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# The Onset of Double-Diffusive Convection in a Superposed Fluid and Porous Layer Under High-Frequency and Small-Amplitude Vibrations

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**Abstract** We numerically simulate the initiation of an average convective flow in a system composed of a horizontal binary fluid layer overlying a homogeneous porous layer saturated with the same fluid under gravitational field and vibration. In the layers, fixed equilibrium temperature and concentration gradients are set. The layers execute high-frequency oscillations in the vertical direction. The vibration period is small compared with characteristic timescales of the problem. The averaging method is applied to obtain vibrational convection equations. Using for computation the shooting method, a numerical investigation is carried out for an aqueous ammonium chloride solution and packed glass spheres saturated with the solution. The instability threshold is determined under two heating conditions—on heating from below and from above. When the solution is heated from below, the instability character changes abruptly with increasing solutal Rayleigh number, i.e., there is a jump-wise transition from the most dangerous shortwave perturbations localized in the fluid layer to the long-wave perturbations covering both layers. The perturbation wavelength increases by almost 10 times. Vibrations significantly stabilize the fluid equilibrium state and lead to an increase in the wavelength of its perturbations. When the fluid with the stabilizing concentration gradient is heated from below, convection can occur not only in a monotonous manner but also in an oscillatory manner. The frequency of critical oscillatory perturbations decreases by 10 times, when the long-wave instability replaces the shortwave instability. When the fluid is heated from above, only stationary convection is excited over the entire range of the examined parameters. A lower monotonic instability level is associated with the development of perturbations with longer wavelength even at a relatively large fluid layer thickness. Vibrations speed up the stationary convection onset and lead to a decrease in the

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wavelength of most dangerous perturbations of the motionless equilibrium state. In this case, high enough amplitudes of vibration are needed for a remarkable change in the stability threshold. The results of numerical simulation show good agreement with the data of earlier works in the limiting case of zero fluid layer thickness.

**Keywords** Double-diffusive convection · Porous medium · Superposed fluid and porous layers · Binary fluid · High-frequency vibrations · Theoretical relationships · bimodal neutral curves

## 1 Introduction

Convective flows in the fluid and fluid-saturated porous layers are driven by the buoyancy force that is provided by inhomogeneities of the fluid density in a gravitational field. A change in density of a binary fluid can be due to two factors—temperature heterogeneity and concentration heterogeneity. They are favorable for excitation of double-diffusive convection (Gershuni and Zhukovitskii 1972; Nield and Bejan 2013; Huppert and Turner 1981). The variation of concentration makes an additional contribution into the resulting buoyancy force and can delay or speed up the onset of convection of a differentially heated fluid. In the case, when thermal and solutal buoyancy forces are opposite to each other, oscillatory convection arises. A fluid motion can occur through long narrow convection cells called salt fingers even if the overall density gradient decreases upward (Nield and Bejan 2013; Huppert and Turner 1981). Salt fingering can be observed in a water column with warm salty water at its upper end and cooler freshwater at its lower end. The phenomenon is due to the fact that salt diffuses much more slowly than heat.

The monotonic and oscillatory instability boundaries in a porous layer saturated with a binary fluid in a gravitational field were found in Gershuni and Zhukovitskii (1972), Nield and Bejan (2013), Nield (1968). They are determined by the analytical expressions, respectively, as $R_m + R_{mc} = 4\pi^2$ and $m\mathrm{Le}_m R_m + R_{mc} = 4\pi^2(1 + m\mathrm{Le}_m)$, where $R_m$ is the Rayleigh number, $R_{mc}$ is the solutal Rayleigh number, $\mathrm{Le}_m$ is the Lewis number, $m$ is porosity of the layer. Frequency $\omega_0$ for the most dangerous oscillatory perturbations with the wave number $k = \pi$ is found from the equation: $m\mathrm{Le}_m \frac{\omega_0^2}{\pi^2} = 4\pi^2 - (R_m + R_{mc})$.

Convection excitation in two- or three-layer systems consisting of the horizontal fluid and porous layers heated from below in a gravitational field has a feature as compared to a single fluid or porous layer. It is the bimodal nature of neutral stability curves. The variation of $d$ (a ratio of layer thicknesses) and other parameters may cause an abrupt change in the character of instability inside layers. For small relative fluid layer thicknesses, the instability is associated with the development of long-wave perturbations penetrating all layers. With the growth of $d$, the shortwave perturbations localized in the fluid layer become the most dangerous. Neutral curves are bimodal for intermediate values of $d$. They have two minima, which correspond to similar values of the critical Rayleigh number for the long-wave and shortwave perturbations. The bimodal nature of neutral stability curves was first revealed in Lyubimov and Muratov (1977), Lyubimov et al. (2002, 2004) for a three-layer system consisting of a fluid layer surrounded by two fluid-saturated porous layers. Later, it was studied in Chen and Chen (1988, 1989), Zhao and Chen (2001), Hirata et al. (2009), Kolchanova et al. (2013) for a two-layer system comprising a fluid layer overlying a porous layer.

The problem of convection excitation in a two-layer system consisting of a fluid layer and a fluid-saturated porous layer in a gravitational field has aroused considerable interest due to its application in solidification of binary alloys or solutions (Chen and Chen 1991; Chen et al. 1994; Tait and Jaupart 1992; Worster 1991, 1992). During directional solidification of a binary solution, a two-phase porous zone (mushy zone) is formed at the interface between the solution and crystal. Concentration of the component of the binary solution deceases from the solution to the crystal. Thus, the conditions favorable for the onset of double-diffusive convection are generated in the solution on cooling from below. As it has been shown in Worster (1992), convection in the solution layer and the solution-saturated mushy zone is associated with two types of instability: shortwave instability (so called boundary-layer mode) and long-wave instability (mushy-layer mode). The shortwave instability leads to the formation of double-diffusive fingers near the interface between the solution and the mushy zone. The long-wave instability causes plume convection inside the mushy zone. One of the ways of controlling convective heat and mass transfer in the solution and porous zone is the vibration.

High-frequency translational vibrations induce an average fluid flow in the gravitational field, provided that density of the fluid is not uniform. Density heterogeneity in non-isothermal single-component fluids is caused by a temperature gradient. Depending on the orientation of the vibration axis with respect to the temperature gradient and acceleration of gravity, translational vibrations can either stabilize or destabilize the quasi-equilibrium state in a fluid layer (Gershuni and Lyubimov 1998; Zen'kovskaya and Simonenko 1966) or in a fluid-saturated porous layer (Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000). In the quasi-equilibrium state, the average fluid velocity equals to zero, but the velocity of high-frequency pulsations can differ from zero. Vertical vibration, when the vibration axis is parallel to the temperature gradient, increases the stability threshold of the mechanical equilibrium of the fluid heated from below (Gershuni and Lyubimov 1998; Zen'kovskaya and Simonenko 1966; Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000).

An average convective fluid flow determines a synchronous response of the fluid to a periodic action in the limiting case of high- frequency and small-amplitude vibrations. The vibration period is considered to be small compared to typical thermal-diffusion and viscous-diffusion times. In this case, the equations for convection can be obtained by the averaging method (Gershuni and Lyubimov 1998; Zen'kovskaya and Simonenko 1966; Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000; Razi et al. 2008, 2009; Lyubimov et al. 2004). According to the method, scalar and vector fields are represented as the sums of the components averaged over the vibration period and pulsation components, and the parameters responsible for the onset of average convection are evaluated.

A direct method of a linear stability analysis is generally used for finite vibration frequencies and amplitudes. With this method, the stability of a periodic solution for the Mathieu equation is determined. At finite frequencies, it is possible to obtain not only the synchronous periodic solutions with a period equal to the vibration period, but also the subharmonic solutions, the period of which is twice as large as the vibration period. The direct method was used in Rrazi et al. (2002, 2005), Govender (2004, 2005a, b, 2008), Maryshev et al. (2013) to determine the equilibrium stability threshold for a single-component fluid in a porous layer heated from below or above in the presence of vertical vibrations. It was shown that when the layer is heated from below, the vibrations significantly stabilize the equilibrium with respect to its synchronous perturbations. However, in the case of subharmonic perturbations, the vibrations produce a rather weak destabilizing effect on the equilibrium. The value of dimensionless frequency, at which a transition from the synchronous to subharmonic periodic solutions takes place, is determined in Rrazi et al. (2002, 2005), Govender (2005b).

The initiation of double-diffusive convection in a rectangular cavity, containing a porous medium saturated with a binary fluid in the presence of high-frequency vertical vibration, was studied in Jounet and Bardan (2001). The vertical temperature and concentration gradients were imposed. Analytical expressions for the stability boundary of mechanical equilibrium in the fluid with respect to monotonic or oscillatory perturbations were found. It was shown that vibration could raise or lower the critical Rayleigh number for the stationary convection onset, depending on the buoyancy ratio and properties of the porous medium. Vibration always delayed the oscillatory convection onset in the case of opposite thermal and solutal buoyancy forces. The effect of vertical vibration on the motionless state in the high-frequency limit was characterized by the vibrational Rayleigh number. For the porous medium saturated with a single-component fluid and subjected to high-frequency vertical vibration, the authors of Bardan et al. (2004) divided the vibrational Rayleigh number into two parts: One of them depended on the temperature difference, and another included amplitude and frequency of vibrations. They obtained a maximum limit value of the vibrational parameter for achieving a stabilizing effect of vibration in the case of a synchronous response. Convection in a double-diffusive porous medium undergoing high-frequency vibration with various orientations was studied in Bardan et al. (2001).

The effect of high-frequency vibrations on the non-isothermal binary fluid in a porous medium in the presence of the Soret effect was investigated in Charrier-Mojtabi et al. (2004, 2005). It was found that transverse vertical vibrations have a stabilizing effect as in the case of a single-component fluid. Destabilization of the fluid quasi-equilibrium is observed in the presence of longitudinal vibrations.

Excitation of an average convective flow in a two-layer system comprising a single-component fluid layer and a fluid-saturated porous layer under the action of the gravitational field and high-frequency vertical vibrations was investigated in Lyubimov et al. (2008, 2015), Kolchanova et al. (2012). A region of parameters responsible for bimodality of neutral stability curves is determined. It was shown that vibrations stabilize the fluid equilibrium and lead to an increase in the wavelength of its most dangerous perturbations. With the growth of vibration intensity, a sharp change in the instability nature (from the shortwave to long-wave modes) is observed. Vibrations significantly delay the onset of convection in the form of shortwave rolls as compared to the initiation of long-wave convective rolls. The shortwave rolls locate in the fluid layer. The long-wave rolls cover both layers. Fluid, while moving through a porous medium, experiences the resistance of porous matrix. Therefore, inertial effects within the porous layer are less pronounced than in the fluid layer free from a porous matrix.

We study the problem of linear stability of mechanical equilibrium in a system of a binary fluid layer and a binary fluid-saturated porous layer under the gravitational field and high-frequency vertical vibrations. At the external boundaries of the system, temperatures and concentrations are assumed to have different constant values. Varying the governing parameters of the two-layer system, we obtain the onset values for stationary and oscillatory average convection under different thermal conditions.

## 2 Governing Equations

Our work is devoted to studying the vibration effect on the convective stability in a system of a horizontal binary layer and a porous layer saturated with the same fluid in a gravitational field (Fig. 1). At the external impermeable boundaries of the system, temperature and concentration

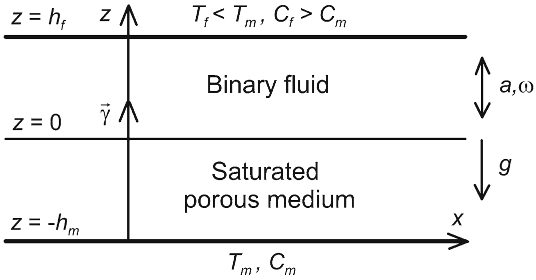



**Fig. 1** Two-layer system under consideration

are assumed to have different constant values. The layers are subjected to transverse vibrations with an amplitude $a$ and a frequency $\omega$.

