



# Storage Coefficients and Permeability Functions for Coal-Bed Methane Production Under Uniaxial Strain Conditions

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## Abstract

The porosity and permeability of coal change with pore pressure, due to changes in effective stress and matrix swelling due to gas adsorption. Three analytical models to describe porosity and permeability change in this context have been presented in the literature, all of which are based on poroelastic theory and uniaxial strain conditions. However, each of the three models provides different results. Review articles have attributed these differences to the use of stress formulations or strain formulations. In this article, the three aforementioned porosity models are used to derive three associated expressions for the storage coefficient. A single mathematical equation for the storage coefficient in an aquifer under uniaxial strain conditions is well established. The storage coefficient represents the volume of fluid released per unit volume of a porous rock following a unit decline in pore pressure. It is shown that only one of the aforementioned three coal-bed methane porosity models leads to the correct equation for the uniaxial strain storage coefficient in the absence of gas sorption-induced strain.

**Keywords** Coal-bed methane · Permeability · Porosity · Storage coefficient · Matrix shrinkage · Rock mechanics

## List of Symbols

- $C_g$  Compressibility of the gaseous methane ( $M^{-1}LT^2$ )
- $C_p$  Seidle's compressibility parameter ( $M^{-1}LT^2$ )
- $E$  Young's modulus ( $ML^{-1}T^{-2}$ )
- $f$  Palmer and Mansoori's model parameter (–)
- $g$  Gravitational acceleration ( $LT^{-2}$ )
- $G$  Shear modulus ( $ML^{-1}T^{-2}$ )
- $k$  Permeability tensor ( $L^2$ )

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$k$	Isotropic permeability ( $L^2$ )
$K$	Bulk modulus ( $ML^{-1}T^{-2}$ )
$k_0$	Reference permeability ( $L^2$ )
$M_g$	Mass of gaseous methane (M)
$m_g$	Mass of gaseous methane per unit bulk volume of coal ( $ML^{-3}$ )
$M_s$	Mass of adsorbed methane (M)
$P_{p0}$	Reference pore pressure ( $ML^{-1}T^{-2}$ )
$P_L$	Langmuir isotherm pressure ( $ML^{-1}T^{-2}$ )
$P_m$	Partial pressure of the adsorbed methane ( $ML^{-1}T^{-2}$ )
$P_p$	Pore pressure ( $ML^{-1}T^{-2}$ )
$S$	Storage coefficient ( $M^{-1}LT^2$ )
$T$	Temperature ( $\Theta$ )
$t$	Time (T)
$V_b$	Bulk volume ( $L^3$ )
$V_m$	Volume of coal mineral ( $L^3$ )
$V_p$	Pore volume ( $L^3$ )
$z$	Elevation (L)
$\alpha$	Biot coefficient ( $-$ )
$\boldsymbol{\epsilon}$	Strain tensor ( $-$ )
$\epsilon_b$	Bulk volume strain ( $-$ )
$\epsilon_L$	Langmuir isotherm strain ( $-$ )
$\epsilon_m$	Mineral volume strain ( $-$ )
$\epsilon_s$	Shrinkage strain associated with methane adsorption ( $-$ )
$\eta$	Permeability exponent ( $-$ )
$\lambda$	Lamé parameter ( $ML^{-1}T^{-2}$ )
$\mu_g$	Dynamic viscosity of gaseous methane ( $ML^{-1}T^{-1}$ )
$\nu$	Poisson's ratio ( $-$ )
$\rho_g$	Density of gaseous methane ( $ML^{-3}$ )
$\rho_s$	Density of the adsorbed methane ( $ML^{-3}$ )
$\boldsymbol{\tau}$	Stress tensor ( $ML^{-1}T^{-2}$ )
$\tau_h$	Hydrostatic stress ( $ML^{-1}T^{-2}$ )
$\tau_m$	Mean stress ( $ML^{-1}T^{-2}$ )
$\phi$	Porosity ( $-$ )
$\phi_0$	Reference porosity ( $-$ )

