

# Experimental Investigation on Non-Darcian Flow in Unbound Graded Aggregate Material of Highway Pavement

Han-Cheng Dan<sup>1,2</sup> · Lin-Hua He<sup>1</sup> · Bo Xu<sup>1</sup>

Received: 8 July 2015 / Accepted: 20 January 2016 / Published online: 1 February 2016  
© Springer Science+Business Media Dordrecht 2016

**Abstract** The unbound graded aggregate base (UGAB) has been widely used in drainage layers of highway pavements. This kind of material is of high permeability and can thus drain efficiently the water infiltrated through cracks in the pavement and reduce the associated water damage. The hydraulic conductivity of UGAB is a key factor describing the water flow behavior in UGAB and thus be considered primarily in the design of highway drainage engineering. In this study, the flow behavior of UGAB material was investigated through constant head permeability experiments. Based on the Reynolds number analysis, it was found that the flow in UGAB material was non-Darcian even though under relatively low hydraulic gradient. Therefore, the Darcy law cannot be directly applied to determine the hydraulic conductivity of UGAB. Furthermore, the empirical expression of coefficients in Forchheimer equation, which can be available to evaluate hydraulic conductivity of UGAB material approximately, is presented incorporating the representative particle size and porosity based on the Ergun equation. Then, it was validated against the aid of normalized objective function analysis. Through comparison analysis, the sequencing of hydraulic conductivity of UGAB material was sorted quantitatively in terms of different gradation and representative particle size ( $d_{50}$  and  $\bar{d}$ ). The results also showed that the  $d_{50}$  ( $\bar{d}$ ) and the content of fine particles ( $<0.075$  mm) are appropriate indexes for the gradation design of UGAB material in highway pavement engineering.

---

✉ Han-Cheng Dan  
danhancheng@csu.edu.cn

Lin-Hua He  
linhuahe\_csu@163.com

Bo Xu  
xubo\_csu@163.com

<sup>1</sup> School of Civil Engineering, Central South University, Changsha 410075, Hunan, China

<sup>2</sup> Post-doctoral Research Center, Guizhou Transportation Planning Survey & Design Academe, Guiyang 550001, Guizhou, China

**Keywords** Unbound graded aggregate · Non-Darcian flow · Forchheimer equation · Hydraulic conductivity · Compaction degree

## 1 Introduction

Currently, many typical bases (e.g., flexible, semi-rigid and rigid bases) have been applied in constructing highway pavement. Compared with other base materials, unbound graded aggregate base (UGAB) which is classified as flexible base cannot only effectively reduce reflection cracks but also can promptly drain water from pavement and ground (Dan et al. 2012a; Lebeau and Konrad 2009). It also has excellent drainage capacity to drain the free water infiltrating from pavement surface and plays a significant role in affecting the strength and stability of pavement structure to large extent (Dan et al. 2015; Haider et al. 2014). Generally, the drainage of the unbound graded material has been required that the discharge should be equal to or greater than the inflow and the layer thickness should not be less than the flow depth in the engineering design of highway. Thus, properly determining the hydraulic conductivity is a precondition to reasonably design drainage layer and adequately ensure strength and stability of pavement structure (Dan et al. 2012b, c, 2013). It is well known that the unbound graded aggregate material which is classified to porous media has large percentage air voids. Currently, the studies on hydraulic conductivity of unbound graded aggregate base are mostly based on the Darcy law (Li et al. 2010; Ji 2002). However, when the flow velocity increases in porous media, the Darcy law may be invalid and the flow becomes non-Darcian (Bordier and Zimmer 2000; Soni et al. 1978; Sedghi-Asl et al. 2014; Moutsopoulos et al. 2009). Hence, it is not reasonable to use the linear relationship between hydraulic gradient and flow velocity to describe the flow behavior in the experiment of unbound graded aggregate material.

The concern of an appropriate form to highlight characteristics for non-Darcian flow is the link between hydraulic gradient and flow velocity. A quadratic form of the nonlinear relationship was proposed firstly by Forchheimer (1901), and then, it is verified by Venkataraman and Rao (1998) and Bordier and Zimmer (2000) through laboratory experiments. In general, the Reynolds number was introduced to identify whether the flow in porous media is laminar or turbulent flow. Bear (1972) pointed out that the nonlinear flow occurs because of the inertial force, and Reynolds number is the key to distinguish the flow regime whether is laminar flow or not. Furthermore, a power form relationship was presented as well by Izbash and Filtracii (1931). Besides, some other researchers (e.g., Ergun 1952; Kovacs 1981; Fand and Thinakaran 1990; Kadlec and Knight 1996; Sidiropoulos et al. 2007; Sedghi-Asl and Rahimi 2011) presented some different empirical equation based on the quadratic form relationship for better description of non-Darcian flow. Tek (1957), Wright (1968) and de Vries (1979) indicated that the critical values of Reynolds number for describing the flow regime are indeterminable in practice. Through experimental investigation, Zeng and Grigg (2006) revised the Forchheimer number as a criterion to distinguish non-Darcian flow in porous media. Moreover, many experimental studies have been carried out to investigate the nonlinear relationship between hydraulic gradient and flow velocity in porous media. Soni et al. (1978) filled the column with glass balls and river sand and proposed that the relationship between hydraulic gradient and flow velocity can be divided into the prelinear, linear, postlinear relation on the basis of experimental investigation. Yamada et al. (2005) provided the experimental data by designing a bar flume which can adjust slope to obtain different hydraulic gradients. Moutsopoulos et al. (2009) proposed semi-empirical relations and analyzed the hydraulic behavior of bidisperse media. Sedghi-Asl et al. (2014) also adopted different hydraulic gradients to assess the flow behavior and evaluated the validity of four

widely used head-loss equations by using experiment data. Both of their results indicated that the Reynolds number increases with the hydraulic gradient increase. Recently, [Salahi et al. \(2015\)](#) studied the relation between Reynolds number and diameter of materials to verify accuracy of the well-known nonlinear relationships in detail through a packed column test. The non-Darcian flow in the coarse rockfill material in embankment dam has been studied by [Ferdos et al. \(2015\)](#), and the Reynolds number has a close connection with the grain diameters distribution through a series of laboratory experiments.

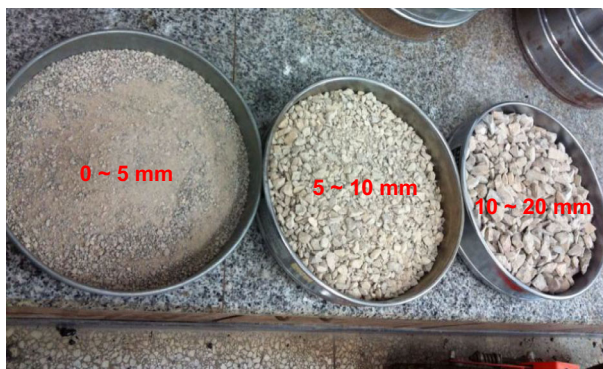
From the above researches, it can be found that the Reynolds number was adopted to identify the flow condition in porous media, and the grain diameter and porosity were used in the equations to evaluate the hydraulic conductivity of porous media. In the most of experiments, the investigated materials are commonly uniform particles. Little attention has been paid on UGAB materials, and the permeability property of UGAB materials needs to be studied further based on non-Darcian law. Particularly in pavement engineering, the mixture design (gradation variety) and compaction degree for UGAB material are significant to ensure its drainage performance and strength, which directly influence the stability and durability of pavement structure.

