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The Onset of Convection in a Heterogeneous Porous Medium with Vertical Throughflow

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Abstract The effect of vertical heterogeneity of permeability, on the onset of convection in a horizontal layer of a saturated porous medium, uniformly heated from below but with a non-uniform basic temperature gradient resulting from vertical throughflow, is studied analytically using linear stability theory. It is found that, to first order, a linear variation of the reciprocal of permeability with depth has no effect on the critical value of the Rayleigh number Ra_c based on the harmonic mean of the permeability, but a quadratic variation increasing in the upwards direction leads to a reduction in Ra_c.

Keywords Permeability heterogeneity · Throughflow · Thermal instability · Horizontal layer

List of Symbols

a	Dimensionless horizontal wavenumber
c_a	Acceleration coefficient
$f(z)$	Function characterizing the basic temperature gradient, defined by Eq. 25
$g(z)$	Function characterizing the reciprocal of the permeability, defined by Eq. 25
$\mathbf{g}$	Gravitational acceleration
H	Dimensional layer depth
k_m	Effective thermal conductivity of the porous medium
K	Permeability of the porous medium
P^*	Pressure

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P	Dimensionless pressure, $P^* K/\mu\alpha_\mathrm{m}$
Q	Péclet number defined by Eq. 14
Ra	Rayleigh–Darcy number
t^*	Time
t	Dimensionless time, $t^*\alpha_\mathrm{m}/\sigma H^2$
T^*	Temperature
T	Dimensionless temperature, $(T^* - T_0)/(T_1 - T_0)$
T_0^*	Temperature at the upper wall
T_1^*	Temperature at the lower wall
(u, v, w)	Dimensionless Darcy velocity components, $(u^*, v^*, w^*)H/\alpha_\mathrm{m}$
V_0	Throughflow velocity
$\mathbf{v}$	Dimensionless Darcy velocity, $\frac{(\rho c)_\mathrm{f} H}{k_\mathrm{m}}$
$\mathbf{v}^*$	Dimensional Darcy velocity, (u^*, v^*, w^*)
(x, y, z)	Dimensionless Cartesian coordinates, $(x^*, y^*, z^*)/H$; z is the vertically upward coordinate
(x^*, y^*, z^*)	Cartesian coordinates

Greek Symbols

α_m	Thermal diffusivity of the porous medium, $\frac{k_\mathrm{m}}{(\rho c_P)_\mathrm{f}}$
γ_a	Acceleration coefficient defined by Eq. 10
γ	Permeability linear heterogeneity parameter defined by Eq. 36
δ	Permeability quadratic heterogeneity parameter defined by Eq. 40
μ	Viscosity of the fluid
ρ	Fluid density
$(\rho c)_\mathrm{f}$	Heat capacity of the fluid
$(\rho c)_\mathrm{m}$	Effective heat capacity of the porous medium
σ	Thermal capacity ratio defined by Eq. 6

Superscripts

*	Dimensional variable
′	Perturbation variable

Subscript

b	Basic solution

1 Introduction

The Horton-Rogers-Lapwood (HRL) problem, which is concerned with the onset of convection in a horizontal layer of a porous medium uniformly heated form below, is a classical one. The effects of heterogeneity of permeability and thermal conductivity have been widely studied. The early studies are surveyed in Section 6.13 of Nield and Bejan (2006). A particularly interesting paper is that by Braester and Vadasz (1993). More recent work involving weak heterogeneity, both horizontal and vertical, was surveyed by Nield (2008), while moderate and strong heterogeneity has been studied by Nield and Simmons (2007), Nield and Kuznetsov (2008), Nield et al. (2009, 2010), Kuznetsov et al. (2010), Kuznetsov et al. (2011) and Simmons et al. (2010).

The effect of vertical throughflow on the HRL problem has also been extensively studied, and the bulk of this work has been surveyed in Section 6.10 of Nield and Bejan (2006). Significant recent papers are those by Shivakumara and Nanjundappa (2006), Hill (2007), Hill et al. (2007), Brevdo (2009), Brevdo and Ruderman (2009a,b) and Barletta et al. (2010).

As far as we know, the interaction of heterogeneity and throughflow has not been studied previously. The general problem, with both vertical and horizontal heterogeneity, is complicated because the heterogeneity of permeability affects the basic solution. In the present paper we consider only vertical heterogeneity (property variation in the vertical direction) and we investigate only first order effects of heterogeneity.

