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The Transient Flow Analysis of Fluid in a Fractal, Double-Porosity Reservoir

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Abstract The mathematical model for transient fluid flow in porous media is based in general on mass conservation principle. Because of the small compressibility of formation fluid, the quadratic term of pressure gradient is always ignored to linearize the non-linear diffusion equation. This may result in significant errors in model prediction, especially at large time scale. In order to solve this problem, it may be necessary to keep the quadratic term in the non-linear equations. In our study, the quadratic term is reserved to fully describe the transient fluid flow. Based on this rigorous treatment, the mathematical models are established to analyze the transient flow behavior in a double porosity, fractal reservoir with spherical and cylindrical matrix. In addition, Laplace transformation method is employed to solve these mathematical models and the type curves are provided to analyze the pressure transient characteristics. This study indicates that the relative errors in calculated pressure caused by ignoring the quadratic term may amount to 10 % in a fractal reservoir with double porosity, which can't be neglected in general for fractal reservoirs with double porosity at large time scale.

Keywords Fractal reservoir · Double porosity · Quadratic gradient · Transient flow · Laplace transform

1 Introduction

Fractal geometry is an effective tool to describe complex phenomenon, especially to scale the non-uniformity and non-sequence of porous media. If the fractal theory is used to study the

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fluid flow behavior in porous media, the geometry of porous media can be described precisely, and characterization of flow processes in porous media can be enhanced. Thus, the fractal approach has many advantages over the traditional method to characterize complicated and highly heterogeneous reservoirs. Among the early studies using the fractal method in porous media application, Chang and Yortsos (1990) developed a mathematical model to analyze the transient flow behavior of Newtonian fluid in fractal reservoirs. Chakrabarty et al. (1993a, b) presented another mathematical model for characterizing the behavior of non-Newtonian fluid in fractal reservoirs. These fluid flow models of fractal reservoirs make it convenient and easy to conduct pressure transient analysis for understanding flow in complex reservoirs (Beier 1990; Aprilian et al. 1993). Because of known flow geometry and parameters of fractal porous media, exact solutions of the flow models can often be obtained to analyze pressure characteristics for different flow models (Tong 1997; Ge et al. 2003; Zhang and Tong 2008).

The quadratic term of pressure gradient is traditionally ignored in almost all analytic mathematical models and solutions in the literature for slightly compressible fluid flow in reservoirs. There are very few studies to examine such an approximation and its applicability. Nevertheless, there have been some studies concerning this approach (Mattews and Russel 1967), especially for low permeability reservoirs. Consequently, the quadratic term was considered in the mathematical model of single-phase flow (Finjord and Adanoy 1989), and the effect of combined compressibility coefficient on each term was discussed. Laplace transformation was employed to solve the nonlinear diffusion equation with the quadratic term, and the analytic solution of bottom-hole pressure was obtained for the transient flow in porous media (Wang and Dusseault 1991). A higher-order derivative of pressure was used to quantitatively evaluate the effect of the quadratic term in diffusion equation (Chakrabarty et al. 1993a, b). In addition, on the basis of Green function, a modified method of logarithmic pressure transformation was presented to solve the diffusion equation with the quadratic term (Jelmert 1996; Jelmert and Vik 1996). There are still several related studies (Braeuning et al. 1998; Tong et al. 2002, 2004; Tong 2003; Xue and Tong 2008; Gonzales et al. 2008; Nie and Jia 2009), focusing on the solution method of the nonlinear diffusion equation with quadratic term, and coefficients of fluid compressibility, wellbore storage, and skin considered in their mathematical models. It was concluded from these studies that the linearization by ignoring the quadratic term could result in errors.

To characterize the fluid flow behavior in fractured porous media, the double-porosity model was introduced by Barenblatt et al. (1960). Then, an improved mathematical model was developed by Warren and Root (1963). The mathematical model for pressure transient analysis in naturally fractured reservoir was proposed by Swaano (1976) and Najurieta (1980). Furthermore, the theory of type-curve analysis was presented to determine the size of fracture and matrix in naturally fractured reservoir (Bourdet and Gringarten 1980). Tong and Liu (2003) presented a pseudo-steady fracture-matrix flow model for double-porosity fractal reservoirs with quadratic term taken into account for and derived the analytic solutions by Laplace Transformation method.

