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The Effect of Pulsating Throughflow on the Onset of Convection in a Horizontal Porous Layer

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Abstract In this paper, we present an analytical study on the onset of convection in a horizontal layer of a saturated porous medium, uniformly heated from below but with a non-uniform basic temperature gradient resulting from a pulsating vertical throughflow. We applied two methods to analyze this problem: the frozen profile and the averaged equations. We found that the two approaches lead to essentially the same results and discussed physical implications of this finding, in particular, with respect to fundamental differences between the throughflow modulation and the temperature modulation problems.

Keywords Pulsating throughflow · Porous medium · Thermal instability · Horizontal layer

Nomenclature

a	Dimensionless horizontal wavenumber
c_a	Acceleration coefficient
D	$\mathrm{d}/\mathrm{d}z$
$f(z)$	Function characterizing the basic temperature gradient, defined by Eq. (28)
g	Gravity
$\mathbf{g}$	Gravitational vector
H	Dimensional layer depth
k_m	Effective thermal conductivity of the porous medium
K	Permeability of the porous medium
P^*	Pressure, excess over hydrostatic
P	Dimensionless pressure, $P^* K/(\mu\alpha_m)$

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Q	Péclet number defined by Eq. (13a)
Ra	Rayleigh–Darcy number defined by Eq. (6)
t^*	time
t	Dimensionless time, $t^*\alpha_m/(\sigma H^2)$
T^*	Temperature
T	Dimensionless temperature, $(T^* - T_0)/(T_1 - T_0)$
T_0	Temperature at the upper wall
T_1	Temperature at the lower wall
(u, v, w)	Dimensionless Darcy velocity components, $(u^*, v^*, w^*)H/\alpha_m$
V_0	Mean throughflow velocity
$\mathbf{v}$	Dimensionless Darcy velocity, $\frac{(\rho c)_f H}{k_m}\mathbf{v}^*$
$\mathbf{v}^*$	Dimensional Darcy velocity, (u^*, v^*, w^*)
(x, y, z)	Dimensionless Cartesian coordinates, $(x^*, y^*, z^*)/H$; z is the vertically upward coordinate
(x^*, y^*, z^*)	Cartesian coordinates

Greek symbols

α_m	Thermal diffusivity of the porous medium, $\frac{k_m}{(\rho c)_f}$
β	Volumetric expansion coefficient of the fluid
γ_a	Acceleration coefficient defined by Eq. (11)
εV_0	Amplitude of velocity pulsations
μ	Viscosity of the fluid
ρ	Fluid density
ρ_0	Fluid density at temperature T_0
$(\rho c)_f$	Heat capacity of the fluid
$(\rho c)_m$	Effective heat capacity of the porous medium
σ	Thermal capacity ratio defined by Eq. (7)
Ω	Angular frequency
ω	Dimensionless angular frequency defined by Eq. (13b)

Superscripts

*	Dimensional variable
′	Perturbation variable

Subscripts

b	Basic solution

1 Introduction

There is a substantial literature on the effects of thermal modulation (whether of temperature or heat flux) or gravity modulation (vertical vibration, g-jitter) on the onset of convection in a horizontal fluid-saturated porous layer that is heated from below. This literature is surveyed in Sections 6.11.3 and 6.24 of the book by Nield and Bejan (2013). More detailed reviews have been made by Rees et al. (2008), Govender (2008), Pedram Razi et al. (2005), and Pedramrazi et al. (2008).

However, we are not aware of any similar study of the effect of modulation of the magnitude of vertical throughflow. In the present paper we investigate the situation where the throughflow pulsates with time about a mean that can be of either sign or zero. The new problem is more complicated than the old ones. Whereas the temperature modulation and the gravity modulation lead directly to an oscillation of the effective buoyancy force, the throughflow acts indirectly, and a convection problem is involved from the outset. With the new problem the oscillation enters in a nonlinear manner in the general case, and this potentially provides both difficulty and interest.

Such problems involving modulation lead to a differential equation system in which a coefficient is periodic in time. The system is commonly studied using one of three approaches: Floquet theory, averaged equations, or frozen profile. The first two approaches are appropriate for the case of high frequency and small amplitude. The third approach, in which it is assumed that perturbations grow quickly in comparison with how quickly the throughflow velocity changes, is more flexible.