2 Analysis

Single-phase flow in a saturated porous medium is considered. Asterisks are used to denote dimensional variables. We consider a horizontal layer occupying $0 \leq z^* \leq H$, where the z^*-axis is in the upward vertical direction. Uniform temperatures T_0 and T_1 are imposed at the upper and lower boundaries, respectively. Within this layer the permeability is assumed to be vertically heterogeneous and is denoted by $K(z^*)$. Consistent with this assumption we suppose that there is a uniform basic flow with velocity V_0 in the z-direction.

The Darcy velocity is denoted by $\mathbf{v}^* = (u^*, v^*, w^*)$. The Oberbeck–Boussinesq approximation is invoked and local thermal equilibrium is assumed. The equations representing the conservation of mass, thermal energy, and Darcy's law take the form (see, for example, Eqs. 6.3–6.6 of Nield and Bejan (2006)):

$$\nabla^* \cdot \mathbf{v}^* = 0, \tag{1}$$

$$(\rho c)_{\mathrm{m}} \frac{\partial T^*}{\partial t^*} + (\rho c)_{\mathrm{f}} \mathbf{v}^* \cdot \nabla^* T^* = k_{\mathrm{m}} \nabla^2 T^*, \tag{2}$$

$$c_{\mathrm{a}} \rho_0 \frac{\partial \mathbf{v}^*}{\partial t^*} = -\nabla^* P^* - \frac{\mu}{K(z^*)} \mathbf{v}^* + \rho_0 [1 - \beta(T^* - T_0)] \mathbf{g}. \tag{3}$$

Here $(\rho c)_{\mathrm{m}}$ and $(\rho c)_{\mathrm{f}}$ are the heat capacities of the overall porous medium and the fluid, respectively, μ is the fluid viscosity, k_{m} is the effective thermal conductivity of the porous medium, c_{a} is the acceleration coefficient, ρ_0 is the fluid density at temperature T_0, and β is the volumetric expansion coefficient, while P^* is the excess of pressure over the reference hydrostatic value.

We introduce dimensionless variables by defining

$$\mathbf{x} = \frac{\mathbf{x}^*}{H}, \tag{4a}$$

$$\mathbf{v} = \frac{(\rho c)_{\mathrm{f}} H}{k_{\mathrm{m}}} \mathbf{v}^*, \tag{4b}$$

$$t = \frac{k_{\mathrm{m}}}{(\rho c)_{\mathrm{m}} H^2} t^*, \tag{4c}$$

$$T = \frac{T^* - T_0}{T_1 - T_0}, \tag{4d}$$

$$P = \frac{(\rho c)_{\mathrm{f}} K_{\mathrm{H}}}{\mu k_{\mathrm{m}}} P^*, \tag{4e}$$

where K_{H} is the harmonic mean value of $K(z^*)$.

We also define a Rayleigh number Ra by

$$Ra = \frac{(\rho c)_{\mathrm{f}} \rho_0 g \beta K_{\mathrm{H}} H (T_1 - T_0)}{\mu k_{\mathrm{m}}} \tag{5}$$

and the heat capacity ratio

$$\sigma = \frac{(\rho c)_{\mathrm{m}}}{(\rho c)_{\mathrm{f}}}. \tag{6}$$

The governing equations then take the form

$$\nabla \cdot \mathbf{v} = 0, \tag{7}$$

$$\frac{\partial T}{\partial t} + \mathbf{v} \cdot \nabla T = \nabla^2 T, \tag{8}$$

$$\gamma_{\mathrm{a}} \frac{\partial \mathbf{v}}{\partial t} = -\nabla P - \frac{K_{\mathrm{H}}}{K(z^*)} \mathbf{v} + Ra \left[T - \frac{1}{\beta (T_1 - T_0)} \right] \mathbf{e}_z. \tag{9}$$

where

$$\gamma_{\mathrm{a}} = \frac{c_{\mathrm{a}} \rho_0 k_{\mathrm{m}} K_{\mathrm{H}}}{\sigma \mu (\rho c)_{\mathrm{f}} H^2}. \tag{10}$$