## 1 Introduction

There are several previously published derivations of analytical models to describe how the porosity and permeability of coal change due to changes in pore pressure. Knowledge about how porosity and permeability evolve is important to help estimate the productivity of coal-bed methane production wells (Liu and Harpalani 2013) in addition to forecasting potential hydromechanical impacts on surrounding geological formations during coal-bed methane production (Wu et al. 2018). Permeability of coal is mostly attributed to the cleats within a given coal formation. As pore pressure is reduced, the effective stress is increased, which leads to a reduction in coal cleat apertures and hence a reduction in coal porosity and permeability. However, reductions in pore pressure also lead to desorption of gas from the coal matrix, which in turn leads to shrinkage of the coal matrix, an increase in coal cleat apertures, and an increase in coal cleat permeability. Of interest in the present work are the

differences between the associated analytical models of Palmer and Mansoori (1998), Shi and Durucan (2004), and Cui and Bustin (2005). These three models all claim to satisfy uniaxial strain conditions, with gas sorption-induced strain (GSIS) treated as analogous to thermal expansion. The fact that the three models provide different results has been attributed to Palmer and Mansoori (1998) adopting a strain formulation, whereas Shi and Durucan (2004) and Cui and Bustin (2005) adopted a stress formulation (Gu and Chalaturnyk 2006; Palmer 2009; Liu and Harpalani 2013; Li et al. 2017). This explanation is unsatisfactory, firstly because it does not explicitly explain the difference between the models of Shi and Durucan (2004) and Cui and Bustin (2005), and secondly, because, if the same theoretical assumptions have been made, the final results should be the same regardless of whether a stress-based or strain-based formulation is used.

Analytical solutions for porosity change due to pore pressure under uniaxial strain conditions have also been derived in the context of fluid production/injection in aquifers (e.g., Gambolati et al. 2000; Jaeger et al. 2007; Zimmerman 2017; Andersen et al. 2017). Associated authors presented a single common equation for the storage coefficient, which they rigorously derived from poroelastic theory. The storage coefficient represents the volume of fluid released per unit volume of a porous rock following a unit decline in pore pressure. In this article, we derive equations for this aforementioned storage coefficient using the three different porosity models of Cui and Bustin (2005), Palmer and Mansoori (1998), and Shi and Durucan (2004). We look at the limits of these equations for when there is no GSIS and compare these with the widely accepted uniaxial strain storage coefficient associated with fluid movement in aquifers (e.g., Gambolati et al. 2000; Jaeger et al. 2007; Zimmerman 2017; Andersen et al. 2017).

The outline of the article is as follows. A modified form of Hooke's law is presented, which incorporates pore pressure and GSIS. It is then shown how to relate GSIS to the mass of adsorbed gas within the coal. A mass conservation equation is derived for gas migration in coal, which leads to a general expression for the aforementioned storage coefficient in terms of fluid compressibility, porosity, bulk strain, and GSIS. A general equation is developed to describe the associated change in porosity. Expressions for stress and bulk strain are derived assuming uniaxial strain conditions. Finally, three storage coefficients are derived using the porosity models of Cui and Bustin (2005), Palmer and Mansoori (1998), and Shi and Durucan (2004). Implications of the results are then discussed in the context of permeability modeling.

## 2 Mathematical Model

### 2.1 Incorporation of Adsorption into Hooke's law

In the context of coal-bed methane production, the following modified form of Hooke's law is generally adopted (consider Jaeger et al. 2007, p. 179)

$$d\boldsymbol{\varepsilon} = \frac{1}{2G}d\boldsymbol{\tau} - \left[ \frac{\nu}{2G(1+\nu)}\text{trace}(d\boldsymbol{\tau}) + \frac{\alpha dP_p}{3K} - \frac{d\varepsilon_s}{3} \right] \mathbf{I} \quad (1)$$

and

$$d\boldsymbol{\tau} = 2Gd\boldsymbol{\varepsilon} + [\lambda\text{trace}(d\boldsymbol{\varepsilon}) + \alpha dP_p - Kd\varepsilon_s] \mathbf{I} \quad (2)$$

where  $\boldsymbol{\varepsilon}$  (–) is the strain tensor,  $\boldsymbol{\tau}$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the stress tensor,  $G$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the shear modulus,  $\nu$  (–) is Poisson's ratio,  $\lambda$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the Lamé parameter,  $K$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the bulk modulus,  $\alpha$  (–) is the Biot coefficient,  $P_p$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the pore pressure, and  $\varepsilon_s$  (–) is

a shrinkage strain associated with methane adsorption, typically described using a Langmuir isotherm of the form (e.g., Ye et al. 2014)

$$\varepsilon_s = \frac{\varepsilon_L P_m}{P_m + P_L} \tag{3}$$

where  $P_m$  ( $ML^{-1}T^{-2}$ ) is the partial pressure of the adsorbed methane and  $\varepsilon_L$  (–) and  $P_L$  ( $ML^{-1}T^{-2}$ ) are empirical parameters.