For the gravity modulation problem, the time-averaged method has been extensively used (see Pedramrazi et al. 2008). However, as Govender (2008) pointed out, subharmonic modes are overlooked when this method is used.

For the temperature modulation problem, Chhuon and Caltagirone (1979) compared (i) results obtained with Floquet theory, (ii) results obtained with frozen profile approach, and (iii) experimental results. They found that the frozen profile approach gave a better fit with experiments than did the Floquet theory (see Figure 6.13 of Nield and Bejan 2013). (A reviewer commented that perhaps the conclusions of Chhuon and Caltagirone (1979) are correct, given their experiments, but the fact remains that, given Darcy's law, the method of averaging yields approximate results, while Floquet theory is an exact theory. Both may, of course, be subject to numerical errors.)

In the present paper we first follow the frozen profile approach. This is relatively simple and direct, and it is pertinent to the low-frequency case that is applicable to hydrological situations. We have in mind convection driven by a salinity gradient in the bed of a tidal basin.

Then we explore the consequences of adopting the averaged equations approach.

2 Analysis

2.1 Basic Equations

Single-phase flow in a saturated porous medium of permeability K is considered. Asterisks are used to denote dimensional variables. We consider a horizontal layer occupying $0 \leq z^* \leq H$, where the z^*-axis is in the upward vertical direction. Uniform temperatures T_0 and T_1 are imposed at the upper and lower boundaries, respectively. Consistent with this assumption we suppose that there is a uniform basic flow with velocity V_0 in the z-direction.

The Darcy velocity is denoted by $\mathbf{v}^* = (u^*, v^*, w^*)$. The Oberbeck-Boussinesq approximation is invoked and local thermal equilibrium is assumed. The equations representing the conservation of mass, Darcy's law, and conservation of thermal energy take the form (see, for example, Eqs. (6.3)–(6.6) of Nield and Bejan 2013):

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{1}$$

$$c_a \rho_0 \frac{\partial \mathbf{v}^*}{\partial t^*} = -\nabla^* P^* - \frac{\mu}{K}\mathbf{v}^* - \rho_0 g[1 - \beta(T^* - T_0)]\mathbf{e}_z, \tag{2}$$

$$(\rho c)_m \frac{\partial T^*}{\partial t^*} + (\rho c)_f \mathbf{v}^* \cdot \nabla^* T^* = k_m \nabla^2 T^*. \tag{3}$$

Here $(\rho c)_m$ and $(\rho c)_f$ are the heat capacities of the overall porous medium and the fluid, respectively, μ is the fluid viscosity, g is gravity, K is the permeability, k_m is the effective thermal conductivity of the porous medium, c_a is the acceleration coefficient, ρ_0 is the fluid density at temperature T_0, $\mathbf{v}^*$ is the Darcy velocity, T^* is the temperature, t^* is the time, and β is the volumetric expansion coefficient of the fluid, while P^* is the excess of pressure over the reference hydrostatic value.

We assume that there is upward throughflow with constant mean value V_0, amplitude εV_0 and angular frequency Ω, so that

$$\mathbf{v}^* = V_0\left(1 + \varepsilon \cos \Omega t^*\right). \tag{4}$$

We introduce dimensionless variables by defining

$$\mathbf{x} = \frac{\mathbf{x}^*}{H}, \quad \mathbf{v} = \frac{(\rho c)_f H}{k_m}\mathbf{v}^*, \quad t = \frac{k_m}{(\rho c)_m H^2} t^*,$$

$$T = \frac{T^* - T_0}{T_1 - T_0}, \quad P = \frac{(\rho c)_f K}{\mu k_m} P^*. \tag{5a,b,c,d,e}$$