The basic steady-state solution is given by $\mathbf{v}_{\mathrm{b}}$, T_{b}, P_{b}, where

$$\mathbf{v}_{\mathrm{b}} = Q \mathbf{e}_z \tag{11}$$

$$Q \frac{\mathrm{d}T_{\mathrm{b}}}{\mathrm{d}z} = \frac{\mathrm{d}^2 T_{\mathrm{b}}}{\mathrm{d}z^2}, \tag{12}$$

$$\nabla P_{\mathrm{b}} = -\frac{K_{\mathrm{H}}}{K(z^*)} Q \mathbf{e}_z + Ra \left[T_{\mathrm{b}} - \frac{1}{\beta (T_1 - T_0)} \right] \mathbf{e}_z. \tag{13}$$

where Q is a Péclet number defined as

$$Q = \frac{(\rho c)_{\mathrm{f}} H V_0}{k_{\mathrm{m}}}. \tag{14}$$

Equation 12 can now be solved subject to appropriate boundary conditions, which here take the form

$$T_{\mathrm{b}} = 0 \ \text{ at } \ z = 0, \quad T_{\mathrm{b}} = 1 \ \text{ at } \ z = 1. \tag{15}$$

The solution is

$$T_{\mathrm{b}} = \frac{1 - e^{Qz}}{1 - e^{Q}}. \tag{16}$$

Then Eq. 13 can be solved to give P_{b}.

We now perturb this basic solution and write

$$\mathbf{v} = \mathbf{v}_b + \mathbf{v}', \quad P = P_b + P', \quad T = T_b + T' \tag{17}$$

so that, on linearizing the equations, we have

$$\nabla \cdot \mathbf{v}' = 0, \tag{18}$$

$$\frac{\partial T'}{\partial t} + \mathbf{v}' \cdot \nabla T_b + \mathbf{v}_b \cdot \nabla T' = \nabla^2 T', \tag{19}$$

$$\gamma_a \frac{\partial \mathbf{v}'}{\partial t} = -\nabla P' - \frac{K_H}{K(z^*)} \mathbf{v}' + Ra T' \mathbf{e}_z. \tag{20}$$

Operating on Eq. 20 with $\mathbf{e}_z \cdot$ curl curl and using Eq. 18 we get

$$\left(\frac{K_H}{K(z^*)} + \gamma_a \frac{\partial}{\partial t} \right) \nabla^2 w' = Ra \nabla_H^2 T', \tag{21}$$

where ∇_H^2 denotes the horizontal Laplacian operator. In writing Eq. 21 we have assumed that the heterogeneity is weak in the sense that the variation of K over the layer is small compared with its mean value, so that $[H/K(z^*)]\, \mathrm{d}K/\mathrm{d}z^*$ is small compared with unity. A similar assumption was made by Nield and Kuznetsov (2007). In terms of normal modes, one can write

$$(w', T') = [W(z), \Theta(z)] \exp(st + ilx + imy), \tag{22}$$

which can be substituted into Eqs. 19 and 21, to obtain

$$(g + \gamma_a s)(D^2 - a^2)W + g' DW = -a^2 Ra \Theta \tag{23}$$

$$-fW = (D^2 - a^2 - QD - s)\Theta \tag{24}$$

where

$$a = (l^2 + m^2)^{1/2}, \quad D \equiv \frac{\mathrm{d}}{\mathrm{d}z}, g(z) = \frac{K_H}{K(z)}, \quad f(z) = -\frac{\mathrm{d}T_b}{\mathrm{d}z}, \tag{25}$$

and the prime on g denotes a derivative with respect to z. In the special problem that is investigated we have

$$f(z) = \frac{Q e^{Qz}}{e^Q - 1}. \tag{26}$$

Equations 21 and 22 constitute a pair of coupled ordinary differential equations which can then be solved subject to appropriate boundary conditions on W and Θ. In the case of impermeable constant-temperature boundaries one has

$$W = 0, \quad \Theta = 0 \quad \text{at } z = 0, 1. \tag{27}$$

In the case of impermeable constant-flux (and hence zero perturbation flux) boundaries one has

$$W = 0, \quad D\Theta = 0 \quad \text{at } z = 0, 1. \tag{28}$$

If the temperature gradient has constant sign then oscillatory disturbances are ruled out and one can take $s = 0$.

