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On the Construction of an Experimentally Based Set of Equations to Describe Cocurrent and Countercurrent, Two-Phase Flow of Immiscible Fluids Through Porous Media

Ramon G. Bentsen · Japan Trivedi

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Abstract Generalized flow equations developed for two-phase flow through porous media contain a second term that enables proper account to be taken of capillary coupling between the two flowing phases. In this study, a partition concept, together with a novel capillary pressure equation for countercurrent flow, have been introduced into Kalaydjian's generalized flow equations to construct modified flow equations which enable a better understanding of the role of capillary coupling in horizontal, two-phase flow. With the help of these equations it is demonstrated that the reduced flux observed in countercurrent flow, as compared to cocurrent flow, can be explained by the reduction in the driving force per unit volume which comes about because of capillary coupling. Also, it is shown experimentally that, because fluids flow through a void space reduced in magnitude due to the presence of immobile irreducible and residual saturations, the capillary coupling parameter should be defined in terms of a reduced porosity, rather than in terms of porosity. Moreover, it is shown statistically that the countercurrent relative permeability curve is proportional to the cocurrent relative permeability curve, the constant of proportionality being the capillary coupling parameter. Finally it is suggested that one can eliminate the need to determine experimentally countercurrent relative permeability curves by making use of an equation constructed for predicting the magnitude of the capillary coupling parameter.

Keywords Capillary coupling · Two-phase flow · Countercurrent flow · Flow equations · Capillary pressure

R. G. Bentsen (✉) · J. Trivedi
Department of Civil and Environmental Engineering, School of Mining and Petroleum Engineering, University of Alberta, 3-112 Markin CNRL-NREF, Edmonton, AB T6G 2W2, Canada
e-mail: ramon.bentsen@ualberta.ca

J. Trivedi
e-mail: jtrivedi@ualberta.ca

List of symbols

Variables

a	Parameter in defining equation for R_{12} which must be determined experimentally
b	Parameter in pressure equation
c	Parameter in pressure equation
d	Parameter in pressure equation
k_i	Effective permeability for phase $i, i = 1, 2$
k_{ij}	Generalized effective permeability for phase $i, i, j = 1, 2$
k_{rw}	Relative permeability to water
k_{rwc}	Corrected relative permeability to water
k_{rwin}	Interpolated value of relative permeability to water
L	Length of porous medium
p_i	Pressure for phase $i, i = 1, 2$
$\tilde{P}_{\text{c}}$	$p_2 - p_1$ = macroscopic capillary pressure
R_{12}	$1 - a(1 - S)$ = function relating the pressure gradient in phase 1 to that in phase 2
S	Normalized saturation in the wetting phase
S_{or}	Residual oil saturation
S_{wi}	Irreducible water saturation
v_i	Darcy velocity of phase $i, i = 1, 2$
x	Horizontal distance in the direction of flow
z	Vertical distance in direction of flow

Greek Letters

α	Capillary coupling parameter
α_{c}	Capillary coupling parameter defined in terms of porosity
α_{cr}	Capillary coupling parameter defined in terms of reduced porosity
α_i	Capillary coupling parameter for phase $i, i = 1, 2$
α_{ij}	Generalized partition coefficient for phase $i, i, j = 1, 2$
ϕ	Porosity
ϕ_{r}	Reduced porosity
δ_{ij}	Relative difference for phase $i, i, j = 1, 2$
λ_i	Effective mobility of phase $i, i = 1, 2$
λ_{ij}	Generalized effective mobility of phase $i, i, j = 1, 2$
μ_i	Viscosity of phase $i, i = 1, 2$
ψ_i	$p_i + \rho_i gz$ = flow potential of phase $i, i = 1, 2$

Subscripts

co	Cocurrent flow
cn	Countercurrent flow

Superscripts

$\circ$	Steady-state, cocurrent flow
$*$	Steady-state, countercurrent flow

1 Introduction

One-dimensional, immiscible, horizontal, two-phase flow may be cocurrent or countercurrent. The type of flow which takes place depends upon the relative magnitude of the viscous and capillary forces acting on the two phases. Usually the balance of forces acting on the two phases is such that they both flow in the same direction (cocurrent flow). However, situations can arise where the balance of forces is such that the two-phases flow in opposite directions (countercurrent flow). In the early investigations of Buckley and Leverett (1942) and Leverett et al. (1942), the importance of such countercurrent flow in porous media was recognized and described.

When water-flooding water-wet, fractured matrix reservoirs (Blair 1964; Bourbiaux and Kalaydjian 1990), countercurrent flows also play a significant role. In such a water flood, the water imbibed from a fracture into the matrix results in the countercurrent expulsion of oil into the fractures surrounding the block (Blair 1964). The reservoir produces its oil almost independently from each neighboring block due to the combined effects of gravity and capillarity. Both cocurrent and countercurrent flow are involved in spontaneous imbibition of this kind. The ratio of the gravity to the capillary forces, and the conditions at the boundaries of each block, dictates the relative amount of each type of flow that takes place.

It is of interest to reservoir engineers to be able to predict immiscible, two-phase displacements in which spontaneous imbibition plays a significant role (Blair 1964; Bourbiaux and Kalaydjian 1990). In order to be able to make such predictions, one must be able to specify the relative permeability curves that pertain to countercurrent flow. One approach to specifying such curves is to assume that the steady-state curves that apply to cocurrent flow also apply to countercurrent flow. The validity of taking such an approach is doubtful (Lelièvre 1966; Lefebvre du Prey 1978; Bourbiaux and Kalaydjian 1990; Bentsen and Manai 1991; Bentsen 2001, 2005a,b).

Countercurrent relative permeability curves may also be determined in the laboratory (Lelièvre 1966; Bourbiaux and Kalaydjian 1990; Bentsen and Manai 1991; Bentsen 2005a,b). The laboratory approach for determining such curves is not easily implemented because the techniques used to measure such curves are fraught with both experimental and computational difficulties. Consequently, very few experimental determinations of countercurrent relative permeabilities, as compared to cocurrent relative permeabilities, have been reported in the literature.

The experiments needed to measure countercurrent relative permeability are difficult to design and carry out. Such is the case because it is usual to use vertical displacements so that use can be made of the gravitational potential gradients that occur in such displacements. Consequently, two problems arise. First, countercurrent flow can take place only over a limited range of rates (Lelièvre 1966). Second, the total potential drop across the length of the core is small, typically being of the order of 1 kPa, because the gravitational potential difference across the length of the core depends on $\Delta\rho g L$. As a consequence, difficulties in accurately measuring the flow rates and potential gradients needed to calculate the associated relative permeabilities using Darcy's law can arise.