The value of  $\varepsilon_L$  is negative in this context because  $\varepsilon$  is a positive compression strain. The absolute value of  $\varepsilon_L$  represents the maximum possible volumetric expansion strain that can be incurred due to gas adsorption. The  $P_L$  parameter represents the value of  $P_m$  at which  $\varepsilon_s = \varepsilon_L/2$ .

### 2.2 Linking Adsorption Mass to Adsorption Strain

The shrinkage due to gas sorption is thought to be mathematically analogous to volumetric strain associated with temperature change. An important point to note in this context is that (Grimvall 1999, p. 296)

$$\frac{\partial \varepsilon_m}{\partial T} = \frac{\partial \varepsilon_b}{\partial T} \tag{4}$$

where  $T$  ( $\Theta$ ) is temperature and  $\varepsilon_m$  (–) and  $\varepsilon_b$  (–) are mineral volume and bulk volume strains, respectively, found from

$$d\varepsilon_m = -\frac{dV_m}{V_m} \tag{5}$$

and

$$d\varepsilon_b = -\frac{dV_b}{V_b} \tag{6}$$

where  $V_m$  ( $L^3$ ) is volume of coal mineral contained within a given bulk volume,  $V_b$  ( $L^3$ ). Also note that

$$V_b = V_m + V_p \tag{7}$$

where  $V_p$  ( $L^3$ ) is the pore volume.

Given the mathematical analogy between gas sorption and temperature, and noting that  $\varepsilon_b = \text{trace}(\boldsymbol{\varepsilon})$ , Eqs. (1), (3), and (4) suggest that

$$\frac{\partial \varepsilon_m}{\partial P_m} = \frac{\partial \varepsilon_b}{\partial P_m} = \frac{\partial \varepsilon_s}{\partial P_m} = \frac{\varepsilon_s^2 P_L}{\varepsilon_L P_m^2} \tag{8}$$

An incremental increase in the mass of adsorbed methane,  $\delta M_s$  (M), will result in an incremental increase in the coal–mineral volume of  $\delta V_m = \delta M_s / \rho_s$  ( $L^3$ ) where  $\rho_s$  ( $ML^{-3}$ ) is the density of the adsorbed methane, which is assumed to be constant. It follows that the associated incremental increase in mineral strain  $d\varepsilon_m = -\delta M_s / (\rho_s V_m)$ . Equation (8) therefore suggests that

$$\frac{dM_s}{\rho_s V_b} = -(1 - \phi)d\varepsilon_s \tag{9}$$

where  $\phi = V_p / V_b$  (–) is the porosity.

### 2.3 Mass Conservation Statement

Following similar ideas to those presented by Jaeger et al. (2007, p. 184), a mass conservation statement for methane within a deformable mass of coal can take the form

$$\frac{\partial \zeta}{\partial t} + \frac{1}{\rho_g} \nabla \cdot (\rho_g \mathbf{q}) = 0 \quad (10)$$

where  $t$  (T) is time and (consider Jaeger et al. 2007, p. 180)

$$d\zeta = \frac{dM_g + dM_s}{\rho_g V_b} \quad (11)$$

and

$$\mathbf{q} = -\frac{\mathbf{k}}{\mu_g} \nabla (P_p + \rho_g g z) \quad (12)$$

where  $M_g$  (M) is the mass of gaseous methane contained within the bulk volume of coal,  $V_b$  ( $L^3$ ),  $\rho_g$  ( $ML^{-3}$ ) is the density of gaseous methane,  $\mathbf{k}$  ( $L^2$ ) is permeability,  $\mu_g$  ( $ML^{-1}T^{-1}$ ) is the dynamic viscosity of gaseous methane,  $g$  ( $LT^{-2}$ ) is gravitational acceleration, and  $z$  (L) is elevation.