How to handle fracture-matrix flow or interaction is the key for simulation of flow through fractured media. Pseudo-steady flow assumption, since introduced by Warren and Root (1963), has been used by almost all flow models, including various types of double-porosity reservoirs and fractal media. To provide a better understanding of the transient flow behavior in a fractal reservoir, this article uses the fully transient flow model to handle fracture-matrix flow in a double-porosity fractal reservoir with spherical and cylindrical matrices, and the fractal dimensionality and fractal exponent are also accounted for in the mathematical model. The numerical solution of the semi-analytic solutions is obtained by Laplace transform and Stehfest inversion scheme. In addition, the effect of quadratic term on pressure transient behavior is discussed by type-curve analysis. Finally, the sensitivity analysis is conducted to evaluate the effect of channel flow factor, ratio of storage coefficient, fractal exponent, fractal dimensionality, and compressibility coefficient. This study can be used for well test analysis for a fractal, double-porosity reservoir.

2 Transient Flow Models with Spherical Matrix Block

Naturally fractured reservoirs typically are represented by the two-scale (fracture/matrix) model of Warren and Root. The fracture network is assumed to be connected and equivalent to a homogeneous medium of Euclidean geometry. According to naturally fractured reservoirs with multiple property scales and a non-Euclidean fracture network, fractal geometry is a natural candidate for the representation of such systems. We present a formulation for a fractal fracture network embedded into a Euclidean matrix as follows.

The matrix block and fractures of the double-porosity model is illustrated in Fig. 1, i.e., the matrix system consists of uniformly packed, identical spheres. On the surface of spheres, the pressure is equal to fracture pressure. The fluid in the matrix block provides the source item for the fracture system. The fractal dimensionality is d_f, embedded in Euclidean matrix block. Based on the fractal character of fractures,

$$k_\mathrm{f} = k_1 \left(\frac{r}{r_\mathrm{w}}\right)^{d_\mathrm{f}-\theta-d}, \quad \phi_\mathrm{f} = \phi_1 \left(\frac{r}{r_\mathrm{w}}\right)^{d_\mathrm{f}-d}, \quad \beta = d_\mathrm{f} - \theta - 1 \tag{1}$$

where k_1 is the permeability in the fracture ($r = r_\mathrm{w}$), and ϕ_1 is the porosity in the fracture ($r = r_\mathrm{w}$)

The differential form of the continuous equation can be written as follows (Tong and Cai, 2002):

$$\frac{\partial(\rho\phi_\mathrm{f})}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(r\rho v) - q = 0 \tag{2}$$

In Eq. (2), q is the flow rate per unit volume of matrix into fractures.

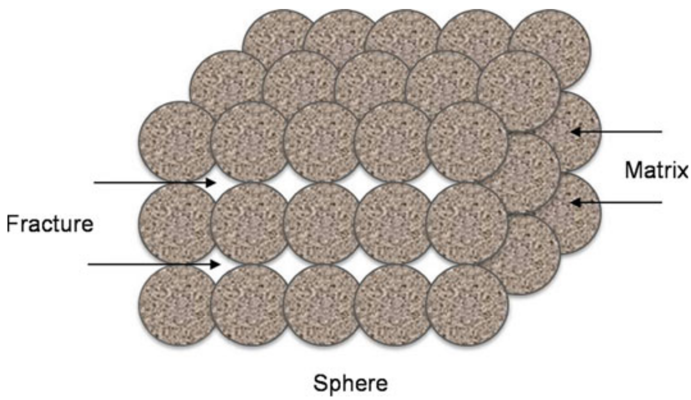

Fig. 1 Double porosity model with spherical matrix block

As to the spherical matrix block, its volume is $\frac{4\pi r_1^3}{3}$, and the spherical surface area is $4\pi r_1^2$, so

$$v = r_1 q/3. \tag{3}$$

Based on the Darcy law

$$v\Big|_{r=r_1} = -\frac{k_2}{\mu}\frac{\partial p_2}{\partial r}\Big|_{r=r_1} \tag{4}$$

Inserting Eq. (3) into Eq. (4), we have,

$$q = -\frac{3}{r_1}\frac{k_2}{\mu}\;\frac{\partial p_2}{\partial r}\Big|_{r=r_1} \tag{5}$$

Inserting Eq. (1) and Eq. (5) into Eq. (2),

$$\frac{\partial^2 p_1}{\partial r^2} + \frac{\beta}{r}\frac{\partial p_1}{\partial r} + c\left(\frac{\partial p_1}{\partial r}\right)^2 + \frac{\mu}{k_1}\left(\frac{r}{r_\mathrm{w}}\right)^\theta q = \frac{\mu c_{t1}}{k_1}\left(\frac{r}{r_\mathrm{w}}\right)^\theta \frac{\partial p_1}{\partial t} \tag{6}$$