In this study, a series of constant head permeability experiments are conducted in this paper to investigate the flow behavior in UGAB materials. To quantify the hydraulic conductivity reasonably, the proper estimation equations are presented based on easily obtained parameters which were commonly used in the literature ([Ergun 1952](#); [Kovacs 1981](#); [Fand and Thinakaran 1990](#); [Herrera and Felton 1991](#); [Kadlec and Knight 1996](#); [Sedghi-Asl et al. 2014](#)). Furthermore, the effect of material properties on the hydraulic conductivity of UGAB materials is discussed in detail.

## 2 Experimental

### 2.1 Experimental Materials

In the experimental test, the UGAB material was composed of three varieties of unbound aggregate material. The ranges of the particle size are 0–5, 5–10 and 10–20 mm, respectively (Fig. 1). Each aggregate material was sieved and grain size distribution curves after mix design are illustrated in Fig. 2. Based on the recommended range (gradation band) of aggregate



**Fig. 1** Raw unbound aggregate materials

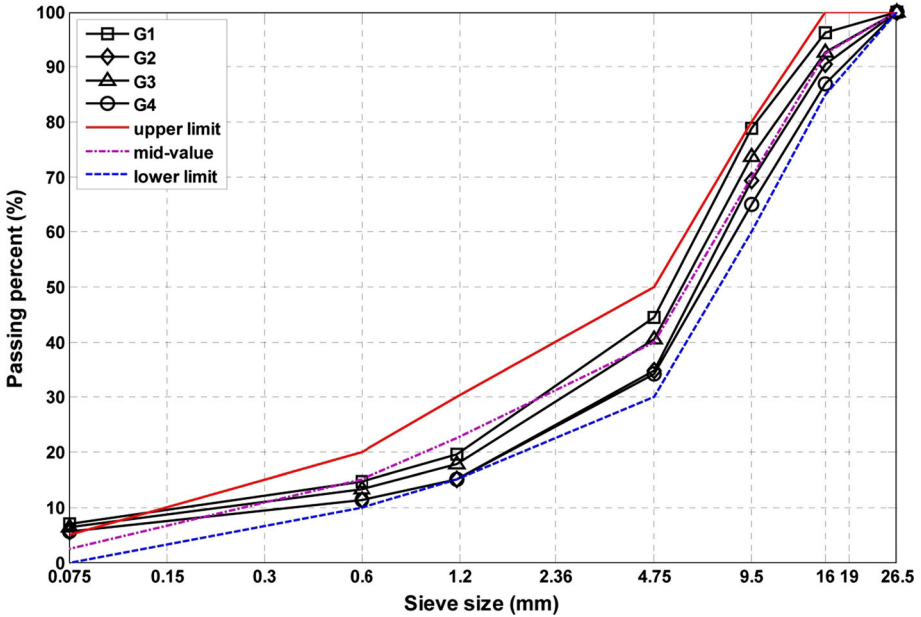


Fig. 2 Particle size distribution of graded aggregate materials

Table 1 Mix design and geometric characteristics of four kinds of the unbound graded aggregate base materials

Gradation variety	Proportion of aggregate materials (%)			$d_{30}$ (mm)	$d_{50}$ (mm)	$d_{60}$ (mm)	$C_u$	$C_c$	Maximum dry density ( $\text{g/cm}^3$ )
	0–5 mm	5–10 mm	10–20 mm						
G1	30.0	63.0	7.0	2.7	5.5	6.9	24.7	3.7	2.251
G2	20.0	63.1	16.9	3.9	6.8	8.2	17.5	3.9	2.276
G3	26.5	60.5	13.0	3.1	6.1	7.5	22.1	3.7	2.283
G4	21.0	55.5	23.5	4.0	7.2	8.8	18.4	3.8	2.255

passing percent through sieve size for UGAB according to SDHAP (*Specifications for Design of Highway Asphalt Pavement*) (MOTPRC 2006), the proportion of aggregate consist of three varieties of unbound aggregate material that are obtained through sieving test and shown in Table 1. In order to characterize the uniformity of the grain size distribution, two dimensionless coefficients  $C_u$  (uniformity coefficient) and  $C_c$  (coefficient of gradation) are calculated as follows (Yamada et al. 2005)

$$C_u = \frac{d_{60}}{d_{10}} \tag{1}$$

and

$$C_c = \frac{d_{30}^2}{d_{60}d_{10}} \tag{2}$$

where  $d_x$  [L] is the grain size of the  $x\%$  particles passing the corresponding sieve by weight. Uniform samples are characterized by values of the two coefficients close to unity. In order to investigate the porosity of each sample, it is calculated by the following equations

$$e = \frac{G_s}{\rho} - 1 \quad (3)$$

$$n = \frac{e}{e + 1} \quad (4)$$

where  $e$  [–] is porosity ratio;  $G_s$  [M/L<sup>3</sup>] is specific gravity of soil;  $\rho$  [M/L<sup>3</sup>] is the compaction density and  $n$  is porosity, [–]. Generally speaking, the investigation material is applied to construct base course in highway pavement, of which the porosity is restrict by the standard. According to Test Methods of Soils for Highway Engineering (JTG E40-2007) (MOTPRC 2007), the porosity of UGAB must be lower than 20% to ensure the strength of base course.

Subsequently, some parameters of UGAB materials are obtained through test according to TMAHE (*Test Methods of Aggregate for Highway Engineering*) (MOTPRC 2005) as shown in Table 1.

It is worth mentioning here that G1 is close to the gradation upper limit of unbound graded aggregate for constructing base of highway pavement, and G4 approximates to the gradation lower limit. G2 and G3 are located in the region of G1 and G4. It can be seen from Table 1 that the ascending order of different characteristic particle sizes ( $d_{30}$ ,  $d_{50}$  and  $d_{60}$ ) can be sorted as G1, G3, G2 and G4. It indicates different grain size distribution either.

## 2.2 Experimental Method

The most general approach in laboratory for testing the hydraulic conductivity of UGAB material is constant head permeability experiment because of its high permeability and high flow velocity (Li et al. 2008). The test equipment is TST-70 permeameter (with diameter and height of 10 and 45 cm, respectively) and its schematic diagram is shown in Fig. 3, which was filled with test material and compacted according to *Test Methods of Soils for Highway Engineering* (JTG E40-2007) (MOTPRC 2007). In this paper, the compaction degree was taken into consideration, and the four samples with different compaction degree were compacted for each graded material under condition of optimum moisture content. After compaction, the height of test sample was measured and the density after compaction can be obtained. Accordingly, the compaction degree of test material can be obtained based on the equation below

$$K = \frac{\rho}{\rho_{\max}} \times 100\% \quad (5)$$

where  $\rho$  is the compaction density [M/L<sup>3</sup>] and  $\rho_{\max}$  is the maximum dry density [M/L<sup>3</sup>];  $K$  represents compaction degree [–].

The sample of packed column was set up vertically which is shown in Fig. 3. The sample was saturated and the hydraulic gradient was obtained through adjusting the water head difference between the top and bottom of the column. The vertical length and head difference can be measured, and the hydraulic gradient can be thus calculated.