In the context of coal-bed methane, the permeability is generally taken to be a scalar quantity found from (Palmer and Mansoori 1998; Seidle et al. 1992; Shi and Durucan 2004; Cui and Bustin 2005)

$$k = k_0 (\phi/\phi_0)^\eta \quad (13)$$

where  $k_0$  is the value of  $k$  when  $\phi = \phi_0$ ,  $\phi_0$  is a reference value of the porosity and  $\eta$  (–) is an empirical exponent (Pyrak-Nolte et al. 1998), generally assumed to be three (due to an association with the so-called match stick model, Seidle et al. 1992).

Let  $m_g = M_g/V_b$  ( $ML^{-3}$ ) be the mass of gaseous methane per unit bulk volume of coal. It follows that

$$d\zeta = \frac{dm_g}{\rho_g} - \frac{m_g d\varepsilon_b}{\rho_g} + \frac{dM_s}{\rho_g V_b} \quad (14)$$

Given Eq. (9) and also that  $m_g = \phi \rho_g$ , it can be further stated that

$$d\zeta = \phi C_g dP_p + d\phi - \phi d\varepsilon_b - (1 - \phi) \frac{\rho_s}{\rho_g} d\varepsilon_s \quad (15)$$

where  $C_g = \rho_g^{-1} \partial \rho_g / \partial P_p$  ( $M^{-1}LT^2$ ) is the compressibility of the gaseous methane.

The storage coefficient,  $S$  ( $M^{-1}LT^2$ ), is found from (Green and Wang 1990)

$$S \equiv \frac{\partial \zeta}{\partial P_p} = \phi C_g + \frac{\partial \phi}{\partial P_p} - \phi \frac{\partial \varepsilon_b}{\partial P_p} - (1 - \phi) \frac{\rho_s}{\rho_g} \frac{\partial \varepsilon_s}{\partial P_p} \quad (16)$$

### 2.4 Determining the Change in Porosity

Given that  $\phi = V_p/V_b$  where  $V_p$  and  $V_b$  are both functions of  $P_p$ ,  $P_m$  and the mean stress,  $\tau_m = \text{trace}(\boldsymbol{\tau})/3$ , the change in porosity can be found from

$$\frac{d\phi}{\phi} = (C_{pp} - C_{bp}) dP_p + (C_{bc} - C_{pc}) d\tau_m + \left( \frac{\partial \varepsilon_b}{\partial P_m} - \frac{\partial \varepsilon_p}{\partial P_m} \right) dP_m \quad (17)$$

where

$$d\varepsilon_p = -\frac{dV_p}{V_p} \quad (18)$$

and

$$\begin{aligned} C_{bc} &= \frac{\partial \varepsilon_b}{\partial \tau_m}, \quad C_{bp} = -\frac{\partial \varepsilon_b}{\partial P_p}; \\ C_{pc} &= \frac{\partial \varepsilon_p}{\partial \tau_m}, \quad C_{pp} = -\frac{\partial \varepsilon_p}{\partial P_p}. \end{aligned} \tag{19}$$

A result from Eq. (8) is that

$$\frac{\partial \varepsilon_b}{\partial P_m} = \frac{\partial \varepsilon_p}{\partial P_m} \tag{20}$$

Furthermore (Jaeger et al. 2007, p. 173),

$$\phi(C_{pp} - C_{bp}) = \frac{\alpha - \phi}{K} \tag{21}$$

$$\phi(C_{bc} - C_{pc}) = \frac{\phi - \alpha}{K} \tag{22}$$

from which it follows that

$$d\phi = \frac{(\alpha - \phi)(dP_p - d\tau_m)}{K} \tag{23}$$

### 2.5 Imposing Uniaxial Strain Conditions

Following Palmer and Mansoori (1998), Shi and Durucan (2004), and Cui and Bustin (2005), uniaxial strain conditions are assumed whereby  $\varepsilon_{xx} = \varepsilon_{yy} = 0$  and  $\varepsilon_b = \varepsilon_{zz}$ . Equation (2) reveals that under such conditions

$$d\varepsilon_b = \frac{d\tau_{zz} - \alpha dP_p + K d\varepsilon_s}{2G + \lambda} \tag{24}$$

and, furthermore, that

$$d\tau_{xx} = d\tau_{yy} = \frac{\lambda d\tau_{zz}}{2G + \lambda} + \frac{2G(\alpha dP_p - K d\varepsilon_s)}{2G + \lambda} \tag{25}$$

and (note that  $3\lambda = 3K - 2G$ )

$$d\tau_m = \frac{3K d\tau_{zz} + 4G(\alpha dP_p - K d\varepsilon_s)}{3(2G + \lambda)} \tag{26}$$