We introduce dimensionless quantities, for fracture medium: $r_\mathrm{D} = r/r_\mathrm{w}$; for matrix: $r_{\mathrm{D}1} = r/r_1$; other dimensionless quantities are defined as follows:

$$t_\mathrm{D} = \frac{k_1 t}{(\phi_1 c_{t1} + \phi_2 c_{t2})\,\mu r_\mathrm{w}^2},\quad p_{\mathrm{D}i} = \frac{2\pi k_1 h\,(p_0 - p_i)}{\mu q},\quad \omega = \frac{\phi_1 c_{t1}}{\phi 1 c_{t1} + \phi_2 c_{t2}}$$
$$\lambda = 15\frac{k_2 r_\mathrm{w}^2}{k_1 r_1^2},\quad \alpha = \frac{\mu q c}{2\pi k_1 h}$$

The flow equations are then written in dimensionless form as follows:

$$\frac{\partial^2 p_{\mathrm{D}1}}{\partial r_\mathrm{D}^2} + \frac{\beta}{r_\mathrm{D}}\frac{\partial p_{\mathrm{D}1}}{\partial r_\mathrm{D}} - \alpha\left(\frac{\partial p_{\mathrm{D}1}}{\partial r_\mathrm{D}}\right)^2 - \frac{\lambda}{5} r_\mathrm{D}^\theta \frac{\partial p_{\mathrm{D}2}}{\partial r_{\mathrm{D}1}}\Big|_{r_{\mathrm{D}1}=1} = \omega r_\mathrm{D}^\theta \frac{\partial p_{\mathrm{D}1}}{\partial t_\mathrm{D}} \tag{7}$$

$$\frac{\partial^2 p_{\mathrm{D}2}}{\partial r_{\mathrm{D}1}^2} + \frac{2}{r_{\mathrm{D}1}}\frac{\partial p_{\mathrm{D}2}}{\partial r_{\mathrm{D}1}} - \alpha\left(\frac{\partial p_{\mathrm{D}2}}{\partial r_{\mathrm{D}1}}\right)^2 = \frac{15\,(1-\omega)}{\lambda}\frac{\partial p_{\mathrm{D}2}}{\partial t_\mathrm{D}} \tag{8}$$

Initial condition is

$$p_{\mathrm{D}1}|_{t_\mathrm{D}=0} = p_{\mathrm{D}2}|_{t_\mathrm{D}=0} = 0 \tag{9}$$

Inner boundary conditions are

$$\frac{\partial p_{\mathrm{D}2}}{\partial r_{\mathrm{D}1}}\Big|_{r_{\mathrm{D}1}=0} = 0 \tag{10}$$

$$p_{\mathrm{D}2}|_{r_{\mathrm{D}1}=1} = p_{\mathrm{D}1} \tag{11}$$

$$C_\mathrm{D}\frac{\partial p_{\mathrm{D}1}}{\partial t_\mathrm{D}}\Big|_{r_\mathrm{D}=1} - r_\mathrm{D}^\beta \frac{\partial p_{\mathrm{D}1}}{\partial r_\mathrm{D}}\Big|_{r_\mathrm{D}=1} = 1 \tag{12}$$

Outer boundary condition is

$$\lim_{r_\mathrm{D}\to\infty} p_{\mathrm{D}1} = 0 \tag{13}$$

We substitute the following transform:

$$p_{\mathrm{D}i} = -\frac{1}{\alpha}\ln(1 - x_i)\quad (i = 1,\ 2) \tag{14}$$

In Eqs. (7)–(13), we have

$$\frac{\partial^2 x_1}{\partial r_D^2} + \frac{\beta}{r_D}\frac{\partial x_1}{\partial r_D} - \frac{\lambda}{5} r_D^\theta \left.\frac{\partial x_2}{\partial r_{D1}}\right|_{r_{D1}=1} = \omega r_D^\theta \frac{\partial x_1}{\partial t_D} \tag{15}$$

$$\frac{\partial^2 x_2}{\partial r_{D1}^2} + \frac{2}{r_{D1}}\frac{\partial x_2}{\partial r_{D1}} = \frac{15\,(1-\omega)}{\lambda}\frac{\partial x_2}{\partial t_D} \tag{16}$$