As shown in Fig. 3, the experiment setup includes water supply (Part I), water head measurement (Part II) and discharge collection (Part III). The elevation of Part III can be changed to produce various water head difference. By adjusting the height of discharging tube, hydraulic gradients ( $i$ ) were correspondingly calculated through reading the difference values ( $\Delta h_1$  and  $\Delta h_2$ ) of piezometric tubes when the flow becomes steady. Then, the hydraulic gradient can be calculated by

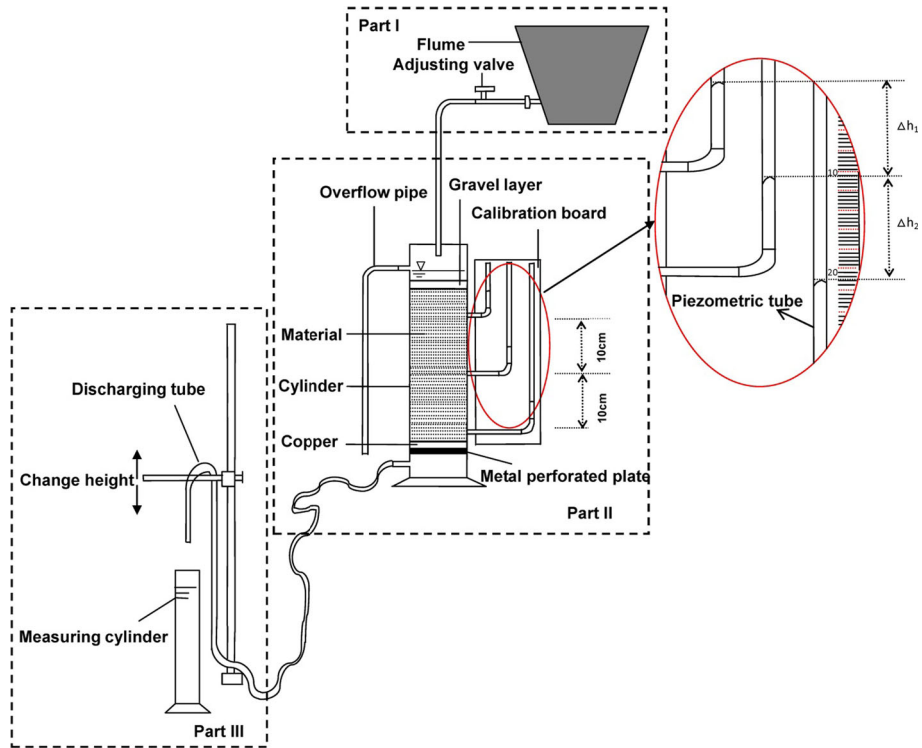


Fig. 3 Schematic diagram of experimental equipment and method

$$i = \frac{\Delta h_1 + \Delta h_2}{20 \text{ cm}} \tag{6}$$

The seepage water was collected by a cylinder (see Fig. 3). The water volume in the cylinder was monitored to determine the steady seepage discharge. For minimizing errors, seepage discharge was measured twice for each hydraulic gradient case and the mean value was adopted. The flow velocities ( $v$ ) responding to each various hydraulic gradient were calculated based on the quantity of seepage discharge. Water temperature in the experiment was measured and around 26 °C. The experimental data and relationships among the four kinds of UGAB materials under various compaction degrees were collected.

### 3 Results and Discussion

#### 3.1 Flow Regime in UGAB

The experimental data were collected under various conditions, and the Reynolds number was evaluated in order to investigate the flow regime in UGAB material.

Generally, the Reynolds number ( $Re$ ) can be defined as follows

$$Re = \frac{vd}{\nu} \tag{7}$$

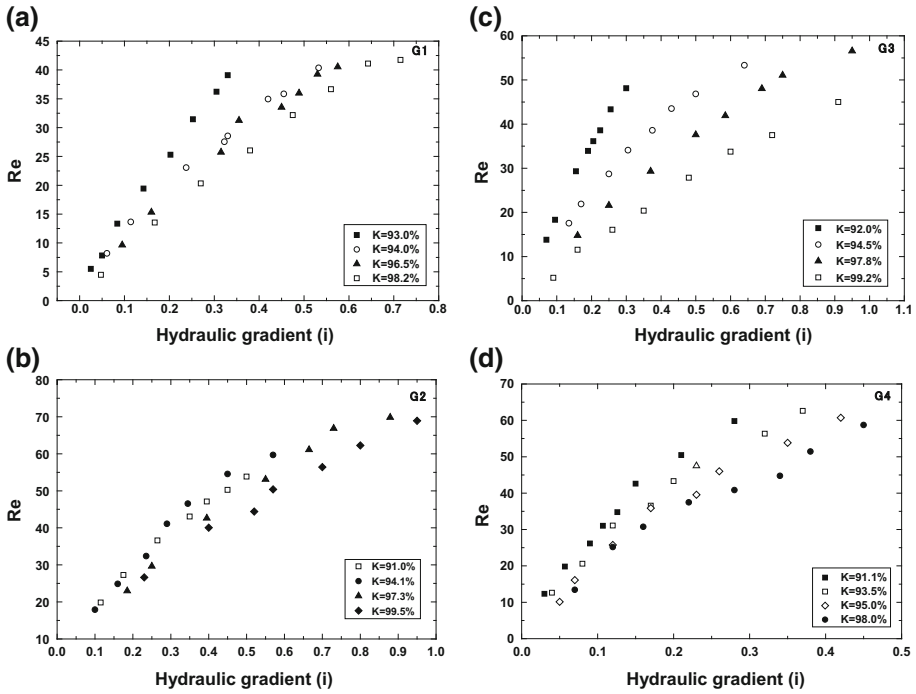


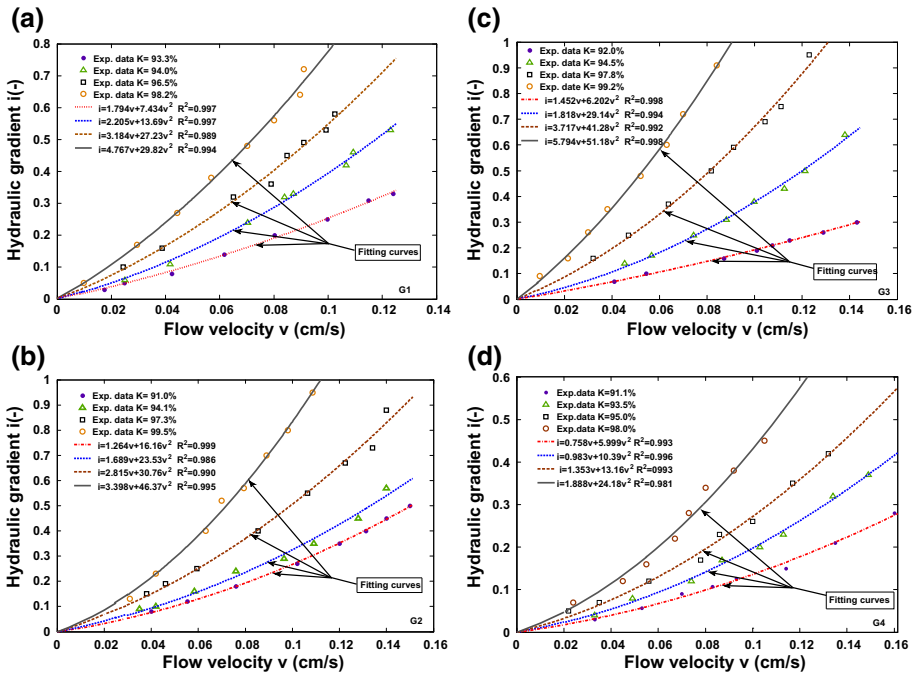
Fig. 4 Relationship between hydraulic gradient and Reynolds number for each material

where  $d$  is a characteristic length of porous media [L],  $v$  is flow velocity [L/T],  $n$  is the porosity [-] and  $\nu$  is the kinematic viscosity [L<sup>2</sup>/T]. The characteristic length is usually represented by particle diameter such as  $d_{30}$ ,  $d_{50}$ ,  $d_{60}$  (Sidiropoulo et al. 2007; Moutsopoulos et al. 2009).