The fluid flows in each of the layers are described by the basic convection equations in the Boussinesq approximation (Gershuni and Zhukovitskii 1972). Fluid filtration through a porous layer obeys the Darcy law (Nield and Bejan 2013). The deviation of the fluid density $\rho$ from its mean value $\rho_0$ is caused by thermal and concentration inhomogeneities and given by the formula: $\rho = \rho_0 (1 - \beta_T T + \beta_C C)$, where $T$ and $C$ are the deviations of the temperature and concentration for the heavier component of the binary fluid from their mean values, $\beta_T = -\frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial T}\right)_P$ is the volumetric expansion coefficient, $\beta_C = \frac{1}{\rho_0}\left(\frac{\partial\rho}{\partial C}\right)_P$ is the coefficient, which determines the dependence of fluid density on concentration. The porous matrix performs rigid body oscillations with a cavity and does not deform. The mass, momentum, energy balance equations, and the diffusion equation for an incompressible fluid are written in the reference frame associated with an oscillating cavity and take the following form in the fluid layer:

$$\mathrm{div}\mathbf{v} = 0, \tag{1}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\,\mathbf{v} = -\frac{1}{\rho_f}\nabla p_f + \nu_f \Delta\mathbf{v} + (\beta_T T - \beta_C C)\left(g - a\omega^2\cos\omega t\right)\boldsymbol{\gamma}, \tag{2}$$

$$\frac{\partial T}{\partial t} + (\mathbf{v}\,\nabla)\,T = \chi_f \Delta T, \tag{3}$$

$$\frac{\partial C}{\partial t} + (\mathbf{v}\nabla)\,C = D_f \Delta C, \tag{4}$$

in the porous layer:

$$\mathrm{div}\mathbf{u} = 0, \tag{5}$$

$$\frac{1}{m}\frac{\partial \mathbf{u}}{\partial t} = -\frac{1}{\rho_f}\nabla p_m - \frac{\nu_f}{K}\mathbf{u} + (\beta_T \vartheta - \beta_C S)\left(g - a\omega^2\cos\omega t\right)\boldsymbol{\gamma}, \tag{6}$$

$$b\frac{\partial \vartheta}{\partial t} + (\mathbf{u}\,\nabla)\,\vartheta = \chi_{eff}\Delta\vartheta, \tag{7}$$

$$m\frac{\partial S}{\partial t} + (\mathbf{u}\,\nabla)\,S = D_m \Delta S. \tag{8}$$

Here, we use the following notation: $\mathbf{v}$ is the fluid velocity in the fluid layer, $\mathbf{u}$ is the fluid filtration velocity in the porous layer, $T$ and $C$ are the deviations of temperature and concentration for the heavier binary fluid component from their mean values in the fluid layer, $\vartheta$ and $S$ are the deviations of temperature and concentration for the heavier binary fluid component from their mean values in the porous layer, $p$ is pressure without the hydrostatic pressure, $\rho_f$ is the fluid density, $\nu_f$ is fluid viscosity, $D_f$ and $D_m = mD_f$ are diffusion coefficients in the fluid and porous layers, $m$ is porosity, $K$ is permeability of the porous layer, $b = (\rho C)_m/(\rho C)_f$ is the ratio of heat capacities in the porous and fluid layers, $\chi_f = \kappa_f/(\rho C)_f$ is thermal conductivity in the fluid layer, $\chi_{eff} = \kappa_m/(\rho C)_f$ is the effective thermal conductivity in the porous layer (the ratio of thermal conductivity of fluid-saturated porous medium to heat capacity of fluid), index $f$ is for fluid, and index $m$ is for fluid-saturated porous medium.

The external boundaries of the two-layer system are solid and impermeable and have different constant temperatures and concentrations. Boundary conditions are given as

$$z = h_f: \qquad \mathbf{v} = 0, \quad T = T_f, \quad C = C_f, \tag{9}$$

$$z = -h_m: \qquad u_z = 0, \quad \vartheta = T_m, \quad S = C_m. \tag{10}$$

The continuity condition for temperatures, heat fluxes, concentrations, and mass fluxes are fulfilled at the interface between the binary fluid and porous layers. We prescribe the conditions of equality for normal velocity components and the continuity condition for normal stresses, and the condition of zero tangential component of the fluid velocity (Lyubimov and Muratov 1977):

$$\begin{gathered} z = 0: \quad T = \vartheta, \;\; \kappa_f \frac{\partial T}{\partial z} = \kappa_m \frac{\partial \vartheta}{\partial z}, \;\; C = S, \\ D_f \frac{\partial C}{\partial z} = D_m \frac{\partial S}{\partial z}, \;\; v_z = u_z, \;\; p_f = p_m, \;\; v_x = 0. \end{gathered} \tag{11}$$

To solve the system of Eqs. (1)–(8) with boundary conditions (9)–(11), we represent the velocity, pressure, temperature and concentration fields as the sums of their components $\left(\mathbf{v},\ p_f, T,\ C,\ \mathbf{u},\ p_m,\ \vartheta,\ S\right)$ averaged over the vibration period and oscillation (pulsation) components $\left(\tilde{\mathbf{v}},\ \tilde{p}_f, \tilde{T},\ \tilde{C},\ \tilde{\mathbf{u}},\ \tilde{p}_m,\ \tilde{\vartheta},\ \tilde{S}\right)$. Let us differentiate between the amplitudes of the pulsation velocity and pressure components in the fluid layer: $\tilde{\mathbf{v}} = \mathrm{Re}\left(\mathbf{V}\, e^{i\omega t}\right)$, $\tilde{p}_f = \mathrm{Re}\left(P_f\, e^{i\omega t}\right)$, and in the porous layer: $\tilde{\mathbf{u}} = \mathrm{Re}\left(\mathbf{W}\, e^{i\omega t}\right)$, $\tilde{p}_m = \mathrm{Re}\left(P_m\, e^{i\omega t}\right)$. We apply the averaging method to obtain a closed system of equations for average and pulsation components (Gershuni and Lyubimov 1998; Zen'kovskaya and Simonenko 1966; Zen'kovskaya 1992; Bardan and Mojtabi 2000; Jounet and Bardan 2001; Lyubimov et al. 2008).

We assume that the fields oscillate rapidly in comparison with characteristic viscous-diffusion, thermal-diffusion and mass-diffusion times. Neglecting compressibility of the fluid, we obtain the assigned frequency range, which determines the high-frequency limit: $\min\left(\frac{\nu_f}{h_f^2},\ \frac{\chi_f}{h_f^2},\ \frac{D_f}{h_f^2}\right) \ll \omega \ll \frac{c}{h_f}$, where $c$ is sound velocity in the fluid. According to the estimate made for an aqueous ammonium chloride solution layer with thickness of 1 sm, this range is $1.2 \cdot 10^{-2} \ll \omega \ll 1.5 \cdot 10^5\ \mathrm{s}^{-1}$ (Chen et al. 1994; Peppin et al. 2008; Bejan 2013). This allows us to neglect the viscous term in the momentum equation for pulsation components in the fluid layer. In a porous layer, this range significantly reduces because the characteristic viscous-diffusion time in porous media is determined by the pore size $\sqrt{K}$ rather than by the layer thickness (Nield and Bejan 2013). Since permeability of porous matrices is usually low, the momentum equation for pulsation components retains a viscous term (Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000; Lyubimov et al. 2008, 2015; Kolchanova et al. 2012). The ratio $\nu_f m/K\omega$ of the viscous term to acceleration can be small only if $\omega \gg \nu_f m/K$. For an ammonium chloride solution-saturated layer of packed glass spheres with the permeability $K$ about $10^{-5}\mathrm{sm}^2$, porosity $m = 0.4$, and thickness $h_m = 1$ sm, we have the range $5 \times 10^2 \ll \omega \ll 1.5 \times 10^5\ \mathrm{s}^{-1}$. Thus, for moderately high vibration frequencies we use the relation for frequency $\min\left(\frac{\chi_{eff}}{h_m^2}, \frac{D_m}{h_m^2}\right) \ll \omega \ll \frac{c}{h_m}$ and preserve a viscous term in the momentum equation for pulsations.

The vibration amplitude is small in comparison with the layer thicknesses, i.e., $a \ll \min\left(\frac{h_f}{\beta_T \theta_f}, \frac{h_f}{\beta_C \Theta_f}\right)$. Here, $\theta_f$ and $\Theta_f$ are the temperature and concentration differences at the boundaries of the fluid layer. We neglect the nonlinear terms in equations for pulsations.

Taking into account the approximations described above, we arrive at the closed system of equations for average components and amplitudes of pulsation components in the fluid layer:

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\,\nabla)\,\mathbf{v} = -\frac{1}{\rho_f}\nabla p_f + \nu_f \Delta \mathbf{v} + g\,(\beta_T T - \beta_C C)\,\boldsymbol{\gamma}$$
$$-\frac{a\omega}{2}\mathrm{Re}\,\{i\,(\mathbf{V}\nabla)\,(\beta_T T - \beta_C C)\}\,\boldsymbol{\gamma}, \tag{12}$$
$$\frac{\partial T}{\partial t} + (\mathbf{v}\,\nabla)\,T = \chi_f \Delta T, \tag{13}$$
$$\frac{\partial C}{\partial t} + (\mathbf{v}\,\nabla)\,C = D_f \Delta C, \qquad \mathrm{div}\mathbf{v} = 0, \tag{14}$$
$$i\omega \mathbf{V} = -\frac{1}{\rho_f}\nabla P_f - a\omega^2\,(\beta_T T - \beta_C C)\;\boldsymbol{\gamma}, \qquad \mathrm{div}\mathbf{V} = 0, \tag{15}$$

in the porous layer:

$$0 = -\frac{1}{\rho_f}\nabla p_m - \frac{\nu_f}{K}\mathbf{u} + g\,(\beta_T \vartheta - \beta_C S)\,\boldsymbol{\gamma} - \frac{a\omega}{2}\mathrm{Re}\left\{i\,(\mathbf{W}\nabla)\left(\frac{\beta_T \vartheta}{b} - \frac{\beta_C S}{m}\right)\right\}\boldsymbol{\gamma}, \tag{16}$$
$$b\frac{\partial \vartheta}{\partial t} + (\mathbf{u}\,\nabla)\,\vartheta = \chi_{eff}\Delta\vartheta, \tag{17}$$
$$m\frac{\partial S}{\partial t} + (\mathbf{u}\,\nabla)\,S = D_m \Delta \mathrm{S}, \qquad \mathrm{div}\mathbf{u} = 0, \tag{18}$$
$$\frac{i\omega}{m}\mathbf{W} = -\frac{1}{\rho_f}\nabla P_m - \frac{\nu_f}{K}\mathbf{W} - a\omega^2\,(\beta_T \vartheta - \beta_C S\,)\,\boldsymbol{\gamma}, \qquad \mathrm{div}\mathbf{W} = 0. \tag{19}$$

The system of Eqs. (12)–(19) is supplemented by the boundary conditions for pulsation component amplitudes:

$$z = h_f: \qquad V_z = 0, \tag{20}$$
$$z = 0: \quad V_z = W_z, \;\; P_f = P_m, \tag{21}$$
$$z = -h_m: \qquad W_z = 0, \tag{22}$$

Boundary conditions for average components are similar to those prescribed for full fields (9)–(11).

Note that in Eqs. (12), (15) and (16), (19) we do not use a Helmholtz decomposition of the average temperature and concentration gradients to express the pulsation variables as a function of the average variables as it is applied in previous studies (Gershuni and Lyubimov 1998; Jounet and Bardan 2001). The Helmholtz decomposition is convenient to obtain the final system of vibrational convection equations that excludes the pulsation variables, but in our work we preserve the equations for pulsations.