The problems associated with using vertical, countercurrent flow to determine countercurrent relative permeabilities can be mitigated, provided it is possible to use a countercurrent, horizontal flow experiment to determine these relative permeabilities. Bentsen and Manai (1991) have carried out a set of experiments in which both cocurrent and countercurrent relative permeabilities were determined using a horizontal core holder. However, questions have arisen concerning the validity of the set of equations used to interpret the data. This was particularly the case with respect to how capillary pressure should be defined for horizontal, countercurrent flow. Thus the purpose of this article is to construct a set of equations, which is consistent with the experimental data acquired by Bentsen and Manai (1991), which can be used to determine both cocurrent and countercurrent relative permeabilities. A further goal is to show how the data acquired in a steady-state, cocurrent flow experiment can be used to determine the relative permeabilities which pertain to countercurrent flow, thus eliminating the need to undertake difficult countercurrent flow experiments to determine them.

2 Theory

2.1 Introduction

On the basis of experimental results presented in the literature (Lelièvre 1966; Bourbiaux and Kalaydjian 1990; Bentsen and Manai 1991, 1993), it appears that the mobilities determined in a countercurrent flow experiment are less than those determined for the same sand fluid system, in a cocurrent flow experiment. Such a result cannot be explained, if one assumes that the conventional flow equations (Muskat 1982) describe correctly two-phase flow through porous media. That is, one must resort to more sophisticated flow equations, such as those constructed by Kalaydjian (1987), or others (de la Cruz and Spanos 1983; Whitaker 1986) to explain such a result.

In Muskat's flow equations (Muskat 1982), the flux is taken to be proportional to one driving force, the pressure gradient (or potential gradient in vertical flow) acting across the phase. On the other hand, in the more sophisticated flow equations, Muskat's equation is modified to include a cross, or coupling term, that is proportional to the pressure (or potential) gradient of the other phase. The need for such a modification is argued usually on the basis of symmetry, or results arising out of irreversible thermodynamics (Katchalsky and Curran 1975). Moreover, it is postulated usually that the coupling effect arises from the interfacial contact between the wetting and non-wetting fluids.

The following analysis is confined to the stable, collinear, horizontal flow of two immiscible, incompressible fluids through a water-wet, isotropic, and homogeneous porous medium where phase 1 is the wetting phase and phase 2 is the non-wetting phase. Moreover, it is assumed that that any viscous coupling, if it exists, is negligibly small (Bentsen 2001). Finally, it is assumed that the flow equations are linear. While there is experimental evidence in the petroleum literature to support such a contention (Fulcher et al. 1985), more recent experimental results (Sinha et al. 2013; Sinha and Hansen 2012; Rassi et al. 2011; Tallakstad et al. 2009a,b; Avraam and Payatakes 1995) suggest that the flow equations are highly nonlinear. When the flow equations are nonlinear, the effective permeabilities, rather than being functions of saturation only, are also dependent on the capillary number (ratio of the viscous forces to the capillary forces), which, in turn, depends on the velocity of the flowing fluids. While, the fact that the flow equations are nonlinear does not appear to have a significant impact on the form of the equations constructed in this article, at least for high values of the capillary number (Sinha and Hansen 2012), it is expected that the nonlinearity of the flow equations has resulted in the introduction of an unknown amount error into the experimentally determined relative permeability curves used in Sect. 2.5.2 to test the theory developed in the article.

2.2 Basic Equations

2.2.1 Flow Equations

Kalaydjian (1987) has shown that, consistent with the assumptions made above, the generalized flow equations for the flow of two continuous phases may be written as

$$v_1 = -\lambda_{11}\frac{\partial p_1}{\partial x} - \lambda_{12}\frac{\partial p_2}{\partial x}, \tag{1}$$

and

$$v_2 = -\lambda_{21}\frac{\partial p_1}{\partial x} - \lambda_{22}\frac{\partial p_2}{\partial x}, \tag{2}$$

where $\lambda_{ij} = k_{ij}/\mu_i,\ i = 1, 2$.

Moreover, the conventional flow equations, again consistent with the same assumptions made above, may be written as

$$v_i = -\lambda_i\frac{\partial p_i}{\partial x}, \quad i = 1, 2, \tag{3}$$

where $\lambda_i = k_i/\mu_i, \quad i = 1, 2$.

2.2.2 Capillary Pressure Equations

In the cocurrent flow experiments carried out by Bentsen and Manai (1991), it was determined that, once steady-state flow was achieved, pressures were distributed linearly along the length of the core, and that saturation was invariant with distance, except for minor perturbations due to local heterogeneities in porosity and/or permeability. Moreover, it was found that, when the experiments were conducted in an approximately meter long core holder, the measured plots of pressure versus distance for the two flowing phases were not quite parallel. Such a result arises because, in long core experiments, the pressure of the non-wetting phase, measured at the inlet end of the core holder is due to two factors: the capillarity of the porous medium and the hydrodynamic effects (Bentsen 1994). As a consequence, the difference in pressure between the non-wetting and wetting phases varies along the length of the core holder, which is inconsistent with the fact the saturation measured was invariant with distance. The size of this problem is illustrated by calculating the difference in pressure, measured at the inlet end of the core holder, $p_2 - R_{12}p_2 = a(1 - S)p_2$. For $a = 0.05$ and $p_2 = 40$ kPa, values used by Manai (1991), this pressure difference varies from 2 kPa at $S = 0$ to 0 kPa at $S = 1$. This problem can be overcome by removing the contribution of the hydrodynamic effects to the non-wetting pressure measured at the inlet end of the core holder. This was achieved by multiplying p_2 by R_{12}, where R_{12} is a weak function of normalized saturation that corrects for the impact of the hydrodynamic effects on the non-wetting pressure profile. As a consequence, the two linear plots of pressure versus distance became parallel. A plot of the resulting pressure profiles is shown in Fig. 1. The equations for the two pressure profiles are, respectively,

$$R_{12}p_2 = b + c - \frac{(b - d)}{L}x \tag{4}$$

and

$$p_1 = b - \frac{(b - d)}{L}x. \tag{5}$$

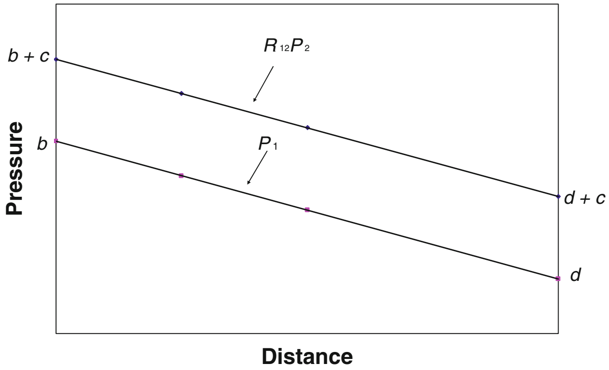

Fig. 1 Plot of pressure versus distance for cocurrent flow

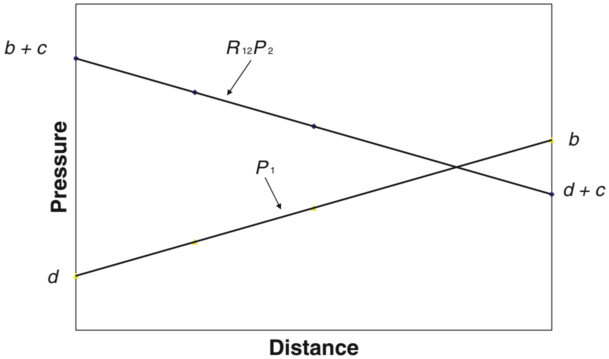

Fig. 2 Plot of pressure versus distance for countercurrent flow

Subtracting Eq. (5) from Eq. (4) yields

$$R_{12}p_2 - p_1 = c(S) = P_c. \tag{6}$$

Note that multiplying p_2 by R_{12} removes the contribution of the hydrodynamic effects from the difference in pressures, leaving only the contribution of the capillarity of the porous medium. Consequently, P_c, as defined by Eq. (6), is the conventionally defined macroscopic pressure. Taking the partial derivative of Eq. (6), with respect to x, one obtains

$$R_{12}\frac{\partial p_2}{\partial x} - \frac{\partial p_1}{\partial x} = \frac{\partial P_c}{\partial x}. \tag{7}$$