In general, the maximum values of hydraulic gradient in the experimental are relatively high. Nevertheless, the hydraulic gradient can approximately reach 0.3 due to the pavement slope in highway engineering; therefore, the hydraulic gradient in experiment should not be less than 0.3. In practice, Darcy’s law is valid as long as the Reynolds number based on average grain diameter does not exceed some values between 1 and 10 (Bear 1972). In this paper, the range of Reynolds numbers have been obtained and shown in Fig. 4. It can be seen that the non-Darcian flow in our experiment is obvious even though under lower hydraulic gradient (0.1).

Additionally, in previous studies, the relationship between hydraulic gradient and flow velocity can be adopted to judge the flow is Darcian or non-Darcian. If the relationship is linear, the flow regime is available for Darcy law. If not linear, the flow is regarded as non-Darcian. Currently, many empirical equations have been summarized from experiments for the water flow behavior in porous media. Izbash equation and Forchheimer equation are well known and typical (Venkataraman and Rao 1998; Bordier and Zimmer 2000; Yamada et al. 2005; Sidiropoulo et al. 2007). Izbash equation, which is known as power law for  $i - v$  correlation, is convenience of calculation and empirical. While the quadratic relationship (Forchheimer equation) is compared to Izbash equation, it is popularly used for analysis of the flow behavior of porous media due to its clear physical meaning, and it is relatively more reasonable in many experimental studies (Ma and Ruth 1993; Sidiropoulo et al. 2007; Li et al. 2008). Therefore, the Forchheimer’s flow theory is employed in analyzing the permeability





**Fig. 5** Comparison between fitted (based on the Forchheimer equation) and measured results

of unbound graded aggregate material in this paper, and the Forchheimer formula can be expressed as follows,

$$i = av + bv^2 \tag{8}$$

where  $a$  [T/L] and  $b$  [T<sup>2</sup>/L<sup>2</sup>] are constants;  $v$  [L/T] is flow velocity and  $i$  [-] is hydraulic gradient. Researches (Ma and Ruth 1993; Sidiropoulo et al. 2007) showed that the flow velocity ( $v$ ) is small enough, and the value of  $bv^2$  term (the inertial term) can be ignored comparing the  $av$  term (viscous force term) and Forchheimer equation is yielded to the linear form of Darcy law. Apparently, the hydraulic conductivity of material can be derived by the linear relationship.

In order to judge the goodness of fitting and regression, some indexes are adopted. In general,  $R^2$  is widely used in statistics for describing the goodness of fitting and regression (Ihaka and Gentleman 1996). As more independent variables are added to the regression model, unadjusted  $R^2$  will generally increase. In this study, adjusted  $R^2$  ( $\overline{R^2}$ ) is used to discount such an effect, i.e.,

$$\overline{R^2} = 1 - (1 - R^2) \frac{n - 1}{n - p - 1} \tag{9}$$

where  $n$  is the number of predictor variables and  $p$  is the total number of observations/simulations. In comparison with unadjusted  $R^2$ , adjusted  $R^2$  can go up or down depending on whether the additional variable adds to the explanatory power of the model or not.

Accordingly, the experimental results are shown in Fig. 5 and Table 2. It can be seen from Table 2 that the correlation coefficients both  $R^2$  and  $\overline{R^2}$  of regression equations are



**Table 2** Forchheimer equation coefficients for the unbound graded aggregate material

Gradation variety	Compaction (%)	Porosity (%)	$a$	$b$	$R^2$	$\overline{R^2}$
G1	93.3	16.4	1.79	7.43	0.997	0.997
	94.0	15.8	2.21	13.69	0.997	0.996
	96.5	13.5	3.18	27.23	0.989	0.988
	98.2	12.0	4.77	29.82	0.994	0.993
G2	91.0	17.7	1.26	16.16	0.999	0.993
	94.1	14.9	1.69	23.53	0.986	0.999
	97.3	12.0	2.82	30.76	0.990	0.985
	99.5	10.0	3.40	46.73	0.995	0.989
G3	92.0	17.4	1.45	6.20	0.998	0.998
	94.5	15.2	1.82	29.14	0.994	0.993
	97.8	12.2	3.72	41.28	0.992	0.991
	99.2	10.9	5.79	51.18	0.997	0.997
G4	91.1	18.6	0.76	6.00	0.993	0.992
	93.5	16.5	0.94	10.39	0.996	0.995
	95.0	15.1	1.35	13.16	0.993	0.991
	98.0	12.4	1.89	24.18	0.981	0.978

basically  $>0.99$ , and the difference between  $R^2$  and  $\overline{R^2}$  is extremely small. That's to say, the relationship between hydraulic gradient and flow velocity of the UGAB material can be well described through the Forchheimer equation. Therefore, the  $i - v$  relationships for various UGAB materials with different compaction degree are not linear, and this kind of nonlinearity will become more obvious when the compaction degree decreases. It can be validated by the first derivation of fitting equations in Fig. 5 (the parameters of  $a$  and  $b$  grow up with the compaction degree decreases).

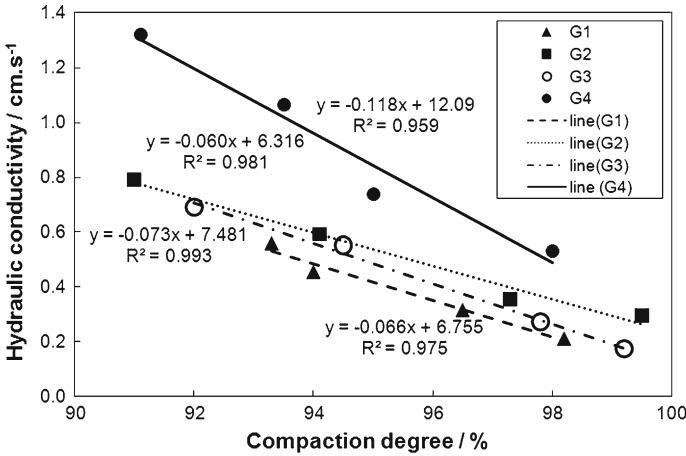
It is worthy of noting here that the maximum values of hydraulic gradient in Fig. 5d are relatively smaller than Fig. 5a–c. It is because the gradation of G4 (shown in Fig. 5d) has higher hydraulic conductivity. In the experiment, the flow velocity in the G4 case can quickly reach a high value for a relatively lower hydraulic gradient. In order to measure the flow velocity, a relatively lower hydraulic gradient is adopted.

In summary, the experimental results demonstrate that the Darcy's law cannot be used directly to determine the hydraulic conductivity of UGAB material.