Let us investigate the stability of equilibrium state in a binary fluid. In the equilibrium state, there is no average fluid flow, the temperature and concentration gradients are constant and vertical and are defined as $\nabla T_0(z) = -A_f \boldsymbol{\gamma}$, $\nabla C_0(z) = B_f \boldsymbol{\gamma}$ in the fluid layer and as $\nabla \vartheta_0(z) = -A_m \boldsymbol{\gamma}$, $\nabla S_0(z) = B_m \boldsymbol{\gamma}$ in the porous layer. After prescribing small perturbations of the equilibrium, we obtain the following system of equations for equilibrium perturbations in the fluid layer

$$\frac{\varepsilon}{\mathrm{Pr}_m}\frac{\partial \mathbf{v}}{\partial t} = -\nabla p_f + \varepsilon \Delta \mathbf{v} + (R_m T - R_{mc} C)\,\boldsymbol{\gamma} + p_v\left(\kappa b R_1^2 + m^2 R_2^2\right)\boldsymbol{\gamma}\,\mathrm{Re}\,\{i\mathbf{V}\cdot\boldsymbol{\gamma}\}, \tag{23}$$

$$\frac{\kappa}{b}\frac{\partial T}{\partial t} - \kappa^2\,(\mathbf{v}\cdot\boldsymbol{\gamma}) = \Delta T, \tag{24}$$

$$m\,\mathrm{Le}_m \frac{\partial C}{\partial t} + m^2\,(\mathbf{v}\cdot\boldsymbol{\gamma}) = \Delta C, \qquad \mathrm{div}\mathbf{v} = 0, \tag{25}$$

$$i\Omega \mathbf{V} = -\nabla P_f - \left(T - \frac{R_{mc}}{R_m} C\right)\boldsymbol{\gamma}, \qquad \mathrm{div}\mathbf{V} = 0, \tag{26}$$

in the porous layer

$$0 = -\nabla p_m - \mathbf{u} + (R_m \vartheta - R_{mc} S)\,\boldsymbol{\gamma} + p_v\left(R_1^2 + R_2^2\right)\boldsymbol{\gamma}\,\mathrm{Re}\,\{i\mathbf{W}\cdot\boldsymbol{\gamma}\}, \tag{27}$$

$$\frac{\partial \vartheta}{\partial t} - (\mathbf{u}\cdot\boldsymbol{\gamma}) = \Delta\vartheta, \tag{28}$$

$$m\frac{\partial S}{\partial t} + (\mathbf{u}\,\nabla)\,S = D_m \Delta \mathrm{S}, \qquad \mathrm{div}\mathbf{u} = 0, \tag{29}$$

$$\frac{i\Omega}{m}\mathbf{W} = -\nabla P_m - \mathbf{W} - \left(\vartheta - \frac{R_{mc}}{R_m} S\right)\boldsymbol{\gamma}, \qquad \mathrm{div}\mathbf{W} = 0, \tag{30}$$

with the boundary conditions

$$z = d: \qquad \mathbf{v} = 0, \quad V_z = 0, \quad T = 0, \quad C = 0, \tag{31}$$

$$z = -1: \qquad u_z = 0, \quad W_z = 0, \quad \vartheta = 0, \quad S = 0, \tag{32}$$

$$z = 0: \quad T = \vartheta, \quad \frac{\partial T}{\partial z} = \kappa\frac{\partial \vartheta}{\partial z}, \quad C = S, \quad \frac{\partial C}{\partial z} = m\frac{\partial S}{\partial z}, \quad v_z = u_z,$$
$$p_f = p_m, \quad v_x = 0, \quad V_z = W_z, \quad P_f = P_m. \tag{33}$$

The problem (23)–(33) includes the following dimensionless parameters: the Rayleigh number $R_m = \frac{g\beta_T A_m h_m^2 K}{\nu_f \chi_{eff}}$, the solutal Rayleigh number $R_{mc} = \frac{g\beta_C B_m h_m^2 K}{\nu_f D_m}$, the Prandtl number $\mathrm{Pr}_m = \frac{b\nu_f}{\chi_{eff}}$, the Lewis number $\mathrm{Le}_m = \frac{\chi_{eff}}{bD_m}$, the Darcy number $\varepsilon = \frac{K}{h_m^2}$, the ratio of thicknesses for the fluid and porous layers $d = \frac{h_f}{h_m}$, the ratio of thermal conductivities for the porous and fluid layers $\kappa = \frac{\kappa_m}{\kappa_f}$, the vibrational parameter $p_v = \frac{\eta^2\varepsilon}{2\Omega \mathrm{Pr}_m}$. Here, $\eta = \frac{a\omega^2}{g}$ is the dimensionless amplitude of vibrations, $\Omega = \frac{\omega K}{\nu_f}$ is the dimensionless frequency of vibrations. Coefficients $R_1$ and $R_2$ are determined from the relations $R_1^2 = R_m^2$ and $R_2^2 = \frac{R_m R_{mc}}{m\,\mathrm{Le}_m}$, respectively.

Scales for length $h_m$, time $bh_m^2/\chi_{eff}$, average components of the velocity $\chi_{eff}/h_m$, temperature $A_m h_m$, concentration $B_m h_m \chi_{eff}/D_m$, and pressure $\rho_f \nu_f \chi_{eff}/K$, amplitudes of pulsation components for velocity $a\omega^2 \beta_T A_m h_m K/\nu_f$ and pressure $a\omega^2 \rho_f \beta_T A_m h_m^2$ are chosen in accordance with Nield and Bejan (2013), Lyubimov et al. (2008).

## 3 Solution Method

The system of Eqs. (23)–(30) with boundary conditions (31)–(33) admits a solution that is periodic along the $x$-axis (Gershuni and Zhukovitskii 1972; Nield and Bejan 2013). Consider normal periodic perturbations of the equilibrium and differentiate between the perturbation amplitudes of the average and pulsation components of fields in the fluid and porous layers:

$$\left(\mathbf{v},\ p_f, T,\ C,\ \mathbf{V},\ P_f\right) = \left(\hat{\mathbf{v}},\ \hat{p}_f, \hat{T},\ \hat{C},\ \hat{\mathbf{V}},\ \hat{P}_f\right) \cdot \exp\{\lambda t + ikx\}, \tag{34}$$

$$\left(\mathbf{u},\ p_m, \vartheta,\ S,\ \mathbf{W},\ P_m\right) = \left(\hat{\mathbf{u}},\ \hat{p}_m, \hat{\vartheta},\ \hat{S},\ \hat{\mathbf{W}},\ \hat{P}_m\right) \cdot \exp\{\lambda t + ikx\}, \tag{35}$$

where $\lambda = \lambda_r + i\omega_0$ is a complex increment of perturbations and $k$ is the wave number of perturbations. Parameter $\lambda_r = \mathrm{Re}\,(\lambda)$ characterizes the rate of perturbation growth, and parameter $\omega_0 = \mathrm{Im}\,(\lambda)$ determines the frequency of perturbations with the wave number $k$.

Taking into consideration expressions (34), (35), we arrive at the eigenvalue problem for perturbation amplitudes. A numerical solution was obtained by the shooting method (Lobov et al. 2004). As a result, we obtained the stability boundary of equilibrium with respect to the monotonic ($\lambda_r = \omega_0 = 0$) and oscillatory ($\lambda_r = 0$, $\omega_0 \neq 0$) perturbations and constructed the neutral curves and stability maps for the problem parameters, at which the perturbations did not grow or decay.

The calculations were carried out for a two-layer system comprising a layer of an aqueous ammonium chloride solution and a layer of densely packed glass spheres. The properties of ammonium chloride solution and glass are presented in Table 1 (Chen et al. 1994; Peppin et al. 2008; Bejan 2013; Prasad 1993).

The porous medium is assumed to be homogeneous. Heat capacity per unit volume of the solution-saturated porous medium $(\rho C)_m$ and its thermal conductivity $\kappa_m$ are written as the sums of coefficients for the solid and fluid phases having the corresponding volume fractions (Nield and Bejan 2013; Bejan 2013):

$$(\rho C)_m = m(\rho C)_f + (1 - m)\,(\rho C)_s, \tag{36}$$

$$\kappa_m = m\kappa_f + (1 - m)\,\kappa_s, \tag{37}$$

Average porosity of the system of densely packed glass spheres is equal to 0.4 (Katto and Matsuoka 1967; Glukhov and Putin 1999). Its permeability is given by the Carman–

**Table 1** Properties of ammonium chloride solution and glass

| Parameter | Symbol | Value | Unit |
|---|---|---|---|
| Kinematic viscosity of solution | $\nu_f$ | $1.2 \times 10^{-6}$ | $\mathrm{m^2/s}$ |
| Solution density | $\rho_f$ | 1050 | $\mathrm{kg/m^3}$ |
| Thermal conductivity of solution | $\kappa_f$ | 0.54 | $\mathrm{W/(m \cdot K)}$ |
| Heat capacity of solution | $C_f$ | $3.5 \times 10^3$ | $\mathrm{J/(kg \cdot K)}$ |
| Diffusivity in the solution | $D_f$ | $1.7 \cdot 10^{-9}$ | $\mathrm{m^2/s}$ |
| Glass density | $\rho_s$ | 2500 | $\mathrm{kg/m^3}$ |
| Thermal conductivity of glass | $\kappa_s$ | 1.10 | $\mathrm{W/(m \cdot K)}$ |
| Heat capacity of glass | $C_s$ | 770 | $\mathrm{J/(kg \cdot K)}$ |

Kozeny formula (Carman 1937; Fand et al. 1987): $K = \frac{D^2 m^3}{180(1-m)^2}$, where $D$ is the diameter of spheres. The sphere diameter is of 10 times smaller than the porous layer thickness.

The calculated values of the dimensionless parameters, which were used in the description of a two-layer system, are as follows: $\kappa = 1.6$, $b = 0.7$, $\mathrm{Pr}_m = 3.6$, $\mathrm{Le}_m = 491$, $\varepsilon = 10^{-5}$, $m = 0.4$. In calculations, the dimensionless frequency of vibrations $\Omega = 0.1$ was also assumed to be a constant quantity. The two-layer system was heated in two directions—from below and above.

## 4 Numerical Results

### 4.1 A Fluid-Saturated Porous Layer ($d = 0$)

#### *4.1.1 Heating from Below ($R_m > 0$)*

To check numerical results and make detailed analysis of the linear stability problem for the mechanical equilibrium of fluid, we compared the numerical results with the analytical solutions in the limiting case of zero fluid layer thickness ($d = 0$). The analysis was carried out for a porous medium with a set of parameters mentioned above. The excitation of convection in a binary fluid saturating a porous layer in a static gravitational field was investigated in Nield and Bejan (2013), Nield (1968). Temperature and concentration were assumed to have different constant values at the solid boundaries of the layer. The authors obtained the analytical solutions for neutral critical perturbations, belonging to the lower level of equilibrium instability with respect to monotonic perturbations (Nield and Bejan 2013; Nield 1968):

$$R_m + R_{mc} = \frac{\left(\pi^2 + k^2\right)^2}{k^2}, \tag{38}$$

and with respect to oscillatory perturbations:

$$m\mathrm{Le}_m R_m + R_{mc} = \frac{\left(\pi^2 + k^2\right)^2}{k^2}\left(1 + m\mathrm{Le}_m\right), \tag{39}$$

The frequency of oscillatory perturbations $\omega_0$ with the wave number $k$ is given by the equation:

$$m\mathrm{Le}_m \frac{\omega_0^2}{k^2} = \frac{\left(\pi^2 + k^2\right)^2}{k^2} - \left(R_m + R_{mc}\right). \tag{40}$$

Figure 2a,b shows neutral curves of the monotonic and oscillatory instabilities with a change in the solutal Rayleigh number. The instability regions are located above these curves. Neutral curves have one minimum at $k = \pi$. In this case, we get $\frac{\left(\pi^2+k^2\right)^2}{k^2} = 4\pi^2$. A state of neutral stability is a marginal state of the system, when small perturbations of the motionless state (stable state) do not grow or decay. There is a set of critical parameters that separate the stable and unstable states of the system and therefore determine a criterion for the convection onset. Stationary or oscillatory convection is caused by buoyancy forces provided by density inhomogeneity in the gravitational field. A density gradient of binary fluid can be due to temperature and concentration gradients. If the buoyancy forces due to temperature and concentration gradients are equal ( $R_m > 0$, $R_{mc} > 0$), stationary convection arises and its onset is speed up as compared to the thermal convection onset in a single-component fluid