We also define a Rayleigh–Darcy number (shortened to Rayleigh number in what follows) Ra by

$$\mathrm{Ra} = \frac{(\rho c)_f \rho_0 g \beta K H (T_1 - T_0)}{\mu k_m} \tag{6}$$

and the heat capacity ratio

$$\sigma = \frac{(\rho c)_m}{(\rho c)_f}. \tag{7}$$

The governing equations then take the form

$$\nabla \cdot \mathbf{v} = 0, \tag{8}$$

$$\gamma_a \frac{\partial \mathbf{v}}{\partial t} = -\nabla P - \mathbf{v} + \mathrm{Ra}\left[T - \frac{1}{\beta(T_1 - T_0)}\right]\mathbf{e}_z, \tag{9}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T, \tag{10}$$

where the dimensionless acceleration coefficient is defined by

$$\gamma_a = \frac{c_a \rho_0 k_m K}{\sigma \mu (\rho c)_f H^2}. \tag{11}$$

We also have the basic flow $\mathbf{v}_b$, P_b, T_b, where

$$\mathbf{v}_b = Q\,[1 + \varepsilon \cos \omega t]\,\mathbf{e}_z. \tag{12}$$

Thus ε is the dimensionless amplitude of the pulsation, while Q is a Péclet number and ω is a dimensionless angular frequency defined as

$$Q = \frac{(\rho c)_f H V_0}{k_m}, \quad \omega = \frac{(\rho c)_m H^2 \Omega}{k_m}. \tag{13a,b}$$

Then Eqs. (9) and (10) give

$$\nabla P_b + \varepsilon \gamma_a Q \omega \sin(\omega t) \mathbf{e}_z = -Q(1 + \varepsilon \cos \omega t)\mathbf{e}_z + \mathrm{Ra}\left[T_b - \frac{1}{\beta(T_1 - T_0)}\right]\mathbf{e}_z, \tag{14}$$

$$Q\,[1 + \varepsilon \cos \omega t]\,\frac{dT_b}{dz} + \varepsilon \gamma_a Q \omega \sin(\omega t) T_b = \frac{d^2 T_b}{dz^2}. \tag{15}$$

Equation (15) can now be solved subject to appropriate boundary conditions, which here take the form

$$T_b = 1 \text{ at } z = 0,\ T_b = 0 \text{ at } z = 1. \tag{16}$$

At this stage we assume that $\varepsilon \gamma_a \omega$ is small compared with unity. For a regular porous medium (one whose Darcy number K/H^2 is small), the value of γ_a will be small, and hence this is then a relatively weak constraint on the magnitude of the frequency ω.

The solution is then

$$T_b = \frac{\exp[Q(1 + \varepsilon \cos \omega t)] - \exp[Q(1 + \varepsilon \cos \omega t) z]}{\exp[Q(1 + \varepsilon \cos \omega t)] - 1}. \tag{17}$$

Then Eq. (14) can be solved to give P_b if that is required.

2.2 Perturbation Analysis, Frozen Profile

We now involve the frozen profile assumption. In the expressions for the basic solution (Eqs. (12) and (17)) we write t_0 for t. For shorthand we introduce

$$\hat{Q} = Q(1 + \varepsilon \cos \omega t_0). \tag{18}$$

We now perturb this basic solution and write

$$\mathbf{v} = \mathbf{v}_b + \mathbf{v}',\ P = P_b + P',\ T = T_b + T', \tag{19}$$

where the primed quantities are functions of $\mathbf{x}$ and t, so that, on linearizing the equations, we have

$$\nabla \cdot \mathbf{v}' = 0, \tag{20}$$

$$\gamma_a \frac{\partial \mathbf{v}'}{\partial t} = -\nabla P' - \mathbf{v}' + \mathrm{Ra} T' \mathbf{e}_z, \tag{21}$$

$$\frac{\partial T'}{\partial t} + \mathbf{v}' \cdot \nabla T_b + \mathbf{v}_b \cdot \nabla T' = \nabla^2 T'. \tag{22}$$