$$x_1|_{t_D=0} = x_2|_{t_D=0} = 0 \tag{17}$$

$$\left.\frac{\partial x_2}{\partial r_{D1}}\right|_{r_{D1}=0} = 0 \tag{18}$$

$$x_2|_{r_{D1}=1} = x_1 \tag{19}$$

$$\left.\left(C_D\frac{\partial x_1}{\partial t_D} + \alpha x_1 - \frac{\partial x_1}{\partial r_D}\right)\right|_{r_D=1} = \alpha \tag{20}$$

$$\lim_{r_D\to\infty} x_1 = 0 \tag{21}$$

Through Laplace transform,

$$\overline{P}_D(r_D,\, s) = \int_0^\infty \mathrm{e}^{-s\tau} p_D(r_D,\, \tau)d\tau \tag{22}$$

the solutions of the equation in the Laplace space can be obtained as

$$\bar{x}_1 = \frac{\alpha r_D^{\frac{1-\beta}{2}} K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sf\,(s)}r_D^{\frac{\theta+2}{2}}\right)}{s\left[(C_D s+\alpha)K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sf\,(s)}\right) + \sqrt{sf\,(s)}K_{\frac{1-\beta}{\theta+2}-1}\left(\frac{2}{\theta+2}\sqrt{sf\,(s)}\right)\right]} \tag{23}$$

where $f\,(s) = \omega + \frac{\lambda}{5s}\left\{\sqrt{\frac{15(1-\omega)s}{\lambda}}cth\sqrt{\frac{15(1-\omega)s}{\lambda}} - 1\right\}$

At the wellbore, Eq. (23) can be simplified as

$$\bar{x}_w\,(s) = \frac{\alpha K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sf\,(s)}\right)}{s\left[(C_D s+\alpha)K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sf\,(s)}\right) + \sqrt{sf\,(s)}K_{\frac{1-\beta}{\theta+2}-1}\left(\frac{2}{\theta+2}\sqrt{sf\,(s)}\right)\right]} \tag{24}$$

$$p_{wD} = \frac{1}{\alpha}\ln\left\{1 - L^{-1}\left[\bar{x}_w\,(s)\right]\right\} \tag{25}$$

Using Stehfest (1970a,b) inversion to Eq. (24), we can evaluate the dimensionless pressure from Eq. (25).

3 Results and Discussion

In the following analysis, we will use the analytic solution to discuss the nonlinear pressure transient. Figure 2 demonstrates the variation of nonlinear dimensionless bottom-hole pressures with time for different values of α. It shows that α affects the pressure curve in the entire flow process. The impact of α is relatively small initially. As time increases, the effect of α becomes larger and larger. As seen in Fig. 2, the pressure curves become very different with different α. The smaller the α is, the larger the dimensionless pressure is.