In the absence of gas adsorption (i.e.,  $\varepsilon_L = 0$  and/or  $\partial \varepsilon_s / \partial P_p = 0$ ), uniaxial strain conditions (as described above) allow Eq. (16) to reduce to (Gambolati et al. 2000; Jaeger et al. 2007; Zimmerman 2017; Andersen et al. 2017)

$$\lim_{\varepsilon_L \rightarrow 0} S = \phi C_g + \frac{(1 - \alpha)(\alpha - \phi)}{K} + \frac{\alpha^2}{2G + \lambda} \tag{27}$$

Note that the  $\tau_{zz}$  term is not present in Eq. (27) because it is also assumed that  $\tau_{zz}$  is only dependent on the weight of the overburden and therefore independent of  $P_p$ .

### 2.6 Application of the Cui and Bustin Model

The assumptions listed above are consistent with those originally adopted by Cui and Bustin (2005). Indeed, substituting Eq. (26) into Eq. (23) leads to (Cui and Bustin 2005)

$$d\phi = (\alpha - \phi) \left[ \left( \frac{1 - \alpha}{K} + \frac{\alpha}{2G + \lambda} \right) dP_p - \frac{d\tau_{zz}}{2G + \lambda} + \frac{4G d\varepsilon_s}{3(2G + \lambda)} \right] \tag{28}$$

Substituting Eqs. (24) and (28) into Eq. (15) then leads to

$$d\zeta = \left[ \phi C_g + \frac{(1 - \alpha)(\alpha - \phi)}{K} + \frac{\alpha^2}{2G + \lambda} \right] dP_p - \frac{\alpha d\tau_{zz}}{2G + \lambda} - \left[ \frac{[3K + 4(1 - \alpha)G]}{3(2G + \lambda)} - \frac{(\rho_g - \rho_s)(1 - \phi)}{\rho_g} \right] d\varepsilon_s \tag{29}$$

such that Eq. (16) reveals that

$$S = \phi C_g + \frac{(1 - \alpha)(\alpha - \phi)}{K} + \frac{\alpha^2}{2G + \lambda} - \left[ \frac{[3K + 4(1 - \alpha)G]}{3(2G + \lambda)} - \frac{(\rho_g - \rho_s)(1 - \phi)}{\rho_g} \right] \frac{\partial \varepsilon_s}{\partial P_p} \tag{30}$$

Note that Eq. (30) exactly reduces to Eq. (27) when  $\varepsilon_L = 0$ .

### 2.7 Application of the Palmer and Mansoori Model

Palmer and Mansoori (1998) apply an alternative porosity equation, originally proposed in an unpublished correspondence from Nigel Higgs, which reads as follows

$$-d\phi = \left[ \frac{1}{2G + \lambda} - (1 - \phi)(1 - \alpha) \frac{f}{K} \right] (d\tau_{zz} - dP_p) + \left[ \frac{1}{2G + \lambda} - \frac{(1 - \phi)}{K} \right] ((1 - \alpha)dP_p + K d\varepsilon_s) \tag{31}$$

where  $f(-)$  is described as a “fraction” that ranges from zero to one. Based on a discussion by Moor et al. (2015), Zimmerman (2017) suggested that  $f$  is thought to relate to the ratio of isotropic strain to deviatoric strain within the mineral grains of the rock of concern. Note that Palmer and Mansoori (1998) state that Eq. (31) can be derived by assuming uniaxial strain conditions, as described above.

Eq. (31) can be rearranged to get

$$d\phi = \left[ \frac{(1 - \alpha)(1 - \phi)(1 - f)}{K} + \frac{\alpha}{2G + \lambda} \right] dP_p - \left[ \frac{1}{2G + \lambda} - (1 - \alpha)(1 - \phi) \frac{f}{K} \right] d\tau_{zz} - \left[ \frac{K}{2G + \lambda} - (1 - \phi) \right] d\varepsilon_s \tag{32}$$