Operating on Eq. (21) with $\mathbf{e}_z \cdot$ curl curl and using Eq. (20) we get

$$\left(1 + \gamma_a \frac{\partial}{\partial t}\right) \nabla^2 w' = \mathrm{Ra} \nabla_H^2 T', \tag{23}$$

where ∇_H^2 denotes the horizontal Laplacian operator. In terms of normal modes, one can write

$$(w', T') = [W(z), \Theta(z)] \exp(st + ilx + imy), \tag{24}$$

which can be substituted into Eqs. (22) and (23), to obtain

$$(1 + \gamma_a s)\left(D^2 - a^2\right) W = -a^2 \mathrm{Ra} \Theta, \tag{25}$$

$$-fW = \left(D^2 - a^2 - \hat{Q}D - s\right)\Theta, \tag{26}$$

where

$$a = \left(l^2 + m^2\right)^{1/2}, D \equiv \frac{\mathrm{d}}{\mathrm{d}z}, \quad f(z) = -\frac{\mathrm{d}T_b}{\mathrm{d}z}. \tag{27}$$

In the special problem that is investigated we have

$$f(z) = \frac{\hat{Q}\mathrm{e}^{\hat{Q}z}}{\mathrm{e}^{\hat{Q}} - 1}. \tag{28}$$

Equations (25) and (26) constitute a pair of coupled ordinary differential equations which can then be solved subject to appropriate boundary conditions on W and Θ. In the case of impermeable constant-temperature boundaries, one has

$$W = 0, \Theta = 0 \text{ at } z = 0, 1. \tag{29}$$

If the temperature gradient has constant sign, then oscillatory disturbances are ruled out and one can take $s = 0$.

The differential equation system consisting of Eqs. (25), (26) and (29) then constitutes an eigenvalue problem in which Ra can be regarded as the eigenvalue.

For the case of small Q an approximate expression, useful for an investigation of trends as parameters are varied in a new situation, for the value of Ra can be found by using a single-term Galerkin expansion. We write $W = AW_1$, $\Theta = B\Theta_1$, with constants A and B, where W_1 and Θ_1 are trial functions satisfying the boundary conditions, and substitute into Eqs. (25) and (26) to obtain two residuals. These can then be made orthogonal to W_1 and Θ_1, respectively, to give two equations from which the ratio B/A can be eliminated. This results in the equation

$$\mathrm{Ra} = \frac{\langle W_1(D^2 - a^2)W_1\rangle \left\{\langle \Theta_1(D^2 - a^2)\Theta_1\rangle - Q\langle \Theta_1 D\Theta_1\rangle\right\}}{a^2\langle W_1\Theta_1\rangle\langle fW_1\Theta_1\rangle} \tag{30}$$

where $\langle(\cdot)\rangle \equiv \int_0^1 (\cdot)\mathrm{d}z$.

2.3 Time—Averaged Formulation

We now return to Eqs. (1)–(3), concentrating on the case of high frequency and small amplitude pulsation, and we follow the presentation of Pedram Razi et al. (2005). Under these conditions two time scales are pertinent, and we divide the fields into two parts. For the first part (slow time) the characteristic time is large compared with the pulsation period. The second part (fast time) varies rapidly with time and is periodic with period $\tau = 2\pi/\omega$. Thus we write

$$\begin{aligned} \mathbf{v}^*(M, t^*) &= \bar{\mathbf{v}}^*(M, t^*) + \mathbf{v}'^*(M, \omega t^*), \\ T^*(M, t^*) &= \bar{T}^*(M, t^*) + T'^*(M, \omega t^*), \\ P^*(M, t^*) &= \bar{P}^*(M, t^*) + P'^*(M, \omega t^*). \end{aligned} \tag{31}$$

Here the average of a given function $f(M, t^*)$ is defined as

$$\bar{f}(M, t^*) = \frac{1}{\tau}\int_{t^*-\tau/2}^{t^*+\tau/2} f(M, s^*)\mathrm{d}s^*. \tag{32}$$

Here M denotes the spatial domain of interest.