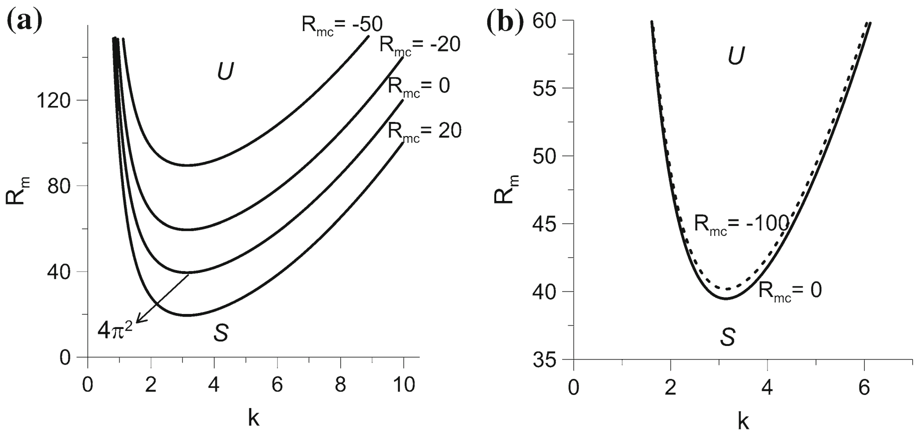



**Fig. 2** Neutral curves of the equilibrium stability of a binary fluid in a porous layer ($d = 0$) heated from below in the absence of vibrations at different values of the solutal Rayleigh number $R_{mc}$. Solid lines denote the monotonic instability, and dashed lines correspond to the oscillatory instability. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region

(Fig. 2a). If the temperature and concentration gradients make opposite contributions into the resulting buoyancy force ($R_m > 0$, $R_{mc} < 0$), convention can be stationary or oscillatory, depending on the parameters of the system (Fig. 2a, b). The critical Rayleigh number for the stationary convection onset increases as the absolute value of the solutal Rayleigh number $R_{mc}$ rises (Fig. 2a) (Nield and Bejan 2013). As it can be seen from the plots, the monotonic instability mode that is responsible for stationary convection onset is most sensitive to a change in the concentration gradient (Fig. 2a). The oscillatory mode does not practically change with variation of the parameter $R_{mc}$. Neutral curves of the oscillatory instability for the selected porous medium with $m\mathrm{Le}_m = 196 \gg 1$ (Fig. 2b) are close to the neutral curves of the monotonic instability constructed at $R_{mc} = 0$ (Fig. 2b). Indeed, from relation (39), it follows that $R_m = \frac{(\pi^2+k^2)^2}{k^2}\left(\frac{1}{m\mathrm{Le}_m}+1\right) - \frac{R_{mc}}{m\mathrm{Le}_m} \approx \frac{(\pi^2+k^2)^2}{k^2}$.

A stability map of the fluid equilibrium in the absence of vibrations is constructed for the most dangerous neutral perturbations with the wave number $k = \pi$, when the solutal Rayleigh number varies (Fig. 3a). The obtained numerical data correspond to the analytical solutions in Nield and Bejan (2013). The boundary of stability with respect to monotonic perturbations is given by the expression

$$R_{mc} + R_m = 4\pi^2, \tag{41}$$

and with respect to oscillatory perturbations is defined as

$$R_m m\mathrm{Le}_m + R_{mc} = 4\pi^2 (1 + m\mathrm{Le}_m). \tag{42}$$

Frequency of perturbations is determined by the equation

$$m\mathrm{Le}_m \frac{\omega^2}{\pi^2} = 4\pi^2 - (R_{mc} + R_m). \tag{43}$$

We plot the frequency versus the solutal Rayleigh number in Fig. 3b. The stationary convection in the form of rolls with the axes elongated along the porous layer occurs when the temperature and concentration gradients play a destabilizing role ($R_m > 0$, $R_{mc} > 0$). The range of

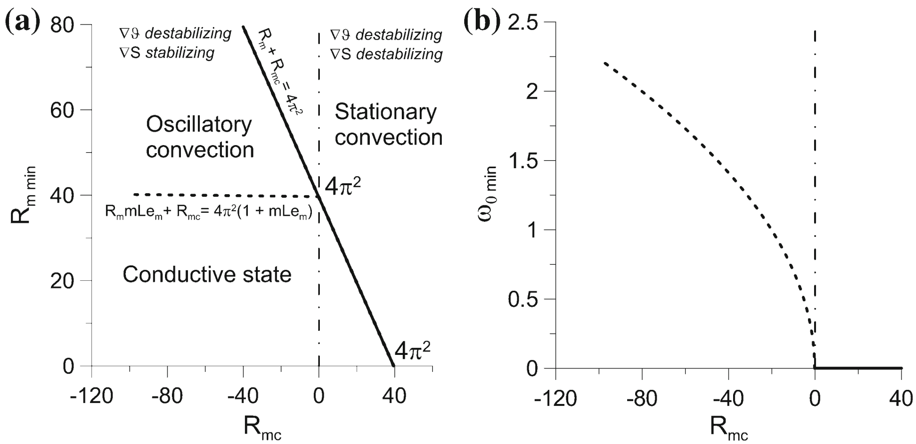



**Fig. 3** **a** The equilibrium stability map for a binary fluid in a porous layer ($d = 0$) heated from below in the absence of vibrations at the most dangerous neutral perturbations with the wave number $k = \pi$. **b** Frequency of the most dangerous oscillatory perturbations of equilibrium with $k = \pi$. Solid lines denote the monotonic instability mode, and dashed lines correspond to the oscillatory instability mode

**Table 2** Comparison of the critical Rayleigh number and frequency for the porous layer as calculated by the analytical solutions (41)–(43) (Nield and Bejan 2013) and the present computational scheme

| $R_{mc}$ | $k_{\min}$ | $R_{m\ \min}$ | $\omega_{0\ \min}$ | $R_{m\ \min}$ (Nield and Bejan 2013) | $\omega_{0\ \min}$ (Nield and Bejan 2013) |
|---|---|---|---|---|---|
| *Stationary convection onset values* | | | | | |
| − 80 | 3.142 | 119.477 | 0 | 119.489 | 0 |
| − 60 | 3.142 | 99.477 | 0 | 99.489 | 0 |
| − 40 | 3.142 | 79.477 | 0 | 79.489 | 0 |
| − 20 | 3.142 | 59.478 | 0 | 59.489 | 0 |
| 0 | 3.142 | 39.478 | 0 | 39.489 | 0 |
| 10 | 3.142 | 29.478 | 0 | 29.489 | 0 |
| 20 | 3.142 | 19.478 | 0 | 19.489 | 0 |
| 39.4 | 3.142 | 0.078 | 0 | 0.089 | 0 |
| *Oscillatory convection onset values* | | | | | |
| − 80 | 3.142 | 40.086 | 1.9975 | 40.097 | 1.9977 |
| − 60 | 3.142 | 39.984 | 1.7292 | 39.995 | 1.7293 |
| − 40 | 3.142 | 39.882 | 1.4107 | 39.893 | 1.4108 |
| − 20 | 3.142 | 39.781 | 0.9949 | 39.792 | 0.9950 |
| − 0.21 | 3.142 | 39.680 | 0.01992 | 39.691 | 0.01989 |

parameters shown in Fig. 3a, at which the stationary convection is excited, corresponds to the monotonic instability of equilibrium. When the fluid is heated from below ($R_m > 0$), the oscillatory convection is typical for the stabilizing concentration gradient of the heavier binary fluid component ($R_{mc} < 0$). It is to be noted that the plots are constructed for the selected range of parameters, corresponding to a porous layer of glass balls saturated with an aqueous solution of ammonium chloride (see Table1). Critical values for the Rayleigh number, wave number and frequency are summarized in Table 2.

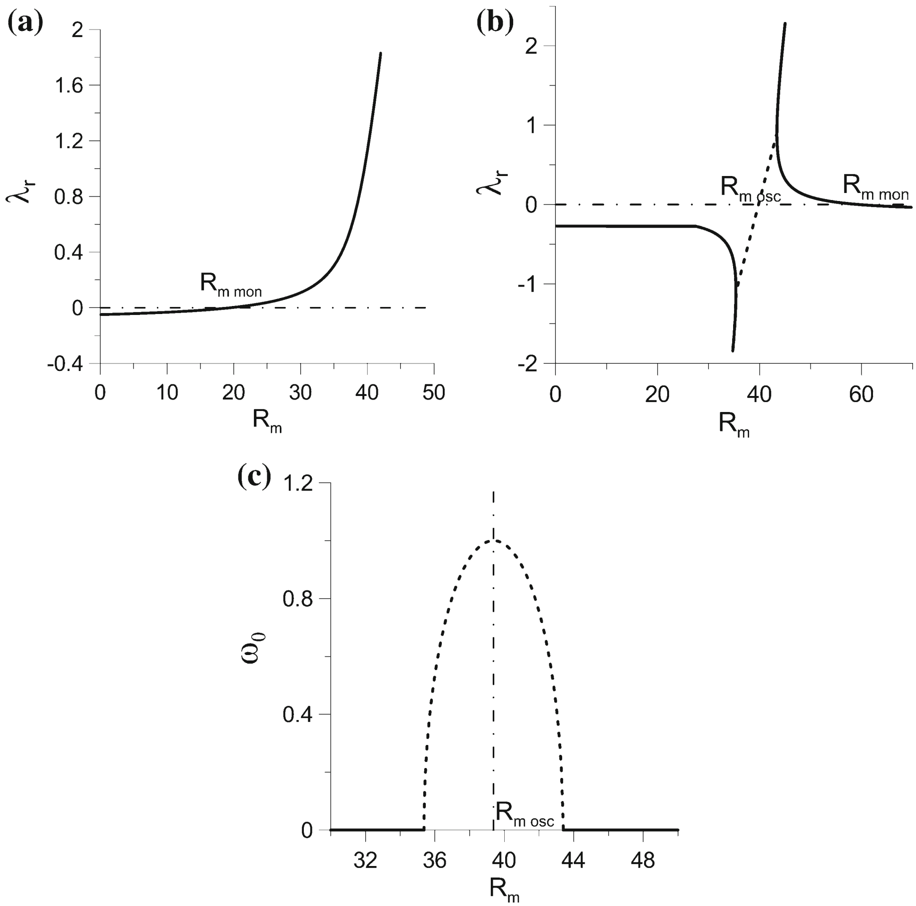



**Fig. 4** The real $\lambda_r$ and imaginary $\omega_0$ parts of the increment $\lambda$ of perturbations with the wave number $k = \pi$ versus the Rayleigh number at different values of the solutal Rayleigh number $R_{mc}$: **a** $R_{mc} = 20$, **b** $R_{mc} = -20$, **c** $R_{mc} = -20$. Solid lines denote the monotonic instability mode, and dashed lines correspond to the oscillatory instability mode. The instability region corresponds to $\lambda_r > 0$, and the stability region is found at $\lambda_r < 0$

The plots of complex increment of perturbations with the wave number $k = \pi$ versus the Rayleigh number at $R_{mc} = 20$ and $R_{mc} = -20$ in the absence of vibrations are presented in Fig. 4a–c. The critical values of the Rayleigh number for neutral non-decaying and non-increasing perturbations are found at zero real part of the increment, i.e., $\lambda_r = \mathrm{Re}\,(\lambda) = 0$. For $\lambda_r < 0$, all perturbations of equilibrium decay and the fluid is motionless. For $\lambda_r > 0$, all perturbations of equilibrium increase and the fluid equilibrium loses its stability. The oscillatory instability is possible at $R_{mc} = -20$, when an imaginary part of the perturbation increment is not zero, i.e., $\omega_0 = \mathrm{Im}\,(\lambda) \neq 0$ (Fig. 4b, c).