Substituting Eqs. (24) and (32) into Eq. (15) then leads to

$$d\zeta = \left[ \phi C_g + \frac{(1 - \alpha)(1 - \phi)(1 - f)}{K} + \frac{\alpha(1 + \phi)}{2G + \lambda} \right] dP_p - \left[ \frac{1 + \phi}{2G + \lambda} - (1 - \alpha)(1 - \phi) \frac{f}{K} \right] d\tau_{zz} - \left[ \frac{K(1 + \phi)}{2G + \lambda} - \frac{(\rho_g - \rho_s)(1 - \phi)}{\rho_g} \right] d\varepsilon_s \tag{33}$$

such that Eq. (16) leads to

$$S = \phi C_g + \frac{(1 - \alpha)(1 - \phi)(1 - f)}{K} + \frac{\alpha(1 + \phi)}{2G + \lambda} - \left[ \frac{K(1 + \phi)}{2G + \lambda} - \frac{(\rho_g - \rho_s)(1 - \phi)}{\rho_g} \right] \frac{\partial \varepsilon_s}{\partial P_p} \tag{34}$$

As previously shown by Zimmerman (2017), regardless of the choice of value of  $f$ , Eqs. (33) and (34) only reduce to Eqs. (29) and (30) when  $\phi = 0$  and  $\alpha = 1$ . It follows that Eq. (31) is invalid when  $\phi > 0$  or  $\alpha < 1$ .

### 2.8 Application of the Shi and Durucan Model

Under hydrostatic stress conditions,  $d\boldsymbol{\tau} = dP_p\mathbf{I} \equiv d\tau_h\mathbf{I}$  (Jaeger et al. 2007, p. 182) where  $\tau_h$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the hydrostatic stress. In this context, Seidle et al. (1992) suggest that the change in porosity can be described by a pore compressibility parameter,  $C_p$  ( $\text{M}^{-1}\text{LT}^2$ ), defined by

$$C_p = -\frac{1}{\phi} \frac{\partial\phi}{\partial\tau_h} \tag{35}$$

Shi and Durucan (2004) further suggest that such an equation should also hold under uniaxial strain conditions and state that

$$d\phi = -\phi C_p (d\tau_{xx} - \alpha dP_p) \tag{36}$$

Substituting Eq. (25) into Eq. (36) then leads to

$$d\phi = -\frac{\phi C_p [\lambda(d\tau_{zz} - \alpha dP_p) - 2GK d\epsilon_s]}{2G + \lambda} \tag{37}$$

and substituting Eqs. (24) and (37) into Eq. (15) gives us

$$d\zeta = \left[ \phi C_g + \frac{\phi\alpha(C_p\lambda + 1)}{2G + \lambda} \right] dP_p - \frac{\phi(C_p\lambda + 1)d\tau_{zz}}{2G + \lambda} - \left[ \frac{\phi K(1 - 2C_p G)}{2G + \lambda} + \frac{\rho_s}{\rho_g}(1 - \phi) \right] d\epsilon_s \tag{38}$$

such that Eq. (16) leads to

$$S = \phi C_g + \frac{\phi\alpha(C_p\lambda + 1)}{2G + \lambda} - \left[ \frac{\phi K(1 - 2C_p G)}{2G + \lambda} + \frac{\rho_s}{\rho_g}(1 - \phi) \right] \frac{\partial\epsilon_s}{\partial P_p} \tag{39}$$

Shi and Durucan (2004) suggest that  $C_p$  should be obtained by calibration to experimental data. However, a theoretical expression for  $C_p$  can be obtained by matching the  $\partial\epsilon_s/\partial P_p$  terms in both Eqs. (30) and (39), which leads to

$$C_p = \frac{2(\alpha - \phi)}{3\phi K} = \frac{2(C_{pp} - C_{bp})}{3} \tag{40}$$

Unfortunately, substituting Eq. (40) back into Eq. (39) and then setting  $\partial\epsilon_s/\partial P_p = 0$  do not lead to the storage coefficient expression given in Eq. (27). The reason is that there are inconsistencies in the theoretical derivations of both Seidle et al. (1992) and Shi and Durucan (2004). The first problem is that Shi and Durucan (2004) derive an expression for  $\tau_{xx}$  based on uniaxial strain conditions and then apply this to an equation that has been derived assuming hydrostatic stress conditions. The next problem is that under hydrostatic conditions,  $d\tau_m = dP_p$ . It follows from Eq. (23) that  $d\phi = 0$ . Therefore, Eq. (35) is not theoretically valid under hydrostatic conditions. Instead, Eq. (35) should be thought of as a heuristic function with an unspecified theoretical basis.