In the steady-state countercurrent flow experiments carried out by Bentsen and Manai (1991), it was found again that the pressures were distributed linearly along the length of the core, and that saturation was invariant with distance. Moreover, it was found that it was again necessary to multiply the non-wetting pressures by R_{12}. However, it was found that the non-wetting and wetting pressure profiles were orthogonal to one another, rather than being parallel, as was the case for cocurrent flow. A plot of the pressure profiles for countercurrent flow is depicted in Fig. 2. The countercurrent equation for the non-wetting pressure profile is the same as it was for cocurrent flow (see Eq. 4). However, the equation for the wetting pressure profile differs and may be written as

$$p_1 = d + \frac{(b-d)}{L}x. \tag{8}$$

In the countercurrent flow experiments carried out by Bentsen and Manai (1991), it was found that the sum of the pressures, rather the difference in pressures, was constant along the length of the core. Thus, adding Eq. (4) to Eq. (8) leads to

$$R_{12}p_2 + p_1 = b + c + d. \tag{9}$$

In the countercurrent flow experiments, the non-wetting phase was injected at $x = 0$, while the wetting phase was injected at $x = L$. Moreover, it was found that, when the difference in the inlet pressures between the two flowing fluids was plotted versus saturation, the countercurrent data fell, within experimental error, on the same capillary pressure curve as did the cocurrent data. Thus, if one subtracts Eq. (8), evaluated at $x = L$, from Eq. (4), evaluated at $x = 0$, there results

$$R_{12}p_2(0) - p_1(L) = c(S) = P_c. \tag{10}$$

Thus, in view of Eq. (10), Eq. (9) may be written as

$$R_{12}p_2 + p_1 = b + d + P_c. \tag{11}$$

Taking the partial derivative of Eq. (11) with respect to x leads to

$$R_{12}\frac{\partial p_2}{\partial x} + \frac{\partial p_1}{\partial x} = \frac{\partial P_c^*}{\partial x}, \tag{12}$$

where it has been assumed that the parameters b and d are independent of x, and where the superscript * indicates that the capillary pressure gradient applies to countercurrent flow. In the study by Bentsen (2005b), it was determined that, when two fluids are flowing in opposite directions, the effect of capillary coupling is to reduce the magnitude of the capillary pressure gradient by the factor α_c; that is, $\partial P_c^*/\partial x = \alpha_c \partial P_c/\partial x$. Hence, upon introducing this relationship into Eq. (12), one obtains

$$R_{12}\frac{\partial p_2}{\partial x} + \frac{\partial p_1}{\partial x} = \alpha_c \frac{\partial P_c}{\partial x}. \tag{13}$$

2.3 Modified Flow Equations

The introduction of Eq. (7) into Eqs. (1) and (2) yields, respectively,

$$v_1 = -\left(\lambda_{11} + \frac{\lambda_{12}}{R_{12}}\right)\frac{\partial p_1}{\partial x} - \frac{\lambda_{12}}{R_{12}}\frac{\partial P_c}{\partial x} \tag{14}$$

and

$$v_2 = -(\lambda_{22} + R_{12}\lambda_{21})\frac{\partial p_2}{\partial x} + \lambda_{21}\frac{\partial P_c}{\partial x}. \tag{15}$$

Based on the experimental results presented by Bentsen and Manai (1991, 1993), it can be inferred that (Bentsen 1998b)

$$\lambda_{ij} = \alpha_{ij}\lambda_i^0, \quad i = 1,\ 2, \tag{16}$$

where the α_{ij} are generalized partition coefficients for phase i, $i, j = 1,\ 2$, and where the $\lambda_i^0, i = 1,\ 2$ are the mobilities determined in a horizontal, steady-state, cocurrent flow experiment. Note that, in view of Eq. (16), and the definition for mobility, the generalized effective permeabilities are proportional to the conventional effective permeabilities, the constant of proportionality being the partition coefficients. Upon introducing Eq. (16) into Eqs. (14) and (15), one obtains, respectively,

$$v_1 = -\lambda_1^0 \left[\left(\alpha_{11} + \frac{\alpha_{12}}{R_{12}} \right) \frac{\partial p_1}{\partial x} + \frac{\alpha_{12}}{R_{12}} \frac{\partial P_c}{\partial x} \right] \tag{17}$$

and

$$v_2 = -\lambda_2^0 \left[(\alpha_{22} + R_{12}\alpha_{21}) \frac{\partial p_2}{\partial x} - \alpha_{21} \frac{\partial P_c}{\partial x} \right]. \tag{18}$$

In countercurrent flow, the pressure gradients act in opposite directions. Hence, upon introducing Eqs. (13) and (16) into Eqs. (1) and (2), one obtains, respectively,

$$v_1^* = -\lambda_1^0 \left[\left(\alpha_{11} - \frac{\alpha_{12}}{R_{12}} \right) \frac{\partial p_1}{\partial x} + \alpha_c \frac{\alpha_{12}}{R_{12}} \frac{\partial P_c}{\partial x} \right] \tag{19}$$

and

$$v_2^* = -\lambda_2^0 \left[(\alpha_{22} - R_{12}\alpha_{21}) \frac{\partial p_2}{\partial x} + \alpha_c \alpha_{21} \frac{\partial P_c}{\partial x} \right]. \tag{20}$$

Moreover, the conventional flow equations for countercurrent flow may be written as

$$v_i^* = -\lambda_i^* \frac{\partial p_i}{\partial x}, i = 1, 2, \tag{21}$$

where the λ_i^*, $i = 1, 2$ are the mobilities measured in a steady-state countercurrent flow experiment. Based on the experimental results presented by Bentsen and Manai (1991, 1993), it is postulated that

$$\lambda_i^* = \alpha_i \lambda_i^0, i = 1, 2 \tag{22}$$

where the α_i are the capillary coupling parameters for phase i, $i = 1, 2$ which can be determined experimentally provided cocurrent and countercurrent mobilities have been measured. Note that, while, in general, the α_i may be functions of saturation, the experimental results presented in Sect. 2.5.2 indicate that the α_i are constants which are independent of saturation.