Substitution into Eqs. (1)–(3) gives rise to two coupled systems of equations. First we have the mean flow equations

$$\nabla^* \cdot \bar{\mathbf{v}}^* = 0, \tag{33}$$

$$c_a \rho_0 \frac{\partial \bar{\mathbf{v}}^*}{\partial t^*} = -\nabla^* \bar{P}^* - \frac{\mu}{K} \bar{\mathbf{v}}^* - \rho_0 g [1 - \beta(\bar{T}^* - T_0)] \mathbf{e}_z, \tag{34}$$

$$(\rho c)_m \frac{\partial \bar{T}^*}{\partial t^*} + (\rho c)_f \bar{\mathbf{v}}^* \cdot \nabla^* \bar{T}^* + (\rho c)_f \overline{\mathbf{v}'^* \cdot \nabla^* T'^*} = k_m \nabla^{*2} \bar{T}^*. \tag{35}$$

Then we have equations for the oscillatory part of the flow

$$\nabla^* \cdot \mathbf{v}'^* = 0, \tag{36}$$

$$c_a \rho_0 \frac{\partial \mathbf{v}'^*}{\partial t^*} = -\nabla^* P'^* - \frac{\mu}{K} \mathbf{v}'^* + \rho_0 g \beta T'^* \mathbf{e}_z, \tag{37}$$

$$\begin{aligned}(\rho c)_m \frac{\partial T'^*}{\partial t^*} &+ (\rho c)_f \mathbf{v}'^* \cdot \nabla^* \bar{T}^* + (\rho c)_f \bar{\mathbf{v}}^* \cdot \nabla^* T'^* + (\rho c)_f \mathbf{v}'^* \cdot \nabla^* T'^* \\ &- (\rho c)_f \overline{\mathbf{v}'^* \cdot \nabla^* T'^*} = k_m \nabla^{*2} T'^*.\end{aligned} \tag{38}$$

We now need to apply some scale analysis to obtain a closed set of equations for the time-averaged fields. Our starting assumption, based on Eq. (4), is that

$$\mathrm{O}(\mathbf{v}'^*) \approx \varepsilon V_0, \quad \mathrm{O}\left(\frac{\partial \mathbf{v}'^*}{\partial t^*}\right) \approx \varepsilon \Omega V_0. \tag{39}$$

First we look at the oscillatory momentum equation. We make the assumption, appropriate for a layer of infinite horizontal extent, that

$$\mathrm{O}(\bar{T}^* - T_0) \approx T_1 - T_0 \equiv \Delta T, \quad \mathrm{O}\left(\frac{\partial(\cdot)}{\partial t^*}\right) \approx \Omega(\cdot), \quad \mathrm{O}\left(\frac{\partial(\cdot)}{\partial z^*}\right) \approx \frac{1}{H}(\cdot). \tag{40}$$

We also assume that $T'^* \ll \Delta T$. Then the orders of magnitudes of the relevant terms are as follows:

$$\text{Inertia:} \quad \mathrm{O}\left(c_a \rho_0 \frac{\partial \mathbf{v}'^*}{\partial t^*}\right) \approx c_a \rho_0 \varepsilon \Omega V_0, \tag{41a}$$

$$\text{Buoyancy:} \quad \mathrm{O}\left(\rho_0 \beta T'\right) \approx \rho_0 \beta \Delta T, \tag{41b}$$

$$\text{Friction:} \quad \mathrm{O}\left(\frac{\mu}{K} \mathbf{v}'^*\right) \approx \left(\frac{\mu}{K} \varepsilon V_0\right). \tag{41c}$$

We assume that the inertial and buoyancy terms are in balance, so that

$$c_a \varepsilon \Omega V_0 \approx \beta \Delta T. \tag{42}$$

We also assume a high pulsation frequency so that the pulsation time scale, $1/\Omega$, is small in comparison with the hydrodynamic time-scale, $c_a \rho_0 K/\mu$, and that means that we can neglect the viscous friction term in Eq. (37).

In a similar manner we look at the terms in the oscillatory thermal energy equation. The relevant terms have the following orders of magnitude:

$$\text{Transient:} \quad \mathrm{O}\left((\rho c)_m \frac{\partial T'^*}{\partial t^*}\right) \approx (\rho c)_m T'^* \Omega, \tag{43a}$$

$$\text{Convective:} \quad \mathrm{O}\left((\rho c)_f \mathbf{v}'^* \cdot \nabla^* \bar{T}\right) \approx (\rho c)_f \varepsilon V_0 \frac{\Delta T}{H}, \tag{43b}$$

$$\text{Diffusive:} \quad \mathrm{O}\left(k_m \nabla^{*2} T'^*\right) \approx k_m \frac{T'^*}{H^2}. \tag{43c}$$