The differential equation system consisting of Eqs. 23, 24 and 27, or Eqs. 23, 24 and 28, then constitutes an eigenvalue problem with Ra as the eigenvalue.

An approximate expression for the value of Ra can be found by using a single-term Galerkin expansion. We write $W = AW_1$, $\Theta = B\Theta_1$, with constants A and B, where W_1 and Θ_1 are trial functions satisfying the boundary conditions, substitute into Eqs. 23 and 24 to obtain two residuals. These can then be made orthogonal to W_1 and Θ_1, respectively, to give two equations from which the ratio B/A can be eliminated. This results in the equation

$$Ra = \frac{\langle [gW_1(D^2 - a^2)W_1 + g'W_1DW_1]\rangle\{\langle \Theta_1(D^2 - a^2)\Theta_1\rangle - Q\langle \Theta_1 D\Theta_1\rangle\}}{\mathrm{a}^2\langle W_1\Theta_1\rangle\langle fW_1\Theta_1\rangle} \tag{29}$$

where $\langle(\cdot)\rangle \equiv \int_0^1 (\cdot)\mathrm{d}z$.

One general feature of this result can be noted immediately. The effect of the permeability heterogeneity expressed through the function $g(z)$ appears only in a single factor in the numerator of Eq. 29. Likewise the effect of the throughflow via the basic temperature gradient, expressed through the parameter Q and the function $f(z)$, appears only in another factor in the numerator and in the denominator of Eq. 29. Thus, the two effects are essentially decoupled at the present level of approximation. Furthermore, at this level of approximation, the factor involving $g(z)$ is of the form $\langle gW_1(D^2 - a^2)W_1\rangle + \langle g'W_1DW_1\rangle$ which is independent of Θ_1 and hence independent of the thermal boundary conditions.

3 Results for Special Cases

3.1 Constant temperature boundaries

For the case of constant-temperature boundaries one can take $W_1 = \sin \pi z$, $\Theta_1 = \sin \pi z$. This choice leads to

$$Ra = \frac{(\pi^2 + a^2)^2}{a^2} \frac{\langle [g \sin^2 \pi z - \{\pi/(\pi^2 + a^2)\}g' \sin \pi z \cos \pi z]\rangle}{\langle f \sin^2 \pi z\rangle}. \tag{30}$$

In the case of a homogeneous porous medium and in the absence of throughflow one has $f = g = 1$ and

$$Ra = \frac{(\pi^2 + a^2)^2}{a^2}. \tag{31}$$

As the wave number a varies this takes the minimum value $4\pi^2$ when $a = \pi$. These are the familiar values of the critical Rayleigh number and the corresponding critical wavenumber in this case.

In a homogeneous porous medium in the presence of throughflow, one has

$$\langle f \sin^2 \pi z\rangle = \frac{2\pi^2}{Q^2 + 4\pi^2} \tag{32}$$

and so

$$Ra = \frac{(\pi^2 + a^2)^2}{a^2}\left[1 + \frac{Q^2}{4\pi^2}\right]. \tag{33}$$

In this case the critical Rayleigh number is

$$Ra_\mathrm{c} = 4\pi^2 + Q^2. \tag{34}$$

As one would expect from the symmetry of the problem, this result does depend on the sign of Q. The effect of throughflow in either vertical direction is stabilizing.

Further, now

$$I_{\mathrm{K}} \equiv \langle gW_1(D^2-a^2)W_1\rangle + \langle g'W_1DW_1\rangle = -\int_0^1 (\pi^2+a^2)g(z)\sin^2\pi z\mathrm{d}z + \int_0^1 \pi g'(z)\sin\pi z\cos\pi z\mathrm{d}z \tag{35}$$

