The effects of 3D-representation instruction on composite-solid surface-area learning for elementary school students

Yao-Ting Sung · Pao-Chen Shih · Kuo-En Chang

Abstract Providing instruction on spatial geometry, specifically how to calculate the surface areas of composite solids, challenges many elementary school teachers. Determining the surface areas of composite solids involves complex calculations and advanced spatial concepts. The goals of this study were to build on students' learning processes for basic and composite solids and employ Google SketchUp, an Internet resource tool, to develop and implement surface-area instructional and learning strategies (SAILS) for composite solids, and then measure its effect on learning achievement and attitudes. The fifth-grade students ($N = 111$) who were enrolled in this study were divided into an experimental and a control group. The experimental group ($N = 56$) received SAILS instruction, whereas the control group ($N = 55$) received traditional instruction. The results indicated that students who received SAILS instruction exhibited better performance on both immediate and maintained surface-area learning achievement tests compared to those who received traditional instruction; furthermore, this effect was more prominent among boys than girls. Low- and moderate-ability students who received SAILS instruction exhibited significantly greater improvement of attitudes toward learning mathematics compared to those receiving traditional instruction with physical teaching aids.

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Landscape architecture and interior design objects encountered in everyday surroundings rarely are simple cubes or rectangles. Instead, their complex shapes can be described as a composite of solid objects. Therefore, being able to calculate the surface areas of composite solids is useful in certain everyday tasks such as estimating the amount of paint required for a house or the number of tiles necessary for tile installation. The study of surface area is an important three-dimensional (3D) geometry topic in elementary school, and many countries include 3D geometry in curriculum standards, such as the American Common Core State Standards Initiative (http://www.corestandards.org) and the British Assessing Pupils' Progress (http://www.education.gov.uk) scheme.

Learning how to calculate the surface area of 3D objects involves an integration of spatial geometry and quantity, which students often find difficult (Battista and Clements 1996; Ben-Chaim et al. 1985) because many students cannot successfully convert two-dimensional (2D) views into 3D images. According to Piaget's theory of cognitive development, these elementary school students are in the concrete operational period, wherein they learn by physical features such as shapes, colors, and sizes (Mayer 1987; Piaget and Inhelder 1956). Thus, most elementary school students only have the ability to summarize the properties of a picture or image, but are unable to perform further abstract comparisons. In other words, when a representation is not sufficiently concrete, students are more likely to experience learning difficulties (Chang et al. 2007; van Hiele 1986).

Composites solids are objects that are composed of two or more basic solids (e.g., cubes, cuboids, cylinders, balls, cones, or spheres). One basic solid can be fused (by adding or subtracting) to another basic solid to form a new composite solid, such as the solids in Fig. 1. This current study focuses on composite solids composed of the basic solids of cubes and cuboids, including concave and convex objects (Fig. 2). The surface area of a composite solid is the sum of the surface of all the coverage area. The calculation of surface area is often included under geometry and number and quantity topics in elementary school mathematics syllabi, and involves the integration of space and area concepts (Battista 2003; French 2004). For elementary and middle school students, the learning of surface area generally centers on basic solids, or composite solids composed of basic solids. These objects are considered simpler forms of composite solids.

Surface-area learning in 3D composite solids

Learning the areas of geometric figures generally begins with figures in 2D planes (e.g., squares or rectangles) followed by 3D figures (including basic and composite solids, with a cube or cuboid typically being as the basic solids). The elements used in learning how to calculate the areas of 3D figures include elements that form a cube, mastering unfolding and perspective (see-through), grasping parallel or perpendicular concepts, and understanding how to calculate the surface areas of basic and composite solids.

Battista (2003) contended that children's conceptions of area and volume involved the following four main mental processes:

(1) Forming and using mental models: Constructing 2D or 3D images from imagination or memory to understand or infer encountered situations.
(2) Spatial structuring: Using identification, association, and organization to make the elements (e.g., a dot or line) that can be used to create abstract objects in developing forms and compositions; for example, a rectangle can be considered as having
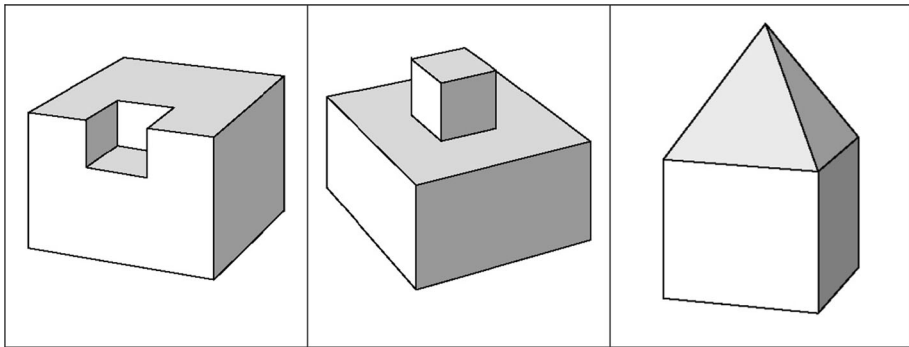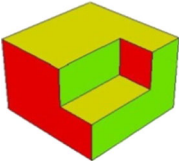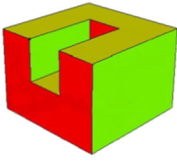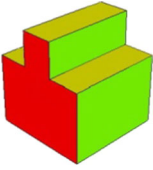
Fig. 1 Examples of composite solids
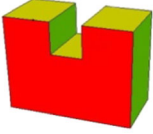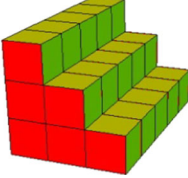
Fig. 2 Composite solids used in this study

    numerous rows and columns. However, this step also requires Steps 3 and 4 below to be conducted.

(3)   Unit location: if 2D or 3D figures (e.g., a rectangle or cuboid) are represented as rows and columns, then each row/column can be further divided into equal subunits, and the area or volume of the figure is the composite of these subunits. The unit location process involves verifying the subunit location in the rows/columns of the 2D plane/3D space, which can generally be accomplished by identifying where the subunit is located within the rows/columns.

(4)   Organizing the composites into a hierarchy: rows/columns are formed by subunits and subsequently form rectangles/squares; the rectangle composites then form cuboids. Spatial structures are formed through this process.

    Although Battista's conceptions focused primarily on the learning of area and volume, his research can give insight on the complicated cognitive processes that occur while

children calculate surface area of composite solids. According to Battista (Battista 1999; Battista and Clements 1996), students' reasoning about 3D cube arrays is a mental act of spatial structuring, through which children construct an organization or form for an object, or set of objects, by establishing units, establishing the relationships between units, recognizing that a subset of the units can generate the whole set, and then reconstructing the composite solid using the subsets.

Children usually encounter difficulties during those processes, such as lack of perspective thinking and abstraction, and therefore conceive 3D configurations in terms of local and separate faces. Consequently, children may construct an array as a medley of independent prism faces, confuse the faces of cubes with the number of cubes and therefore become unable to enumerate the number of basic solids in an array. This further makes them unable to coordinate the different orthogonal views (e.g., front, top, and right sides) of the same object and therefore also unable to recognize the same object from different views.

Teaching of the surface-area concepts of the composite solids

The use of multiple representations to support mathematics learning is widespread in traditional classroom teaching or computer-based learning environments (Ainsworth et al. 2002; Brenner et al. 1997). To build multiple representations in 3D geometrical concepts and enhance students' learning effects, physical manipulatives (e.g., boxes or building blocks) are usually used as teaching aids for forming representations when teaching the concept of basic-solid surface area. For example, teachers teach students how to convert between plane images in a 2D space and objects in a 3D space through observations and manipulations by cutting open a box, creating a flat pattern or expanded diagram, and applying the same colors to corresponding or symmetric rectangle sides (Fig. 3). The total surface area of the cuboid is derived from the sum of the sides or the simple formula of summing the three colored sides and multiplying by two. Students who have difficulty understanding spatial concepts may also be able to derive correct answers by memorizing formulas. By deconstructing and reconstructing expanded diagrams, students learn that the six planes on the three axes can be unfolded into a single plane before summing the surface area. Using building blocks or paper boxes is advantageous because students can establish a relationship between the mental models of 2D images and 3D objects by employing top- and side-view clues from the manipulation of simple objects. They can also understand basic-solid surface-area concepts when observing the sides of a 3D object laid flat on a 2D surface.