### 3.2 Hydraulic Conductivity of UGAB

As we know, hydraulic conductivity is a material property which is independent of flow regime in porous media. It can be easily evaluated by Darcy equation under laminar flow condition. Forchheimer (1901) added a second order of the velocity term to represent the microscopic inertial effect, and corrected the Darcy equation into the Forchheimer equation. As shown in Eq. (8), the viscous term ( $av$ ) has been defined, and the parameter  $a$  can be evaluated by (Zeng and Grigg 2006)

$$a = \frac{\mu}{\rho g k} \quad (10)$$



**Fig. 6** Relationship between compaction degree and hydraulic conductivity

where  $k [L^2]$  is the intrinsic permeability;  $\rho [M/L^3]$  is the density of fluid and  $g [L/T^2]$  is the gravitational acceleration;  $\mu [M/(LT)]$  is the coefficient of dynamic viscosity of fluid. Therefore, hydraulic conductivity  $K_S [L/T]$  can be obtained according to the following expression (Zanker 1972)

$$K_S = \frac{\rho g k}{\mu} \tag{11}$$

Accordingly, the relationship between  $K_S$  and  $a$  can be given as follows,

$$K_S = \frac{1}{a} \tag{12}$$

Therefore, hydraulic conductivity can be calculated by using Forchheimer equation. Based on Eq. (12), the calculated results are presented in Fig. 6 and it can be seen that the hydraulic conductivity of UGAB materials falls down with the increase of compaction degree. The relationship between hydraulic conductivity and compaction degree is approximately linear. For instance, for G1 gradation, the hydraulic conductivity decreases linearly from 0.557 to 0.209 cm/s when compaction degree grows from 93.3 to 98.2%; while the hydraulic conductivity of G4 falls down sharply from 1.319 to 0.530 cm/s when compaction degree goes up from 91.1 to 98.0 %. Moreover, the hydraulic conductivities of other gradated materials (G2 and G3) are located in the regions of (0.294, 0.791 cm/s) and (0.172, 0.689 cm/s), respectively corresponding to the compaction degree regions of (91, 99.5 %) and (92, 99.2 %).

It can also be seen that Table 2 indicates both the coefficients of  $a$  and  $b$  in Forchheimer equation increase with compaction degree increases. In fact, the change of compaction degree will affect the porosity of material, and their relationship is in an inverse proportion. Similarly, Sedghi-Asl et al. (2014) found that values of  $a$  and  $b$  decrease with porosity increases for non-Darcian flow of porous media. Forchheimer (1901) summarized the nonlinear equation after experimental investigation but without discussing the influence factor for  $a$  and  $b$ . Ergun (1952) presented the form of nonlinear coefficients as

$$a = \frac{150v(1 - n)^2}{gn^3d^2}, \quad b = \frac{1.75(1 - n)}{gn^3d} \tag{13}$$

where  $n [-]$  is porosity of the porous medium,  $d [L]$  is grain diameter,  $g [L/T^2]$  is gravity acceleration, and  $\nu [L^2/T]$  is kinematic viscosity of the fluid. Kovacs (1981) obtained a similar expression and it can be expressed by

$$a = \frac{144\nu(1-n)^2}{gn^3d^2}, \quad b = \frac{2.4(1-n)}{gn^3d} \tag{14}$$

Kadlec and Knight (1996) suggested that  $a$  and  $b$  can be estimated by the following expressions respectively

$$a = \frac{255\nu(1-n)}{gn^{3.7}d^2}, \quad b = \frac{2(1-n)}{gn^3d} \tag{15}$$

From the above expressions for  $a$  and  $b$ , it can be known that values of  $a$  and  $b$  are commonly determined by grain diameter and porosity. To some extent, compaction degree has a close relationship with the porosity of UGAB material. The particle size distribution is determined by the mixture design of unbound graded aggregate material.

As mentioned before, various empirical equations for  $a$  and  $b$  have been derived (Ergun 1952; Kovacs 1981; Fand and Thinakaran 1990; Kadlec and Knight 1996; Sedghi-Asl and Rahimi 2011). However, for the UGAB material, it is difficult to evaluate the representative diameter of particles due to various gradations and the porosity is not constant but changeable. In this paper, geometric parameters of the investigated materials (see Table 1) are employed to evaluate the coefficient  $a$ . In order to select the optimized empirical formulas, the normalized objective function (NOF) criterion is applied here. The NOF is the ratio of the Root Mean Square Error to overall mean  $X$  of the experimental data, defined as (Moutsopoulos et al. 2009)

$$NOF = \frac{RMSE}{X} \tag{16}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (x_i - y_i)^2}{N}} \tag{17}$$

$$X = \frac{1}{N} \sum_{i=1}^N x_i \tag{18}$$

where  $x_i$  are the experimental values of  $a$  and  $b$  in Table 2;  $y_i$  are the values calculated by Eqs. (13)–(15);  $N$  is the total number of values. In order to obtain an excellent match, NOF should be close to 0.0. If NOF is  $< 1.0$ , it means that the theoretical method is still reliable. The calculated results are listed in Tables 3 and 4.

It can be seen from Tables 3 and 4 that the Ergun equation seems to be relatively reliable in which  $d_{50}$  is the relatively adequate value of the diameter for the estimation of the coefficient  $a$  and  $b$ . However, the NOF are larger than 1.0 (see in Tables 3, 4) in most cases and the maximum is 5.258. Therefore, the expression form presented by Ergun (1952), Kovacs (1981) and Kadlec and Knight (1996) is inappropriate for estimating hydraulic conductivity of UGAB materials.

According to above analysis, it seems adequate to use  $d_{50}$  to determine the coefficient of Forchheimer equation based on the NOF analysis. However, some researches claimed that the average diameter of particle can be used to characterize the non-uniformity of material. Therefore, in order to select the reasonable empirical equation to estimate the hydraulic conductivity of UGAB material, the form of Ergun formula with respect to porosity and particle

**Table 3** Evaluation of coefficient *a* in the Forchheimer equation through NOF analysis

Geometry parameter	NOF											
	Eq. (13) Ergun				Eq. (14) Kovacs				Eq. (15) Kadlec and Knight			
	G1	G2	G3	G4	G1	G2	G3	G4	G1	G2	G3	G4
<i>d</i> <sub>30</sub>	1.099	1.120	0.848	0.902	1.012	1.038	0.769	0.826	2.468	2.523	1.888	4.048
<i>d</i> <sub>50</sub>	0.567	0.439	0.633	0.488	0.587	0.461	0.653	0.511	3.075	5.258	3.251	3.693
<i>d</i> <sub>60</sub>	0.748	0.618	0.807	0.670	0.761	0.635	0.820	0.685	1.585	3.349	1.737	2.173

**Table 4** Evaluation of coefficient *b* in the Forchheimer equation through NOF analysis

Geometry parameter	NOF											
	Eq. (13) Ergun				Eq. (14) Kovacs				Eq. (15) Kadlec and Knight			
	G1	G2	G3	G4	G1	G2	G3	G4	G1	G2	G3	G4
<i>d</i> <sub>30</sub>	0.992	0.990	1.051	1.023	0.949	0.960	1.022	0.988	0.975	0.978	1.040	1.010
<i>d</i> <sub>50</sub>	1.051	1.025	1.090	1.066	1.031	1.008	1.075	1.046	1.044	1.019	1.084	1.058
<i>d</i> <sub>60</sub>	1.063	1.033	1.066	1.075	1.046	1.019	1.085	1.059	1.056	1.028	1.093	1.069

**Table 5** Particle distribution of the *i*th sized aggregates

<i>d<sub>i</sub></i> (mm)	<i>w<sub>i</sub></i> (%)			
	G1	G2	G3	G4
21.3	1.95	4.75	3.65	6.6
12.8	10.6	15.3	13.15	17.55
7.13	25.8	27.85	26.05	26.3
2.97	29.55	27.2	27.9	24.95
0.89	14.9	11.7	13.6	11.4
0.338	6.35	4.65	5.7	4.7
$\bar{d}$ (mm)	5.2	6.4	5.9	7.0
<i>d</i> <sub>50</sub> (mm)	5.5	6.8	6.1	7.2

size is referenced and the average diameter ( $\bar{d}$ ) and *d*<sub>50</sub> are selected as the representative particle size for estimating the coefficient of Forchheimer equation, respectively. The average diameter ( $\bar{d}$ ) can be obtained by sieve analysis (see in Fig. 2) and can be written by (Herrera and Felton 1991)

$$\bar{d} = \frac{\sum_{j=1}^n d_j \times w_j}{\sum_{j=1}^n w_j} \tag{19}$$

where *j* is *j*th sized aggregates caught in between two succeeding sieves; *d<sub>j</sub>* means average diameter of *j*th sized aggregates obtained by averaging the sieve opening of two succeeding sieves and *w<sub>j</sub>* is percentage (%) by weight of the *j*th sized aggregates. The results are shown in Table 5.