For example, if we introduce a first-order Taylor expansion for the dimensionless reciprocal of the permeability (normalized to unit mean), of the form

$$g(z) = \frac{1+\gamma z}{1+\gamma/2}, \tag{36}$$

so that γ is a parameter expressing the degree of heterogeneity, then

$$I_{\mathrm{K}} = -\frac{(\pi^2+a^2)}{4}, \tag{37}$$

and so

$$Ra = \frac{(\pi^2+a^2)^2}{a^2}\left[1+\frac{Q^2}{4\pi^2}\right]. \tag{38}$$

For values of γ smaller than unity, $a_{\mathrm{c}} \approx \pi$, and approximately

$$Ra_{\mathrm{c}} = (4\pi^2+Q^2). \tag{39}$$

We conclude that Ra_{c} is independent of γ for the linear variation expressed by Eq. 36. Hence we need to proceed to examine a quadratic variation of the form

$$g(z) = \frac{1+\gamma z+\frac{\delta}{2}z^2}{1+\frac{\gamma}{2}+\frac{\delta}{6}}. \tag{40}$$

Then, assuming that γ and δ are small compared with unity, one obtains

$$Ra_{\mathrm{c}} = (4\pi^2+Q^2)\left(1-\frac{\delta}{4\pi^2}\right). \tag{41}$$

Thus, if δ is positive then the critical Rayleigh number is decreased by a small amount. Thus, in this case the effect of heterogeneity is slightly destabilizing.

3.2 Constant flux boundaries

For the case of constant-flux boundaries an appropriate choice of trial functions is $W_1 = z-z^2$, $\Theta_1 = 1$. This choice leads to

$$Ra = 12\frac{\langle g(z-z^2)[1+(1/2)a^2(z-z^2)]+(1/2)g'(z-z^2)(2z-1)\rangle}{\langle f(z-z^2)\rangle}. \tag{42}$$

In the case of a homogeneous porous medium and no throughflow one has $f = g = 1$ and

$$Ra = 12+\frac{6a^2}{5}. \tag{43}$$

As the wave number a varies this takes the minimum value 12 when $a = 0$. These are the familiar values of the critical Rayleigh number and the corresponding critical wavenumber in this case.

For a homogeneous porous medium with throughflow one can use the result

$$\langle f(z - z^2) \rangle = \frac{Q \coth(Q/2) - 2}{Q^2} \tag{44}$$

to obtain

$$Ra_c = \frac{2Q^2}{Q \coth(Q/2) - 2}, \tag{45}$$

in accord with the result reported by Nield (1987).

For small values of Q this gives the approximation

$$Ra_c = 12 + \frac{Q^2}{5}. \tag{46}$$

In the case where the permeability distribution is given by Eq. 40 and $\gamma \ll 1$, $\delta \ll 1$, then approximately

$$Ra_c = \left(12 + \frac{Q^2}{5}\right)\left[1 - \frac{\delta}{4\pi^2}\right]. \tag{47}$$

The effects of throughflow and permeability heterogeneity are qualitatively similar for the different thermal boundary conditions.

4 Conclusions

We have applied linear stability theory to investigate the onset of convection in the presence of vertical throughflow and vertical heterogeneity of permeability. We have investigated two cases of thermal boundary conditions, namely both boundaries at constant temperature and both boundaries at constant heat flux. It has been shown that, to a first-order approximation, the effects of vertical throughflow and vertical heterogeneity of permeability are in each case independent of each other. In the case of the symmetric boundary conditions being considered here, throughflow in either directions results in increased stability. The effect of vertical heterogeneity of permeability is more subtle. A linear variation of the reciprocal of permeability does not affect the critical value of a Rayleigh number based on the harmonic mean of the permeability and a quadratic variation can lead to either an increase or decrease in the critical value. The present results are consistent with known results for weak heterogeneity presented by Nield (2008). There it was reported that linear variation produces a second-order effect but quadratic variation produces a first-order effect.

However, it should be noted that a harmonic mean of any variable is less than or equal to the arithmetic mean of a variable (with equality if there is no variation). This means that if one worked in terms of the usual average of the permeability one would underpredict the value of the critical temperature gradient required to produce convection, and when one applied ones theory to an experimental situation it would appear that the permeability variation had resulted in a more stable system.

The first-order Galerkin approximation that we have employed can be expected to yield an upper bound on the value of the critical Rayleigh number. It is known that in the classical Rayleigh–Bénard problem (with a fluid clear of solid material and with rigid conducting boundary conditions) the estimated critical Rayleigh number is about 3% too large, and in practice this error is acceptable since the experimental error in an experimental determination of the critical Rayleigh number using the Schmidt–Milverton technique is at least 8%. In the present situation we can expect a similar accuracy for the approximation if γ and Q are both small in comparison with unity.

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