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A Note on Local Thermal Non-Equilibrium in Porous Media Near Boundaries and Interfaces

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Abstract Recent work on what has been called the phenomenon of heat flux bifurcation, that occurs at a boundary of a porous medium, or at an interface with a fluid clear of solid material, when a two-temperature model for the porous medium is employed, is discussed. An alternative interpretation of the situation, one in which the physics of the problem is emphasized, is presented.

Keywords Local thermal non-equilibrium · Interfaces · Boundary conditions

1 Introduction

This note is a response to a group of papers by Yang and Vafai (2010, 2011a, b, c) on what the authors call 'the phenomenon of heat flux bifurcation in porous media'. They analysed the details of the thermal boundary conditions at the interface between a porous medium and a fluid clear of solid material in situations where there is local thermal non-equilibrium (LTNE), that is where the temperature distribution in the solid phase of a porous medium is different from the distribution in the fluid phase. They considered five sets of boundary conditions and obtained analytical solutions for the fluid temperature and the solid temperature for each set. They treated the five cases in a more-or-less even handed manner, and provided little discussion of the merits on physical grounds of their five models.

In the opinion of the present author, the approach and terminology employed by Yang and Vafai is misleading. For one thing, there is no bifurcation in the sense that a single parameter exists such a bifurcation of a solution occurs as the value of that parameter passes through a certain critical value. Rather, Yang and Vafai treat a more general situation, one in which the solution of the differential equation system that arises is undetermined until further information is available to determine how the total heat flux is split between the fluid and solid

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phases. Such a situation was previously noted by Nield and Kuznetsov (1999), a paper that treated forced convection in a plane channel occupied by a porous medium bounded by solid slabs. The indeterminacy arises because the standard model for local thermal non-equilibrium involves two thermal energy equations which are coupled thermally in a simplified manner, namely by a term proportional to the difference between the temperatures in the two phases with a coefficient (the interphase heat transfer coefficient) that is assumed to be constant. Since one then has a system consisting of two second-order differential equations, one needs at an interface four boundary conditions, two involving the temperature and two involving the heat flux. The conservation of energy imposes just one heat flux condition, and hence another condition must be sought. This depends on the choice of model. The indeterminacy is a generic phenomenon, and so the matter of heat flux splitting can arise in various situations. Thus, it is not surprising that Yang and Vafai (2011b) reported that they had identified 'two primary types of heat flux bifurcations', one of which was confined to a transient situation.

In the remainder of this note, attention is focussed on the analysis in Yang and Vafai (2011c), the most recent of their four papers. The relative merits of the five models on physical grounds are discussed.

2 Selection of Model

Yang and Vafai (2011c) considered flow in the x-direction in a composite channel in which the region between $y = 0$ and $y = H_1$ is occupied by a porous medium and that between $y = H_1$ and $y = H$ occupied by a fluid clear of solid material. They employed as energy equations

$$k_{\mathrm{f,eff}} \frac{\partial^2 T_{\mathrm{f}}}{\partial y^2} + h_{\mathrm{i}} \alpha \left(T_{\mathrm{s}} - T_{\mathrm{f}}\right) = \rho c_{\mathrm{p}} u \frac{\partial T_{\mathrm{f}}}{\partial x}, \tag{1}$$

$$k_{\mathrm{s,eff}} \frac{\partial^2 T_{\mathrm{s}}}{\partial y^2} - h_{\mathrm{i}} \alpha \left(T_{\mathrm{s}} - T_{\mathrm{f}}\right) = 0. \tag{2}$$

Here $\mathrm{T_f}$ and T_{s} are the temperatures in the fluid and solid phases, $k_{\mathrm{f,eff}}$ and $k_{\mathrm{s,eff}}$ are the effective fluid and solid thermal conductivities, defined by

$$k_{\mathrm{f,eff}} = \varepsilon k_{\mathrm{f}}, \tag{3}$$

$$k_{\mathrm{s,eff}} = (1 - \varepsilon) k_{\mathrm{s}}, \tag{4}$$

and ρ and c_{p} are the density and specific heat of the fluid, h_{i} is the interstitial heat transfer coefficient, and α is the interfacial area per unit volume of the porous medium. For thermal boundary conditions, they adopt the following in turn.

2.1 Model A

$$T_{\mathrm{f}} \Big|_{y=H_1^-} = T_{\mathrm{s}} \Big|_{y=H_1^-} = T_{\mathrm{f}} \Big|_{y=H_1^+}, \tag{5}$$

$$k_{\mathrm{f,eff}} \frac{\partial T_{\mathrm{f}}}{\partial y} \Big|_{y=H_1^-} + k_{\mathrm{s,eff}} \frac{\partial T_{\mathrm{s}}}{\partial y} \Big|_{y=H_1^-} = k_{\mathrm{f}} \frac{\partial T_{\mathrm{f}}}{\partial y} \Big|_{y=H_1^+} = q_{\mathrm{i}}, \tag{6}$$

where q_{i} is the heat flux at the interface and k_{f} is the thermal conductivity in the clear fluid.