2.4 Determination of Partition Coefficients

On the basis of Eq. (17), it appears that the total pressure force per unit volume available to act on a unit volume of phase 1, $\partial p_1/\partial x$, may be partitioned into two components: a phase component, $\alpha_{11}\partial p_1/\partial x$, and a coupling (capillary) component, $\alpha_{12}/R_{12}\partial p_1/\partial x$, that arises because of the introduction of the capillary pressure equation (Eq. 7) into Eq. (1). Because the pressure forces per unit volume, $\partial p_1/\partial x$ and $\partial p_2/\partial x$, act in the same direction, the phase and coupling components of the total pressure force per unit volume must also act in the same direction. Similar comments also can be made with respect to Eq. (18), the equation that determines the flux of phase 2. Moreover, the partition coefficients α_{ij} may be viewed (see Eqs. 27, 28) as being the fraction of the pressure force per unit volume of phase j that is available to act on a unit volume of phase i, $i, j = 1, 2$.

The capillary pressure gradient, $\partial P_c/\partial x$, under conditions of steady-state flow, is identically equal to zero. Moreover, the flux of a given phase may be defined by Kalaydjian's flow equations or by the conventional flow equations. Hence, for steady-state, cocurrent flows, Eqs. (17) and (18) may be combined with Eq. (3) to obtain

$$\alpha_{11} + \frac{\alpha_{12}}{R_{12}} = 1 \tag{23}$$

and

$$\alpha_{22} + R_{12}\alpha_{21} = 1. \tag{24}$$

Under conditions of steady-state, countercurrent flow, the capillary pressure gradient, $\frac{\partial P_c}{\partial x}$, is identically equal to zero. Moreover, the flux of a given phase may be defined by Kalaydjian's flow equations or by the conventional countercurrent flow equations. Hence, by combining Eqs. (19) and (20) with Eq. (21), and by introducing Eq. (22) into the resulting equations, it may be shown that, for steady-state countercurrent flow,

$$\alpha_{11} - \frac{\alpha_{12}}{R_{12}} = \alpha_1 \tag{25}$$

and

$$\alpha_{22} - R_{12}\alpha_2 = \alpha_2 \tag{26}$$

To ensure that the capillary pressure gradient for countercurrent flow is consistent with that for cocurrent flow, one more condition must be imposed. Introducing Eq. (16) into Eqs. (1) and (2), and setting the resulting equations equal to Eq. (3), respectively, yields

$$\alpha_{11}\frac{\partial p_1}{\partial x} + \alpha_{12}\frac{\partial p_2}{\partial x} = \frac{\partial p_1}{\partial x} \tag{27}$$

and

$$\alpha_{21}\frac{\partial p_1}{\partial x} + \alpha_{22}\frac{\partial p_2}{\partial x} = \frac{\partial p_2}{\partial x}. \tag{28}$$

Multiplying Eq. (28) through by R_{12}, adding the resulting equation to Eq. (27), and collecting like terms, one obtains

$$R_{12}\frac{\partial p_2}{\partial x} + \frac{\partial p_1}{\partial x} = (\alpha_{12} + R_{12}\alpha_{22})\frac{\partial p_2}{\partial x} + (\alpha_{11} + R_{12}\alpha_{21})\frac{\partial p_1}{\partial x}. \tag{29}$$

Equating the coefficients of like terms yields

$$\alpha_{11} + R_{12}\alpha_{21} = 1 \tag{30}$$

and

$$\alpha_{12} + R_{12}\alpha_{22} = R_{12}. \tag{31}$$

Note that only one of Eqs. (30) and (31) is needed; that is use of either of these two equations leads to the same result.

Equations (23), (24), (25), (26) and either Eq. (30) or (31) comprise a system of five equations involving six unknowns: α_{11}, α_{12}, α_{21}, α_{22}, α_1, and α_2. This system of equations may be solved for five of the unknowns in terms of the sixth unknown (Bentsen 2001) to obtain

$$\alpha_{11} = \alpha_{22} = \frac{1+\alpha}{2}, \tag{32}$$

$$\alpha_{12} = \frac{R_{12}(1-\alpha)}{2}, \tag{33}$$

and

$$\alpha_{21} = \frac{1-\alpha}{2R_{12}}, \tag{34}$$

where

$$\alpha_1 = \alpha_2 = \alpha. \tag{35}$$

The introduction of Eqs. (32)–(34) into Eqs. (17) and (18) leads, for one-dimensional, horizontal, cocurrent flow, to

$$v_1 = -\lambda_1^0 \left(\frac{\partial p_1}{\partial x} + \frac{1-\alpha}{2} \frac{\partial P_c}{\partial x} \right) \tag{36}$$

and

$$v_2 = -\lambda_2^0 \left(\frac{\partial p_2}{\partial x} - \frac{1-\alpha}{2R_{12}} \frac{\partial P_c}{\partial x} \right). \tag{37}$$

Note that, if $\alpha = 1$ (no interfacial coupling), and/or if $\partial P_c/\partial x = 0$, the conventional equations for one-dimensional, horizontal, cocurrent, two-phase flow, are obtained.

The introduction of Eqs. (32)–(34) into Eqs. (19) and (20) yields, for one-dimensional, horizontal, countercurrent flow

$$v_1^* = -\lambda_1^0 \left(\alpha \frac{\partial p_1^*}{\partial x} + \alpha_c \frac{1-\alpha}{2} \frac{\partial P_c}{\partial x} \right) \tag{38}$$

and

$$v_2^* = -\lambda_2^0 \left(\alpha \frac{\partial p_2^*}{\partial x} + \alpha_c \frac{1-\alpha}{2R_{12}} \frac{\partial P_c}{\partial x} \right). \tag{39}$$

Assuming that $\alpha_c = \alpha$ enables Eqs. (38) and (39) to be written as

$$v_1^* = -\lambda_1^* \left(\frac{\partial p_1^*}{\partial x} + \frac{1-\alpha}{2} \frac{\partial P_c}{\partial x} \right) \tag{40}$$

and

$$v_2^* = -\lambda_2^* \left(\frac{\partial p_2^*}{\partial x} + \frac{1-\alpha}{2R_{12}} \frac{\partial P_c}{\partial x} \right). \tag{41}$$

where λ_i^*, $i = 1, 2$ is defined by Eq. (22). Again note that if $\alpha = 1$ (no interfacial coupling) and/or if $\partial P_c/\partial x = 0$, the conventional equations for one-dimensional, horizontal, countercurrent flow are obtained.

2.5 Determination of Capillary Coupling Parameter

Two methods are available for estimating the magnitude of the capillary coupling parameter α_c: a theoretical method and an experimental method.

