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ORIGINAL PAPER

A Functional Form for Wear Depth of a Ball and a Flat Surface

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Abstract Formulae are derived from first principles which predict the wear depth of a ball and a flat surface through time as they slide against each other, in relation to any phenomenological law for wear volume, and taking into account the effect of component geometry. The equations can be fit using experimental wear volume data from ball-on-flat tribometers. The formulae remove previous limiting approximations made in the literature and extend to the prediction of the wear depth of both contacting surfaces. The wear model accords with a previous model that is validated by pin-on-disc testing of a steel/steel contact. The current paper uses the formulae derived to predict the wear depth of a diamond-like carbon (DLC) coating and a steel ball as they slide against each other in deionised water. An Archard equation is used to predict the wear volume of each surface; however, a DLC coating is known to form a transfer layer which reduces the rate of wear, and since this scenario does not obey Archard's law directly, a time-dependent-specific wear rate is used to fit a semi-empirical model to experimental results. The final model predicts the wear depth of the ball and flat accurately.

Keywords Wear · Archard · DLC

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1 Introduction

Estimations of wear in real-life applications are often based on experimental testing in the laboratory, under accelerated test conditions and on idealised test geometries. Whilst accelerated test conditions are necessary to provide data in an allowable time frame, the use of an idealised geometry such as a ball-on-flat contact may provide an erroneous assessment of wear by disregarding geometric effects on the evolution of wear depth, and as such care needs to be taken in the interpretation of experimental data.

A common assessment of wear is Archard's wear law [1, 2] that estimates the total volume of wear as a function of sliding distance d and normal load N. Assuming that wear occurs in hemispherical volumes at each asperity contact that the contact pressure at an asperity contact equals the yield pressure of the softer material, and that the area of contact is a constant, Archard derived the following expression for the wear volume w_{V} of either surface.

$$w_{\mathrm{V}} = kNd \tag{1}$$

In Eq. 1, the specific wear rate k is a constant which is unique to every tribological scenario and material pair. In the case of mild wear, the specific wear rate is usually given over the range $10^{-8} - 10^{-4}\,\mathrm{mm}^3/\mathrm{Nm}$.

Archard's wear law [1, 2] predicts linearity between the volume of wear and the product of load and sliding distance, but for many materials, this has been shown not to be the case. For example, transitions between wear mechanisms or changes in surface chemistry may affect the evolution of wear volume with respect to time. In the case of diamond-like carbon (DLC) coatings, a transfer layer composed of wear debris from the DLC coating is often known to adhere to the counterface material, which limits contact between the DLC coating and the counterface, thus lowering the specific wear rate as the contact ensues [3].

An important consideration is that wear depth is dependent on the area of contact between surfaces, and whilst the microscopic wear volume of an asperity contact may occur at some fixed pace according to a fundamental wear law, the wear depth may vary non-linearly due to a larger number of asperity contacts as the apparent contact area increases. For example, in a pin-on-disc test, once the head has worn away, the area of contact must remain constant, and so linearity might be assumed between wear depth and sliding distance, but in a ball-on-flat contact, the area of contact will increase monotonically from the initial Hertzian value upwards [4], and so a non-linear prediction of wear depth with time may be more appropriate.

Prior to Archard's estimation of wear volume in a tribological contact [1, 2], Preston [5] suggested that the rate of change of wear depth should vary proportionally to the contact pressure P and the sliding velocity v. In Eq. 2, w_{D} denotes the wear depth of either surface, and k denotes the specific wear rate.

$$\frac{\mathrm{d}w_{\mathrm{D}}}{\mathrm{d}t} = kPv \tag{2}$$

In order to compare the wear models of Archard [1, 2] and Preston [5], the relationship between wear depth and wear volume must be known. In this paper, a general formulation is considered where wear volume $w_{\mathrm{V}} = w_{\mathrm{V}}(w_{\mathrm{D}}, A)$ is written as a function of wear depth w_{D} and contact area A. The rate of change of wear volume (Eq. 3) is then determined by two terms; the first is related to the rate of change of wear depth, and the second is related to the rate of change of the contact area (which is ignored in the Archard and Preston formulations of wear).