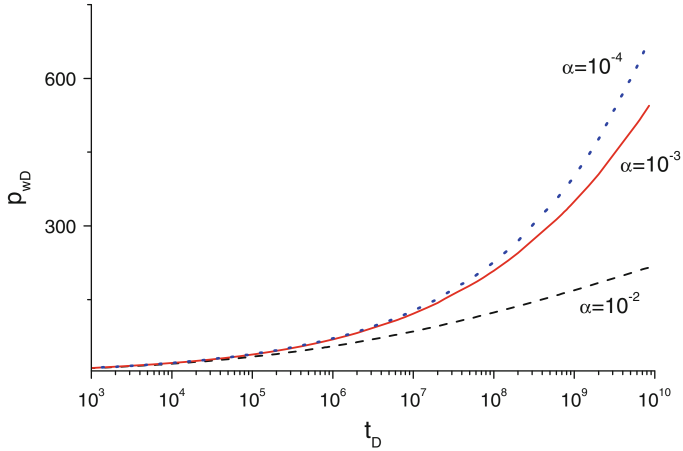

Fig. 2 The dimensionless pressure versus time based on the magnitude of α

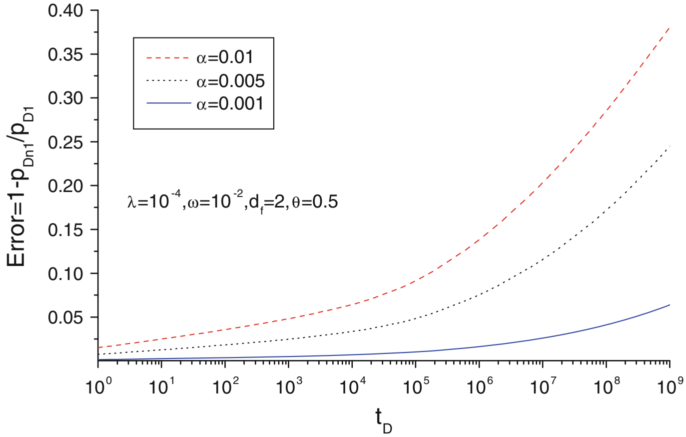

Fig. 3 Pressure error distribution versus time depending on α

The nonlinear and linear (as plotted In Fig. 3) solutions show very small differences in the beginning. However, controlled by the magnitude of α, the difference increases with time. In order to quantify the difference between the linear and nonlinear pressure solutions, we use the following term (Chakrabarty et al. 1993a, b):

$$\varepsilon = 1 - \frac{p_{\mathrm{Dnl}}}{p_{\mathrm{Dl}}} \tag{26}$$

where p_{Dnl} and p_{Dl} are dimensionless nonlinear and linear solutions, respectively. From Eq. (26), it can be seen that the bigger the term "ε" is, the larger the difference between the linear and nonlinear pressure solutions is. The variation of pressure with each parameter is discussed as follows.

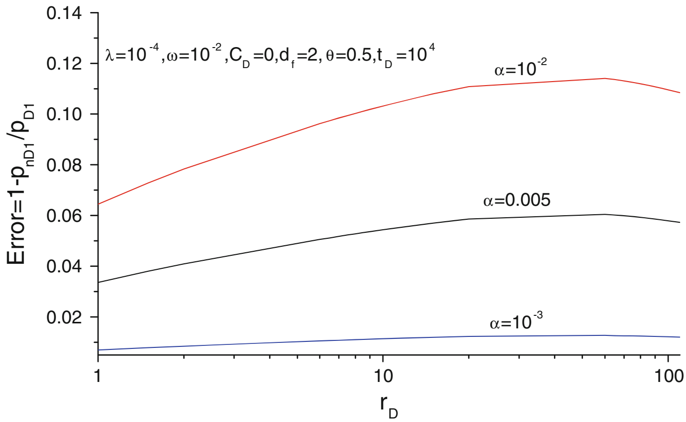

Fig. 4 Errors in calculated pressure with different α values

3.1 Effect of Combination of Compression Coefficient α

Figure 3 displays the error distribution of pressures from linear solution and nonlinear solution versus time. Initially, α has little impact on the error distribution. As the time increases, such influence increases up to 40 %. This means that we cannot ignore the quadratic gradient term in the fractal reservoir; otherwise, there will be a huge error.

Figure 4 shows the spatial distribution, indicating that errors will increase with the increase in radial distance. When they reach the maximum value, the curve will be flat or start to decline. For a long time or a large distance from the wellbore, the errors could be higher up to 10 %. If α is large, then the error may be greater for injection of slightly compressible fluid than for the formation with low permeability.

3.2 Effect of Fractal Reservoir Parameters

According to a naturally fractured reservoir with a non-Euclidean fracture network, Fig. 5 shows the error distribution of pressures, calculated using linear and nonlinear solutions, which are impacted by the parameter,d_f throughout the process. Initially, the influence of d_f is very small. As the time increases, the influence increases and as shown on the plot, the smaller the value of d_f is, the more rapidly the error will increase, say up to about 70 %. The matrix is a Euclidean object within which the fracture network is embedded. If the fracture network is also Euclidean, then its dimension is 2 in the dual-porosity case. The smaller the value of d_f of fractal structure is, the greater will be the increase in error; it may cause too significant errors to the predicted pressure to allow us ignore the quadratic pressure gradient term in a fractal reservoir with double porosity.

Figure 6 shows the influence of θ on error distribution. Initially, there is little effect on errors of pressure. As time increases, however, errors increase sharply. The greater value of θ leads to a quicker increase in errors and starts earlier. The upper-most curve of error distribution of Fig. 6 corresponds to $\theta = 0.7$; the maximum error can be up to 60 %. Therefore, the results show that we cannot ignore the quadratic gradient term in a fractal reservoir. The curve and comparison are similar to the error distribution profile as obtained with α (Fig. 4).