2.5.1 Theoretical Method

By replacing the multiphase porous medium with three overlapping continua, and by undertaking the analysis within the confines of a representative elementary volume (Bear 1972), Bentsen (2005b) was able to show that the capillary coupling parameter is defined by

$$\alpha_c = 1 - \phi. \tag{42}$$

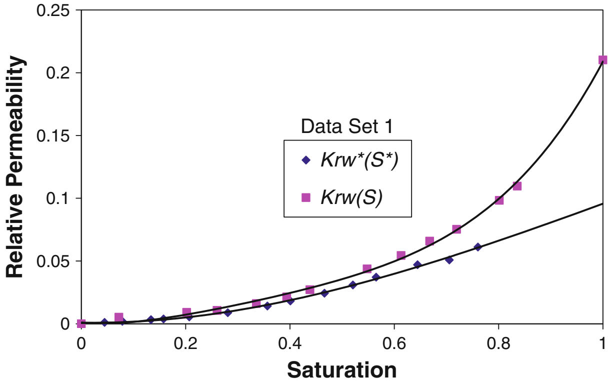

Fig. 3 Countercurrent and cocurrent wetting phase relative permeabilities for dataset 1

Equation (42) was constructed (Bentsen 2005b) under the assumption that the entire pore volume was available for fluid flow. Strictly speaking such is not the case because the portion of the pore volume occupied by the residual oil saturation and the irreducible water saturation is not available for the fluid flow purposes. Hence, it may be preferable to define capillary coupling in terms of a reduced porosity, ϕ_r. Thus, it is postulated that

$$\alpha_{cr} = 1 - \phi_r, \tag{43}$$

where $\phi_r = (1 - S_{wi} - S_{or})\phi$.

2.5.2 Experimental Method

Implementing the experimental method for estimating the magnitude of the capillary coupling parameter requires experimental data. Two different sets of data are available for this purpose: the data taken by Manai (1991) and the data collected by Lelièvre (1966).

Manai's Data Manai (1991) undertook two sets of steady-state experiments in a long (1 m), horizontal core holder in which both cocurrent and countercurrent flow took place. In situ measurements of saturation, and of both the wetting and the non-wetting phase pressures, were taken along the length of the core. This data, together with the use of Darcy's law, enabled the determination of the relative permeability curves which pertain to both cocurrent and countercurrent flow. A typical set of cocurrent ($k_{rw}(S)$) and countercurrent ($k^*_{rw}(S^*)$) relative permeability curves, for the wetting phase, is shown in Fig. 3.

Lelièvre's Data Lelièvre (1966) undertook a series of steady-state experiments in a long (1.2 m) vertical core holder in which both cocurrent and countercurrent flow took place. In situ measurements of saturation, and of the wetting phase potential gradient, were taken along the length of the core. Knowing the densities of both flowing phases, and the potential gradient of the wetting phase, enabled the non-wetting potential gradient to be calculated. This data, together with the use of Darcy's law, permitted the determination of the relative permeability curves which pertain to both cocurrent and countercurrent flow. A summary of the equipment, materials, and techniques used, estimated errors in the measured data, experimental procedure, and some of Lelièvre's results are available in the literature (Bentsen 2005a).

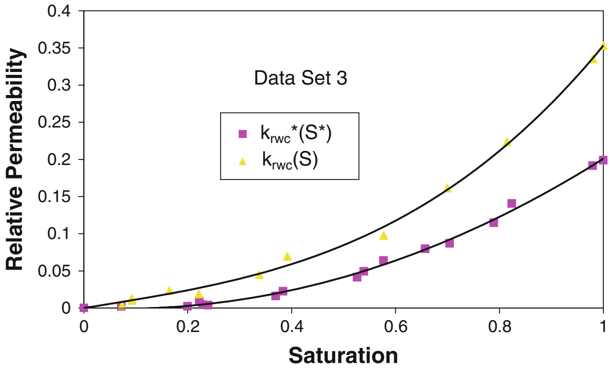

Fig. 4 Cocurrent and countercurrent wetting phase relative permeabilities for dataset 3

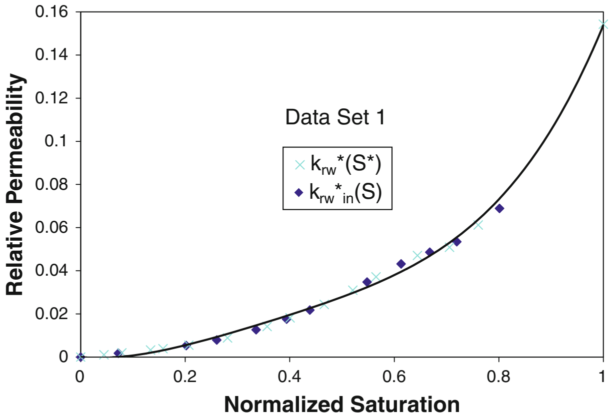

Fig. 5 Comparison of interpolated data with original data for countercurrent relative permeability for dataset 1

Lelièvre (1966) used the classical form of Darcy's law, rather than modified generalized flow equations (Bentsen 2005a), to calculate the relative permeabilities. Consequently, a correction must be made to the experimentally determined relative permeability curves to remove the model error incurred by failing to account properly for the effect of interfacial coupling on the total driving force acting on the flowing phases. Equations constructed by Bentsen (2005a) were used to correct the relative permeability curves measured by Lelièvre (1966). A typical set of corrected wetting phase cocurrent relative permeability curves is shown in Fig. 4.

Experimental Procedure Based on Eq. (22), the countercurrent curve in Fig. 3 (or in Fig. 4) should be proportional to the cocurrent curve, the constant of proportionality being α_1. Because the experimental points for the two curves were not measured at the same saturation, linear interpolation was used to obtain estimates of data points on the countercurrent curve which pertained to the same saturation as those on the cocurrent curve. A comparison of the interpolated data points ($k^*_{\text{rwin}}(S)$) on the countercurrent curve with those measured ($k^*_{\text{rw}}(S)$) is depicted in Fig. 5.

A trial value for α_1 was now estimated. Then each of the interpolated data points was divided by α_1 and the result subtracted from the appropriate cocurrent data point. Finally, these differences were squared and then summed over the number of data points on the curve.

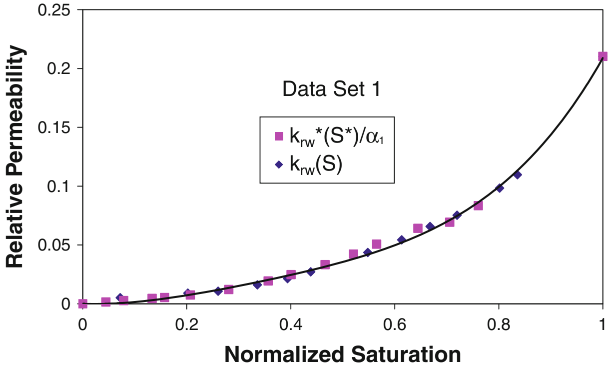

Fig. 6 Comparison of cocurrent relative permeability and countercurrent relative permeability divided by α_1

Table 1 Summary of parameters

Parameter	Dataset 1	Dataset 2	Dataset 3
K, darcies	20.5	20.4	230
S_{wi}	0.09	0.095	0.0
$(S_{or})_{co}$	0.18	0.175	0.051
$(S_{or})_{cn}$	0.18[a]	0.175[a]	0.03[a]
ϕ	0.358	0.3536	0.43
α_1	0.734	0.706	0.569
α_2	0.753	0.756	0.611
α_c	0.642	0.646	0.57
$(\alpha_{cr})_{co}$	0.739	0.742	0.592
$(\alpha_{cr})_{cn}$	0.739	0.742	0.583
δ_{r1}, %	0.68	4.85	2.4
δ_{r2}, %	−1.89	−1.89	−4.8