Yang and Vafai (2011c) introduce this model by saying that: 'When the heat transfer between the fluid and solid phases at the interface is large enough, these temperatures are equal at the interface'.

2.2 Model B

$$T_{\mathrm{f}}\Big|_{y=H_1^-} = T_{\mathrm{f}}\Big|_{y=H_1^+}, \tag{7}$$

$$k_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial y}\Big|_{y=H_1^+} = q_{\mathrm{i}}, \tag{8}$$

$$k_{\mathrm{f,eff}}\frac{\partial T_{\mathrm{f}}}{\partial y}\Big|_{y=H_1^-} = \beta q_{\mathrm{i}}, \tag{9}$$

$$k_{\mathrm{s,eff}}\frac{\partial T_{\mathrm{s}}}{\partial y}\Big|_{y=H_1^-} = (1-\beta) q_{\mathrm{i}}, \tag{10}$$

where β is the ratio of the heat flux for the fluid region to the total heat flux at the interface.

Yang and Vafai (2011c) introduce this model by saying that 'For most cases, the heat transfer between the fluid and solid phases at the interface is not large enough, thus their temperatures are not equal at the interface. Therefore, a interface thermal parameter, β, is introduced to evaluate the total heat flux distribution between the solid and fluid phases at the interface in Model B'.

They then refined this model, with the ratio being given by the following alternative expressions:

$$\text{Model B1}: \beta_1 = \frac{k_{\mathrm{f,eff}}}{k_{\mathrm{f,eff}} + k_{\mathrm{s,eff}}}. \tag{11}$$

$$\text{Model B2}: \beta_2 = \frac{k_{\mathrm{f}}}{k_{\mathrm{f}} + k_{\mathrm{s}}}. \tag{12}$$

$$\text{Model B3}: \beta_3 = \varepsilon, \tag{13}$$

where ε is the porosity and k_{f} and k_{s} are the fluid and solid thermal conductivities, respectively.

The fact that no preference of any particular one of the three sub-models B1, B2, B3 has been given means that an important question has not been addressed by Yang and Vafai. The question is, how is q_{i} controlled? Is it not something to be determined as part of the solution (though it need not be explicitly calculated), which in turn depends on the whole problem?

2.3 Model C

$$T_{\mathrm{f}}\Big|_{y=H_1^-} = T_{\mathrm{f}}\Big|_{y=H_1^+}, \tag{14}$$

$$k_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial y}\Big|_{y=H_1^+} = q_{\mathrm{i}}, \tag{15}$$

$$k_{\mathrm{f,eff}}\frac{\partial T_{\mathrm{f}}}{\partial y}\Big|_{y=H_1^-} = q_{\mathrm{i}} - h_{\mathrm{int}}\left(T_{\mathrm{f}}\Big|_{y=H_1^-} - T_{\mathrm{s}}\Big|_{y=H_1^-}\right), \tag{16}$$

$$k_{\mathrm{s,eff}}\frac{\partial T_{\mathrm{s}}}{\partial y}\Big|_{y=H_1^-} = h_{\mathrm{int}}\left(T_{\mathrm{f}}\Big|_{y=H_1^-} - T_{\mathrm{s}}\Big|_{y=H_1^-}\right). \tag{17}$$

In Yang and Vafai (2011c), this model was introduced with the explanation: 'The temperatures of fluid and solid phases are considered not to be equal at the interface, and the heat flux jump interfacial condition presented by Ochoa-Tapia and Whitaker (1997) is utilized as the basis for Model C, in which a interface heat transfer, h_{int}, is introduced to calculate the heat exchange between fluid and solid phases at the interface'.

The results of Ochoa-Tapia and Whitaker (1997) were based on the assumption of what they called local gradient equilibrium, i.e. that the temperature gradients in the two phases of the porous medium can be equated even when the two temperatures are not equal. As far as the present author is aware, no physical justification has been given for the assumption of local gradient equilibrium.

It is the contention of the present author that one should distinguish between the heat transfer in the bulk of the porous medium (which is governed by the two thermal energy equations, and so depends on the value of the interphase heat transfer coefficient) and the heat transfer across the interface (which is affected by what happens on the other side of the interface, i.e. outside the porous medium). The flux splitting at the interface is largely controlled by the overall situation. For example, if the porous medium is bounded by a solid with high thermal conductivity (a constant-temperature boundary), then Model A is obviously applicable. In the context of a forced convection problem, the Model A formulation has already been applied by Nield and Kuznetsov (2011).

Much the same is true if the neighbouring region is a fluid of high conductivity. If the region is a solid of very low conductivity (an insulating boundary), then there is no boundary flux to be divided between the two phases. More generally, if the solid boundary is controlled by a constant flux, the natural result is that just across the interface in the porous medium the flux is also constant. Thus, the flux is split so that the flux in the fluid phase is the same as in the solid phase. That means that the interfacial heat transport is divided between the fluid and solid phases in the ratio of ε to $(1 - \varepsilon)$. In other words, in these circumstances, Model B3 is an appropriate model. It was employed by Nield and Kuznetsov (1999). It appears that there is no physical support for either Model B1 or Model B2, and not much for Model C

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