However, the benefits of physical teaching aids are somewhat limited when teaching composite-solid surface area both from the perspectives of teaching efficiency and learning effects. First, from the perspective of teaching efficiency, there are numerous types of composite solids, which means that obtaining and managing physical teaching aids for all such objects is difficult and inconvenient; in addition, the limited availability of teaching aids usually means that all students cannot be given the opportunity to experiment with or manipulate these aids. Further, if concrete objects are adopted as the primary tool to portray 3D images, the complexity of varying shapes can cost teachers extra time posing problems to students, which reduces teaching efficiency.

Second, from the perspective of learning effects, although representations of concrete manipulative objects may be useful for learning the surface area of basic solids, the spatial concepts involved in several composite solids require dynamic representation processes to

Fig. 3 A picture of an unfolding cuboid with identical colors assigned to symmetric sides
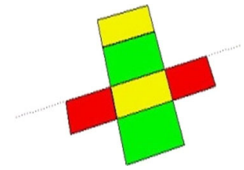
create appropriate mental images. It is more difficult to apply the cutting box method when calculating the surface area of a composite solid, such as those in Fig. 2. If students do not understand the concept of spatial shifts, filling-in, or conservation, they cannot apply formulas to derive correct answers (Battista 2003; French 2004). Battista (Battista and Clements 1996; Battista 1999) proposed that the mental processes of spatial structuring, coordination of different viewing perspectives, and the integration of pieces of images into a coherent mental model, are required for children to enumerate 3D cube arrays (i.e., cuboids), which are important basic solids and are also important for calculating surface areas. However, simply manipulating the physical cubes is not sufficient for children to represent the shapes of the basic solid from different perspective views for spatial structuring, nor can it illustrate the relationships among cubes which build the basic solid in different orthogonal views for coordination. Furthermore, the lack of clear unit markings on physical objects makes it difficult to enumerate and calculate the area of each side. Third, during the process of manipulating physical objects, as students have to figure out the related components of the solids by themselves, there is a lack of feedback, such as the accuracy of enumeration or prediction, for the results of students' interaction with the objects. This may reduce the motivation and learning effects of using physical manipulatives.

Applying virtual manipulatives to teaching about the surface area of composite solids

The recent emergence of information-technology-based virtual manipulatives presents a possible solution for these difficulties. Many studies have indicated that virtual manipulatives are mathematics teaching tools with great potential that enable students to acquire and create mathematical knowledge (Moyer et al. 2008; Reimer and Moyer 2005; Steen et al. 2006). Further, manipulating visual representations and mental images can allow students to develop spatial operation skills (Christou et al. 2007; Clements and Battista 1992; Clements 1999). So visual 3D virtual manipulatives could improve students' spatial abilities and enable them to link 3D objects with 2D representations, which in turn would allow more effective instruction in surface area calculations.

Currently, the most common virtual manipulatives used in geometry instruction are Geometer's Sketchpad (GSP), MagicBoard, National Library of Virtual Manipulatives (NLVM), and GeoGebra geometric figure-sketching software. The functions of these manipulatives are listed in Table 1, which indicates that these software options have various limitations in terms of the functions they provide. They focus primarily on the exploration of plane geometry (i.e., circumference, area, and angles), and are not entirely applicable for learning about composite-solid surface area. The focus of the present study was composite-solid surface-area concepts, and so an alternative tool, Google SketchUp, was adopted to develop an assistive teaching model for 3D representations (see the "Method" section).

Table 1 Comparison of the functions of Google SketchUp and other common virtual manipulatives

	GSP	MagicBoard	NLVM	GeoGebra	SketchUp
Building of 2D images	V	V	V	V	V
Building of 3D images	△	V	V	△	V
Building of composite solids		V	V		V
Virtual stacking of building blocks		V	V		V
Object dragging and zooming	V	V	V	V	V
Perspective-drawing development			V		V
Three-view drawing display					V
Rotating 3D objects	△		V	△	V
Unfolded view or expanded diagram					V
Calculation of area or surface area	V	V		V	V
Size-marking capability					V
Coloring capability					V
Archiving capability	V			V	V

"V" denotes the presence of the listed functions; "△" means that these functions are not easily accessed

Individual differences in technology-enhanced 3D concepts learning

Studies (Barbieri and Light 1992; Blaye et al. 1991; Li and Ma 2010; Volman and van Eck 2001) have been conducted involving students of both genders and with varying mathematical abilities to determine the effects of introducing technology into education. The simulation of real experiences afforded by virtual technology, including the introduction of a 3D virtual operating environment, can enable learning-disadvantaged students to increase their understanding of abstract concepts (Braddock et al. 2004; Cromby et al. 1996; Lannen et al. 2002). These methods not only render abstract concepts more tangible, but also enable students to actively participate in learning rather than simply engaging in passive observation (Pantelidis 1995). Research has shown that students with low spatial abilities experience difficulty developing internal representations of 3D geometric structures (Garg et al. 2002). Furthermore, students with higher spatial abilities are better at constructing dynamic mental models when observing and manipulating 3D geometric images (Huk 2006).

Previous studies have demonstrated that generally boys benefit more from the integration of technology into educational programs (Barbieri and Light 1992; Blaye et al. 1991). However, there are scarce studies investigating if boys benefit more from 3D-representation mathematics programs- specifically. Rafi et al. (2008) showed that the spatial ability of male students improved after receiving 3D-representation instruction compared to traditional instruction, whereas the results for female students were similar irrespective of the instruction method implemented. More research is needed to examine the learning effects of 3D-representation mathematics programs on boys and girls.

Studies (Aliasgari et al. 2010; Battista 1990; Geary and DeSoto 2001; Mayer-Smith et al. 2000; Maria and Garcia 2009) have shown that applying information technology to mathematical concept learning improves students' learning attitudes. After studying in technology-integrated learning environments, most students recognize the importance of mathematics learning as well as its pragmatic value. They also feel less anxiety about learning abstract concepts and are more confident in their ability to learn mathematics. The

use of computer-based visual aids to represent difficult concepts can improve students' mathematics learning attitudes and motivation, significantly increasing their enthusiasm for and investment in learning mathematical concepts (Pantelidis 1995). Previous studies have indicated that using technology that includes area and plane geometry (e.g., GSP, MagicBoard, and GeoGebra) in education improves the attitudes of students toward mathematics (Braddock et al. 2004). Some studies (Huk et al. 2003) have found that students with low mathematical abilities exhibited significant positive changes in learning attitudes after receiving 3D visual-representation instruction. Learning composite-solid surface-area concepts is more difficult than some other geometric topics, with more students becoming anxious and withdrawn when studying this subject. However, how employing technology in composite-solid surface-area instruction influences students' mathematics learning attitudes has not been determined. Therefore, the present study explores changes in the mathematics learning attitudes of high-grade-level elementary school students after receiving technology-assisted instruction.

Purposes of the current study

Based on the above introduction about the current status quo of teaching and learning surface-area concepts of composite solids, the current study aims to address several issues. Firstly, despite the popular use of virtual manipulatives in 2D geometry learning, scarce research has investigated using virtual manipulatives for 3D geometry learning. The first purpose of this study aims to help remedy this dearth by developing a program called Surface Area Instructional and Learning Strategies (SAILS), which is a Google SketchUp-based program, for learning composite-solid surface area calculation, and by evaluating the learning effect of SAILS. Secondly, previous studies have shown distinct differences in technology-assisted learning with regard to how male and female students learn complex 2D geometrical concepts (Battista 1990; Geary and DeSoto 2001; Mayer-Smith et al. 2000) and boys tend to benefit more than girls in learning spatial concepts through 3D technology (Rafi et al. 2008). However, whether males outperform females in mathematics learning is not conclusive (Lindberg et al. 2010). Currently only a few studies have investigated whether the learning benefits of 3D-representation instruction differ with gender or mathematics ability, therefore more research on this issue is needed to clarify if there is gender difference in learning composite solids. Furthermore, although some research has demonstrated the positive effects of technology-based visual aids for students' mathematics attitudes (Aliasgari et al. 2010; Maria and Garcia 2009), very little research is concerned with student attitude change after technology-based instruction on 3D composite-solid geometry. Therefore, the second purpose of this study is to explore the effects of gender, mathematical ability, and 3D-representation-based instruction on students' composite-solid surface area learning performance and attitude, and examine whether any interaction effect exists between these three factors.