[a]Estimated

Successive values for α_1 were selected until the value of α_1 which minimized the sum of squares was found. This was deemed to be the best estimate for α_1. A comparison of $k_{rw}(S)$ and $k^*_{rw}(S^*)/\alpha_1$ is shown in Fig. 6. The same procedure was used to estimate α_i, $i = 1, 2$, for the remaining three datasets measured by Manai (1991), and for the dataset (Dataset 3) measured by Lelièvre. A summary of the results obtained is presented in Table 1. Also presented in Table 1 are the calculated values for α_c and α_{cr}, together with the basic data needed to calculate these parameters. It is important to note that, while the magnitude of the α_i varied slightly from experiment to experiment, the experimental evidence does not support the idea that the capillary coupling parameter may be a function of saturation. In addition the absolute permeabilities of the cores used in the three sets of experiments are reported in Table 1. Moreover, the relative difference, δ_{ri}, $i = 1, 2$, is reported in the table. The relative difference is defined by

$$\delta_{ri} = \frac{\alpha_{cr} - \alpha_i}{\alpha_{cr}} 100 \tag{44}$$

3 Discussion

3.1 Capillary Pressure

In this study, the capillary pressure equations for both cocurrent and countercurrent flow were constructed on the basis of the linear plots of pressure versus distance measured by Manai. While Eq. (7) is consistent with the cocurrent, capillary pressure gradient equation used in earlier studies (Bentsen 1992, 1994, 1997, 1998a), Eq. (13), the capillary pressure gradient equation for horizontal, countercurrent flow, differs from the equation used in earlier studies (Bentsen 1995, 2005b). This is because the earlier equation did not take proper account of the fact that interfacial coupling acts to decrease the magnitude of the capillary pressure driving force. It is important to note that Eq. (13) applies only to linear, horizontal, countercurrent flow, and not to gravity driven countercurrent flow. In vertical, gravity driven countercurrent flow, the driving forces are the potential gradients. As a consequence, in vertical, gravity driven, countercurrent flow it is the potential gradients that are opposite in sign, while the pressure gradients are not. Rather, a version of Eq. (7), derived using potential gradients, rather than pressure gradients (Bentsen 2003), should be used for gravity driven countercurrent flow.

3.2 Modified Flow Equations

Note that, while the capillary pressure terms in Eqs. (36) and (37) are similar to those constructed in an earlier study (Bentsen 2001), those in Eqs. (38) and (39) are not. There are two reasons for this. First, the form of the capillary pressure gradient equation used in the earlier study was incorrect. Second, proper account was not taken, in the earlier study, of the fact that capillary coupling acts to decrease the magnitude of the capillary pressure driving force when countercurrent flow is taking place. Moreover, if the equations constructed in another earlier study for vertical, cocurrent flow (Bentsen 2005b) are simplified, so that they pertain to horizontal, cocurrent flow, they are also consistent with Eqs. (36) and (37). However, when the countercurrent forms of these equations are simplified, they are inconsistent with Eqs. (38) and (39). In particular, the sign preceding the capillary pressure gradient term in the phase two equations differs. This is because, in the vertical, countercurrent flow study, where the potential gradients were opposite in sign, the capillary pressure gradient was defined by Eq. (7), while in the horizontal, countercurrent flow study; the capillary pressure gradient was defined using Eq. (13). That is, one needs to be careful, when simplifying a vertical flow equation to obtain a horizontal flow equation, if the introduction of error is to be avoided.

3.3 Capillary Coupling

3.3.1 Theoretical Method

In order to be able to estimate the magnitude of the capillary coupling parameter, it is convenient to suppose that it is permissible to replace a multiphase porous medium by three overlapping continua, where each continuum is taken to represent one phase (solid, wetting, and non-wetting phase) that fills the entire porous medium domain. Suppose, also, that at every point in space, one may assign the properties (whether of the medium or of the fluids filling the void space) of any of the continua. Finally suppose that interactions may take place between any two of the continua. The taking of such an approach enables one to estimate an average (or effective) potential for each fluid phase (Bentsen 2005b).

To estimate the average potential for each phase, it is convenient to make use of a representative elementary volume (Bear 1972). Note that, while the size of the REV used here is consistent with the size of a standard REV, the way the fluids are distributed within the REV is nonstandard. When a standard REV is used to construct a continuum, it is supposed the relative amounts of fluids contained within the REV depend on the saturations of the saturating fluids. Because we are not constructing a continuum, but rather a defining equation for the magnitude of the capillary coupling parameters, α_i, $i = 1, 2$, it is supposed that the fractional amounts of wetting and non-wetting fluid contained within the REV do not depend on saturation, but rather on the number of fluids contained within the pore space of the REV. It is supposed that the REV may be portioned into solid and fluid portions and that the fluid portion of the REV may be partitioned into wetting and non-wetting portions. As energy is an additive property, it is possible to add together the energy stored in each portion of the REV, and then divide by the bulk volume of the REV to arrive at an estimate of the average (or effective) energy per unit volume available to drive a particular phase.

When these ideas were first formulated, it was supposed that the entire void space was available for the flow of the two fluids. However, in reality, while the two fluids fill the entire void space, because of the presence of an irreducible wetting phase saturation and a residual non-wetting phase saturation, only a fraction of the total void space is available for the flow of the two fluids. That is to say, it seems preferable to define the capillary coupling parameter in terms of a reduced porosity, rather than in terms of the actual porosity.

As noted above, the total energy per unit volume available to act on a volume element of a given phase, say phase 1, can be partitioned into three parts: the fraction $(1 - \phi_\mathrm{r})$ of the energy per unit volume stored in the immobile (solid phase + irreducible and residual saturations) portion of the REV; the fraction $(\phi_\mathrm{r}/2)$ of the energy per unit volume stored in the phase 1 portion of the REV; and the fraction $(\phi_\mathrm{r}/2)$ of the energy per unit volume stored in the phase 2 portion of the REV. Note that, for reasons of symmetry, it is desirable that the amount of energy available to drive phase 1 that is stored in phase 2 is equal to the amount of energy available to drive phase 2 that is stored in phase 1. This was achieved by making the volumes of wetting and non-wetting fluid equal. The driving force per unit volume for phase 1 can be found by taking the partial derivative of the total energy per unit volume with respect to x. Because the total energy per unit volume has been portioned into three parts, the magnitude of the driving force (a vector) can also be portioned into three parts. In cocurrent flow, where the two pressure gradients are acting in the same direction, the three fractions of the driving force per unit volume sum to one, indicating that, for such flow, the average (or effective) pressure gradient is equal to the imposed pressure gradient. However, in countercurrent flow, where the two pressure gradients are acting in opposite directions, the two-phase fractions of the driving force per unit volume $(\phi_r/2)$ are equal in magnitude and opposite in sign. Consequently, the three fractions of the driving force per unit volume sum to $1 - \phi_\mathrm{r}$. That is, for such flow, the average (or effective) driving force per unit volume for phase 1 is reduced by the factor $1 - \phi_\mathrm{r}$. Similar comments can be made with respect to the phase 2 pressure gradient. Note that, as can be seen in Table 1, the use of a reduced porosity, ϕ_r, to predict the magnitude of the capillary coupling parameter, results in much better agreement between the experimental and predicted estimates of the magnitude of the capillary coupling parameter.