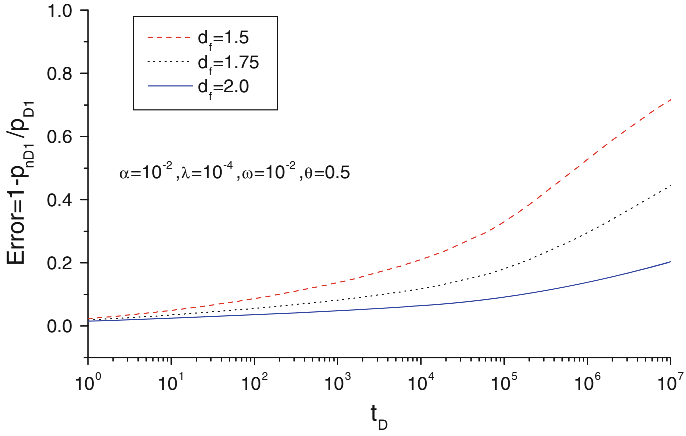

Fig. 5 Errors in calculated pressures, depending on d_f

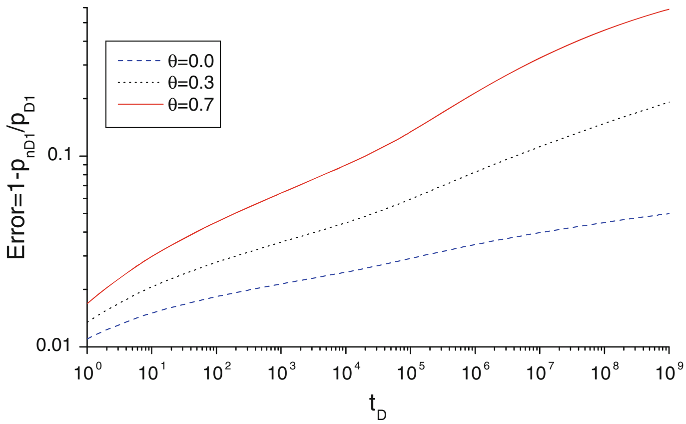

Fig. 6 Errors in calculated pressures, depending on θ

3.3 Effect of Wellbore Storage

In a constant injection case, Fig. 7 shows curves of the nonlinear dimensionless wellbore pressure versus time when $C_D = 0, \ 1000$, the nonlinear solution is obtained with $\alpha = 0.01, \ 0.001$. From Fig. 7, we can find that the difference of nonlinear solutions is obvious at later time with greater C_D, and it also shows that the nonlinear solution's error between different α values can reach 15 % when $t_D = 10^7$, but there is almost no difference initially.

3.4 Effect of Double-Porosity Characteristic Parameters

Figure 8 shows the transient behavior of dimensionless borehole pressure versus time, which is determined using the nonlinear solution with $\omega = 0.1, \ 0.01$, and $\lambda = 0.01, \ 0.0001$. As expected, the influence of ω is reflected initially, while the influence of λ appears during the intermediate time, i.e., transition period.

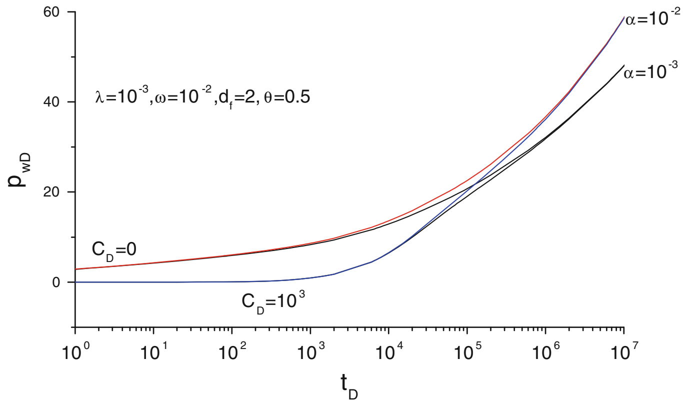

Fig. 7 Comparison between linear and nonlinear solutions with different C_D

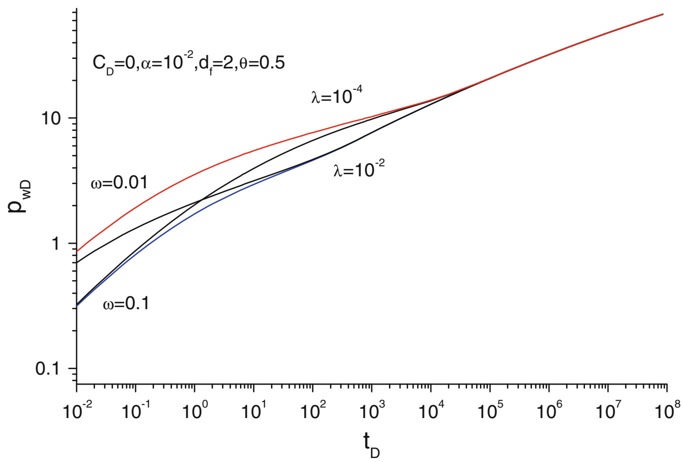

Fig. 8 Transient pressure behavior at well with different double-porosity parameters $\omega,\ \lambda$