The following questions were investigated:

(1) Whether the immediate and maintained learning achievements for composite-solid surface area differed significantly between students who received SAILS instruction and those who received traditional instruction with physical teaching aids.

(2) Whether the effects of SAILS instruction on achievement are moderated by gender and mathematic abilities; that is, whether the effects of SAILS instruction on

composite-solid geometry will show different effects on students with different gender and mathematic abilities.

(3) Whether the attitudes towards mathematics learning differed significantly between students who received SAILS instruction and those who received traditional instruction with physical teaching aids.

(4) Whether the effects of SAILS instruction on attitudes are moderated by gender and mathematic abilities; that is, whether the effects of SAILS instruction on attitudes are different on students with different gender and mathematic abilities.

Method

Participants

The participants of this study were students from four classes at an elementary school in Taiwan. A total of 111 fifth-grade students (61 males and 50 females, average age 10 years and 9 months, SD = 3.45 months) were recruited as the research sample. The four intact classes were randomly assigned to the experimental and control groups because the school did not approve of randomly assigning students of the same classes to different groups due to administration and regular instruction requirements. There were 56 and 55 students in the experimental and control groups, respectively. Based on the standardized elementary school mathematics competency test (ESMCT, see the "Instructional and measuring materials" section), students with ESMCT scores higher than 29 (above the average more than one standard deviation) or lower than 21 (below the average more than one standard deviation) would be categorized as high mathematical ability or low mathematical ability, respectively. Accordingly, 37, 40 and 34 students were categorized as possessing high, moderate, and low mathematical abilities, respectively. The participants had previously studied the identification and 3D stacking of basic solids (cubes and cuboids) in grade 2; the understanding and measurement of square and rectangle areas in grade 3; and the origins and applications of formulae for calculating area, the understanding of cube and cuboid 3D images on 2D planes using 3D geometric objects stacking, and the calculation of areas of composite solids on 2D planes in grade 4.

Research design

A pretest/posttest quasi-experimental factorial design was implemented, in which the instruction methods (3D representation vs. physical-aids teaching), gender and the mathematical abilities (high vs. moderate vs. low) were set as the three independent variables. The experimental group received the SAILS instruction; and the control group received physical-teaching-aid (traditional) instruction. The pretest results for a composite-solid surface-area achievement test (CSSAAT, see next section) and mathematics learning attitudes scale (MaLAS, see "Instructional and measuring materials" section) were set as the covariate. Before the experiment, the CSSAAT-1 and MaLAS pretest were conducted. After the treatments, the CSSAAT-2 and MaLAS were conducted and the results were set as the dependent variables. To further identify maintained learning effects and achievements, a delayed posttest of the CSSAAT-2 was conducted 3 weeks subsequent to the posttest.

Because we used the class as a unit for assigning students to groups, to control for the possible influence of initial differences between achievements in composite-solid surface-area calculation in the experimental and control groups (which would show in the pretest scores of CSSAAT-1), an analysis of co-variance (ANCOVA) was employed as a method of statistical control (instead of random-assignment control). The ANCOVA procedure adjusted the means of the posttest and delayed posttest according to the co-variates, and then the adjusted means of the groups were used for post hoc comparisons to determine if there were significant differences between groups (Black 1999; Rutherford 2011). It is noteworthy that because one of the major goals of this study was to evaluate the change of mathematics learning attitudes before- and after the treatment, if the pretest scores of MaLAS were not significantly different between the experimental and control groups, then a 2 (treatment: experimental vs. control) × 2 (gender: male vs. female) × 3 (mathematic ability: high vs. moderate vs. low) × 2 (testing moment: pretest vs. posttest) four-way mixed design analysis of variance would be conducted to examine the differences among treatment groups, gender, ability, and testing moment.

Instructional and measuring materials

Instruction content

We employed the unit on composite solids in fifth-grade textbooks—which contained a total of five instructional units. The instructional objective as well as the content and concepts to be learned in each unit are explained in Table 2 and Fig. 2. The instructional content for the experimental and control groups were identical.

SAILS: the program of surface area instructional and learning strategies for composite solids

This study utilized the functions of Google SketchUp to develop the SAILS teaching program for helping children learn how to calculate surface areas for composite solids. The functions of representation in Google SketchUp include unfolded or expanded diagrams, three-view drawing, perspective views, coloring capability, building composite-solid models, and automated calculation of surface area. The SAILS program is composed of two essential parts, the first part is teaching four consecutive procedures/strategies, and the second part includes the functions of Google SketchUp, which implements the four procedures/strategies. These procedures, strategies, and related SketchUp functions are illustrated in Table 3, and their detailed descriptions are presented in Appendix A.

As a technology-enhanced program of learning 3D composite solids, SAILS has three features: Firstly, the learning/teaching procedure is derived from theories of surface-area and cube arrays learning. The steps of learning/teaching were designed based on the cognitive processes and possible difficulties children may experience when they try to solve problems of area and volume measurement. SAILS adopted the strategy of presenting the concepts about surface area from simple (e.g., the componential units of a cube) to complicated (e.g., the whole surface area of a concave center), and then decomposing complex concepts into smaller learning units (e.g., decomposing the surface area of a whole cuboid into six colored planes). This step-by-step progressive teaching is helpful for students' learning of complicated concepts (Atkinson et al. 2000; Chang et al. 2002; Eiriksdottir and Catrambone 2009). Secondly, through the assistance of Google SketchUp,

Table 2 Instructional units and contents in this study

Units	Instructional objectives and contents
1. Basic	Calculate the surface area of the 2D composite solids
	Recognize the number of points, line segments, and sides of a cube
	Recognize the corresponding sides of a cuboid
	Recognize the corresponding parallel sides of a cube
2. Conservation	Understand the nature of conservation of a surface area
	Recognize and calculate the surface area of the hidden sides
	Understand the change in the surface area after objects are combined
3. Spatial	Recognize the top view and the side views of an object
	Judge the object's shape from the top view and the side views
4. Estimation	Use mental image manipulation to shift sides and estimate the surface area of a cube
	Use mental image manipulation to shift sides and estimate the surface area of a composite solid
5. Measurement	Use unit area to measure the surface area of a composite solid with concave side(s)
	Use unit area to measure the surface areas of composite solids with a convex center, double convex, a concave center and a convex ladder

Table 3 The procedures, strategies and related Google SketchUp functions in the SAILS

Procedure	Strategies	Exemplar SketchUp functions
1 Establishing a link between 2D and 3D spatial structure representations	Visualization of 3D models	Modeling for building 3D models
	Coordination and structuring of 2D and 3D features	Dragging, rotating, hiding planes, X-ray
2 Decomposing, calculating, and recomposing the area of each side of the basic solids	Unfolding objects and counting the number of sides	Framing, rotating, unfolding
	Enumeration and calculation	Unitization of figures for enumerating
	Recomposing and summing the areas of all sides/faces	
	Verification of the calculation results	Automated calculation of surface area
3 Restructuring the basic solid into a composite solid to establish a link between basic- and composite-solid representations	Unitization by cubes	Unitization of cubes, coloring, virtual building blocks
	Re-organizing and re-locating	Virtual building blocks, modeling
4 Decomposing, calculating, and recomposing the surface area of each side of the composite solids	Identifying the location and number of the sides/faces	Coloring, rotating, X ray
	Calculating the surface area of basic-cuboid sides by filling, shifting and integrating	Filling, shifting, calculating the amount of the surface area
	Calculating the surface area of non-basic-cuboid sides	
	Recomposing, summing, and verifying the surface areas of all sides	Automated calculation of the surface area

SAILS employs multiple representations of 2D patterns and 3D objects, such as perspective, folded and unfolded planes, and rotated views, to help children reduce their difficulties in forming metal images, coordinating different view perspectives, and integrating relationships of components/units within planes and objects. Thirdly, SAILS exploits the advantages of virtual manipulatives to provide each student with rich opportunities to access, manipulate, and speculate about various 3D objects. The operation, conjecturing, and feedback procedures facilitate students' deeper thinking about basic and composite solids and are consistent with the proposed principles of designing simulation environments for effective abstract concepts learning in previous research (Chang et al. 2008; Chen et al. 2011, 2013; Olkun et al. 2005).