The effect of high-frequency vertical vibrations on the convective stability in a porous layer ($d = 0$) saturated with a binary fluid under the action of gravitational field was studied in Jounet and Bardan (2001). The equilibrium temperature and concentration gradients were set constant. Equations (37)–(39) for the boundaries of equilibrium stability with respect to monotonic and oscillatory perturbations involve now an additional term, which depends on the vibration intensity and properties of the porous medium and fluid. The critical parameters for the monotonic instability are related by the following relationship (Jounet and Bardan 2001):

$$R_m = \frac{\left(\pi^2 + k^2\right)^2}{k^2\,(1 + N\,\mathrm{Le})} + P_v R_m^2 \left(1 + \frac{N}{\varepsilon}\right) \frac{k^2}{\left(\pi^2 + k^2\right)}, \tag{44}$$

and for the oscillatory instability by:

$$R_m = \frac{\left(\pi^2 + k^2\right)^2 (1 + \varepsilon \mathrm{Le})}{k^2 \mathrm{Le}\,(\varepsilon + N)} + P_v R_m^2 \left(1 + \frac{N}{\varepsilon}\right) \frac{k^2}{\left(\pi^2 + k^2\right)}. \tag{45}$$

The frequency of oscillatory perturbations is derived from the relation:

$$m\mathrm{Le}_m \frac{\omega_0^2}{k^2} = \frac{\left(\pi^2 + k^2\right)^2}{k^2} - (1 + N\,\mathrm{Le}) \left\{ R_m - P_v R_m^2 \left(1 + \frac{N}{\varepsilon}\right) \frac{k^2}{\left(\pi^2 + k^2\right)} \right\}, \tag{46}$$

Here $\varepsilon = m/b$ is the normalized porosity, $\mathrm{Le} = b\,\mathrm{Le}_m$ is the normalized Lewis number, $N = \frac{R_{mc}}{b\,\mathrm{Le}_m R_m} = \frac{\beta_C B_m}{\beta_T A_m}$ is the buoyancy ratio, and $P_v = \frac{p_v \Omega/m}{(1+\Omega^2/m^2)}$ is a modified vibrational parameter. It is seen from relations (44) and (45) that vibrations can both stabilize and destabilize the binary fluid equilibrium, depending on the sign of the expression: $\left(1 + \frac{N}{\varepsilon}\right)$ (Jounet and Bardan 2001). The obtained numerical data coincide with the results of analytical solutions (44)–(46). Figure 5a–c shows the equilibrium stability maps for the aqueous solution of ammonium chloride saturating a porous layer of glass spheres. When the fluid is heated from below ($R_m > 0$), vibrations stabilize its equilibrium with respect to the monotonic and oscillatory perturbations for all values of the solutal Rayleigh number being considered (Fig. 5a). The wavelength of the most dangerous perturbations increases with the growth of the vibrational parameter $p_v$, which characterizes the vibration intensity (Fig. 5b). The frequency of oscillatory perturbations decreases with increase in (Fig. 5c). The result is analogous to the case of thermal convection in a porous layer saturated with a single-component fluid under the action of gravitational field and high-frequency vertical vibrations (Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000).

Let us now discuss the value of a modified vibrational parameter , for which maximum stability takes place in the limiting case of thermal convection arising in a porous layer saturated with a single-component fluid and subjected to high-frequency vertical vibrations. Solving quadratic equation (44) with $N = 0$, we arrive at the critical Rayleigh number for the onset of stationary convection (Zen'kovskaya 1992; Bardan et al. 2004)

$$R_m = \frac{\left(\pi^2 + k^2\right)}{2k^2 P_v} \left(1 \pm \sqrt{1 - 4P_v\left(\pi^2 + k^2\right)}\right). \tag{47}$$

The critical Rayleigh number does not exit for $P_v > P_{v*}$, where $P_{v*} = 1/4\left(\pi^2 + k^2\right)$.

### *4.1.2 Heating from Above ($R_m < 0$)*

Convection in a binary fluid-saturated porous layer, which is heated from above and is under the action of the static gravitational field, can occur in the presence of the upward-directed concentration gradient of the heavier fluid component ($R_{mc} > 0$) (Nield and Bejan 2013). A fluid flow is caused by the solutal gravitational mechanism of instability (Gershuni and Zhukovitskii 1972; Nield and Bejan 2013). A temperature gradient directed upwards ($R_m <$ 0) suppresses convection induced by a concentration gradient. With an increase in the absolute

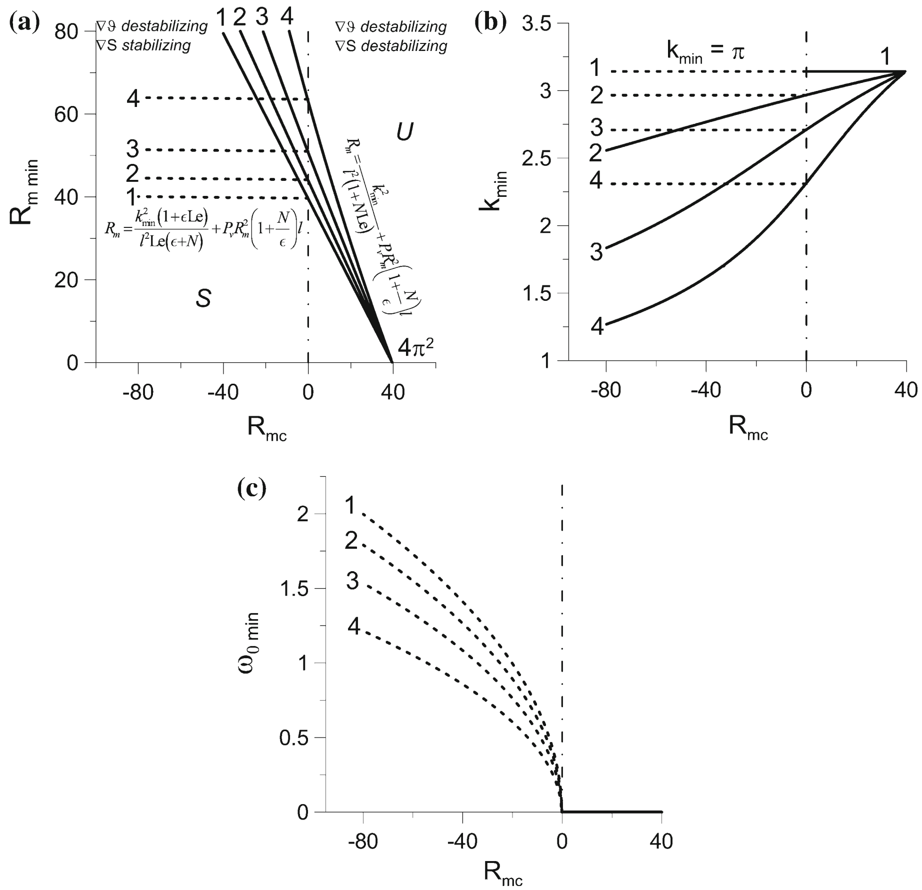



**Fig. 5** The equilibrium stability maps for a binary fluid in a porous layer ($d = 0$) heated from below under the action of gravitational field and high-frequency vertical vibrations at $\Omega = 0.1$ and different values of the vibrational parameter $p_v$: 1 – $p_v = 0$, 2 – $p_v = 0.02$, 3 – $p_v = 0.04$, 4 – $p_v = 0.06$. **a** The minimal critical Rayleigh number, **b** the wave number of the most dangerous perturbations of equilibrium, and **c** the frequency of the most dangerous perturbations. Solid lines denote the monotonic instability mode, and dashed lines denote the oscillatory instability mode. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region. The parameter $l$ in equations is defined by the relation $l = \frac{k_{\min}^2}{\pi^2 + k_{\min}^2}$, where $k_{\min}$ is the wave number of the most dangerous perturbations

value of $R_m$, the fluid equilibrium is stabilized (Fig. 6). A stability boundary of the fluid equilibrium in a porous layer is defined by relations (38)–(40) (Nield and Bejan 2013; Nield 1968). The perturbations with the wave number $k = \pi$ are the most dangerous. In the layer of glass spheres saturated with an aqueous ammonium chloride solution and heated from above, convection is monotonically excited at all values of being considered (Fig. 6). The oscillatory instability is not observed.

Maps of monotonic instability of equilibrium in a binary fluid-saturated porous layer heated from above under the action of gravitational field and high-frequency vertical vibrations are shown in Fig. 7a, b. The maps are constructed at $\Omega = 0.1$ and different values of the vibrational parameter $p_v$. It is seen that for the selected set of parameters the vibrations

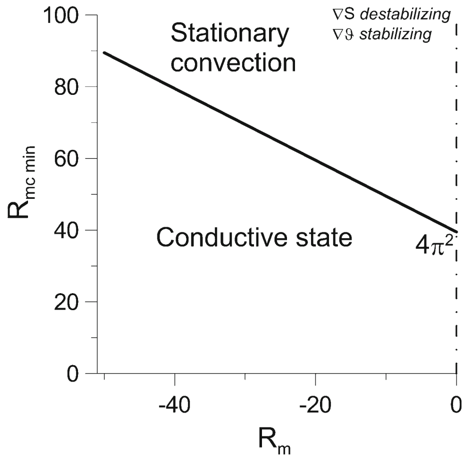



**Fig. 6** The equilibrium stability map for a binary fluid in a porous layer ($d = 0$) heated from above in the absence of vibrations at the most dangerous monotonic perturbations with the wave number $k = \pi$

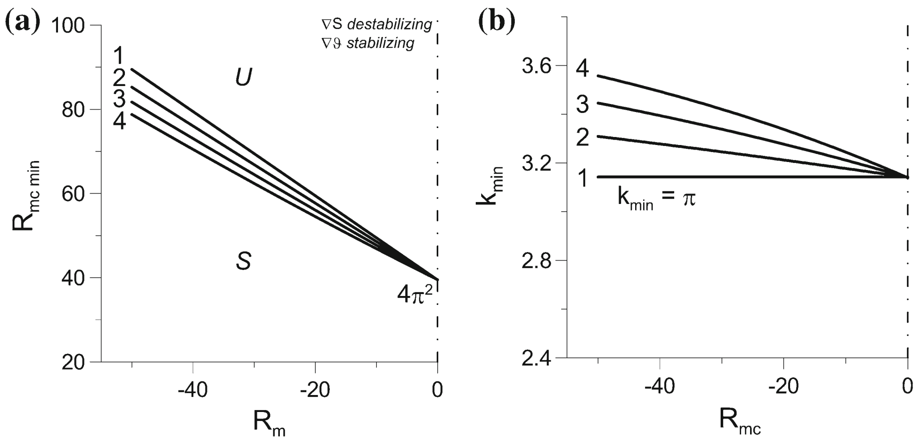



**Fig. 7** The equilibrium stability maps for a binary fluid in a porous layer ($d = 0$) heated from below under the gravitational field and high-frequency vertical vibration at $\Omega = 0.1$ and different values of the vibrational parameter $p_v$: 1 – $p_v = 0$, 2 – $p_v = 0.02$, 3 – $p_v = 0.04$, 4 – $p_v = 0.06$. **a** The minimal critical solutal Rayleigh number and **b** the wave number of the most dangerous monotonic perturbations of equilibrium. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region

produce a destabilizing effect on the fluid equilibrium and reduce the wavelength of its most dangerous perturbations (curves 2–4, Fig. 7a, b). The results of our numerical calculations are consistent with the data analytically obtained using formulas (38)–(46) (Nield and Bejan 2013; Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000; Jounet and Bardan 2001).

## 4.2 Superposed Fluid and Porous Layers

### *4.2.1 Heating from Below ($R_m > 0$)*

Now we turn our attention to the discussion of the convective stability in a two-layer system heated from below and consisting of a binary fluid layer and a fluid-saturated porous layer.