$$\frac{\mathrm{d}w_{\mathrm{V}}}{\mathrm{d}t} = \frac{\partial w_{\mathrm{V}}}{\partial w_{\mathrm{D}}}\frac{\mathrm{d}w_{\mathrm{D}}}{\mathrm{d}t} + \frac{\partial w_{\mathrm{V}}}{\partial A}\frac{\mathrm{d}A}{\mathrm{d}t} \tag{3}$$

If wear volume is considered to be a function of wear depth only, i.e., the contact area is assumed to be a constant, then the second term of Eq. 3 vanishes, and the Archard and Preston formulations can be expressed in a comparable rate form. Of course, in many real-life applications and in many common test geometries such as a ball-on-flat contact, the area of contact cannot be assumed constant, and this has an effect on the prediction of the wear volume of both surfaces.

For a range of component geometries, Kauzlarich and Williams [6] presented an equation to link wear depth with sliding distance for a ball sliding against a flat surface. For a ball-on-flat contact, they assumed that the radius of the wear scar was small in relation to the radius of the ball and derived approximate relations for the wear depth of the ball. They did not consider how the wear to the flat surface would change as a result of the changing contact area, which is a result derived later in this work. Furthermore, Kauzlarich and Williams [6] highlight that wear depth does not necessarily conform to a linear relationship with sliding distance and that the effects of geometry on the wear depth must not be ignored.

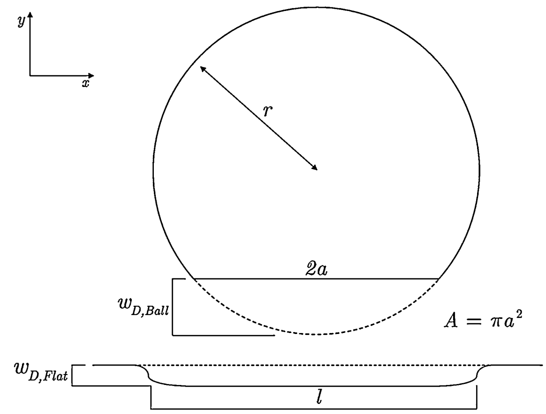

Fig. 1 A schematic of a ball-on-flat contact. The wear depth of the ball $w_{\mathrm{D,Ball}}$ and the wear depth of the flat surface $w_{\mathrm{D,Flat}}$ are labelled, as are the geometric parameters (namely stroke length l, contact radius a, ball radius r, and contact area A). *Dashed lines* represent the original unworn geometry

Departing from the formulation described above, the present paper provides formulae to link wear depth with wear volume of a ball-on-flat contact according to some fundamental law such as Archard's wear law, based on no underlying assumptions other than the surfaces wear uniformly, and obey the governing law to predict wear volume. The proposed formulation removes the asymptotic approximation made in the work of Kauzlarich and Williams [6] that the radius of the wear scar is small in relation to the radius of the ball. Additionally, the phenomenological model is extended to predict the wear depth of the flat surface as a function of the contact area.

The outputs from the wear model derived can be compared to experimental data to assess the fit of the wear law—and in this manner, confidence can be placed in the experimental outputs from testing. The wear model derived in the next section to predict the wear depth of the ball is shown to compare favourably to the work of Kauzlarich and Williams [6] who validated their model against experimental data from pin-on-disk testing of a steel/steel contact [7]. The derived wear model in our work is extended further to predict the wear depth of the flat surface. Experimental data from the ball-on-flat contact of a

DLC coating as it slides against an AISI 440C steel ball in deionised water are used to validate the model. Since the wear of a DLC coating does not obey Archard's wear law due to the growth of a carbonaceous transfer layer, a time-dependent-specific wear rate is used to predict the wear depth of both surfaces.