Instructional activities

The control group was provided with physical teaching aids to support explanations, whereas the experimental group was provided with the virtual manipulatives in SAILS. The main characteristics of the instructional activities in the two groups were as follows:

(1) Teacher, instructional progress, and duration: both the experimental and control groups were guided by the same teacher. Three classes were taught each week for 40 min per class. The program was conducted for 8 classes within 3 weeks, totaling 320 min of teaching. Objectives 1–4 (Table 2) were each taught in one class period, whereas objective 5 was taught over four class periods.
(2) Instructional materials: the experimental and control groups were given the same textbooks (which included flat 3D images) for classroom instruction.
(3) Instructional methods: the teacher followed the same procedures in both experimental and control groups, listed as follows: (A) establishing a link between 2D and 3D spatial structure representations; (B) decomposing, calculating, and recomposing the area of the various sides or planes for the basic solid; (C) restructuring basic solids into a composite solid, and establishing a link between the representation of basic and composite solids; (D) decomposing, calculating, and recomposing the surface areas of various sides for a composite solid.
The control group was provided with traditional physical teaching aids for classroom instruction. The teacher projected flat 3D image problems on the whiteboard and used physical aids to clarify explanations. Students then observed and compared the 3D teaching aids with flat 2D images. For the experimental group, the SAILS program was employed as the teaching method. Students learned in the computer lab and the teacher was not required to project problems on whiteboards for this group; instead, they constructed 3D objects using Google SketchUp and provided verbal explanations. The experimental group students could immediately manipulate 3D objects after the teacher gave an explanation. These students were trained to use the basic functions of the operational tools before the start of the official lessons and instructions. During official instruction, the teacher demonstrated how to use the program before guiding the students in operating the program themselves. To conserve space, we illustrate the differences between the instructional procedures used for each of the two groups through two examples at the URL http://140.122.96.188/.
(4) Assignments: during the official classroom instruction for both the experimental and control groups, exercises, homework assignments and quizzes were administered.

The content and quantity of work were identical for the two groups. The major difference was that the experimental students could receive immediate feedback after their exercises through the functions of Google Sketchup.

Elementary school mathematics competency test (ESMCT)

The ESMCT was developed by the Department of Education, Keelung City (2008), Taiwan. The ESMCT was a standardized achievement test developed for determining elementary students' mathematical learning progress after every school year. The ESMCT included four subtests, numbers and quantity, geometry, algebra, and statistics and probability. There are 10 multiple-choice questions (each with four response alternatives) in each subtest, with each question scoring one point to a maximum possible score of 40. The ESMCT had good reliability and validity, the internal consistency coefficient of Kuder–Richardson reliability was 0.94; the criterion-related validity using 671 students' school mathematical achievement as the criterion was 0.76.

Composite-solid surface-area achievement test (CSSAAT)

The composite-solid surface-area achievement test developed by the researchers incorporated spatial concepts with surface-area learning theory. The test comprised five aspects: basic (4 questions), conservation (3 questions), estimation (4 questions), spatial (2 questions), and measurement (7 questions) concepts, for a total of 20 questions (see Appendix A for examples). A multiple-choice question design was adopted for the test, in which each question had 4 answer options, among which one was the correct answer; the maximum total score was therefore 20 points. Fourteen mathematics experts and experienced elementary and junior high school mathematics teachers were invited to review the questions in order to determine the content validity of the test. After preliminary revisions, test question creation principles were adopted to develop a parallel test, i.e., CSSAAT-2. 110 fifth- and sixth-grade students who had studied this subject for a pilot test were recruited. The test results showed mean pass rates of 0.55 and 0.56, respectively, for CSSAAT-1 and CSSAAT-2, mean discrimination indexes of 0.58 and 0.62, and Kuder-Richardson reliabilities of 0.85 and 0.86. The values were similar, and the two tests exhibited a strong correlation ($r = 0.85$, $p = 0.01$). We employed version 1 as the pretest tool and version 2 the posttest and delayed posttest tool. Each test required a total of 40 min. Students were not allowed to keep the tests, teachers were not allowed to discuss the tests, and test feedback was only given after the delayed posttest was administered.

Mathematics-learning-attitudes scale (MaLAS)

The mathematics-learning-attitudes scale for this study was modified based on the mathematics attitudes content and scale introduced by Reyes (1984) and Fennema and Sherman (1976). The content comprised the following six aspects: (1) mathematics learning confidence (e.g., "I believe I have good mathematics concepts" and "I do not consider learning mathematics to be excessively difficult"). (2) Mathematics preference level (e.g., "I enjoy engaging in mathematics topics" and "studying mathematics makes me happy"). (3) Mathematics learning anxiety (e.g., "the thought of attending mathematics classes makes me depressed" and "I am always nervous when a mathematics test is coming up"). (4) Mathematics learning process (e.g., "I cannot concentrate in mathematics class" and

"my thoughts always wander during mathematics class and I think about topics irrelevant to math"). (5) Mathematics learning method (e.g., "when I come up with incorrect answers for mathematics questions, I always try to understand the reason for my mistake" and "when encountering a question that I do not understand, I think about it by myself first and do not directly ask others for help"). (6) The usefulness of mathematics (e.g., "I believe that mathematics is important for thinking ability training" and "I believe mathematics makes people smart").

Each of the 6 aspects comprised 4 or 5 questions, with a total of 25 questions. A 4-point Likert scale was used, with the participants choosing from 1 (*strongly disagree*), 2 (*disagree*), 3 (*agree*), and 4 (*strongly agree*) for each positive item description, with the reverse applied to each negative item. Higher scores reflected better learning attitudes. Using Cronbach's α coefficients as the reliability indexes, the coefficients were 0.91 and 0.89 for the pretest and posttest, respectively.

Procedure

Prior to treatment, pretest data were collected for the CSSAAT-1 as well as the MaLAS. After 3 weeks of treatment, posttest data were also collected for the CSSAAT-2 and MaLAS. In addition, the delayed posttest data were obtained for the CSSAAT-2 3 weeks after the experiment was concluded. Because the date of administering the test was near the end of the semester, the school could not allocate time for the delayed posttest of MaLAS. Therefore we only had the pre- and post-test scores of the MaLAS.

Results

The effects of SAILS on composite-solid surface-area learning achievement

Table 4 gives the pretest, posttest, and delayed posttest of the CSSAAT for students with different gender and mathematical abilities in the experimental and control groups.

An independent-samples t test was conducted on the CSSAAT-1 scores of experimental and control groups, which found that there was no difference in CSSAAT-1 between the two groups, $t(109) = -0.092$, $p = 0.369$. This indicated that the two groups had comparable beginning composite-solid achievement. To explore whether the immediate and maintained effects of composite-solid surface-area learning achievement differed between students receiving different treatments, and whether the treatment effects were moderated by gender and mathematical abilities, a 2 (treatment: experimental vs. control) × 2 (gender: male vs. female) × 3 (mathematical ability: high vs. moderate vs. low) × 2 (testing moment: posttest vs. delayed posttest) four-factor mixed design analysis of covariance (ANCOVA) was conducted, in which the treatment, gender, and mathematical ability were set as between-subject factors, the testing moment was set as a within-subject factor, the scores of the pretest of CSSAAT-1 as the covariate, the scores of CSSAAT-2 as the dependent variable.