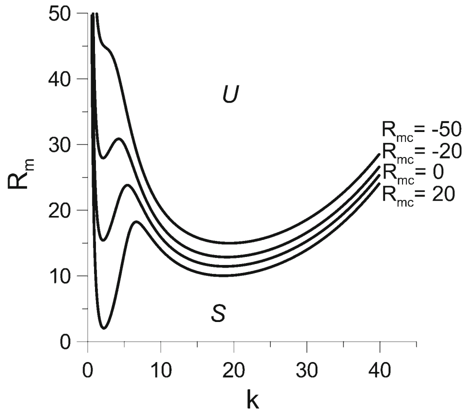



**Fig. 8** Neutral curves for the monotonic equilibrium instability of the binary fluid in a two-layer system heated from below in the absence of vibrations at $d = 0.15$ and different values of the solutal Rayleigh number $R_{mc}$. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region. The instability regions are located above the curves

Figure 8 shows the neutral curves of equilibrium stability of the fluid heated from below in a static gravitational field for the ratio of the fluid layer thickness to that of the porous layer $d = 0.15$ and different values of the solutal Rayleigh number $R_{mc}$. The figure shows the monotonic instability curves. For all values of $R_{mc} < 0$ considered in our study, the oscillatory instability curves practically coincide with the monotonic instability curves plotted at $R_{mc} = 0$. They are not depicted in this figure. In contrast to a porous layer ($d = 0$), in which the perturbations with a fixed wave number $k = \pi$ are the most dangerous in the absence of vibrations, in a two-layer system an abrupt change in the character of instability can occur due to a change of one of the system parameters. This change is caused by a noticeable difference in the wave numbers of the most dangerous perturbations as a result of a change in the value of the system parameter (for example, the solutal Rayleigh number). For $R_{mc} = 20$ (the heavier component of the solution is at the top boundary), the instability is caused by the growth of perturbations with a longer wavelength, spanning the porous and fluid layers (Fig. 8). The destabilizing effect of the upward-directed gradient of concentration of the heavier component ($R_{mc} > 0$) on the fluid equilibrium lowers its stability threshold in comparison with the case of thermal convection ($R_{mc} = 0$) (Fig. 8). For $R_{mc} \leq -20$ (the heavier component of the solution is at the bottom boundary), the perturbations with a shorter wavelength, spreading through the entire fluid layer and slightly penetrating into the porous layer, become the most dangerous (Fig. 8). In this case, an increase in the absolute value of the concentration gradient leads to the growth of the stability threshold. Neutral curves are bimodal at $R_{mc} \to 0$ and $d = 0.15$. They have two minima corresponding to close values of the Rayleigh numbers (Fig. 8).

A transition from the long-wave to shortwave most dangerous perturbations with the variation of the solutal Rayleigh number and the ratio of layer thicknesses can be traced from the stability maps shown in Fig. 9a–c. The boundary of the equilibrium stability with respect to monotonic and oscillatory perturbations in the porous layer saturated by a binary fluid was obtained at $d = 0$ (curves 3, Fig. 9a). It is defined from relations (38)–(40). In this case, convection arises in the form of rolls with dimensionless wavelength of $l = 2\pi/k = 2$, spreading over the entire porous layer. With the growth of the relative fluid layer thickness, the instability can be associated with the development of shortwave perturbations localized in a fluid layer already at $d = 0.15$ (curves 1, Fig. 9a, b). At a fixed value of $d$, the transition from the long-wave to shortwave most dangerous perturbations is due to a change in the solutal Rayleigh number $R_{mc}$. A decrease in $R_{mc}$ causes stabilization of the fluid equilibrium

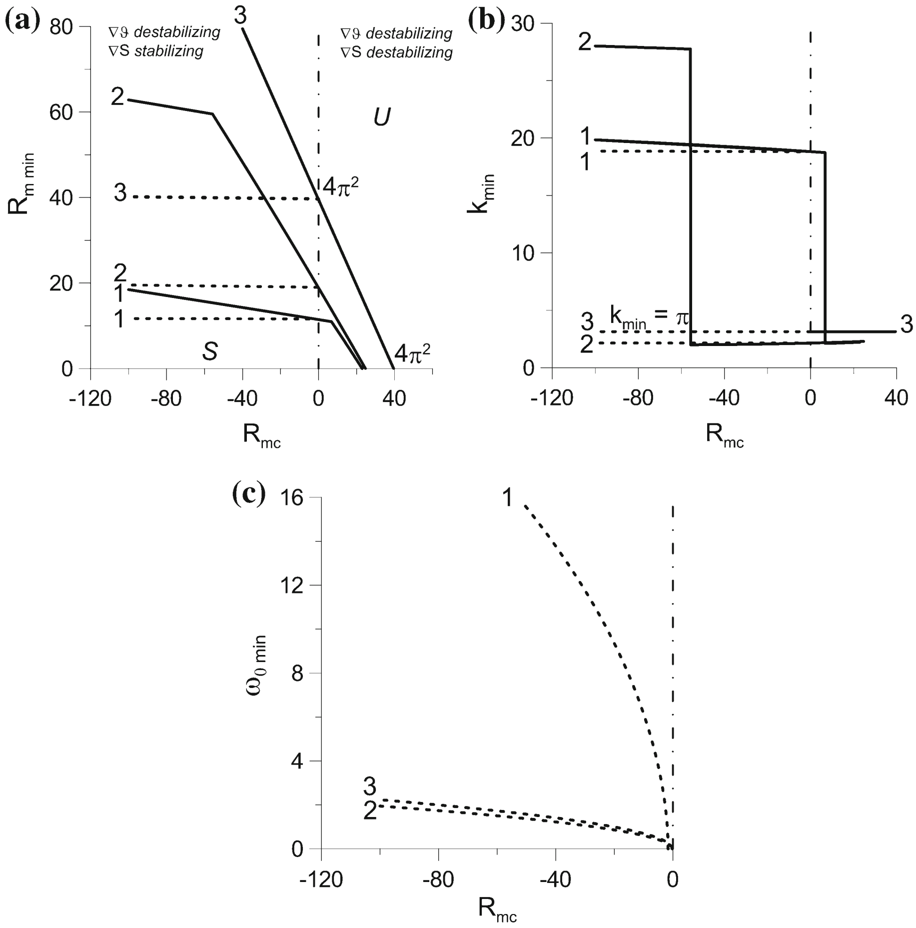



**Fig. 9** The equilibrium stability maps for a binary fluid in a two-layer system heated from below in the absence of vibrations at different values of the ratio of the fluid layer to the porous layer thicknesses $d$: 1 – $d = 0.15$, 2 – $d = 0.10$, 3 – $d = 0$. **a** The minimal critical Rayleigh number, **b** the wave number of the most dangerous perturbations of equilibrium, and **c** the frequency of the most dangerous perturbations. Solid lines denote the monotonic instability, and dashed lines correspond to the oscillatory instability. Symbol *S* is for the stability region, and symbol *U* corresponds to the instability region

and a sharp change in the character of instability (breaks of solid curves 1, Fig. 9a, b). The shortwave perturbations with the wave number $k \approx 20$ occurring in the fluid layer become the most dangerous (curves 1, Fig. 9b).

Depending on the value of $R_{mc}$, the most dangerous perturbations of different wavelengths can arise either in a monotonic or an oscillatory manner. An abrupt change in the character of equilibrium instability with respect to monotonic perturbations caused by a change in their wavelength was detected in Chen and Chen (1988), Hirata et al. (2009). The values of $d$ and $R_{mc}$, at which this alteration takes place, correspond to the breaks of solid curves 1 and 2 in Fig. 9a, b. According to our calculations, for $R_{mc} < 0$ convection in the layers occurs in an oscillatory rather than monotonic fashion (dashed curves in Fig. 9a). It is just due to the presence of the oscillatory instability that a transition from the long-wave to shortwave

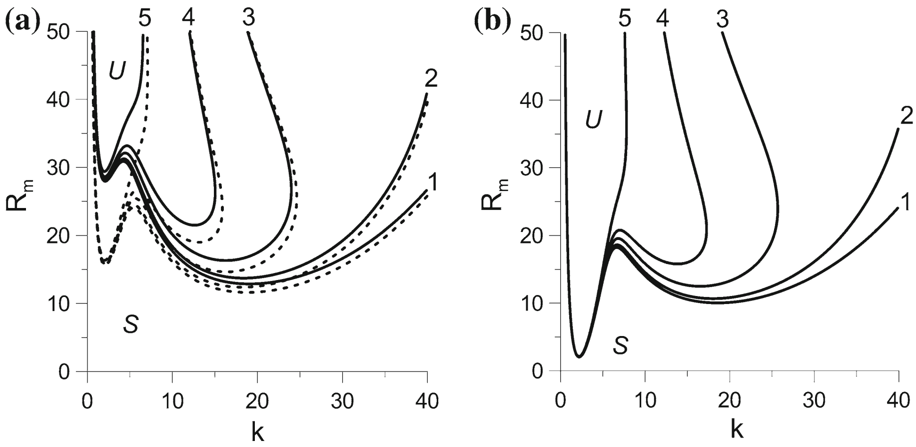



**Fig. 10** Neutral curves of the equilibrium instability for a binary fluid in a two-layer system heated from below under the action of gravitational field and high-frequency vertical vibrations at $d = 0.15$, $\Omega = 0.1$, and different values of the vibrational parameter $p_v$: $1 - p_v = 0, 2 - p_v = 0.001, 3 - p_v = 0.003, 4 - p_v = 0.005$, $5 - p_v = 0.01$, and the solutal Rayleigh number $R_{mc}$: **a** $R_{mc} = -20$, **b** $R_{mc} = 20$. Solid lines denote the monotonic instability, and dashed lines denote the oscillatory instability. Symbol *S* is for the stability region, and symbol *U* corresponds to the instability region

most dangerous perturbations occurs at larger values of $d$ compared to the case of monotonic instability (see the solid and dashed curves 2, Fig. 9a, b). For example, a large-scale oscillatory flow with the wave number $k \approx 2.5$ is excited in the system at $d = 0.1$ and $R_{mc} \le -60$ (dashed curve 2, Fig. 9b). At the same time, when $R_{mc} \le -60$, the monotonic instability is caused by the development of shortwave perturbations with the wave number (solid curve 2, Fig. 9b). The oscillatory shortwave perturbations for the same values of $R_{mc}$ become the most dangerous only at $d = 0.15$ (dashed curves 1, Fig. 9a, b). They have a frequency, which is 10 times greater than the frequency of long-wave oscillatory perturbations in the porous layer (curve 1, Fig. 9c).

Figure 10a, b shows neutral curves for the monotonic and oscillatory instability of fluid equilibrium in a two-layer system at $d = 0.15$, $\Omega = 0.1$, and different values of the vibrational parameter $p_v$. The curves are constructed for downward-directed ($R_{mc} = -20$) and upward-directed ($R_{mc} = 20$) concentration gradients of the heavier component of the binary fluid. It is seen that high-frequency vertical vibrations stabilize the fluid equilibrium and lead to an increase in the wavelength of its most dangerous perturbations at the selected values of $R_{mc}$ (curves 2–5, Fig. 10a, b). Moreover, they have the greatest impact on the shortwave perturbations (regions of the larger wave numbers in Fig. 10a, b). The threshold of equilibrium stability with respect to long-wave perturbations slightly varies with increasing in $p_v$ (regions of the smaller wave numbers in Fig. 10a, b). This can be explained by a different role of inertial effects in the binary fluid and fluid-saturated porous layers. The result obtained is analogous to the vertical vibration effect on the equilibrium of a single-component fluid in a two-layer system including a fluid layer and a fluid-saturated porous layer and heated from below (Lyubimov et al. 2008, 2015; Kolchanova et al. 2012). A distinguishing feature of the convection excitation in a binary fluid in the presence of the vibration field is the onset of the oscillatory mode of equilibrium instability at $R_{mc} = -20$ (dashed curves in Fig. 10a). At all values of the vibrational parameter $p_v$, an average convective fluid flow is of the oscillatory nature. For $R_{mc} = 20$, convection in the system is excited monotonically.

