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ORIGINAL PAPER

Proof of Friction Law: Static Friction is Higher than Dynamic Friction

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Abstract
The paper proves the friction law which states that static friction is higher than dynamic friction by proposing the asynchronous rupture of adhesive junction. It is also shown that dynamic friction coefficient is about 0.5–0.67 of static friction coefficient.

Keywords Friction law · Static friction · Dynamic friction · Kinetic friction

1 Introduction

As one of friction laws it states that the static frictional coefficient is higher than the dynamic frictional coefficient. The higher static frictional coefficient is well confused with the fact that friction coefficient increases with the dwell time, which is explained by the growth of adhesion during the static contact in adhesive friction theory. Here, a new mechanism of the static frictional coefficient higher than the dynamic coefficient is proposed based on the tangential deformation of asperities in sliding contact.

2 Asynchronous Rupture of Adhesive Junctions

Surface contact is limited to the local area of asperity peaks which are various both in shape and height. Figure 1 shows the schematic model of the asperity contacts. The tangential force is different at different asperities because the tangential deflection is different. The difference in the tangential force of the different asperities is due to the difference in the contact starting time for the different asperities. We know that the sliding is a dynamic process of rupture of the adhesive junctions and the formation of the adhesive junctions. The tangential force in the contact area is calculated by the following equation of Hertz's contact [1]:

$$\tau_{\mathrm{i}} = \mu_s p_{i0}\left[1 - \left(\frac{r}{a_{\mathrm{c}}}\right)^2\right]^{\frac{1}{2}}, \tag{1}$$

where μ_{s} is the static frictional coefficient, p_{i0} the maximum contact pressure, and a_{c} the radius of contact area. The contact radius is given by

$$a_{\mathrm{c}} = \left(\frac{3WR}{4E}\right)^{\frac{1}{3}} = \left[\frac{(2-\nu)\pi p_{i0} a_c^2 R}{2G}\right]^{\frac{1}{3}}, \tag{2}$$

that is

$$a_{\mathrm{c}} = \frac{(2-\nu)\pi p_{i0} R}{2G}, \tag{3}$$

where ν is the Poisson ratio, G the shearing modulus, and R the asperity curvature radius.

The tangential deflection of the contact asperity under the tangential force is given by [1]

$$u_{\mathrm{i}} = \frac{(2-\nu)\pi a_{\mathrm{c}} \tau_{i0}}{8G}. \tag{4}$$

The tangential force at the ith contact asperity is given as follows:

$$F_{\mathrm{i}} = \frac{2}{3}\pi a_{\mathrm{c}}^2 \tau_{i0} = \frac{16 G a_c u_i}{3(2-\nu)}. \tag{5}$$

Noticing that the contact radius is dependent only on the normal load we know that the tangential force and deflection are proportional to each other in sliding. The total tangential force is the sum of the tangential force at all the contact asperities. This is the static friction. Before the incipient

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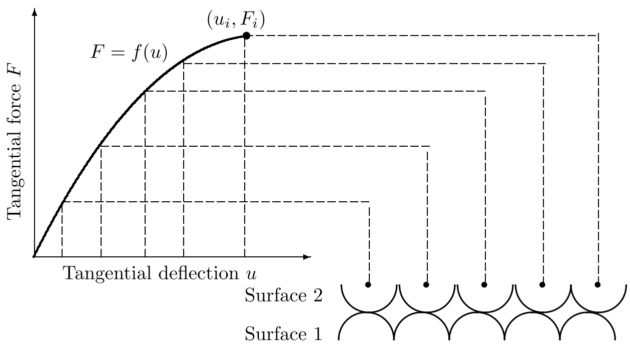

Fig. 1 Schematic model of asynchronous rupture of adhesive junctions

of the rupture of adhesive junctions, the tangential force is less than that given by Eq. (3) and therefore the deflection is also below what is given in Eq. (2). Mindlin [2] gave the following equation before sliding:

$$u = u_{\mathrm{i}}\left[1-\left(1-\frac{F}{F_{\mathrm{i}}}\right)^{\frac{2}{3}}\right]. \tag{6}$$