2 Derivation of the Equations

A schematic of a ball-on-flat contact including the relevant geometric parameters is shown in Fig. 1. For a ball sliding against a flat surface, the actual wear volume of the ball $w_{\mathrm{V,Ball}}$ can be given as a function of the wear depth of the ball $w_{\mathrm{D,Ball}}$ and the contact radius a, by the spherical cap formula [8], as follows.

$$w_{\mathrm{V,Ball}} = \frac{\pi w_{\mathrm{D,Ball}}}{6}\left(3a^2 + w_{\mathrm{D,Ball}}^2\right) \tag{4}$$

Similarly, the wear depth of the ball can be written as a function of the radius of contact a and the radius of the ball r [8].

$$w_{\mathrm{D,Ball}} = r - \sqrt{r^2 - a^2} \tag{5}$$

In this manner, wear depth and wear volume of a ball may be related using geometric parameters. Assuming that the initial contact radius a_0 is Hertzian [4] (where E^* is the reduced modulus, given as a function of the Young's modulus E and Poisson's ratio v of the ball and flat surfaces),

$$\begin{aligned} a_0 &= \left(\frac{3Nr}{2E^*}\right)^{1/3} \\ \frac{1}{E^*} &= \left(\frac{1 - v_{\mathrm{Ball}}^2}{E_{\mathrm{Ball}}} + \frac{1 - v_{\mathrm{Flat}}^2}{E_{\mathrm{Flat}}}\right) \end{aligned} \tag{6}$$

And that the average contact pressure may be given as a ratio of normal load to contact area;

$$P = \frac{N}{A} = \frac{N}{\pi a^2} \tag{7}$$

Then, through substitution of Eqs. 4, 5, and 7 into Eq. 3, the following first-order non-linear ordinary differential equation is derived for the contact radius.

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{\mathrm{d}w_{\mathrm{V}}}{\mathrm{d}t}\frac{\sqrt{r^2 - a^2}}{\pi a^3} \tag{8}$$

This equation describes the rate of change of contact radius a as a product of the rate of change of wear volume w_{V} (determined by experiments and provided by a phenomenological law such as Archard's wear law) and a function determined by geometric considerations. The same differential equation may be derived from either the Archard or Preston formulation of wear. Using Eqs. 4 and 5, the wear depth and wear volume of the ball can be extracted once a is known.

An analytical solution for the contact radius may be found for Eq. 8. Integrating with respect to time yields the general solution to the differential equation, where w_{V} is estimated using Archard's wear law, or some equivalent phenomenological wear law [9].

$$\frac{-\pi}{3}\sqrt{r^2 - a^2}\left(a^2 + 2r^2\right) = w_{\mathrm{V}} \tag{9}$$

Equation 9 can be rearranged to give the contact radius explicitly in terms of geometric parameters and the wear volume of the ball.

$$\begin{aligned} a &= r\sqrt{\xi^{1/3} + \xi^{-1/3} - 1} \\ \xi &= 1 - \frac{3w_{\mathrm{V}}}{2\pi^2 r^6}\left(\sqrt{9w_{\mathrm{V}}^2 - 4\pi^2 r^6} + 3w_{\mathrm{V}}\right) \end{aligned} \tag{10}$$

Supposing that$w_{\mathrm{V}} = kNvt + C$ (using Archard's wear law), where C is the initial volume shrinkage of the sphere (due to elastic deformation), then based on the initial conditions (Eq. 6), the constant C is given as follows:

$$C = \frac{-\pi}{3}\sqrt{r^2 - a_0^2}\left(a_0^2 + 2r^2\right) \tag{11}$$

Using the theory developed here, the wear volume and wear depth of the flat surface can also be evaluated. Assuming that the area of the wear scar is given by the product of the stroke length and the contact diameter, then the wear volume of the flat$w_{\mathrm{V,Flat}}$and the average wear depth of the flat$w_{\mathrm{D,Flat}}$can be related by Eq. 12, where l denotes the stroke length.