It can be seen from Table 5, the value of *d*<sub>50</sub> and  $\bar{d}$  are very close, and the *d*<sub>50</sub> is just a little bit higher than  $\bar{d}$ .

Accordingly, the expression of  $a$  and  $b$  can be, respectively, defined by the following equations

$$a = \frac{A_1 v (1 - n)^2}{g n^{A_2} d_{50}^2}, \quad b = \frac{B_1 (1 - n)}{g n^{B_2} d_{50}} \tag{20a}$$

and

$$a = \frac{C_1 v (1 - n)^2}{g n^{C_2} \bar{d}^2}, \quad b = \frac{D_1 (1 - n)}{g n^{D_2} \bar{d}} \tag{20b}$$

The constants of  $A_i, B_i, C_i$  and  $D_i$  ( $i = 1, 2$ ) in Eqs. (20a) and (20b) will be determined to generalize a uniform equation for  $a$  and  $b$  with nonlinear least square method. Furthermore, in order to highlight the difference between the calculated value and experimental data, the ratio  $\gamma$  is applied and calculated with the definition as below

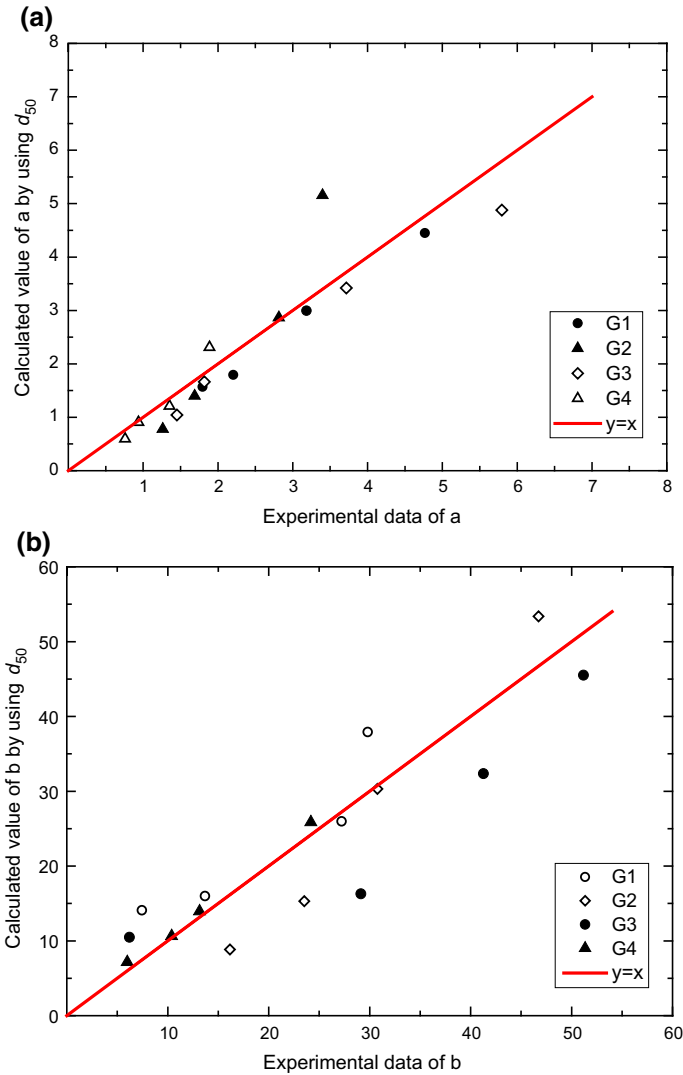
$$\gamma = y/x \tag{21}$$

where  $y$  represents calculated value and  $x$  is experimental value. The ratio of  $y$  and  $x$  should be as close as possible. When the ratio value is  $>1.0$ , it indicates the prediction equation overestimates the experimental data; while  $<1.0$  means that the prediction equation underestimates the experimental data. As shown in Figs. 7 and 8, the scattered points are closer to the regression line which indicates the prediction method being more reliable. With the method of theoretical estimation for the coefficient  $a$  and  $b$  of Forchheimer equation, it can be found that the value of  $\gamma$  are various due to different gradation varieties and compaction degrees. It can also be found in the Figs. 7 and 8 that, whichever the parameters of  $d_{50}$  and  $\bar{d}$  is taken, as a result, the estimation values of  $a$  are more closer to the regression line than that of  $b$ , especially for G1, while, for G4, the estimation values of  $b$  is better. The NOF for evaluating the value of  $b$  with using  $d_{50}$  is most located in 0.08 to 0.29 and that of  $a$  is in 0.10 to 0.40 (see in Table 6) which indicates the presented equation can be applied to approximately estimate the value of  $a$  and hydraulic conductivity as well. Overall, there is no obvious difference for NOF analysis whether  $\bar{d}$  or  $d_{50}$  is adopted, which can be shown in Table 6.

### 3.3 Effect Analysis

#### 3.3.1 Effect of Compaction Degree

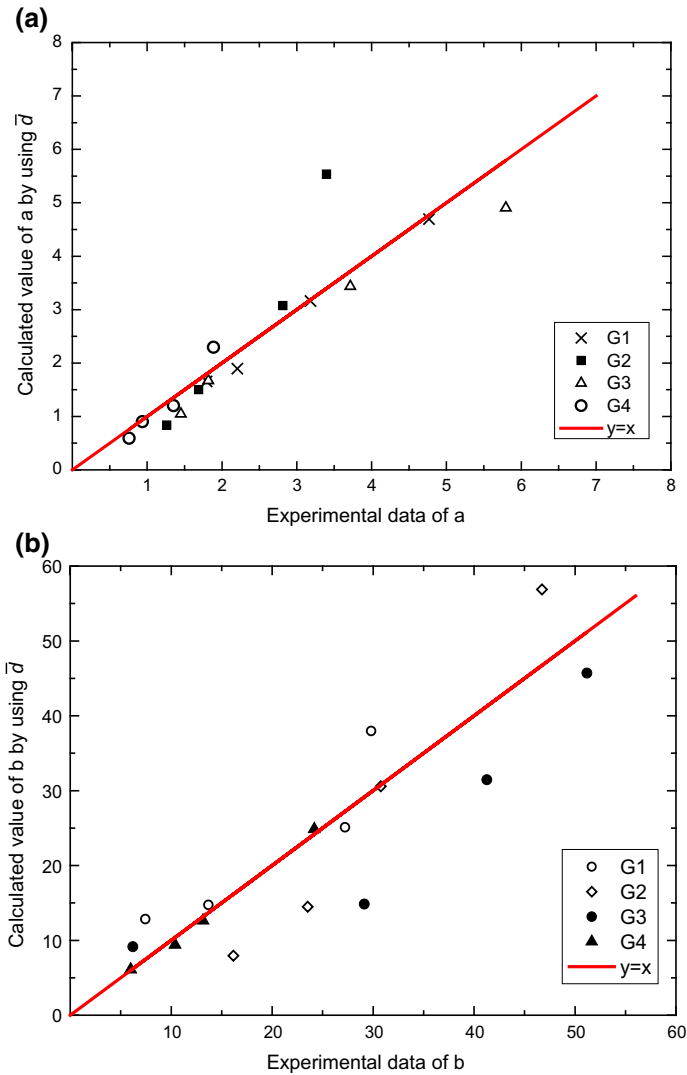
Generally, the structural strength of unbound graded aggregate material is lower than other base materials (e.g., semi-rigid material), and the coarse aggregate plays a significant role of framework and the voids are filled with fine aggregate. If the compaction degree is insufficient, especially in the heavy traffic asphalt pavement, too much residual deformation can be left on the pavement surface, and shear failure and fatigue damage may be easily happen. Therefore, compaction quality needs to be strictly controlled to ensure the strength of the graded aggregate (Li and Yan 2009). In fact, the unbound graded aggregate base material requires a high compaction degree in highway construction. Furthermore, the hydraulic conductivity is also affected by compaction degree in the reverse manner associated with porosity. For instance, at a low compaction degree level, hydraulic conductivity decreases quickly with compaction degree increase (see in Fig. 6). When compaction degree of material reaches a high value, the change of hydraulic conductivity is insignificant because skeleton of material structure is interlocked well. Under this condition, increasing compaction degree