The summary table of ANCOVA analyses was shown in Appendix C. The main effect of testing moment was significant, $F(1, 99) = 7.856$, $MSE = 3.154$, $p = 0.006$, $\eta^2 = 0.074$; the posttest score of CSSAAT-2 ($M' = 9.985$) was significantly higher than the delayed posttest score ($M' = 9.291$). The main effect of gender was not significant, $F(1, 98) = 0.017$, $MSE = 18.19$, $p = 0.90$, $\eta^2 = 0.00$; the main effect of treatment was

Table 4 The descriptive statistics for the CSSAAT scores of students with different gender and mathematical abilities in different treatment groups

Treatment	Gender	Ability	N	Pretest		Posttest		Delayed posttest		Adjusted posttest	Adjusted delayed posttest
				M	SD	M	SD	M	SD	M'	M''
SAILS (experimental)	Female	High	9	7.33	1.22	13.33	2.60	14.00	3.39	13.19	13.94
		Moderate	6	7.50	2.59	7.83	3.43	6.67	2.50	7.65	6.59
		Low	9	5.22	1.48	9.89	3.41	9.44	3.05	10.26	9.60
			24	6.58	1.98	10.67	3.75	10.46	4.20	10.37	10.04
	Male	High	9	7.44	2.3	13.78	4.41	13.11	4.65	13.61	13.04
		Moderate	16	6.63	2.31	11.25	2.44	10.44	3.81	11.28	10.45
		Low	7	4.86	1.21	9.86	3.72	9.43	4.35	10.32	9.62
			32	6.47	2.26	11.66	3.56	10.97	4.28	11.74	11.04
			56	6.52	2.12	11.23	3.64	10.75	4.21	11.05	10.54
Physical aids (control)	Female	High	12	9.08	3.85	12.17	3.04	10.67	3.52	11.60	10.43
		Moderate	7	6.43	1.90	8.14	2.41	9.43	1.72	8.22	9.46
		Low	7	4.71	1.98	7.43	2.88	7.14	2.79	7.92	7.35
			26	7.19	3.45	9.81	3.53	9.38	3.83	9.25	9.08
	Male	High	7	8.86	5.08	11.14	4.78	9.71	3.55	10.64	9.50
		Moderate	11	6.73	2.83	8.55	2.94	6.09	2.70	8.55	6.09
		Low	11	5.64	1.91	6.45	2.88	5.36	2.38	6.73	6.09
			29	6.83	3.36	8.38	3.78	6.69	3.23	8.64	7.03
			55	7.00	3.38	9.05	3.70	7.96	3.46	8.94	8.05

significant, $F(1, 98) = 14.86$, $MSE = 18.19$, $p < 0.001$, $\eta^2 = 0.132$; the SAILS group had higher CSSAAT-2 score ($M' = 10.797$) than the traditional teaching group ($M' = 8.498$).

The main effect of mathematical ability was also significant, $F(2, 98) = 13.69$, $MSE = 18.19$, $p < 0.001$, $\eta^2 = 0.218$. The post hoc comparison using the Bonferroni method showed that the high-ability group had higher CSSAAT-2 score ($M' = 11.996$) than the moderate-ability group ($M' = 8.539$) and the low-ability group ($M' = 8.408$), both $p < 0.001$; but the moderate-ability group did not have higher posttest score than the low-ability group, $p = 0.864$.

The interaction effects in the ANCOVA summary table (Appendix C) showed that the interaction between mathematical ability and treatment was not significant, $F(2, 98) = 1.350$, $MSE = 18.19$, $p = 0.253$, $\eta^2 = 0.043$, meaning that students with different mathematical abilities benefitted similarly from the SAILS program. The only significant effect was the two-way interaction effect of gender by treatment, $F(1, 98) = 4.451$, $MSE = 18.19$, $p = 0.037$, $\eta^2 = 0.043$. Using the post hoc comparisons of the Bonferroni method to make six (=6) pairwise comparisons for the four means (males in experimental and control groups; females in experimental and control groups), the results indicated that males ($M' = 11.387$) outperformed females ($M' = 10.207$) in the experimental group ($p = 0.045$); males in the experimental group ($M' = 11.387$) outperformed males in the control group ($M' = 7.831$), $p < 0.001$. There were also two comparisons with significant differences, females in the experimental group ($M' = 10.207$) outperformed males in the control group ($M' = 7.831$), $p = 0.003$; males in the experimental group ($M' = 11.387$) outperformed females in the control group ($M' = 9.165$), $p = 0.003$.

In summary, the results above demonstrated that the SAILS instruction group performed better in the CSSAAT-2 than the traditional teaching group. Students with higher mathematical abilities demonstrated higher scores in CSSAAT-2 than students with moderate and low mathematical abilities. The posttest scores of CSSAAT-2 were significantly higher than the delayed posttest scores. Furthermore, this learning advantage of SAILS program applied to students with different mathematical abilities, who benefitted similarly from the SAILS program. Moreover, the learning advantage of the SAILS program was more salient in male students, because males demonstrated larger positive differences between SAILS and traditional teaching program both in CSSAAT-2 posttest and delayed posttest.

The effects of SAILS on mathematics learning attitudes

Table 5 lists the descriptive statistics for the pre- and post-test scores of MaLAS for students receiving different treatments. An independent-samples t test was conducted on the MaLAS pretest scores of experimental and control groups, which found that there was no difference in the MaLAS pretest scores between the two groups, $t(107) = -1.744$, $p = 0.084$, indicating that the two groups had comparable beginning mathematical learning attitudes. To understand whether treatments influenced the changes in students' learning attitudes, and whether the influences were moderated by mathematical ability and gender, a 2 (treatment: experimental vs. control) × 2 (gender: male vs. female) × 3 (mathematical ability: high vs. moderate vs. low) × 2 (testing moment: pretest vs. posttest) four-factor mixed design analysis of variance (ANOVA) was conducted, in which the treatment, gender, and mathematical ability were set as between-subject factors, while the testing moment (pre- and post-test) was set as a within-subject factor, and the MaLAS posttest score was the dependent variable. The summary table of ANOVA was shown in Appendix D.

The results showed that the main effect of testing moment was significant, students demonstrated higher learning attitudes scores ($M = 74.02$) after the treatment than before

Table 5 The descriptive statistics for the MaLAS scores of students with different gender and mathematical abilities in different treatment groups

Treatment	Gender	Mathematical ability	N[a]	Pretest		Posttest	
				M	SD	M	SD
SAILS (experimental)	Female	High	9	73.22	13.15	75.70	14.70
		Moderate	6	65.40	8.08	70.67	12.37
		Low	9	69.67	14.25	73.44	13.74
			24	69.43	13.06	73.26	15.69
	Male	High	9	80.40	9.53	82.33	13.86
		Moderate	15	66.94	13.85	77.88	14.50
		Low	7	61.57	15.57	67.91	13.92
			31	69.63	16.10	76.03	14.57
			55	69.53	14.72	74.65	16.32
Physical aids (control)	Female	High	12	77.50	10.81	80.25	7.64
		Moderate	7	73.57	17.94	75.71	12.47
		Low	7	65.26	12.99	66.67	15.54
			26	72.11	13.66	74.21	15.02
	Male	High	7	77.61	10.89	78.83	10.71
		Moderate	11	71.27	13.76	72.18	13.23
		Low	10	69.91	10.39	66.64	14.02
			28	72.93	11.99	72.55	14.44
			54	72.52	12.58	73.38	14.51

[a] Two students did not complete the questionnaire, their data were not included for the analysis

the treatment ($M = 71.01$), $F(1, 97) = 7.66$, $MSE = 45.10$, $p = 0.007$, $\eta^2 = 0.072$. The main effect of treatment was not significant, $F(1, 97) = 1.02$, $MSE = 345.24$, $p = 0.314$, $\eta^2 = 0.010$, indicating that the posttest of MaLAS scores of the experimental and control groups were not different. The main effect of gender was not significant, $F(1, 97) = 0.42$, $MSE = 345.24$, $p = 0.517$, $\eta^2 = 0.004$, indicating that the MaLAS posttest score was not different between male and female students. The main effect of mathematical ability was significant, $F(2, 97) = 6.99$, $MSE = 345.24$, $p = 0.001$, $\eta^2 = 0.124$, indicating that students with different mathematical abilities demonstrated different posttest scores in MaLAS. The post hoc analyses using the Bonferroni method indicated that high mathematical ability students had higher MaLAS posttest scores ($M = 79.90$) than moderate-ability students ($M = 74.12$), and low-mathematical ability students ($M = 68.67$), at $p = 0.020$ and $p = 0.001$, respectively; there were also significant differences between moderate-ability and low-ability students ($p = 0.031$).