$$w_{\mathrm{V,Flat}} = 2alw_{\mathrm{D,Flat}} \tag{12}$$

The wear depth of the flat surface $w_{\mathrm{D,Flat}}$ may be expressed in terms of a measured experimental wear volume W_{V}.

$$w_{\mathrm{D,Flat}} = \frac{W_{\mathrm{V}}}{2al} \tag{13}$$

Estimation of the wear volume of the flatW_{V}allows for extraction of wear depth of the flat surface as a function of time taking into account the changing contact area according to Eq. 13. The advantage of this new theory for the wear depth of each surface is that from a single test run from which wear volume may be calculated, the theory allows for an entire description of wear depth as a function of time which is of great importance to the design engineer interested in changes in tolerance.

3 Results

Consider the reciprocating contact of a DLC coating and a 6-mm diameter AISI 440C steel ball at a 5 N load and

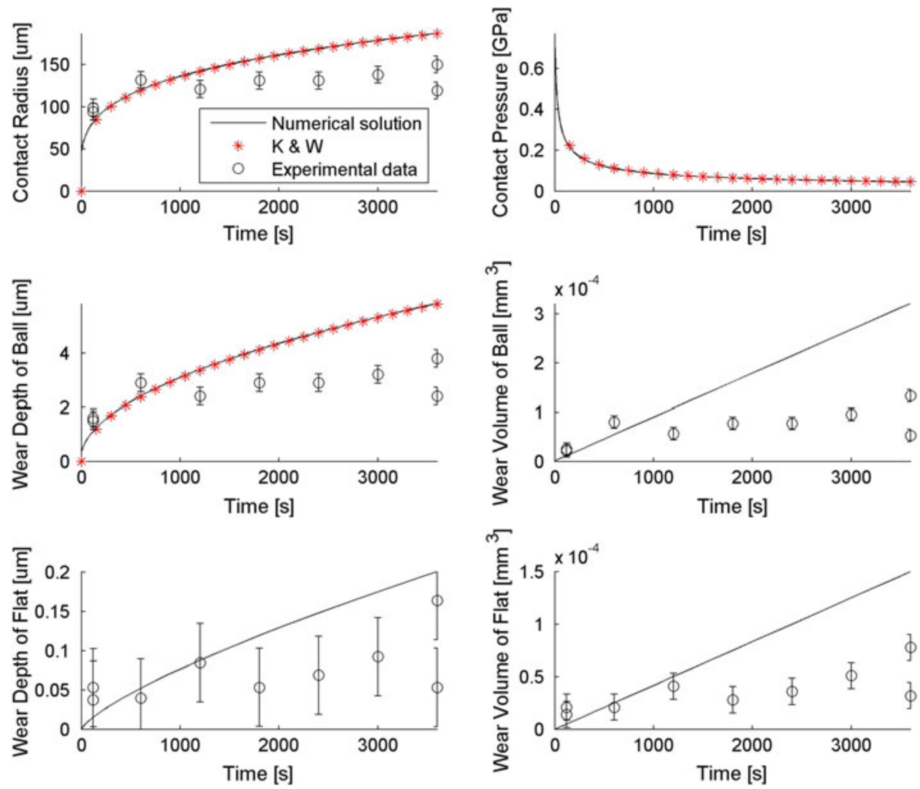

Fig. 2 Contact radius (*top-left*), contact pressure (*top-right*), wear depth of the ball (*middle-left*), wear volume of the ball (*middle-right*), wear depth of the DLC coating (*bottom-left*), wear volume of the DLC coating (*bottom-right*), plotted as a function of time. The *black line* represents the numerical solution, the *red asterisks* represent the model of Kauzlarich and Williams (K & W) [6], and the *black circles* represent experimental data. *Error bars* are shown by *black vertical lines* and represent the error in measurement (Color figure online)

0.02 m/s sliding velocity over a 2 mm stroke. Wear volume data have been collected at 120, 600, 1,200, 1,800, 2,400, 3,000, and 3,600 s. The specific wear rate after 120 s (averaged from the two data points at this time) was calculated to be $1.75 \times 10^{-6}\mathrm{mm}^3/\mathrm{Nm}$ for the DLC coating, and $2.05 \times 10^{-6}\mathrm{mm}^3/\mathrm{Nm}$for the ball, using Eq. 1, and this is used as an input to the wear model described in the previous section.