**Fig. 7** NOF analysis of coefficients  $a$  and  $b$  when  $d_{50}$  is adopted

is hard to reduce the pore space. As discussed previously, the coefficient  $a$  of Forchheimer equation relates to evaluate the hydraulic conductivity of the unbound graded aggregate ( $K_S$ ) in Eq. (12). It can be seen from Fig. 6 the hydraulic conductivity linearly decreases with compaction degree increase for each kind of graded material. For instance, for gradation variety of G2, the hydraulic conductivity of material falls down linearly from 0.791 to 0.294 cm/s when the compaction degree increases from 91.0 to 99.5%, the correlation coefficient is 0.98.

Furthermore, it can be obtained from Fig. 6 that the hydraulic conductivities of four gradations can be evaluated for specified compaction degree. For instance, when the compaction degree is 98% for G1, G2, G3 and G4, the corresponding hydraulic conductivities are 0.29, 0.44, 0.32 and 0.53 cm/s, respectively.



**Fig. 8** NOF analysis of coefficients  $a$  and  $b$  when  $\bar{d}$  is adopted

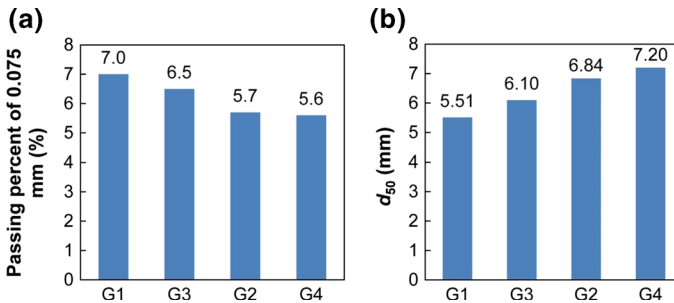
### 3.3.2 Effect of Gradation Variety

The researches of unbound graded aggregate material mainly focus on the mixture design for obtaining its optimum particle size distribution, and making the construction of the unbound graded aggregate base material with high density, high strength and appropriate permeability performance. Accordingly, it is of great importance to choose a rational mixture design (gradation variety) based on requirement in practice. The strength and stability of the unbound graded aggregate are not only influenced by the aggregates and its properties, but also the mixture design. The mixture design directly affects the passing percent of 0.075 mm and accordingly results in the difference of permeability of unbound graded aggregate base mate-



**Table 6** Constant values through NOF analysis with optimization

Parameters	Gradation variety			
	G1	G2	G3	G4
$A_1$	292			
$A_2$	3.0			
NOF <sub>a</sub>	0.10	0.40	0.16	0.19
$B_1$	40			
$B_2$	3.0			
NOF <sub>b</sub>	0.29	0.22	0.27	0.08
$C_1$	275			
$C_2$	3.0			
NOF <sub>a</sub>	0.06	0.48	0.16	0.19
$D_1$	20			
$D_2$	3.3			
NOF <sub>b</sub>	0.26	0.27	0.29	0.05

**Fig. 9** Passing percent of 0.075 mm particle and  $d_{50}$  of each unbound graded aggregate materials

rial (Li and Yan 2009). In this paper, the percent of diameter  $<0.075$  mm of designed gradated materials is shown in Fig. 9a (passing percent of 0.75 mm particles are, respectively, 7.0, 6.5, 5.7 and 5.6% for gradation variety of G1, G3, G2 and G4). It can be seen that under the same compaction degree, the sequencing of hydraulic conductivity of each unbound graded aggregate material can be sorted as  $G1 < G3 < G2 < G4$ . For instance, as shown in Fig. 6, when the compaction degree is 94.5% for G3, hydraulic conductivity coefficient is larger than G1 with compaction degree of 94%. It could be reasonably inferred that the G1 and G3 reach a same compaction level, drainage performance of G3 is better than G1 (It can be obtained from the fitting expressions in Fig. 6). Furthermore, the above order of hydraulic conductivity can be also validated through comparing the magnitude of  $d_{50}$  (take  $d_{50}$  for example) shown in Fig. 9b ( $d_{50}$  are, respectively, 5.51, 6.10, 6.84 and 7.20 mm corresponding to gradation variety of G1, G3, G2 and G4). It can be seen from Fig. 9b that the order of hydraulic conductivity is inversely proportional to  $d_{50}$  ( $d_{50}$  is adopted alternatively). Moreover, this order can be explained by the change of porosity. The voids between large size particles are most filled by diameters  $>0.075$  mm. Under the condition that compaction degree is the same, more quantity of finer particles (e.g.,  $>0.075$  mm) is contained in the unbound graded aggregate material, the pore space will be filled better and accordingly less seepage flow in unit time will be obtained; Moreover, the value of  $d_{50}$  represents an representative parameter of particle size distribution. The larger  $d_{50}$  indicates the magnitude of coarse aggregate is

higher and the content of finer particle is more, which represents the larger pore space of material. Accordingly, the hydraulic conductivity is relatively higher. Therefore,  $d_{50}$  and the content of particles  $>0.075$  mm can be the indexes for controlling the gradation design of unbound graded aggregate base material.

## 4 Conclusions

The flow behavior of the unbound graded aggregate base material is investigated through a series of experiments. The hydraulic conductivity of unbound graded aggregate base material is determined and evaluated through analytical method. Then, the gradation variety (four varieties of gradations) and compaction degree are taken into consideration. Through analysis, the conclusions can be drawn as follows.

- The relationship between hydraulic gradient and flow velocity is nonlinear and can be well described by the Forchheimer equation. It can be found that both the coefficients  $a$  and  $b$  of the Forchheimer equation grow up with the increase of compaction degree of unbound graded aggregate material.
- Based on the Ergun equation, the empirical expression of coefficient  $a$  and  $b$  in the Forchheimer equation are presented and validated through NOF analysis, which can be available to evaluate hydraulic conductivity of unbound graded aggregate base approximately.
- Through comparison analysis, the order of hydraulic conductivity of each unbound graded aggregate material is sorted as  $G1 < G3 < G2 < G4$  under the same compaction degree, which can also be sequenced by the representative particle size ( $d_{50}$  and  $\bar{d}$ ). Therefore, the gradation variety is significant for mixture design to meet the drainage capacity requirement.
- Less percentage of finer particles (e.g.,  $<0.075$  mm) and larger  $d_{50}$  ( $\bar{d}$ ) result in higher hydraulic conductivity. Therefore, the  $d_{50}$  ( $\bar{d}$ ) and the content of particles  $<0.075$  mm can be the indexes used for gradation design of unbound graded aggregate base material.