For the interaction effects, the two-way interaction effect of testing moment by treatment was significant, $F(1, 97) = 5.37$, $MSE = 45.10$, $p = 0.023$, $\eta^2 = 0.051$; the three-way interaction of testing moment by treatment by mathematical ability was also significant, $F(2, 97) = 4.15$, $MSE = 45.10$, $p = 0.019$, $\eta^2 = 0.077$, both suggested that the effect of testing moment was not only moderated by treatment but also moderated by mathematical ability. To examine whether the change of mathematical attitudes (i.e., the effect of testing moment) was moderated by students' mathematical abilities and treatments, this study conducted the simple interaction effect analysis and then used the post hoc pairwise comparisons of the Bonferroni method for the four means (pre- and posttest in the experimental group; pre- and

posttest in the control group) in each ability level. For the high-ability group, the interaction effect between treatment and test was not significant, $p = 0.431$. Further, none of the six ($C_2^4 = 6$) comparisons reached a 0.05 level of significant of difference, which indicates that, for high-ability students, the MaLAS scores between the experimental ($M = 77.92$) and control ($M = 78.64$) groups were not significantly different; the pretest ($M = 77.18$) and posttest ($M = 79.38$) scores were also not significantly different.

For the moderate-ability group, the simple interaction effect between treatment and testing moment was significant, $p = 0.040$. Further, the post hoc pairwise comparisons of the Bonferroni method for the four means (pre- and posttest in experimental and control groups) indicated that in the six comparisons, only the experimental group's posttest score ($M = 74.28$) of MaLAS was significantly higher than the experimental group's pretest score ($M = 66.172$), $p = 0.032$; no other comparisons reached the 0.05 level of significance. These results indicated that for moderate-ability students, the experimental group did not have higher MaLAS posttest scores than the control group, but the experimental group had significant increase in the MaLAS after the treatment.

For the low-ability group, the simple interaction effect between treatment and testing moment was significant, $p = 0.005$. The post hoc pairwise comparisons of the Bonferroni method for the four means (pre- and posttest in experimental and control groups) indicated that that the experimental group's MaLAS scores of posttest ($M = 70.68$) was significantly higher than the pretest of the experimental group ($M = 65.62$, $p = 0.013$); the posttest score of MaLAS of the experimental group ($M = 70.68$) was significantly higher than the posttest of the control group ($M = 66.65$, $p = 0.044$) and the pretest of the control group ($M = 67.99$, $p = 0.048$). No other comparisons reached the 0.05 level of significant difference. These results indicated that for the low-ability group, the SAILS group demonstrated more positive mathematics learning attitudes after treatment, and more positive learning attitudes than the traditional teaching group.

In summary, the results above indicated that students with high mathematical ability had more positive mathematics learning attitudes than students with moderate- and low-mathematical ability; males and females had similar mathematics learning attitudes. The SAILS program had a significant effect on enhancing the mathematics learning attitudes of both male and female students. The SAILS program was able to enhance the low- and moderate-mathematical ability students' mathematics learning attitudes, but this facilitating effect did not appear in students with high mathematical abilities.

Discussion and conclusions

This study expanded the thinking processes and strategies proposed by Battista (Battista and Clements 1996; Battista 1999, 2003) regarding the learning of area concepts and cube arrays and considered the difficulties of understanding composite-solid surface area. We then employed Google SketchUp as an assistive device to develop the SAILS program. The results of this study indicate that fifth-grade students studying under SAILS showed better immediate and maintained effects for learning about composite-solid surface area compared to students receiving traditional instruction. The learning outcomes observed in this study could be attributed to several factors.

Firstly, using various tool functions—particularly rotation, coloring, top view, perspective view, unfolded view, shifting and filling—the SAILS provided multiple representations of basic and composite solids, such as the perspective view of a cube, the unfolded planes of a

cuboid, the dynamic images illustrating the relationships between a concave center composed of separate and connected cubes. Previous research (Ainsworth et al. 2002; Brenner et al. 1997; Pape and Tchoshanov 2001; Yerushalmy 1991) indicated that multiple representations contain complementary information for supporting cognitive processes, or link different representations for constructing a more complete or deeper understanding of learning materials. These multiple representations provided by SAILS enabled students to obtain more precise correspondences when constructing 2D and 3D points, line segments, sides, and other physical features involved in calculating surface area of composite solids. This method also increased the ease and accuracy of constructing mental models for basic or composite solids. Our findings concurred with previous studies (Christou et al. 2007; Moyer et al. 2002) which indicated that applying virtual manipulatives to visual 3D representations can increase students' spatial visualization abilities and their ability to create different views.

Secondly, the instructional strategies of SAILS could have enhanced the assistive effect of virtual manipulatives on student learning. The SAILS approach led students to disassemble and locate the sides of basic/composite solids through functions such as rotation and unfolding, or by locating and disassembling the basic solids within composite solids, which enabled them to clarify the relationships between the sides of a basic solid and the components of a composite solid. The enumeration and calculation strategy taught the students how to calculate the area of individual sides before using recomposing and summing strategies to determine the surface area of the complete object. These steps provided students with a clear objective when disassembling problems, and allowed them to solve problems in a step-by-step and progressive manner. Deconstructing complicated objects into constituent parts might also help students move from a medley of views (Battista and Clements 1999) towards more sophisticated mental models of composite solids.

Finally, actual operation, calculation, and verification processes probably enhanced students' learning achievements about surface areas. The sufficient number of virtual manipulatives meant that students studying under SAILS had access to virtual manipulatives for operations in the learning process. Furthermore, they could verify their results by comparing their calculations with those of computer aids, something difficult with traditional methods. In contrast, the limited quantity of physical teaching aids meant that most of the time students receiving traditional instruction could only observe their teacher's demonstrations. These results also agreed with the findings of some previous studies; that is, when simulation instruction provides students with the opportunity to manipulate aids themselves (Clements and Stephan 2003) and verify their predictions (Chang et al. 2008; Chen et al. 2011; Olkun et al. 2005), their achievements are typically higher than those of students who made only visual observations during the simulation process.

We also found that the effects for 3D-representation education were better than the effects of traditional teaching for high-, moderate-, and low-mathematical ability students. For students with low- and moderate-mathematical abilities, the concrete, individualized, and feedback-available environments mentioned above may have compensated for the limited learning abilities of students with low- and moderate-mathematical abilities, and therefore was especially helpful for enhancing their surface-area learning achievement. Under regular circumstances, when solving simple 3D geometric images in 2D planes, high-ability students receiving traditional instruction can employ mathematical formulae to their calculations to achieve a satisfactory performance without much difficulty. However, because the level of logical thinking and mental image analysis required for composite solids is higher, 3D-representation education likely increases the achievement levels of high-ability students because it improves mental spatial manipulation skills rather than only focusing on formulae.

Although the above explanations for the learning effects of SAILS on students with different levels of achievement needs further verification, this finding is essentially consistent with the notion supported by Braddock et al. (2004) and Garg et al. (2002), who believed that simulations of real-life experiences using virtual technology could increase the understanding of abstract concepts in 3D space among students with low and high mathematical abilities.