4 Transient Flow Model with Cylindrical Matrix Block

The cylinder radius is r_1, the pressure distribution within the matrix block is cylindrically symmetric, as shown in Fig. 9. Similar to the model with spherical matrix block, the differential form of the continuous equation of fracture can be written as

$$\frac{\partial^2 p_1}{\partial r^2} + \frac{\beta}{r}\frac{\partial p_1}{\partial r} + c\left(\frac{\partial p_1}{\partial r}\right)^2 + \frac{\mu}{K_1}\left(\frac{r}{r_w}\right)^\theta q = \frac{\mu c_{t1}\phi_1}{K_1}\left(\frac{r}{r_w}\right)^\theta \frac{\partial p_1}{\partial t} \tag{27}$$

the solution of Eq. (27) in the Laplace space is obtained as followed,

$$\bar{y}_1 = \frac{\alpha r_D^{\frac{1-\beta}{2}} K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sg(s)}\, r_D^{\frac{\theta+2}{2}}\right)}{s\left[(\alpha + C_D s) K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sg(s)}\right) + \sqrt{sg(s)}\, K_{\frac{1-\beta}{\theta+2}-1}\left(\frac{2}{\theta+2}\sqrt{sg(s)}\right)\right]} \tag{28}$$

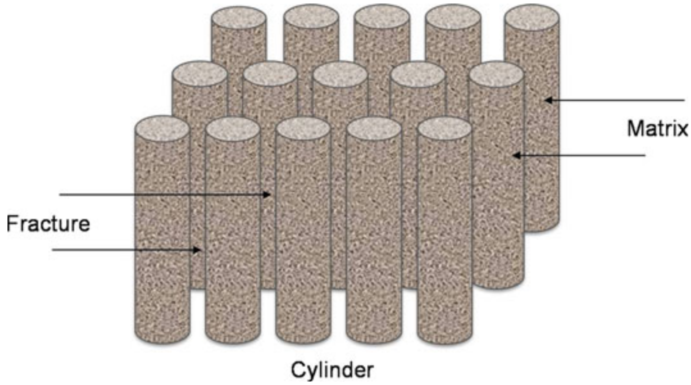

Fig. 9 Double-porosity model with cylindrical matrix block

As to wellbore, Eq. (28) can be simplified as

$$\bar{y}_{\mathrm{w}}(s)=\frac{\alpha K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sg(s)}\right)}{s\left[(\alpha+C_{\mathrm{D}}s)K_{\frac{1-\beta}{\theta+2}}\left(\frac{2}{\theta+2}\sqrt{sg(s)}\right)+\sqrt{sg(s)}K_{\frac{1-\beta}{\theta+2}-1}\left(\frac{2}{\theta+2}\sqrt{sg(s)}\right)\right]} \tag{29}$$

where

$$g(s)=\omega+2\sqrt{\frac{15(1-\omega)s}{\lambda}}\frac{I_1\left(\sqrt{\frac{15(1-\omega)s}{\lambda}}\right)}{I_0\left(\sqrt{\frac{15(1-\omega)s}{\lambda}}\right)}$$

$$p_{\mathrm{wD}}=\frac{1}{\alpha}\ln\left\{1-L^{-1}\left[\bar{x}_{\mathrm{w}}(s)\right]\right\}$$

Figure 10 shows the errors introduced by the linear solution, when compared with the exact, nonlinear solution for constant injection rate production, which is influenced by α during the entire transient flow period. Initially, the difference between the two solutions is small. As time increases, however, the difference becomes significant. The bigger α is, the more larger the difference of error is (when $\alpha = 10^{-2}$, the error reaches about 70 %). For a smaller (when $\alpha = 10^{-3}$), the difference is about 20 %.

Figure 11 shows the error variation with d_{f}, which is similar to those with the sphere matrix block model, that is, as regards the difference in pressures, calculated from the two solutions, it changes with different d_{f} values. The smaller d_{f} is, the larger the error is, which is up to about 90 %.