The reduction in the magnitude of the driving force per unit volume, by the factor $1 - \phi_r$, provides a plausible explanation as to why, in countercurrent flow, the measured relative permeabilities for countercurrent flow are less than those measured for cocurrent flow (Lelièvre 1966; Bourbiaux and Kalaydjian 1990; Bentsen and Manai 1991, 1993). Hence, as suggested by Eqs. (31) and (32), conventional relative permeability curves should be able to be used to predict countercurrent flow, provided the proper flow equations are used in the prediction.

3.3.2 Experimental Method

Manai's Data Three sources of error contribute to the differences between the experimentally determined and the predicted estimates of the magnitude of the capillary coupling parameter: those arising in the original experiments undertaken by Manai (1991) to determine experimentally the relative permeability curves, and those that arise as a consequence of the procedures used to estimate $\alpha_i,\ i = 1,\ 2$. It is important to note that in order to determine the magnitude of the capillary coupling parameter, two distinct experiments must be conducted, one to determine the cocurrent relative permeability curves, and one to determine the countercurrent relative permeability curves. Ideally, the properties of the core and fluids used in these two experiments should be identical. Practically, for experimental reasons, such a goal is very difficult to achieve. In order to ensure that the cores were as close to being the same as possible, the same unconsolidated core and fluids were used for each set of experiments. Moreover, the same irreducible saturation to the wetting phase was put in place at the beginning of each experiment. However, while it was possible to measure directly the end-point permeabilities in the cocurrent flow experiments, these permeabilities were not measured in the countercurrent experiments. Consequently, it is not known whether the residual saturations to the non-wetting phase were the same in the cocurrent and countercurrent experiments. Moreover, it has been observed that over time the wetting characteristics of a core can change. That is, as time goes on the initially water-wet surfaces in the core can become less water-wet because of hydrophobic material transferring from the oil in the core to the sand surfaces. Because the countercurrent flow experiments were conducted after the cocurrent flow experiments, such changes may have resulted in slightly different cores being used in the two experiments. Consequently, for these and possibly other reasons the cores in the cocurrent flow and countercurrent flow experiments were likely not identical. This is thought to be one of the major reasons underlying the differences in magnitude of the experimentally determined capillary coupling parameters reported in Table 1.

Another experimental source of error is the accuracy of the pressures upon which the estimated values of relative permeability are based. It was noticed that the accuracy of the pressure transducers used to measure pressure degraded over time because of changes in the wetting characteristics of the fritted disks through which the pressures were measured. This was a much more serious problem for the wetting phase pressures than it was for the non-wetting phase pressures. Because the pressure in the non-wetting phase is higher than that in the wetting phase, it is unlikely that wetting phase can pass through the non-wetting fritted disk to compromise the accuracy of the non-wetting pressure measurements. Here it is only necessary that the fritted disk have a high permeability so that, when the pressure changes, a new equilibrium pressure is achieved rapidly. With respect to the transducers sampling the wetting phase pressure, obtaining accurate pressures is much more difficult. This is because the fritted disks through which the pressures are measured must meet two criteria. First, in order that the transducer can respond quickly to a change in pressure, the fritted disk must have a high permeability, as was the case for the non-wetting fritted disks. However, because the pressure is higher in the non-wetting phase than it is in the wetting phase, the fritted disk must also have a high displacement pressure, if the passing of non-wetting fluid through the fritted disk, thus compromising the accuracy of the measurement, is to be avoided. A high displacement pressure implies a low porosity, while a high permeability implies a high porosity. The finding of water-wet fritted disks which satisfy both requirements has proven to be a very difficult problem, which has never been completely solved. In the steady-state experiments conducted by Manai (1991), it took about half a day to reach steady-state, after a change in rate took place. Consequently, the requirement that the pressure transducer be able react quickly to a change in pressure is less critical than would be the case for unsteady-state flow. Nevertheless, it is thought that the difficulty in obtaining accurate wetting phase pressures is the likely explanation as to why there is much more variability in the experimentally determined values (see Table 1) for α_1 as compared to those obtained for α_2.

The second source of error arises out of the techniques used to estimate the capillary coupling parameter. As can be seen in Fig. 3 it was not experimentally feasible to measure the cocurrent and countercurrent relative permeabilities at the same saturation. Consequently, linear interpolation was used to estimate countercurrent permeabilities at saturations which corresponded to those on the cocurrent curve. The amount of error introduced into the analysis because of the use of linear interpolation depends on the amount of scatter in the data points, and upon the distance in saturation between the data points used for the interpolation. A further difficulty arose because no data were available for extrapolation purposes in the region beyond the last measured data point. As can be seen in Fig. 5, use of interpolation to estimate data points on the countercurrent curve does not appear to have been a major contributor to differences between the predicted and experimentally determined estimates of the magnitude of α.

The third source of error arises because, when experimentally determining the effective permeabilities, it was assumed that the flow equations were linear, which they are not. In the experiments conducted by Manai, water-wet and oil-wet fritted porous plates were used at the inlet and outlet ends of the core holder to ensure even distribution of fluids across the entire inlet section of the core and to keep the production of oil and water separate at the outlet end of the core. Because, in the countercurrent flow experiments, each end of the core holder acts as both the inlet and the outlet, the use of such porous plates was critical to the successful determination of the countercurrent relative permeabilities. Moreover, since it was necessary not only to keep the pressure measurement errors as small as possible but also to avoid exceeding the displacement pressure of the porous plates, all of the displacements were conducted, as closely as possible, at the same pressure drop across the core, the pressure drop used being just below the displacement pressure. As a consequence, each of the different values of relative permeability measured was obtained at a different value of velocity, rather than at the same velocity, as should have been the case. In as much as the same procedure was used to measure both the steady-state cocurrent and countercurrent relative permeabilities, approximately the same amount of error should have been introduced at each data point on each curve. As a consequence, the non-wetting relative permeability curves were found to be proportional to the wetting phase relative permeability curves, the proportionality constant being the capillary coupling parameter.

Lelièvre's Data Again three sources of error contribute to the differences between the experimentally determined and predicted estimates of the magnitude of the capillary coupling parameter: errors arising in the original experiments undertaken by Lelièvre (1966) to determine the relative permeability curves, and errors arising out of the techniques used to estimate α_1 and α_2. Moreover, issues arising out of the need to conduct two distinct experiments in identical cores are similar to those discussed with respect to the experiments conducted by Manai (1991). For higher rates of flow, the relative error incurred in estimating the volumetric rates of flow was less, in Lelièvre's (1966) experiments, than 1 %. However, for very small rates of flow, the error could be as high as 10 %.