Regarding the gender differences in the benefits of SAILS, this study found that males benefitted more from this technology-based program. One of the reasons for this finding may be related to female students' comparative lack of experience and competence with computer-related tasks (Barbieri and Light 1992; Blaye et al. 1991; Drabowicz 2014; Plumm 2008; Reinen and Plomp 1997; Vekiri and Chronaki 2008; Volman and van Eck 2001). In contrast, male students' more abundant experience and competence with computer technology may have contributed to their better learning performance exhibited in SAILS. Another possible reason may be that traditional teaching methods were not as effective for males as for females. Previous studies (Friedman 1989; Lindberg et al. 2010) have indicated that certain traditional non-computer-based mathematics learning programs or tasks may be not as effective for males as for females, that is, females may perform better than males in certain traditional non-computer-based mathematics programs. However, most technology-based programs did not show this disadvantage of males (Li and Ma 2010; Volman and van Eck 2001). Therefore, our findings about the larger benefits for males from SAILS may have resulted from our traditional program about composite-solid surface-area not being as effective for males.

Regarding mathematics learning attitude, the low- and moderate-ability students who studied under SAILS exhibited greater improvements from pretest to posttest than those students receiving traditional instruction. However, high-ability students did not show similarly significant effects. The reasons for the better learning attitudes exhibited by lower- and moderate-ability students in SAILS could be because the strategies/procedures of SAILS made abstract concepts tangible, and individualized feedback increased learning efficiency throughout instruction, which in turn would increase student interest and investment in learning activities to an appropriate level. This also likely provided low- and moderate-ability students with a greater sense of accomplishment, and the learning results they achieved increased their interest in and improved their attitude toward mathematics (Ma 1997; Ma and Xu 2004). Our findings partially supported previous viewpoints that 3D virtual manipulatives display and represent difficult concepts, which can improve students' learning attitudes towards mathematics or other subjects (Korakakis et al. 2009; Pantelidis 1995).

In addition to enhanced learning achievement, SAILS provides a greater variety of learning experiences than traditional methods because it links technology with mathematics learning. The use of computers allows for a deeper level of participation because each student has a chance to independently create their own composite solids, expand and assemble them, and then after surface area calculations, attain an immediate verification of their results. These features might give students insight into different ways of learning and added confidence for using different methods for learning mathematics. Aliasgari et al. (2010) and Maria and Garcia (2009) also argued that the integration of technology and mathematics means that students receiving technology-integrated instruction would be substantially more aware of the importance of mathematics learning as well as its pragmatic value. These views are consistent with the findings of the present study.

Although high-ability students experienced an identical teaching/learning procedure as low-ability students, we found that the enhancement of high-ability students' performance was significant whereas their attitudes towards mathematics learning did not significantly differ. One of the reasons may be that the existent knowledge and skills of higher ability

students allowed them to complete the complicated tasks of SAILS without as much difficulty as low- and moderate-ability students. Therefore, unlike low- and moderate-ability students, high-ability students' may have felt SAILS was only a small improvement over traditional methods. Thus their level of attitude change was not as obvious as low- and moderate-ability students. The second reason might be that high-ability students had more positive prior attitudes towards mathematics learning, making it more difficult to discern the attitude change of higher-ability students after the treatment.

This study introduced the SAILS program which incorporated learning strategies based on area and volume measurement and the functions of Google SketchUp. The technology-integrated instruction of SAILS produced satisfactory results for improving students' learning of composite solids and mathematics learning attitudes. Despite the positive effects of the SAILS program, there are some restrictions in this study which should be addressed in future research. For example, in order to eliminate class-level influences future research designs may randomly assign individual participants instead of whole classes to groups. Further, this study used the same measurement tool for its posttest and delayed posttest. Although test results were not provided to students until after the delayed posttest, practice and memory effects may still have affected the precision of the measurement of composite solid surface-area learning achievement. Future studies may use tests with different items for their delayed posttest.

In addition, this study focused on the learning of certain types of composite solids, such as concaves and convexes. Future studies may extend SAILS to include more composite solids, such as the combination of cubes with pyramids, for more advanced learners. Furthermore, although low- and moderate-ability students' attitudes towards mathematics learning were shown to increase after students engaged in SAILS, it remains uncertain which specific factor contributed the most to this increase. Future investigations could consider collecting students' learning behaviors (such as recording the use of certain functions of SAILS), attitudes, and achievement during the treatment period to clarify the causal relationships among students' learning behaviors, mathematics attitudes, and learning achievement through methods such as structural equation modeling. Finally, SAILS has several design features such as theory-based teaching procedure and strategies, multiple representations, and individualized access and feedback, that all could have contributed to its learning and attitude outcomes. Future research may focus on the issue of how much each specific feature contributed to the positive gains in learning achievement and attitude.

Appendix A: The procedures, strategies, and SketchUp functions used in the SAILS program

Procedure 1

Establishing a link between 2D and 3D spatial structure representations. Basic solids (e.g., a cuboid) can be illustrated in a 2D plane. However, clues regarding the top, side, and perspective views are lacking, and may thus prevent children from understanding the conversion between plane figures in 2D space and solid objects in 3D space (Battista 1999; Piaget and Inhelder 1956; Piaget et al. 1960). This also increases the difficulty of transforming 3D images into actual 3D mental models. Therefore, the first

essential task is to help students convert 2D graphical representations into 3D mental representations.

There are two instructional strategies in the first procedure. The first, Strategy 1A, involves the visualization of 3D models. Here learners are guided to build a 3D object (Fig. A1) based on 2D or physical object images by using the modeling function of Google SketchUp. They could also employ three colors to represent the three-axis planes in order to identify the sides/faces of the cubes. Strategy 1B includes the coordination and structuring of 2D and 3D features. Using functions such as dragging and rotating, hiding planes (Fig. A2), and X-ray (Fig. A3), learners can manipulate the 3D model and perform tasks such as rotation, displaying side and top views, and hiding axis planes. In this way learners establish the correspondence between points, line segments, and side relationships for both 3D and 2D physical objects.

Procedure 2

Decomposing, calculating, and recomposing the area of each side of the basic solid. Determining the surface area of a basic solid involves the calculation of multiple areas. Therefore, children must first decompose the sides/faces of the basic solids and grasp how many different sides/faces there are as well as their differences and similarities. They should then calculate the area of each side/face using Battista's enumeration method or by applying a general formula, and then sum (re-compose) the areas of multiple sides to arrive at a surface-area value.

There are four instructional strategies in the second procedure. Strategy 2A is unfolding objects and counting the number of sides. Using the frame function options, 3D images can be converted to show top, bottom, front, back, left, right, and isometric views, enabling learners to understand the views of each side of the cube. By unfolding the basic solids, learners can determine the number of sides and side shapes for these solids, as well as the symmetric or corresponding sides (Fig. A4). Strategy 2B is enumeration and calculation. Learners first identify the corresponding or symmetric sides and conduct enumeration or use formulae to calculate the area (Fig. A4). Strategy 2C is recomposing and summing the areas of all sides/faces. Students sum the total surface areas of the six sides of the cube. Finally, strategy 2D is the verification of the calculation results. Learners compared the results they obtained with those provided by Google SketchUp through the calculation functions.

Procedure 3

Restructuring the basic solid into a composite solid to establish a link between basic- and composite-solid representations. Composite solids can be considered as composites of basic solids (Figs. A5, A6, A7), which means that composite solids are deformations resulting from the addition or subtraction of basic solids. However, the similarities and differences between mental models for basic and composite solids can only be understood through unit filling, shifting and conservation.

There are two instructional strategies in the third procedure. The first, strategy 3A, is unitization by cubes. Using unit cubes (in which each side equaled 1 in length, see Fig. A5), basic cuboids are decomposed as composites of unit cubes (Fig. A6). Using the coloring function, learners color the corresponding three pairs of sides red, yellow,

and blue. Using the virtual building blocks function, learners form a basic cuboid by copying and pasting unit cubes. Strategy 3B is re-organizing and re-locating. Cube units are added to or subtracted from the basic solid to produce composite solid (Fig. A7). The relationship between the location of the basic and composite solids is also identified.
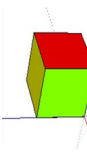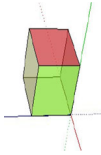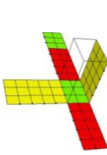
Procedure 4

Decomposing, calculating, and recomposing the surface area of each side of the composite solid. Similar to Procedure 2, in which the surface of a basic solid should be decomposed into several sides, and then recomposed into the surface area of each side, composite solids must first be decomposed and then converted into a composite of one or more basic-solid to facilitate the calculation of each basic solid surface areas. After the surface area of each piece is calculated, the values are summed to obtain the surface area of the composite solid. In addition to possessing the ability to complete this step, children must also know not to repeat the calculation of shifted or filled-in surface areas.