Now we assume that the diffusive term is small in comparison with the transient term. This is so if the pulsation time scale $1/\Omega$ is small in comparison with the diffusive time scale $(\rho c)_m H^2/k_m$. The transient and convective terms are then in balance if

$$\sigma T'^* \Omega \approx \varepsilon V_0 \frac{\Delta T}{H}. \tag{44}$$

Since we are also assuming that $T'^* \ll \Delta T$, Eq. (44) is a restraint on the magnitude of the pulsation amplitude,

$$\varepsilon \ll \frac{\sigma H \Omega}{V_0}. \tag{45}$$

We now apply the results of this scale analysis to the system of Eqs. (33)–(38). We need to relate the oscillatory fields to the mean flow fields. In the present problem, the mean flow equation of continuity and the momentum equation (Eqs. (33) and (34)) do not require modification and that leaves the thermal energy equation (35) to be modified. We are controlling the oscillatory velocity field and so we already know that

$$\bar{\mathbf{v}}^* = V_0 \mathbf{e}_z, \quad \mathbf{v}'^* = \varepsilon V_0 \cos(\Omega t^*) \mathbf{e}_z. \tag{46a,b}$$

We need information about the oscillatory temperature field, T'^*. In accordance with the scale analysis, we can neglect the term on the right-hand side of Eq. (38), which then gives

$$(\rho c)_f \overline{\mathbf{v}'^* \cdot \nabla^* T'^*} = (\rho c)_m \frac{\partial T'^*}{\partial t^*} + (\rho c)_f \varepsilon V_0 \cos \Omega t^* \frac{\partial \bar{T}^*}{\partial z^*} + (\rho c)_f V_0 \left(1 + \varepsilon \cos \Omega t^*\right) \frac{\partial T'^*}{\partial z^*}. \tag{47}$$

Thus Eq. (35) becomes

$$(\rho c)_m \frac{\partial \bar{T}^*}{\partial t^*} + (\rho c)_m \frac{\partial T'^*}{\partial t^*} + (\rho c)_f \bar{\mathbf{v}}^* \cdot \nabla^* \bar{T}^*$$
$$+ (\rho c)_f \varepsilon V_0 \cos \Omega t^* \frac{\partial \bar{T}^*}{\partial z^*} + (\rho c)_f V_0 \left(1 + \varepsilon \cos \Omega t^*\right) \frac{\partial T'^*}{\partial z^*} = k_m \nabla^{*2} \bar{T}.^* \tag{48}$$

Finally we neglect the terms in T'^* to obtain the modified averaged thermal energy equation

$$(\rho c)_m \frac{\partial \bar{T}^*}{\partial t^*} + (\rho c)_f \bar{\mathbf{v}}^* \cdot \nabla^* \bar{T}^* + (\rho c)_f \varepsilon V_0 \cos \Omega t^* \frac{\partial \bar{T}^*}{\partial z^*} = k_m \nabla^{*2} \bar{T}^*. \tag{49}$$

In terms of dimensionless quantities defined in Eq. (5), the time-averaged equations are

$$\nabla \cdot \bar{\mathbf{v}} = 0, \tag{50}$$

$$\gamma_a \frac{\partial \bar{\mathbf{v}}}{\partial t} = -\nabla \bar{P} - \bar{\mathbf{v}} + \mathrm{Ra}\left[\bar{T} - \frac{1}{\beta(T_1 - T_0)}\right] \mathbf{e}_z, \tag{51}$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{\mathbf{v}} \cdot \nabla \bar{T} + Q\varepsilon \cos \omega t \frac{\partial \bar{T}}{\partial z} = \nabla^2 \bar{T}. \tag{52}$$

We see that, when the substitution from Eq. (12) is made, Eqs. (51) and (52) are essentially the same as Eqs. (14) and (15). This means that the basic temperature field is not affected by the type of approach. This implies in turn that the linear stability problem is not affected by the type of approach, and thus the utilization of averaged equations gives essentially the same result as the utilization of the frozen profile approach. Thus the situation for a pulsating velocity field is dramatically different from that for a pulsating gravity field. This feature could have been anticipated. For the former situation there is nothing analogous to the thermo-vibration effect that arises with the latter, where the mean and oscillatory contributions to buoyancy interact directly. The throughflow modifies just the basic temperature gradient and does not itself provide any destabilizing agency. The throughflow modulation problem is also fundamentally different from the temperature modulation problem. For the latter, the modulation of the temperature difference applied at the boundaries produces a change in buoyancy force that is felt throughout the layer. For the former, the modulation of throughflow merely modifies the distribution of buoyancy force within the layer.