### 3 Implications for Permeability Modeling

Partially differentiating Eq. (28) with respect to  $P_p$  leads to

$$\frac{1}{\phi} \frac{\partial \phi}{\partial P_p} = (C_{pp} - C_{bp}) \left[ \left( 1 - \alpha + \frac{\alpha(1 + \nu)}{3(1 - \nu)} \right) + \frac{2E}{9(1 - \nu)} \frac{\partial \varepsilon_s}{\partial P_p} \right] \quad (41)$$

where  $(C_{pp} - C_{bp})$  is defined in Eq. (21) and  $E$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) and  $\nu$  ( $-$ ) are the Young's modulus and Poisson's ratio, respectively.

It is further assumed that  $P_m = P_p$  and  $(C_{pp} - C_{bp})$  is constant. Integrating Eq. (41) with respect to  $P_p$  and substituting the result into Eq. (13) then lead to (Cui and Bustin 2005)

$$\frac{k}{k_0} = \exp \left[ A(P_p - P_{p0}) + B \left( \frac{P_p}{P_p + P_L} - \frac{P_{p0}}{P_{p0} + P_L} \right) \right] \quad (42)$$

where

$$A = \eta(C_{pp} - C_{bp}) \left( 1 - \alpha + \frac{\alpha(1 + \nu)}{3(1 - \nu)} \right) \quad \text{and} \quad B = \frac{2\eta(C_{pp} - C_{bp})E\varepsilon_L}{9(1 - \nu)} \quad (43)$$

and  $P_{p0}$  ( $\text{ML}^{-1}\text{T}^{-2}$ ) is the reference pore pressure at which  $k = k_0$ .

Applying the same set of procedures to Eq. (37) also leads to Eq. (42) but with (Shi and Durucan 2004)

$$A = \frac{\eta C_p \alpha \nu}{1 - \nu} \quad \text{and} \quad B = \frac{\eta C_p E \varepsilon_L}{3(1 - \nu)} \quad (44)$$

The analysis in this article has shown that Eq. (42) in conjunction with Eq. (43), due to Cui and Bustin (2005), is consistent with conventional poroelastic theory and uniaxial strain conditions. In contrast, Eq. (44), due to Shi and Durucan (2004), is theoretically inconsistent. However, in the literature it is found that Eq. (44) is better able to capture the response of experimentally observed permeability data, as compared to Eq. (43) (e.g., Shi et al. 2014; Zeng and Wang 2017). The reason for this is that  $\eta$  is generally restricted to three (to reflect the match stick model). Consequently, Eq. (43) only has one fitting parameter for this purpose, namely  $\varepsilon_L$ . In contrast, Eq. (44) has two fitting parameters  $C_p$  and  $\varepsilon_L$ , and therefore will inevitably be easier to fit to observed data. In practice, the value of  $\eta$  is often unpredictable (Pyrak-Nolte et al. 1998). If  $\eta$  is treated as an additional fitting parameter, Eq. (43) will have the same degrees of freedom as Eq. (44) and the Cui and Bustin (2005) permeability model will be able to fit experimental data in exactly the same way as the Shi and Durucan (2004) model but with the important added advantage that a theoretically consistent storage coefficient can also be derived.

### 4 Conclusions

Theoretical equations have been derived to describe porosity change and storage coefficient in coal-bed methane systems under both general and uniaxial strain conditions. Three equations for the storage coefficient were then derived using the porosity models of Cui and Bustin (2005), Palmer and Mansoori (1998), and Shi and Durucan (2004). For the limiting case when there is no gas adsorption, only the storage coefficient derived using the Cui and Bustin (2005) porosity model correctly reduced to an established expression for the storage coefficient in an aquifer under uniaxial strain conditions. The storage coefficient derived using the Palmer and Mansoori (1998) porosity model was found to provide the correct limiting result only

when the Biot coefficient is assumed to be one and the porosity is assumed to be zero. The storage coefficient derived using the Shi and Durucan (2004) porosity model is found not to have the correct limit because its derivation involves a mixture of hydrostatic and uniaxial stress assumptions. Out of the three models, only the Cui and Bustin (2005) porosity model is shown to rigorously satisfy poroelastic theory under uniaxial strain conditions.

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