Communication Systems department in the Fraunhofer FKIE. His current interests are on tactical communications with focus on networks, software defined radios and reconnaissance of radio systems.