**Acknowledgments** This research has been supported by the National Natural Science Foundation (Grant No. 51248006 and 51308554), the Special Financial Grant from the China Postdoctoral Science Foundation (Grant No. 2013T60865) and the Guizhou Transportation Science and Technology Foundation (Grant No. 2013-121-013) to the corresponding author. The research is also assisted by the Guizhou Science and Technology Department Foundation (Grant No. 20132167).

## References

- Bear, J.: Dynamics of Fluids in Porous Media. Dover publications, Inc., New York (1972)
- Bordier, C., Zimmer, D.: Drainage equations and non-Darcian modeling in coarse porous media or geosynthetic materials. *J. Hydrol.* **228**(3), 174–187 (2000)
- Dan, H.C., Luo, S.P., Li, L., Zhao, L.H.: Boussinesq equation-based model for flow in drainage layer of highway. *J. Cent. South Univ.* **19**(8), 2365–2372 (2012a)
- Dan, H.C., Xin, P., Li, L., Li, L., Lockington, D.: Boussinesq equation-based model for flow in the drainage layer of highway with capillarity correction. *J. Irrig. Drain. Eng. ASCE* **138**(4), 336–348 (2012b)
- Dan, H.C., Xin, P., Li, L., Li, L., Lockington, D.: Capillary effect on flow in the drainage layer of highway pavement. *Can. J. Civ. Eng.* **39**(6), 654–666 (2012c)
- Dan, H.C., Xin, P., Li, L., Li, L.: Improved Boussinesq equation-based model for transient flow in a drainage layer of highway: capillarity correction. *J. Irrig. Drain. Eng. ASCE* **139**(12), 1018–1027 (2013)
- Dan, H.C., He, L.H., Zhao, L.H., Chen, J.Q.: Coupled hydro-mechanical response of saturated asphalt pavement under moving traffic load. *Int. J. Pavement Eng.* **16**(2), 1029–8436 (2015)

- de Vries, J.: Prediction of non-Darcian flow in porous media. *J. Irrig. Drain. Div.* **105**(2), 147–162 (1979)
- Ergun, S.: Fluid flow through packed columns. *Chem. Eng. Prog.* **48**, 89–94 (1952)
- Fand, R.M., Thinakaran, R.: The influence of the wall on flow through pipes packed with spheres. *J. Fluids Eng.* **112**(1), 84–88 (1990)
- Forchheimer, P.: *Wasserbewegung durch Boden*. Zeit. Ver. Deutsch. Ing. **45**, 1782–1788 (1901)
- Ferdos, F., Wörman, A., Ekström, I.: Hydraulic conductivity of coarse rockfill used in hydraulic structures. *Transp. Porous Media* **108**(2), 367–391 (2015)
- Haider, I., Kaya, Z., Cetin, A., et al.: Drainage and mechanical behavior of highway base materials. *J. Irrig. Drain. Eng. ASCE* **140**(6), 195–202 (2014)
- Herrera, N.M., Felton, G.K.: Hydraulics of flow through a rockfill dam using sediment free water. *Trans. ASAE* **34**(3), 871–875 (1991)
- Ihaka, R., Gentleman, R.: R: a language for data analysis and graphics. *J. Comput. Graph. Stat.* **5**(3), 299–314 (1996)
- Izbash, S.O., Filtracii, V.: *Kropnozernstom Materiale*. USSR, Leningrad (1931)
- Ji, Q.K.: Indoor measurement of porous material permeability coefficient. *J. Highw. Transp. Res. Dev.* **19**(2), 31–34 (2002)
- Kadlec, H.R., Knight, L.R.: *Treatment Wetlands*. Lewis Publishers, Boca Raton (1996)
- Kovacs, G.: *Seepage Hydraulics, Developments in Water Science*. Elsevier, Amsterdam (1981)
- Lebeau, M., Konrad, J.: Pavement subsurface drainage: importance of appropriate subbase materials. *Can. Geotech. J.* **46**(8), 987–1000 (2009)
- Li, F.P., Yan, E.H.: *The Applied Technology Guide of Structure Design and Construction for Asphalt Pavements with Asphalt Treated Base and Dense Graded Aggregate Base*. China Communications Press, Beijing (2009)
- Li, J., Huang, G.H., Wen, Z., Zhang, H.B.: Experimental study on non-Darcian flow in two kinds of media with different diameters. *J. Hydraul. Eng.* **39**(6), 726–732 (2008)
- Li, W., Zheng, N.X., Fu, H.W.: Study of drainage performance of graded crushed stone and base drainage system. *J. Highw. Transp. Res. Dev.* **27**(10), 11–16 (2010)
- Ma, H., Ruth, D.W.: The microscopic analysis of high Forchheimer number flow in porous media. *Transp. Porous Media* **13**(2), 139–160 (1993)
- Moutsopoulos, K.N., Papaspyros, I.N.E., Tsihrintzis, V.A.: Experimental investigation of inertial flow processes in porous media. *J. Hydrol.* **374**(3–4), 242–254 (2009)
- MOTPRC: *Test Methods of Aggregate for Highway Engineering (JTG E42-2005)*. China Communications Press, Beijing (2005)
- MOTPRC: *Specifications for Design of Highway Asphalt Pavements (JTG D50-2006)*. China Communications Press, Beijing (2006)
- MOTPRC: *Test Methods of Soils for Highway Engineering (JTG E40-2007)*. China Communications Press, Beijing (2007)
- Salahi, M.B., Sedghi-Asl, M., Parvizi, M.: Nonlinear flow through a packed column experiment. *J. Hydrol. Eng.* (2015). doi:[10.1061/\(ASCE\)HE.1943-5584.0001166](https://doi.org/10.1061/(ASCE)HE.1943-5584.0001166)
- Sedghi-Asl, M., Rahimi, H.: Adoption of Manning's equation to 1D non-Darcy flow problems. *J. Hydraul. Res.* **49**(6), 814–817 (2011)
- Sedghi-Asl, M., Rahimi, H., Salehi, R.: Non-Darcian flow of water through a packed column test. *Transp. Porous Media* **101**(2), 215–227 (2014)
- Sidiropoulou, M.G., Moutsopoulos, K.N., Tsihrintzis, V.A.: Determination of Forchheimer equation coefficients a and b. *Hydrol. Process.* **21**(4), 534–554 (2007)
- Soni, J.P., Islam, N., Basak, P.: An experimental evaluation of non-Darcian flow in porous media. *J. Hydrol.* **38**(3), 231–241 (1978)
- Tek, M.R.: Development of a generalized Darcy equation. *J. Pet. Technol.* **9**(6), 376–377 (1957)
- Venkataraman, P., Rao, P.R.M.: Darcian, transitional, and turbulent flow through porous media. *J. Hydraul. Eng. ASCE* **124**(8), 840–846 (1998)
- Wright, D.E.: Nonlinear flow through granular media. *J. Hydraul. Div.* **94**(9), 851–872 (1968)
- Yamada, H., Nakamura, F., Watanabe, Y., Murakami, M., Nogami T.: Measuring hydraulic permeability in a stream bed using the packer test. *Hydrol. Process.* **19**(13), 2507–2524 (2005)
- Zanker, A.: Nomograph for hydraulic conductivity and intrinsic permeability of water–soil system. *Soil Sci.* **113**(5), 375–377 (1972)
- Zeng, Z., Grigg, R.: A criterion for non-Darcy flow in porous media. *Transp. Porous Media* **63**(1), 57–69 (2006)