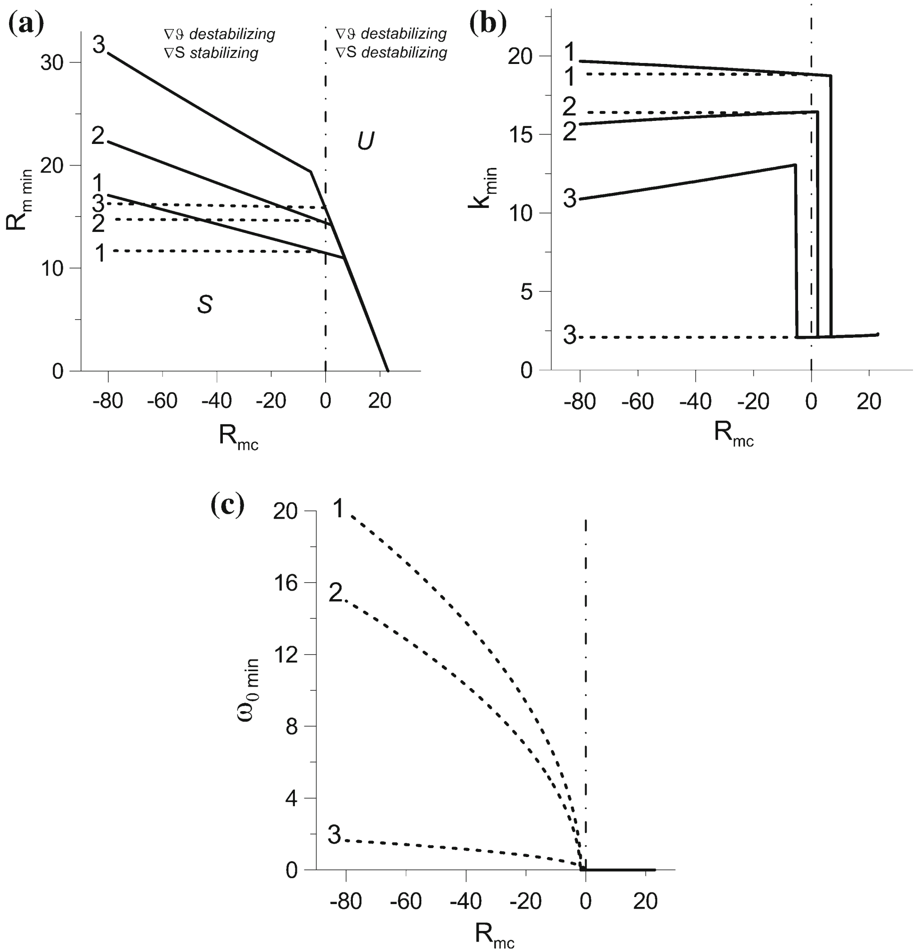



**Fig. 11** The equilibrium stability maps for a binary fluid in a two-layer system heated from below under the gravitational field and high-frequency vertical vibrations at $d = 0.15$, $\Omega = 0.1$, and different values of the vibrational parameter $p_v$: $1 - p_v = 0$, $2 - p_v = 0.003$, $3 - p_v = 0.005$. **a** The minimal critical Rayleigh number, **b** the wave number of the most dangerous perturbations of equilibrium, and **c** the frequency of the most dangerous perturbations. Solid lines denote the monotonic instability, and dashed lines denote the oscillatory instability. Symbol *S* is for the stability region, and symbol *U* corresponds to the instability region

A more detailed analysis of the vibration effect on the occurrence of convection in layers can be performed based on the maps of shown in Fig. 11a–c. The maps are constructed at a fixed ratio of layer thicknesses $d = 0.15$ and dimensionless frequency of vibrations $\Omega = 0.1$ for different values of the vibrational parameter $p_v$. The breaks of monotonic instability curves (solid curves 1–3, Fig. 11a, b) are associated with a transition from the long-wave to shortwave most dangerous perturbations as the solutal Rayleigh number $R_{mc}$ reduces. For $R_{mc} > 0$, the average convection arises monotonically. Increase in the vibration intensity at a fixed value of $R_{mc}$ results in suppression of convection in the fluid layer. The long-wave perturbations spreading through both layers become the most dangerous (solid curves 2 and 3, Fig. 11b). The effect of vibrations on the long-wave perturbations is markedly weaker (Fig. 11a, b).

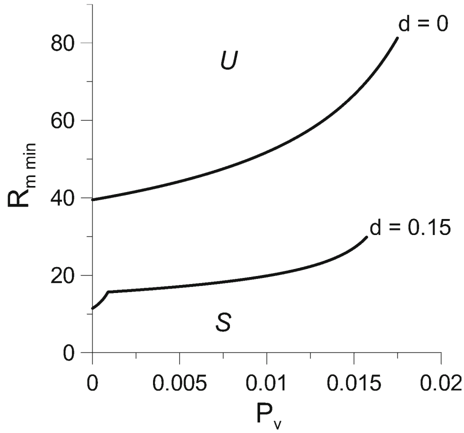



**Fig. 12** The minimal critical Rayleigh number versus the modified vibrational parameter $P_v$ for stationary convection onset in a single-diffusive porous layer ($d = 0$) and superposed pure fluid and porous layers ($d = 0.15$) at $R_{mc} = 0$ and $\Omega = 0.1$. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region

At $R_{mc} < 0$, the instability can be due to the development of either monotonic or oscillatory perturbations of equilibrium (dashed curves 1, Fig. 11a, b). Their wavelength abruptly changes as the vibration intensity grows. Vibrations have a greater impact on the oscillatory instability mode in comparison with the monotonic mode (solid curves 2, 3, Fig. 11a, b). For all $R_{mc} < -5$, a transition to long-wave oscillatory perturbations occurs already at $p_v = 0.005$ (dashed curves 3, Fig. 11a, b). In this transition, the frequency of oscillatory perturbations decreases by a factor of 10 (dashed curve 3, Fig. 11c). In the case of monotonic instability, the shortwave mode remains the most dangerous at $R_{mc} < -5$ and $p_v \leq 0.005$ (solid curves 1, 2, 3, Fig. 11a, b).

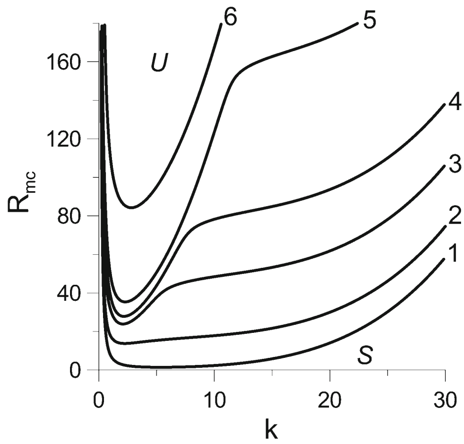
To compare values of a modified vibrational parameter $P_v = \frac{p_v \Omega/m}{(1+\Omega^2/m^2)}$, for which maximum stability takes place in a differently heated single-diffusive porous layer [see, Eq. (47)] and superposed pure fluid and porous layers undergoing high-frequency vertical vibration, we show the minimal Rayleigh number for stationary convection onset versus $P_v$ in Fig. 12. Plots are presented at fixed thickness ratios $d = 0$ and $d = 0.15$. It is seen that a maximum limit for achieving a stabilizing effect in a two-layer system is lower than in a porous layer.

### *4.2.2 Heating from Above ($R_m < 0$)*

The convective instability in a two-layer system heated from above is caused by the solutal gravitational mechanism of convection excitation. Convection in the system can occur in the presence of an upward-directed concentration gradient for the heavier component of the solution ($R_{mc} > 0$). As in the case of a binary fluid-saturated porous layer, the temperature gradient directed upward ($R_m < 0$) stabilizes the fluid equilibrium (curves 2–6, Fig. 13). The convective instability is due to the development of monotonic perturbations of equilibrium at a given set of parameters (see Table 1).

In a two-layer system heated from above, a gradual change in the wavelength of the most dangerous perturbations of fluid equilibrium is observed as the absolute Rayleigh number $|R_m|$ decreases. Neutral curves have one minimum for all values of $R_m$ being considered (Fig. 13). A sharp change in the character of instability accompanied by a jump-wise transition from the long-wave to shortwave perturbations with decreasing $|R_m|$ does not occur even at relatively large values of the ratio of the fluid layer to the porous layer thicknesses ($d = 0.50$) (Fig. 13).

**Fig. 13** Neutral curves of monotonic equilibrium instability for a binary fluid in a two-layer system heated from above in the absence of vibrations at $d = 0.50$ and different values of the Rayleigh number $R_m$: 1 – $R_m = 0$, 2 – $R_m = -1$, 3 – $R_m = -3$, 4 – $R_m = -5$, 5 – $R_m = -10$, 6 – $R_m = -50$. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region



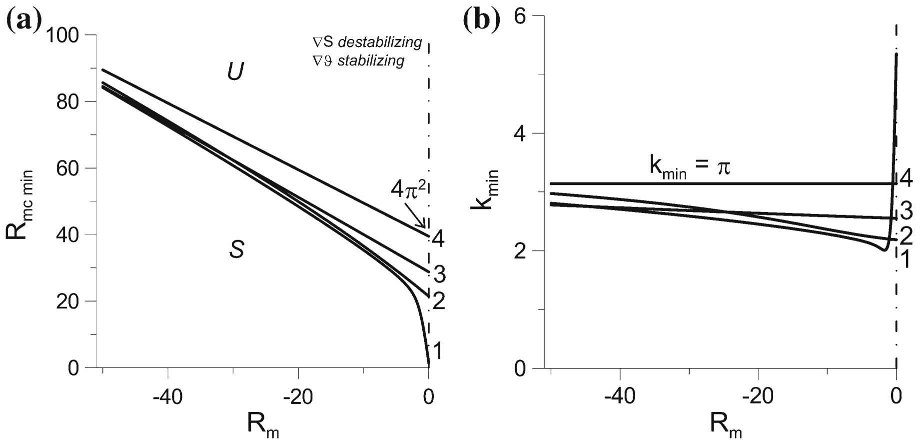




**Fig. 14** The equilibrium stability maps for a binary fluid in a two-layer system heated from above in the absence of vibrations at different values for the ratio of the fluid layer to the porous layer thicknesses $d$: 1 – $d = 0.50$, 2 – $d = 0.20$, 3 – $d = 0.05$, 4 – $d = 0$. **a** The minimal critical solutal Rayleigh number, **b** the wave number of the most dangerous monotonic perturbations of equilibrium. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region

Figure 14a,b represents the equilibrium stability maps for fluid with respect to monotonic perturbations at different values of the ratio of layer thicknesses $d$. An increase in the relative fluid layer thickness destabilizes the equilibrium at all values of $R_m$ (curves 1–3, Fig. 14a). A fluid layer also contributes to destabilization of the equilibrium. Therefore, as its thickness increases, the stability threshold reduces. The wavelength of the most dangerous perturbations monotonically increases as $d$ grows for all $R_m < -3$ (Fig. 14b). For $d = 0.50$ in the range of the Rayleigh numbers $0 < R_m < -3$, the wavelength of critical perturbations is reduced almost by 2 times. However, a sharp change in the instability nature (from the long-wave to shortwave modes) is not observed at $R_m \to 0$.