The matrix is a Euclidean object (i.e., of dimension $d = 2$ for cylindrically symmetric reservoirs) within which the fracture network is embedded. The fracture network is non-Euclidean. Similar to spherical matrix block, the smaller the d_{f} value of the fractal structure is, the greater will be the error increase; it may cause too significant errors to the predicted pressure to allow us ignore the quadratic pressure gradient term in a fractal reservoir with double porosity. Hence, nonlinear term is more important for fractal reservoirs than those of Euclidean geometry.

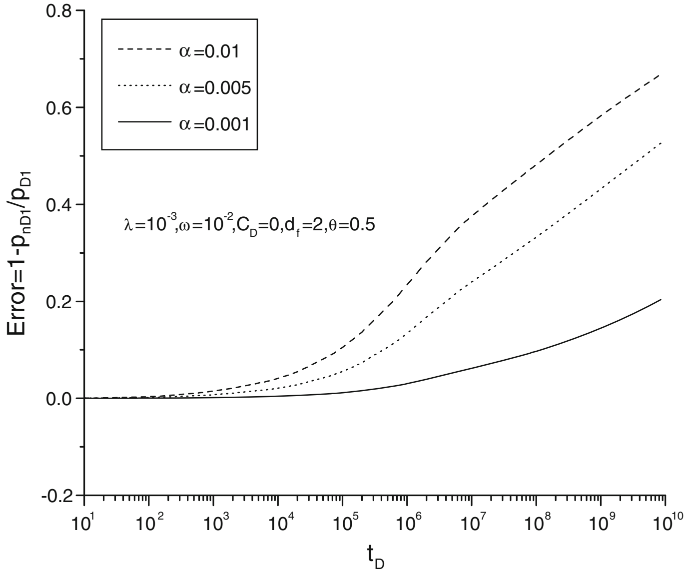

Fig. 10 Errors of pressure with different α values for the cylindrical matrix model

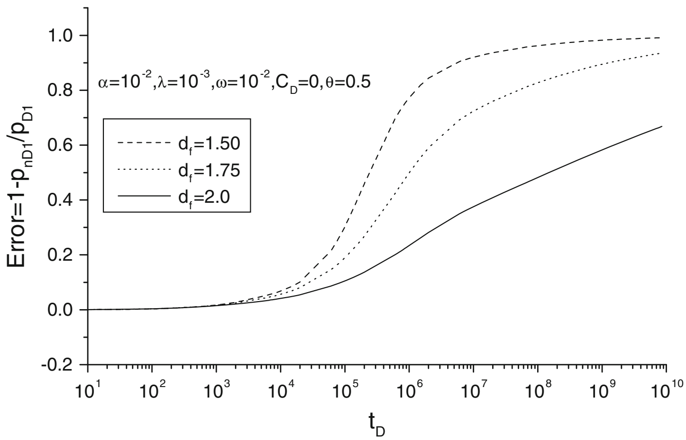

Fig. 11 Errors in calculated pressure with different d_f values for the cylindrical matrix model

5 Concluding Remarks

This article presents exact solutions for transient flow in fractal-fractured reservoir with spherical and cylindrical matrix blocks, the two double-porosity conceptual models. The model accommodates the effect of wellbore storage as well as fractal parameters. Both the fractal reservoir flow models are consistent with the principle of mass conservation without ignoring quadratic pressure gradient terms. A methodology to solve the diffusion with the nonlinear quadratic gradient terms is proposed. The typical pressure transient behavior is analyzed using the analytic solutions. Several important conclusions can be drawn from this study:

(1) The new type curves for transient pressure responses in the fractal double-porosity reservoir with the quadratic pressure gradient term enable us overcome the shortcomings of the traditional type curves used for analyzing well testing data.
(2) We use the analytic solutions to study the transient flow behavior with and without the quadratic pressure gradient terms in our fractal, double-porosity model. The study results indicate that the quadratic gradient term does impact pressure transient. The errors introduced by ignoring the nonlinear quadratic pressure gradient term are directly proportional to the parameter, α, and time. The relative error in pressure predicted by the nonlinear model is very sensitive to the four parameters: $\alpha,\ c_{\mathrm{D}},\ c_{\mathrm{D}}$, and θ. The error increases with time.
(3) It may cause too significant errors to the predicted pressure to allow us ignore the quadratic pressure gradient term in the flow governing equations, especially in a fractal reservoir with double porosity.

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