Transp Porous Med (2011) 87:385–395
DOI 10.1007/s11242-010-9690-9

Influence of Fracture Connectivity and Characterization Level on the Uncertainty of the Equivalent Permeability in Statistically Conceptualized Fracture Networks

Sung-Hoon Ji · Young-Jin Park · Kang-Kun Lee

Received: 1 June 2009 / Accepted: 23 November 2010 / Published online: 8 December 2010

Abstract A statistically conceptualized fracture network is generally used in modeling flow and transport in the discrete fracture network (DFN) approach. To quantify the influence of the fracture connectivity and characterization level on the uncertainty from a statistical conceptualization of fractures, the ensemble mean and variability of the equivalent permeability for stochastically generated fracture networks is analyzed with various percolation parameters (p) for different structures following power law size distributions. The results of Monte Carlo analyses show that statistics of a fracture network can be used to estimate its hydraulic properties with an acceptable level of uncertainty when p is greater than the specific percolation parameter (p_s) where the domain size is expected to become equal to the correlation length of a given fracture network. However, when p is smaller than the p_s, the uncertainty of the hydraulic properties induced from statistical characteristics of fractures is large, thus statistical conceptualization is not recommended. Conditional simulations support them: although we have deterministic information on a significant amount of fractures in the domain, a small number of stochastically generated fractures still produce significant uncertainty in the estimated system properties when p is smaller than p_s. These results suggest that the p_s and correlation length of a fracture network can be criteria to evaluate the applicability of the statistical conceptualization for modeling flow in a given fractured rock.

Keywords Fracture network · Statistical conceptualization · Uncertainty · Specific percolation parameter

S.-H. Ji (✉)
HLW Disposal Research Division, Korea Atomic Energy Research Institute,
1045 Daedeokdaero, Yuseong, Daejeon 305-353, Republic of Korea
e-mail: shji@kaeri.re.kr

Y.-J. Park
Department of Earth and Environmental Sciences, University of Waterloo,
200 University Avenue West, Waterloo, ON, N2L 3G1, Canada

K.-K. Lee
School of Earth and Environmental Sciences, Seoul National University,
599 Gwanakro, Gwanak, Seoul 151-742, Republic of Korea

1 Introduction

Equivalent continuum/discontinuum approaches imitate apparent flow and transport behaviors in fractured rocks using a multiple-porosity (Moench 1984), stochastic continuum (Tsang et al. 1996) or equivalent discrete network (e.g., lattice network) (Long et al. 1982). These approaches may have a fundamental limitation as they are not necessarily predictive in fractured rocks because they assume a fractured rock as an equivalent medium. To represent a fracture network in a mathematical model, a discrete fracture network (DFN) approach was suggested (Dershowitz et al. 1991). It describes the geometry and hydraulic properties of individual fractures and thus can account for flow and transport phenomena through these fractures. It is generally applied to modeling of a block- or smaller-scale groundwater flow and transport phenomena rather than regional-scale ones for computational practicality (Hartley et al. 2006). In this approach, fractures can be populated based on the characterization of media. Fracture network characterizations can be classified as deterministic or statistical according to the type of information collected. The deterministic characterization represents the explicit acquisition of information on the geometry and hydraulic properties for individual fractures, and the statistical characterization indicates the collection of implicit group information such as distribution characteristics of fracture size, orientation, and transmissivity for a population of fractures.

Based on the characterized data, the fracture network for a given domain is conceptualized to imitate the real system. In the real-world applications, the fractures of the network are generally conceptualized statistically because it is impossible to characterize and conceptualize the whole individual fractures deterministically in the system. The DFN for a site is constructed using statistical properties of a fracture system derived from observable samples through the statistical characterization, and several fractures, which are investigated closely and considered as important ones for flow and transport in the system, can be added to it deterministically (e.g., Thury et al. 1994; Hartley et al. 2006). Note that the statistical conceptualization can be considered to be sufficient to infer the flow and transport phenomena if it guarantees similar (not necessarily the same) hydraulic behaviors to the observed ones for different realizations (Bear 1993). When the DFN is constructed from the statistical conceptualization, an infinite number of networks can be created with the same statistical parameters, and the geometrical and hydraulic properties of the network are various among the realizations although their average behavior can be defined. This variability is one of the causes of uncertainty in simulating the hydraulic characteristics of a fracture network with the DFN approach.