Figure 2 shows the predicted contact radius (Eqs. 10 and 11), contact pressure (Eq. 7), wear depth and wear volume of the ball (Eqs. 4 and 5), and wear depth and volume of the flat surface (Eqs. 12 and 13), as a function of time, based on the initial specific wear rate at 120 s. The results are compared to the analytical results of Kauzlarich and Williams [6].

The contact radius (top-left) is predicted by the new model to increase from the initial Hertzian value of 46.5–187 μm after 3,600 s. The model of Kauzlarich and Williams [6] shows a near identical prediction to our model in this case since the assumption that the contact radius is small in relation to the radius of the ball is valid. Since our model compares well with the Kauzlarich and Williams model under different load and sliding velocity—and the Kauzlarich and Williams model was validated against the general case of a steel/steel contact—we can conclude that our model is validated in the general case of a steel/steel contact. Indeed, the model can be trusted in any ball-on-flat contact, which obeys Archards wear law.

In the context of the DLC coating/steel contact, the experimental data for the contact area match the model well initially, but deviate as the test time increases. This is suggested to be due to the growth of a carbonaceous transfer layer as is known for DLC coatings [10] which reduces wear to both surfaces. As a result of the changing contact area, the contact pressure (top-right) is shown to decrease from an initial Hertzian value of 0.73 GPa to a final value of 0.05 GPa after 3,600 s.

The wear depth of the ball (middle-left) is predicted by the new model to be 5.84 μm after 3,600 s. The model overestimates the wear depth considerably, when compared to the average experimental value of 3.1 μm. The model fits the data well initially, however. Similarly, the wear volume of the ball (middle-right) is overestimated by the new model (since this is directly related to the wear depth). A final wear volume of $3.21 \times 10^{-4}\mathrm{mm}^3$is predicted by the new model, in comparison with the experimental observation of $0.93 \times 10^{-4}\mathrm{mm}^3$.

The prediction of wear volume and wear depth of the ball by the model of Kauzlarich and Williams [6] is the same as the prediction of the new model. However, the new model can be extended to predict the wear depth of the DLC coating also. The wear depth of the DLC coating is predicted by the new model to be 0.20 μm after 3,600 s, whereas physical tests suggest that it is approximately half of this in reality. Similarly, the wear volume of the DLC coating is overpredicted by the new model in the latter part of the test. Again, this is suggested to be due to the growth of a transfer layer in the contact.

An observation from the model presented in this work is that the wear depth of the ball is initially calculated to be non-zero (0.36 μm) by Hertzian calculations, and this is due to the elastic deformation of the ball in which Eq. 5 interprets as wear (since the contact width is non-zero initially). The supposed wear depth of the ball calculated by the differential equation is actually a sum of the wear depth plus the elastic deformation of the ball. Since the elastic deformation of the ball tends to be zero as the pressure tends to be zero, this approximation becomes less important as the test goes on.

The wear model is able to predict the wear depth and wear volume of a ball and a flat surface as they slide against each other. However, an input to this model is the specific wear rate of each surface, which must be calculated directly from experiments, and an issue arises in the context of DLC coatings since the experimental specific wear rate varies with time, and as such the Archard-based wear model cannot predict the evolution of wear accurately. A physical interpretation for this is thought to be related to the growth of a transfer layer—which is known to reduce the specific wear rate of a DLC coating [3, 10]. For example, if the development of a transfer layer prevents contact between the DLC coating and steel—then the contact develops from a DLC coating/steel contact with a high-specific wear rate to a DLC coating/transfer layer contact with a low-specific wear rate as the transfer layer grows across the contact. To account for the growth of a transfer layer during the wear of a DLC coating against steel, the model can be extended to include a non-constant-specific wear rate. This extension is also of interest in a general case for any tribological scenario when Archards wear law does not hold, for example due to a transition between wear mechanisms or due to chemical changes at the interface.