3 Results

For the case of constant-temperature boundaries one can take $W_1 = \sin \pi z$, $\Theta_1 = \sin \pi z$. This choice leads to

$$\mathrm{Ra} = \frac{(\pi^2 + a^2)^2}{a^2} \frac{< \sin^2 \pi z >}{< f \sin^2 \pi z >}. \tag{53}$$

In the absence of throughflow one has $f = 1$ and

$$\mathrm{Ra} = \frac{(\pi^2 + a^2)^2}{a^2}. \tag{54}$$

As the wavenumber a varies this takes the minimum value $4\pi^2$ when $a = \pi$. These are the familiar exact values of the critical Rayleigh number and the corresponding critical wavenumber in this case.

In the presence of a small amount of throughflow, one has

$$< f \sin^2 \pi z >= \frac{2\pi^2}{\hat{Q}^2 + 4\pi^2} \tag{55}$$

and so

$$\mathrm{Ra} = \frac{(\pi^2 + a^2)^2}{a^2} \left[1 + \frac{\hat{Q}^2}{4\pi^2} \right]. \tag{56}$$

In this case the critical Rayleigh number is

$$\mathrm{Ra}_c = 4\pi^2 + \hat{Q}^2. \tag{57}$$

This approximate formula gives a value accurate to 1 % when $\hat{Q} = 1$. (Eq. (57) gives 40.784 whereas a more exact value, obtained by methodology described by Barletta et al. (2016), is 40.8751). We infer that the single-term Galerkin approximation is satisfactory for this case when $\hat{Q}$ is less than unity.

As one would expect from the symmetry of the problem, this result does not depend on the sign of $\hat{Q}$. The effect of throughflow in either vertical direction is stabilizing.

As t_0 varies, the value of $(\hat{Q}/Q)^2$ varies between a minimum $(1 - \varepsilon)^2$ and a maximum $(1 + \varepsilon)^2$. The mean of these two values is $1 + \varepsilon^2$, something greater than unity, and so the pulsation has an overall stabilizing effect. The value of Ra_c varies between

$$\mathrm{Ra}_{c\min} = 4\pi^2 + Q^2(1 - \varepsilon)^2 \tag{58}$$

and

$$\mathrm{Ra}_{c\mathrm{maz}} = 4\pi^2 + Q^2(1 + \varepsilon)^2. \tag{59}$$

When Ra is less than $\mathrm{Ra}_{c\min}$, no convection takes place. When Ra is greater than $\mathrm{Ra}_{c\max}$, continuous convection occurs. For intermediate values of Ra, convection occurs for just part of each cycle.

4 Conclusions

We applied the frozen profile and the averaged equations approaches to investigate the effect of pulsating throughflow on instability in a fluid occupying a horizontal fluid-saturated porous layer, which is heated from below. We investigated the situation where the throughflow pulsates with time about a mean that can be of either sign or zero. The frozen profile approach predicts that the effect of throughflow in either vertical direction is stabilizing. [A physical explanation was provide by Nield (1987) and is given in Section 6.10.2 of Nield and Bejan (2013).] An increased amplitude of throughflow oscillations leads to increased stability by an amount that depends on the frequency. The application of the averaged equations approach leads to the same results, and this leads us to conclude that the situation for a pulsating velocity field is dramatically different from that for a pulsating gravity field. The throughflow modulation modifies just the basic temperature gradient and does not itself provide any destabilizing agency. We also established threshold values of the Rayleigh number which separate three possible regimes: (1) no convection takes place, (2) a state in which disturbances grow during the first part of the cycle and decay during the second part and thus substantial convection occurs for just part of each cycle, and (3) convection occurs at a continuous level.

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