For plastic contact we roughly assume that the shear stress τ_{s} is constant in the sliding ring area $a_{\mathrm{s}} \le \sqrt{\xi^2+\eta^2} \le a_{\mathrm{c}}$, while it is zero in non-sliding areas, and the tangential deflection is still elastic. Then the tangential deflection at the center of contact area is given [1]

$$u = \frac{\tau_{\mathrm{s}}}{2\pi G}\int\int_{a_{\mathrm{s}}\le\sqrt{\xi^2+\eta^2}\le a_{\mathrm{c}}}\left[\frac{1-\nu}{(\xi^2+\eta^2)^{1/2}}+\frac{\nu\xi^2}{(\xi^2+\eta^2)^{3/2}}\right]\mathrm{d}\xi\mathrm{d}\eta,$$

which leads to

$$u = u_{\mathrm{i}}\left[1-\left(1-\frac{F}{F_{\mathrm{i}}}\right)^{\frac{1}{2}}\right]. \tag{7}$$

In macroscopic sliding the contact spots are released and recaptured repeatedly, and/or new contact spots are generated. The ruptures of adhesive junctions on the contact surfaces are asynchronous, which as contrary are synchronous at the incipient of the macroscopic sliding. When the number of deflected junctions are uniformly distributed against the tangential force then the dynamic frictional force is half of the static frictional force. Therefore the dynamic friction coefficient is given by

$$\mu_{\mathrm{d}} = 0.5\mu_{\mathrm{s}}. \tag{8}$$

On the other hand if the distribution is uniform against the tangential deflection, then the dynamic frictional force F_{d} is given by

$$\frac{F_{\mathrm{d}}}{F_{\mathrm{i}}} = \frac{\mu_{\mathrm{d}}}{\mu_{\mathrm{s}}} = \frac{1}{u_{\mathrm{i}}}\int_0^{u_{\mathrm{i}}}\frac{F}{F_{\mathrm{i}}}\mathrm{d}u. \tag{9}$$

Substituting Eq. (6) into Eq. (9) for elastic contact with rearrangement gives

$$\begin{aligned}\mu_{\mathrm{d}} &= \frac{2}{3}\mu_{\mathrm{s}}\int_0^{F_{\mathrm{i}}}\frac{F}{F_{\mathrm{i}}}\left(1-\frac{F}{F_{\mathrm{i}}}\right)^{-\frac{1}{3}}\mathrm{d}\left(\frac{F}{F_{\mathrm{i}}}\right)\\ &= \frac{2}{3}\frac{\Gamma(2)\Gamma(\frac{2}{3})}{\Gamma(2+\frac{2}{3})}\mu_{\mathrm{s}} = 0.6\mu_{\mathrm{s}}.\end{aligned} \tag{10}$$

For plastic contact substituting Eq. (8) into Eq. (9) gives

$$\begin{aligned}\mu_{\mathrm{d}} &= \frac{1}{2}\mu_{\mathrm{s}}\int_0^{F_{\mathrm{i}}}\frac{F}{F_{\mathrm{i}}}\left(1-\frac{F}{F_{\mathrm{i}}}\right)^{-\frac{1}{2}}\mathrm{d}\left(\frac{F}{F_{\mathrm{i}}}\right)\\ &= \frac{1}{2}\frac{\Gamma(2)\Gamma(\frac{1}{2})}{\Gamma(2+\frac{1}{2})}\mu_{\mathrm{s}} = 0.67\mu_{\mathrm{s}}.\end{aligned} \tag{11}$$

3 Conclusion

A model of asynchronous rupture of adhesive junctions is proposed, which proves why the static friction is always higher than dynamic friction. The dynamic frictional coefficient is 0.5–0.67 of the static frictional coefficient.

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Data Availability Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Competing interest The authors declare no competing interests.

References

1. Johnson, K.L.: Contact Mechanics, p. 74. Cambridge University press, Cambridge (1996)
2. Mindlin, R.D.: Compliance of elastic bodies in contact. ASME J. Appl. Mech. **16**, 259–267 (1949)