4 Extension to a Time-Dependent Specific Wear Rate

A general case where the specific wear rate depends on time is considered. If wear volume is estimated by Archard's wear law [1, 2] and the specific wear rate is given as $k = k(t)$, then the differential equation for contact area (Eq. 8) can be written as follows.

Table 1 Two special cases of Eq. 14, for linear and exponential specific wear rates

Functional form	Contact area temporal gradient
Linear, $k(t) = \alpha_1 t + \alpha_2$	$\frac{\mathrm{d}a}{\mathrm{d}t} = Nv(2\alpha_1 t + \alpha_2)\frac{\sqrt{r^2-a^2}}{\pi a^3}$
Exponential, $k(t) = \beta_1 e^{\beta_2 t}$	$\frac{\mathrm{d}a}{\mathrm{d}t} = Nv\beta_1(1+\beta_2 t)\mathrm{e}^{\beta_2 t}\frac{\sqrt{r^2-a^2}}{\pi a^3}$

$$\frac{\mathrm{d}a}{\mathrm{d}t} = Nv\left[k + \left(\frac{\mathrm{d}k}{\mathrm{d}t}\right)t\right]\frac{\sqrt{r^2 - a^2}}{\pi a^3} \tag{14}$$

A numerical solution to Eq. 14 will yield estimations of the wear depth and wear volume of a ball and a flat surface (using Eqs. 4, 5, 12, and 13). The numerical solutions presented in this paper have been obtained using the implicit scheme ode 15 s in MatLab R2012a (MathWorks Inc., Natick, MA, USA).

The functional form that is chosen for the specific wear rate is conditioned by the experimental findings. It may be the case that a constant specific wear rate is found for tests of varying sliding distance, suggesting that the analytic results of the previous section are sufficient to describe the evolution of wear. For more complex cases, where the specific wear rate is a function of time, it must be emphasised that care needs to be taken when choosing a functional form for $k(t)$, especially when only a few data points are available. Table 1 considers two special cases for $k(t)$. In these cases, care must be taken when extrapolating the data—since a negative gradient may lead to unrealistic negative wear rates.

Figure 3 shows how the specific wear rate varies with time for both the steel ball and DLC-coated flat surfaces. A linear regression (black line) and an exponential regression (red line) are used to fit the model to the data. For both the ball and flat surfaces, an exponential curve provides the best fit.

The wear model can now be fitted to the data, by numerically solving the appropriate ordinary differential equation (Eq. 14) using an exponential form for k. Figure 4 shows the numerical solution of the new (semi-empirical) wear model (black line) fitted to the experimental data (black circles). The change in contact radius (top-left) fits very well with the experimental data, predicting a final contact radius of 139 μm suggesting a mean contact pressure of 0.08 GPa. The final wear depth of the ball is predicted to be 3.23 μm in comparison with the observed average of 3.20 μm from the experiments. The depth of the DLC coating wear scar is estimated as 0.093 μm in comparison with the observed average of 0.11 μm from the experiments.