There are four instructional strategies in the fourth procedure. Strategy 4A is identifying the location and number of the sides/faces. The number of sides of the composite solid is verified by observing the rotational, top, side, and perspective views. The similarities and differences between the shapes of the sides, as well as their relationships, are also verified. Strategy 4B is calculating the surface area of basic-cuboid sides by filling, shifting and integrating. The filling function of Google SketchUp is employed to fill up the holes in the concave side of a concave object and deform the concave into a conglomerate of basic solids (like Fig. A6); the shifting function is employed to remove the protrusion in the convex side of a convex object and dismantle the convex into basic solids (like Figs. A6, A7). The processes and end-results of filling and shifting will form dynamic images which are helpful for students' understanding of the differences and similarities of the original and deformed solids and their corresponding surfaces. The total surface area of the basic solid is subsequently calculated. For example, after the unit cube in Fig. A7 was shifted, the surface area of the yellow side in Fig. A8 comprised $3 \times 3 = 9$ units of area.

Strategy 4C is calculating the surface area of non-basic-cuboid sides. In addition to the basic-cuboid sides, the surface areas of the concave or convex sides are calculated. For example, in addition to the top-view of Fig. A6, both the red (Fig. A9) and green (Fig. A10) sides comprised $(2 \times 3) + 1 = 7$ units of area. Strategy 4D is recomposing and summing the surface areas of all sides: The total surface area of the composite solid can be obtained by recomposing the surface area of the basic solid and that of the concave/convex composite. For example, the areas of the three colors in Fig. A7 can be summed as $(9 + 7 + 7) + (9 + 7 + 7) = 46$. After the calculation, learners verify the results they obtained with those provided by Google SketchUP through the calculation function.

See Table 6.

Table 6 Figures of Google SketchUP functions

No.	Functions	Figures
A1	Building a basic solid through SketchUp	
A2	The hiding plane function of SketchUp	
A3	The X-ray function of SketchUp	
A4	Adding the grids to the folding of a cuboid's sides to allow for the enumeration of the surface area	
A5	A unit cube	
A6	Decomposing the basic solid into unit cubes	
A7	Reorganizing unit cubes to be a composite solid	
A8	Top view of Fig. A7	
A9	Left-side view of Fig. A7	
A10	Right-side view of Fig. A7	

Appendix B

See Table 7.

Table 7 Exemplar items of the composite-solid surface-area achievement test

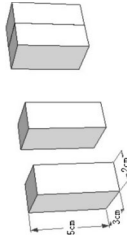| Concepts	Items
Basic	Which descriptions are correct about the cuboid (Fig. B1) on the right?
	(A) 3 sides and 12 line segments
	(B) 6 sides and 12 line segments
	(C) 6 sides and 6 points
	(D) 6 sides and 8 points
	(1) A and B (2) B and C
	(3) C and D (4) B and D
Conservation	There are two cuboids whose length, width, and height are 3, 4, and 5 cm respectively, as Fig. B2
	What will the change to the surface area be if the two cuboids are put together as shown in the figure?
	(1) Increase by 15 square cm
	(2) Decrease by 15 square cm
	(3) Decrease by 30 square cm
	(4) No change
Estimation	Fig. B3 on the right is a composite solid comprised by cubes. The line segment of each cube is 1 cm long. If we add 3 cubes to fill the gap in the center, what is the change to the surface area of the composite solid?
	(1) become larger (2) become smaller (3) no change (4) cannot compare due to insufficient information
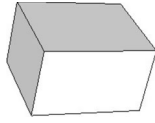
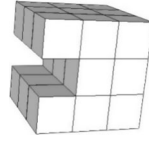Fig. B1: The cuboid

Fig. B2: The combination of two cuboids
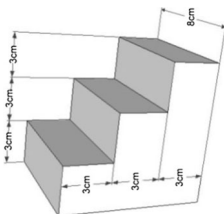
Fig. B3: A double convex

Table 7 continued

Concepts	Items
Spatial	In Fig. B4, Which figure would fit the following side view of the blocks?
	(1) Figures A and B (2) Figures A and D
	(3) Figures B and C (4) Figures C and D
	Fig. B4: A set of figures composed of small cubes
Measurement	What is the surface area of the composite solid in Fig. B5 on the right?
	(1) 144 square cm
	(2) 252 square cm
	(3) 288 square cm
	(4) 396 square cm
	Fig. B5: A concave ladder

Appendix C

See Table 8.

Table 8 Summary table of the four-factor mixed design ANCOVA for the CSSAAT

Source of variance	Type III sum of squares	df	Mean square	F	p	Partial eta squared
Within subjects						
Testing moment	24.775	1	24.775	7.856	0.006	0.074
Testing moment × gender	10.566	1	10.566	3.350	0.070	0.033
Testing moment × treatment	2.457	1	2.457	0.779	0.380	0.008
Testing moment × mathematical ability	0.459	2	0.230	0.073	0.930	0.001
Testing moment × gender × treatment	4.405	1	4.405	1.397	0.240	0.014
Testing moment × gender × mathematical ability	4.085	2	2.043	0.648	0.525	0.013
Testing moment × treatment × mathematical ability	7.909	2	3.955	1.254	0.290	0.025
Testing moment × gender × treatment × mathematical ability	16.752	2	8.376	2.656	0.075	0.051
Error	312.208	99	3.154			
Between subjects						
Intercept	1,887.568	1	1,887.568	103.787	0.000	0.514
Pretest	39.381	1	39.381	2.165	0.144	0.022
Gender	0.305	1	0.305	0.017	0.897	0.000
Treatment	270.174	1	270.174	14.855	0.000	0.132
Mathematical ability	497.891	2	248.945	13.688	0.000	0.218
Gender × treatment	80.949	1	80.949	4.451	0.037	0.043
Gender × mathematical ability	36.566	2	18.283	1.005	0.370	0.020
Treatment × mathematical ability	49.1	2	24.550	1.350	0.264	0.027
Gender × treatment × mathematical ability	50.698	2	25.349	1.394	0.253	0.028
Error	1,782.314	98	18.187			

Appendix D

See Table 9.

Table 9 Summary table of the four-factor mixed design ANOVA for the MaLAS

Source of variance	Type III sum of squares	df	Mean square	F	p	Partial eta squared
Within subjects						
Testing moment	345.519	1	345.519	7.661	0.007	0.072
Testing moment × gender	17.479	1	17.479	0.388	0.535	0.004
Testing moment × treatment	242.243	1	242.243	5.371	0.023	0.051
Testing moment × mathematical ability	50.926	2	25.463	0.565	0.570	0.011
Testing moment × gender × treatment	0.544	1	0.544	0.012	0.913	0.000
Testing moment × gender × mathematical ability	148.141	2	74.071	1.642	0.199	0.032
Testing moment × treatment × mathematical ability	373.883	2	186.941	4.145	0.019	0.077
Testing moment × gender × treatment × mathematical ability	57.573	2	28.786	0.638	0.530	0.013
Error	4,374.603	97	45.099			
Between subjects						
Intercept	1,059,100.025	1	1,059,100.025	3,067.730	0.000	0.969
Gender	146.246	1	146.246	0.424	0.517	0.004
Treatment	353.202	1	353.202	1.023	0.314	0.010
Mathematical ability	4,823.347	2	2,411.674	6.986	0.001	0.124
Gender × treatment	29.971	1	29.971	0.087	0.769	0.001
Gender × mathematical ability	972.640	2	486.320	1.409	0.249	0.028
Treatment × mathematical ability	780.217	2	390.109	1.130	0.327	0.022
Gender × treatment × mathematical ability	1,469.603	2	734.801	2.128	0.124	0.041
Error	33,488.183	97	345.239			

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