Neutral monotonic instability curves at $d = 0.50$, $\Omega = 0.1$, and different values of the vibrational parameter $p_v$ are shown in Fig. 15a, b. Vertical high-frequency vibrations destabilize the fluid equilibrium (curves 2–4, Fig. 15a, b). The wavelength of the most dangerous perturbations of equilibrium decreases as the vibration intensity grows. Note that the change

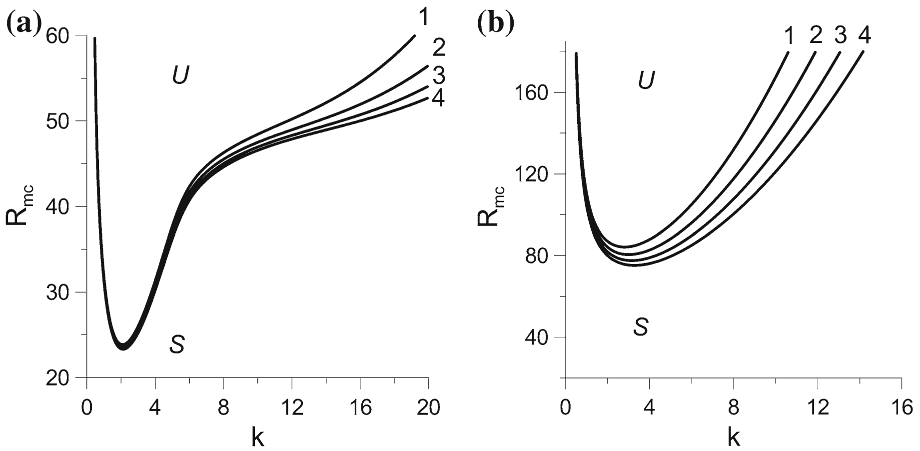



**Fig. 15** Neutral curves of monotonic equilibrium instability for a binary fluid in a two-layer system heated from above under the gravitational field and high-frequency vertical vibrations at $d = 0.50$, $\Omega = 0.1$, and different values of the vibrational parameter $p_v$: $1 - p_v = 0$, $2 - p_v = 0.02$, $3 - p_v = 0.04$, $4 - p_v = 0.06$, and the Rayleigh number $R_m$: **a** $R_{mc} = -3$, **b** $R_{mc} = -50$. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region

in the equilibrium stability boundary with the growth of $p_v$ is more pronounced for the values of parameters falling in the region of large critical wave numbers of perturbations (Fig. 15a, b). The greater is the absolute value of $|R_m|$, the stronger is the vibration effect on the stability threshold (Fig. 15b).

Based on the maps of fluid equilibrium stability, constructed at fixed values of $d = 0.50$ and $\Omega = 0.1$, one can trace a change in the stability threshold caused by a change in the vibrational parameter $p_v$ and Rayleigh number $R_m$ (Fig. 16a, b). For a relatively large fluid layer thickness $d = 0.50$ (a ratio of the fluid layer thickness to that of the porous layer is 1:2), a wave number (or wavelength) of the most dangerous perturbations of equilibrium changes non-monotonically in the range of Rayleigh numbers $0 < R_m < -3$, which is characteristic of the systems in a static gravitational field (Fig. 14b). The same behavior is also observed in the presence of the vibrational acceleration (Fig. 16b). It is due to the fact that at small values of the Rayleigh number $R_m$, destabilization of equilibrium by vibrations is poorly pronounced. In the range of Rayleigh numbers $R_m < -3$, the wave number of critical perturbations monotonically diminishes with decreasing $|R_m|$ at all values of the vibrational parameter $p_v$.

When the fluid is heated from above, the average convection appears in the form of rolls with a large value of the wavelength, which spread through both layers (small wave numbers $k \approx 3$ of the most dangerous perturbations of equilibrium, Fig. 16b). In this case, in order to achieve a considerable change in the stability threshold of fluid equilibrium in the layers, we should apply large vibrational accelerations (values of the vibrational parameter $p_v \approx 0.08$) in contrast to the case of a static gravitational field (Fig. 16a). When the fluid is heated from below, the shortwave instability can occur [large wave numbers $k \approx 20$ of the most dangerous perturbations of equilibrium (Fig. 11b)]. The onset of this instability should be related to the development of perturbations in the fluid layer. Here, the effect of vibrations on the shortwave instability is more pronounced than on the long-wave instability. It is due to a different role of the inertial effects in the fluid and porous layers (Lyubimov et al. 2008, 2015; Kolchanova et al. 2012). Unlike the case of a static gravitational field, a noticeable

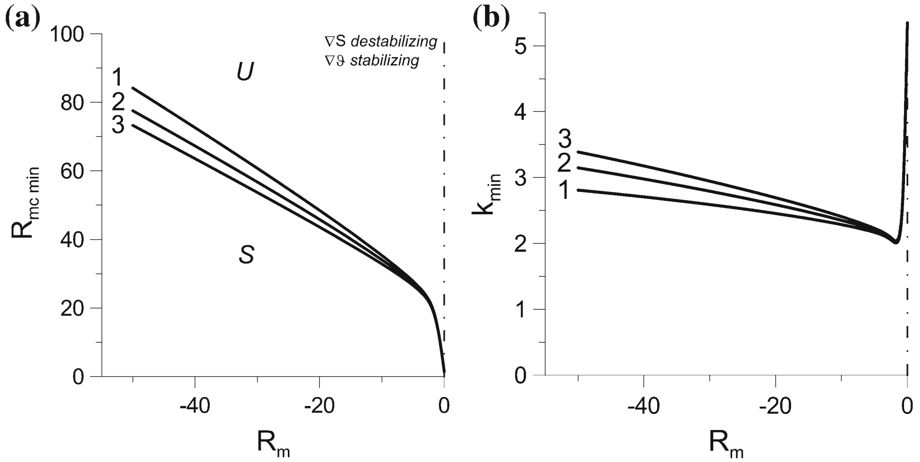



**Fig. 16** The equilibrium stability maps for a binary fluid in a two-layer system heated from above under the gravitational field and high-frequency vertical vibrations at $d = 0.50$, $\Omega = 0.1$, and different values of the vibrational parameter $p_v$: $1 - p_v = 0$, $2 - p_v = 0.04$, $3 - p_v = 0.08$. **a** The minimal critical solutal Rayleigh number and **b** the wave number of the most dangerous monotonic perturbations of equilibrium. Symbol $S$ is for the stability region, and symbol $U$ corresponds to the instability region

change in the stability threshold of the fluid equilibrium in the layers is observed at the values of the vibration parameter $p_v \approx 0.005$ (Fig. 11a), which is an order of magnitude lower than the values required for the layers heated from above.

## 5 Conclusions

We investigated a linear stability problem for a system comprising a horizontal binary fluid layer and a porous layer saturated with the fluid in a gravitational field. Temperature and concentration at the external solid boundaries of the system were assumed to have constant, different values. A binary fluid saturated the porous layer and filled the space of the cavity above the layer. As a porous medium, we used a system of glass spheres, the diameter of which was 10 times smaller than the thickness of the porous layer they form. The saturating fluid was an aqueous solution of ammonium chloride. The layers were subjected to high-frequency vertical vibration. The porous medium did not deform. The spheres were fixed and oscillated together with the cavity.

The vibrational convection equations in the fluid and porous layers were derived using the averaging method (Gershuni and Lyubimov 1998; Zen'kovskaya and Simonenko 1966; Zen'kovskaya 1992; Zen'kovskaya and Rogovenko 1999; Bardan and Mojtabi 2000; Jounet and Bardan 2001; Lyubimov et al. 2008, 2015; Kolchanova et al. 2012). A vibration period was assumed to be small compared to the characteristic viscous-diffusion, thermal-diffusion and mass-diffusion timescales in the layers. Filtration of the binary fluid in the porous layer was described by the Darcy law. The convection equations were written in the Boussinesq approximation (Gershuni and Zhukovitskii 1972; Nield and Bejan 2013). A stability problem was numerically solved by the shooting method (Lobov et al. 2004). We determined the average convection onset under two different heating conditions, when the system is heated from below (the Rayleigh number $R_m > 0$) or above ($R_m < 0$).

### 5.1 Heating from Below

A concentration gradient in addition to a temperature gradient makes a contribution into the resulting buoyancy force that drives double-diffusive convection in a system heated from below under a static gravitational field. In the case of equal contributions ($R_m > 0$, $R_{mc} > 0$), stationary convection arises and the monotonic instability of the mechanical equilibrium state develops. In the opposite case ($R_m > 0$, $R_{mc} < 0$), oscillatory convection can occur (Nield and Bejan 2013).

A distinguishing feature of the problem is a bimodal nature of neutral stability curves (Lyubimov and Muratov 1977; Lyubimov et al. 2002, 2004; Chen and Chen 1988, 1989; Zhao and Chen 2001; Hirata et al. 2009; Kolchanova et al. 2013). There is a range of the system parameters (numbers $R_{mc}$ and $R_m$, the ratio of layer thicknesses $d$), in which the neutral curves have two minima. They correspond to the stability threshold with respect to perturbations of different wavelengths. Variation of the governing parameters causes an abrupt change in the character of instability: from the long-wave to shortwave modes, or vice versa. The convective flow in the form of long-wave rolls arises in both layers. The shortwave rolls locate mainly in the fluid layer. Their wave numbers differ approximately by 10 times. The long-wave and shortwave instabilities can develop either in a monotonic or oscillatory manner, depending on the relation between the solution parameters.

Vibrations stabilize the motionless equilibrium state and lead to an increase in the wavelength of its most dangerous perturbations in the whole range of the parameters studied. A sharp change in the character of instability (transition from the shortwave to long-wave modes) can be observed with increasing the solutal Rayleigh number $R_{mc}$ or the vibrational parameter $p_v$. As in the case of a single-component fluid, vibrations substantially delay the shortwave convection onset in comparison with the onset of long-wave convection (Lyubimov et al. 2008, 2015; Kolchanova et al. 2012). This is due to a difference in the role of inertial effects between the fluid and porous layers. For the thickness ratio $d = 0.15$ a noticeable change in convection onset values, as compared to the case of a static gravitational field, is observed already at $p_v \approx 0.005$.

In case of opposite thermal and buoyancy forces ($R_m > 0$, $R_{mc} < 0$), the instability can be initiated by both the development of the monotonic and oscillatory perturbations. Their wavelength abruptly changes with the growth of $p_v$. Vibrations produce a stronger effect on the oscillatory convection onset as compared to the onset of stationary convection. For all $R_{mc} < -5$, a transition from the shortwave to long-wave oscillatory perturbations occurs already at $p_v = 0.005$. The frequency of oscillatory perturbations at this transition decreases by a factor of 10. In the case of monotonic instability, the shortwave instability mode at the same parameters remains the most dangerous.

### 5.2 Heating from Above

When the fluid is heated from above, stationary convection is caused by the destabilizing concentration gradient ($R_m < 0$, $R_{mc} > 0$) in a static gravitational field. An example of the situation can be salt fingering in a water column with warm salty water at its upper end and cooler freshwater at its lower end or convection excitation in the binary solution solidified from below.

For superposed ammonium chloride solution layer and porous layer of glass spheres, stationary convection arises and the monotonic instability develops in the form of rolls with large enough wavelength. For all the values of $d$ and $R_m$ being considered, no oscillations are observed. A sharp change in the character of instability, accompanied by a tenfold jump in the perturbation wavelength with the growth of $R_m$, does not occur even at relatively large fluid layer thicknesses ($d = 0.50$). The wavelength of the most dangerous perturbations changes gradually. However, this change can be non-monotonic. For $d = 0.50$, it increases with $R_m$ for all $R_m < -3$. In the range of the Rayleigh numbers $0 < R_m < -3$, the perturbation wavelength decreases almost by half with increasing $R_m$.

Vibrations speed up the stationary convection onset and cause a decrease in the wavelength of most dangerous perturbations of the motionless equilibrium state. In comparison with the case of a static gravitational field, a noticeable change in the instability threshold in the layers with $d = 0.50$ is observed at the value $p_v \approx 0.08$, which is an order of magnitude greater than in the case of the fluid heated from below.

In the paper, a good agreement between the numerical simulation results and well-known analytical results was obtained in the limiting case of zero fluid layer thickness ($d = 0$). Convection of a binary fluid in a porous layer in the absence and presence of high-frequency vertical vibration was studied in Nield and Bejan (2013), Nield (1968), Jounet and Bardan (2001).

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