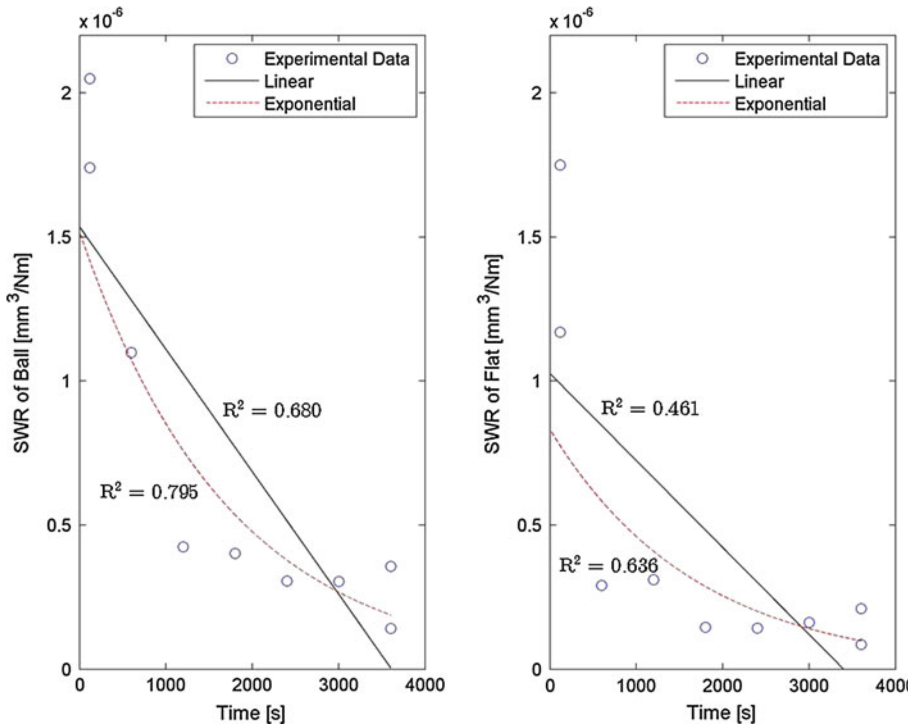

Fig. 3 Specific wear rate (SWR) plotted as a function of time for the ball (*left*), and the DLC coating (*right*). Experimental data are indicated by *blue circles*. The linear regression is given as a *black line*, and the exponential regression is given as a *dashed red line* (Color figure online)