The potential gradients were determined by measuring the piezometric heights in a series of manometers. For non-wetting saturations up to 70 %, the relative error in the measurement of the gradient was of the order of 2 %. For non-wetting saturations ranging from 70 to 90 %, some of the potentials were perturbed. Usually, however, there remained a sufficient number of unperturbed potentials to determine the potential gradient of the wetting phase. No estimate of the error incurred in determining the potential gradient was provided for this range of saturations. In Lelièvre's steady-state experiments, it took about 12 h to reach stabilization, subsequent to a change in flow rate. Moreover, because the non-wetting potential gradients in Lelièvre's (1966) experiments ($\sim$ 10,000 Pa/m) were about one quarter of the non-wetting pressure gradients ($\sim$ 40,000 Pa/m) in Manai's (1991) experiments, maintaining the integrity of the wetting phase semi-permeable disks was likely easier in the Lelièvre experiments.

The non-wetting saturation was deduced from the attenuation in light flux that took place as a monochromatic beam of light passed through the core. The relative error in the measurement of the non-wetting saturation was estimated to be of the order of 5 %. No estimate of the residual saturation to the non-wetting phase was available for the countercurrent run.

As before, a second source of error arises out of the methods used to determine the capillary coupling parameter. The amount of error introduced into the analysis because of the use of linear interpolation depends upon the amount of scatter in the data points, and upon the distance in saturation between the data points used for the interpolation. As can be seen by comparing Fig. 3 with Fig. 4 there was somewhat more scatter in the Lelièvre data (Fig. 4) as compared to the Manai data (Fig. 3). This was particularly the case for the non-wetting countercurrent flow data. This provides a possible explanation for why the relative error is larger for α_2 as compared to α_1 in Dataset 3 in Table 1.

As was the case in Manai's experiments, Lelièvre (1966), when determining his relative permeability curves, assumed the flow equations were linear, which they are not. Because the same procedure was followed when determining the cocurrent and countercurrent relative permeability curves, it is again expected that approximately the same amount of error was introduced at each data point on the two curves. Consequently, the non-wetting and wetting curves were again found to be proportional, the constant of proportionality being the capillarity coupling parameter.

It is of interest to know whether the functions $k^*_{\text{rw}}(S)$ and $k^*_{\text{rwin}}(S)$ are the same. To determine whether such is the case, two approaches may be taken. First, one can plot the two sets of data and visually determine whether they are the same (see Fig. 6). However, such an approach is subjective in nature, and depends on the opinion of the analyst. Moreover, it becomes increasingly unreliable as the amount of scatter in the data increases. Second, one can undertake a statistical analysis of the data to decide whether the two curves are the same. Such an approach is objective in nature, and does not depend on the opinion of the analyst. In addition, it provides a quantitative estimate of the degree to which one is justified in saying that the two datasets fall on the same curve. As a consequence, because the authors believe the second approach is a superior way for ascertaining whether two datasets fall on the same curve, a statistical analysis was used to decide whether the two functions were the same. The first step in such an analysis is the demonstration that the population correlation coefficients for the two relative permeability curves differ from zero. Because the mean (μ) and standard deviation (σ) for the relative permeability data was unknown, and because of the small number of data points for each curve, Student's t test was used to show that, at the 0.05 level, the population coefficients of correlation for the relative permeability curves were different from zero. The next step is to test the hypothesis that the correlation coefficients obtained from the two relative permeability datasets are the same. Because these coefficients differed from zero, the sampling distributions of the correlation coefficient r for the two curves are skewed, and use must be made of Fischer's z transformation, when testing whether the two sample correlation coefficients differ significantly from each other. First, each correlation coefficient is converted into a z score using Fischer's r-to-z transformation. Then these z scores were combined, making use of the sample sizes for each curve, to obtain the z statistic. Using the z statistic, it was found that, at the 0.05 level, there was no significant difference between the sample correlation coefficients for $k^*_{\mathrm{rw}}(S)$ and $k^*_{\mathrm{rwin}}(S)$. In view of the fact that the sample correlation coefficients, at the 0.05 level, did not differ significantly, it can be inferred that $k^*_{\mathrm{rw}}(S)$ and $k^*_{\mathrm{rwin}}(S)$ are the same function. Similar statistical analyses were used to show that $k_{\mathrm{rw}}(S)$ and $k^*_{\mathrm{rw}}(S^*)/\alpha_1$(see Fig. 5) are the same function.

The question now arises as to whether the capillary coupling parameters, α_1 and α_2 are equal to each other (that is equal to α), and whether $\alpha = \alpha_{\mathrm{cr}}$. In an attempt to answer this question, a statistical analysis, similar to that described in the previous paragraph, was carried out for all of the remaining datasets. In each case the cocurrent relative permeability curve, and the countercurrent relative permeability curve divided by the appropriate value of α, was found to be the same function. Because the relative differences between α_{cr} and α_1 (or α_2) are all less than 5 % (see Table 1), and because the cocurrent relative curves and the countercurrent relative permeability curves divided by the appropriate value of α all appear to be the same function, it appears to be reasonable to suppose that differences between the experimental and theoretical estimates of α arise, not because of problems with the assumptions underlying Eqs. (26) and (34), but rather because of experimental errors in the basic data used to determine the relative permeability curves, and because of error introduced when using linear interpolation to estimate the data points needed to compare, at the same saturation, the countercurrent curves with the cocurrent curves. That is it appears to be reasonable to suppose that $\alpha_1 = \alpha_2 = \alpha_{\mathrm{cr}}$, and that, as a consequence, one can avoid the difficult problems of experimentally determining the magnitude of the capillary coupling parameter by making use of Eq. (43) to predict its magnitude.

4 Concluding Remarks

In this article, a partitioning concept developed in an earlier article (Bentsen 1998b) was introduced into Kalaydjian's flow equations (Kalaydjian 1987) to construct modified flow equations for linear, horizontal, two-phase cocurrent, and countercurrent flow through an isotropic, homogeneous porous medium. In constructing these equations, a more appropriate, experimentally based, capillary pressure gradient equation for countercurrent flow was used rather than the one that had been used in an earlier study (Bentsen 2001). With the aid of these equations, it is shown that the reduced flux observed in countercurrent flow, as compared to cocurrent flow, can be explained by the reduction in the driving force per unit volume brought about by capillary coupling. Moreover, it is demonstrated experimentally that, because the fluids flow through a void space reduced in magnitude because of the presence of immobile irreducible and residual saturations, the capillary coupling parameter should to be defined in terms of a reduced porosity, rather than in terms of the porosity as was done in an earlier study (Bentsen 2005b). In addition, it is demonstrated statistically that the countercurrent relative permeability curves are proportional to the cocurrent relative permeability curves, the constant of proportionally being the capillary coupling parameter. Finally, it is argued that one can avoid the undertaking of difficult to design and undertake countercurrent flow experiments to obtain countercurrent relative permeability curves by making use of an equation constructed for predicting the magnitude of the capillary coupling parameter.

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