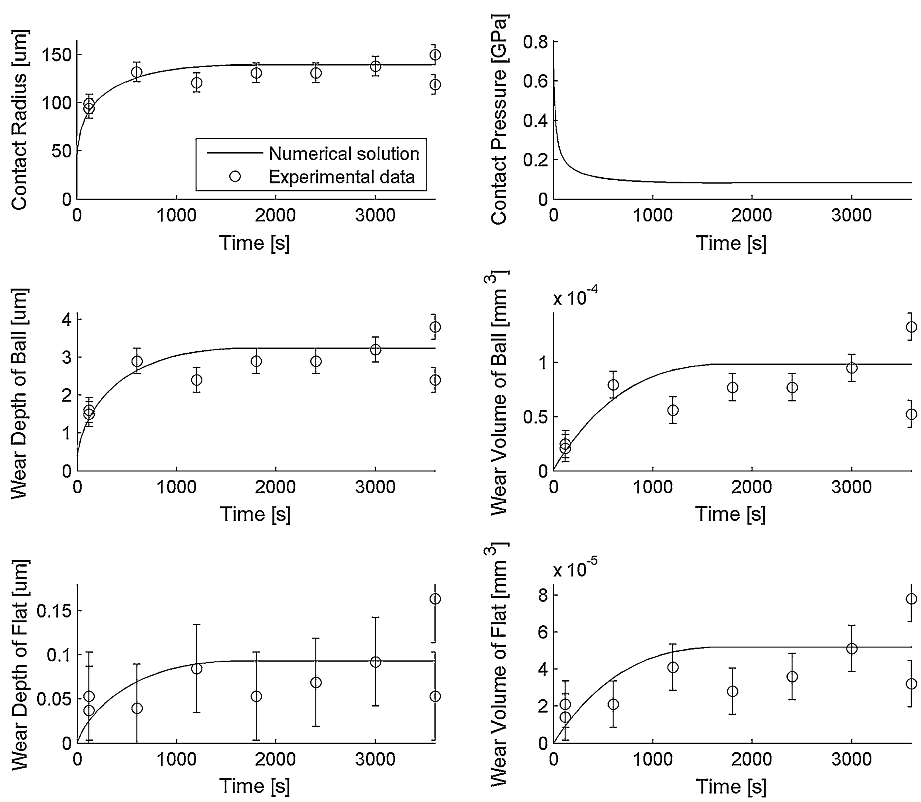

Fig. 4 Contact radius (*top-left*), contact pressure (*top-right*), wear depth of the ball (*middle-left*), wear volume of the ball (*middle-right*), wear depth of the DLC coating (*bottom-left*), wear volume of the DLC coating (*bottom-right*), plotted as a function of time. The specific wear rate of the ball and flat surfaces is assumed to decrease exponentially. The *black line* represents the numerical solution, and the *black circles* represent the experimental data. *Error bars* are shown by *black vertical lines* and represent the error in measurement

5 Conclusion

- The Archard and Preston formulations were generalised to include the change in contact area as the ball-on-flat test progressed. An equation was derived (Eq. 10) to predict the change in contact radius with time for any phenomenological wear law. From this, wear depth and wear volume of a ball and a flat surface were extracted.

- Experimental tests of an AISI 440C steel ball against a DLC coating showed that the original model with a constant specific wear rate did not predict the wear accurately. This was due to the assumption that the specific wear rate was constant, whereas experimentally, the specific wear rate was observed to vary with time. This was due to the formation of a transfer layer, the effects of which are not included for in Archard's wear law.
- Assuming a specific wear rate varies in time, and an exponential model was fitted to the data for the ball and flat surfaces based on experimental observations. Wear volumes and wear depths were predicted accurately as a result with less than a 5 % deviation from the experimental data. The assumption that the specific wear rate varies in time may, in a general case, be due to a transition between wear mechanisms or due to chemical changes.
- The model features a high degree of modularity as it can accommodate arbitrary functional forms to calculate wear volume. It is hoped that the flexibility of the model presented in the paper will be exploited by tribologists to further put its descriptive and predictive power to the test.
- Care should be taken when fitting functional forms for the specific wear rate. Critically, a need for several data points was highlighted to allow confidence in the choice of fitting a functional form for the specific wear rate.
- Possible extensions to this work include a consideration of a line contact or elliptic contact, following a similar methodology to the point contact discussed in this work.

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References

1. Archard, J.F.: Contact and rubbing of flat surfaces. J. Appl. Phys. **24**(8), 981–988 (1953). doi:10.1063/1.1721448
2. Archard, J.F., Hirst, W.: The wear of materials under lubricated conditions. Proc. R. Soc. Mech. Eng. A **236**, 397–410 (1956)
3. Erdemir, A., Donnet, C.: Tribology of Diamond-Like Carbon Films. Springer, New York (2008)
4. Hertz, H.: Über die berührung fester elastischer Körper (On the contact of rigid elastic solids). In: Schott, J.A. (ed.) Miscellaneous Papers, vol. 92. Macmillan, London (1896)
5. Preston, F.W.: The theory and design of plate glass polishing machines. J. Soc. Glass Technol. **11**, 214–256 (1927)
6. Kauzlarich, J.J., Williams, J.A.: Archard wear and component geometry. Proc. Inst. Mech. Eng. Part J J. Eng. Tribol. **215**(4), 387–403 (2001). doi:10.1243/1350650011543628
7. Podra, P., Andersson, S.: Simulating sliding wear with finite element method. Tribol. Int. **32**(2), 71–81 (1999). doi:10.1016/S0301-679x(99)00012-2
8. Harris, J.W., Stocker, H.: Spherical segment (spherical cap). In: Harris, J.W., Stocker, H. (eds.) Handbook of Mathematics and Computational Science, p. 107. Springer-Verlag, New York (1998)
9. Bryant, M.D., Khonsari, M.M., Ling, F.F.: On the thermodynamics of degradation. Proc R Soc A **464**(2096), 2001–2014 (2008). doi:10.1098/rspa2007.0371
10. Erdemir, A., Donnet, C.: Tribology of diamond-like carbon films: recent progress and future prospects. J. Phys. D Appl. Phys. **39**(18), 311–327 (2006). doi:10.1088/0022-3727/39/18/R01