The hydraulic characteristic of a fracture network has been studied through the concept of equivalent permeability. The percolation theory and the effective medium theory explain the relation between the conductivity and the site- or bond-occupancy probability in a lattice network with a power and a linear laws, respectively (Stauffer and Aharony 1991; Kirkpatrick 1973). By expanding them to a fracture network, the relations between the equivalent permeability (K_{eqv}) and the percolation parameter (p), that was suggested to describe the connectivity of a fracture network (de Dreuzy et al. 2001), at various conditions have been studied. Mourzenko et al. (2004) elucidated the influences of three-dimensional power law distributed fracture sizes on the equivalent permeability using numerical approaches, and they proposed two analytical formulas, which are fitted well to the numerical results over a wide range of percolation parameters, based on the percolation theory and effective medium theory. Their results suggest that both of theories govern the relation between K_{eqv} and p in the range $0.4 < p - p_{\mathrm{c}} < 10$ but the application of the percolation theory is expected to be limited for $p > 20$. Then, a complexity such as variable apertures, spatially correlated fractures, and porous matrix was added. de Dreuzy et al. (2000) studied the influence of lognormal distributed apertures on K_{eqv} of the two-dimensional power law distributed fracture networks, and they found that a power law still governs the relation between K_{eqv} and p but the effect of aperture distribution is dependent on the power law size exponent and the correlation between the fracture size and aperture. de Dreuzy et al. (2004) shows using the numerical results with two-dimensional hypothetical fracture networks that the more spatially correlated fracture network has the less equivalent permeability although the broader fracture size distributed network the larger equivalent permeability. The equivalent permeability of three-dimensional fractured porous media with power law size distribution was investigated numerically by Bogdanov et al. (2007), and they introduced two generic formulas for un-percolated and percolated fracture networks to estimate the equivalent permeability with percolation parameter, hydraulic conductivities of fracture and matrix, and fracture shape. These theoretical studies were conducted using statistically conceptualized fracture networks, but the uncertainty from the conceptualization has not been investigated in previous researches to the best of our knowledge. Note that most of real-world DFN applications also have not considered the uncertainty from the statistical conceptualization (e.g., Thury et al. 1994; Hartley et al. 2006; Surrette and Allen 2008). Then, the uncertainty of a system's behavior estimated from the statistical conceptualization can be expected to be reduced when the deterministic explicit data for a part of a fracture system is incorporated. This effect also has not been quantified to the best of our knowledge.

In this study, we evaluate the applicability of the statistical conceptualization of various fracture networks by quantifying the influences of the fracture connectivity and characterization level on the uncertainty of the hydraulic properties of the statistically conceptualized networks. We define different network structures with various fracture connectivities following power law fracture size distributions, and then analyze the ensemble mean and variability of the equivalent permeability of the networks. From the ensemble behaviors of the equivalent permeability, we discuss when the statistical conceptualization can be applied to modeling groundwater flow in a fracture network. We then analyze the effects of the deterministic explicit data for a part of a fracture network on the uncertainty of the estimated system properties.

2 Approaches

There is increasing field evidence that fracture trace lengths and fracture sizes are distributed according to a power law (Bonnet et al. 2001), defined as

$$n(R) = \alpha R^{-a} \quad \text{for} \quad R \in [R_{\min}, R_{\max}] \tag{1}$$

where, $n(R)dR$ is the probability of a fracture having a size in the range of $[R, R + dR]$, α is a normalization factor, a is a characteristic power law exponent, and $R_{\min}$ and $R_{\max}$ are the lower and upper cutoffs of the fracture size, respectively. In this study, the fracture size (R) is defined as the radius of a circle of area equal to the area of a given polygonal fracture following Dershowitz et al. (1998).

Power law size distributions for fracture networks can represent various network structures, ranging from the networks composed of constant size fractures to those with fractures much larger than the domain size: when a approaches infinity, the power law length distribution becomes a delta function, resulting in the constant fracture size of $R_{\min}$, while when a equals one, all fractures are truncated by the domain boundaries. In general, when the characteristic exponent $a < 1.5$, the connectivity and the transport properties of fracture networks are dominated by the large fractures near $R_{\max}$, which is generally much larger than the domain size (Bour and Davy 1998) (e.g., Fig. 1a), while when $a > 4$, fractures of lower cutoff size (1/10 of the domain size in this study) are dominant in the networks (Fig. 1c). In the intermediate case, fractures with various sizes contribute to network properties (Fig.. 1b).

Fracture networks with power law exponents of 1.0, 2.5, 3.5, and 4.5 are considered in this study for the simulations of different network structures. Fractures of a hexagonal shape, inscribed in circles, are generated in a three-dimensional domain ($L \times L \times L$). The fracture location is assumed to be statistically uniform in the domain following a Poisson process, and the orientation is assumed not to have any preferential direction.

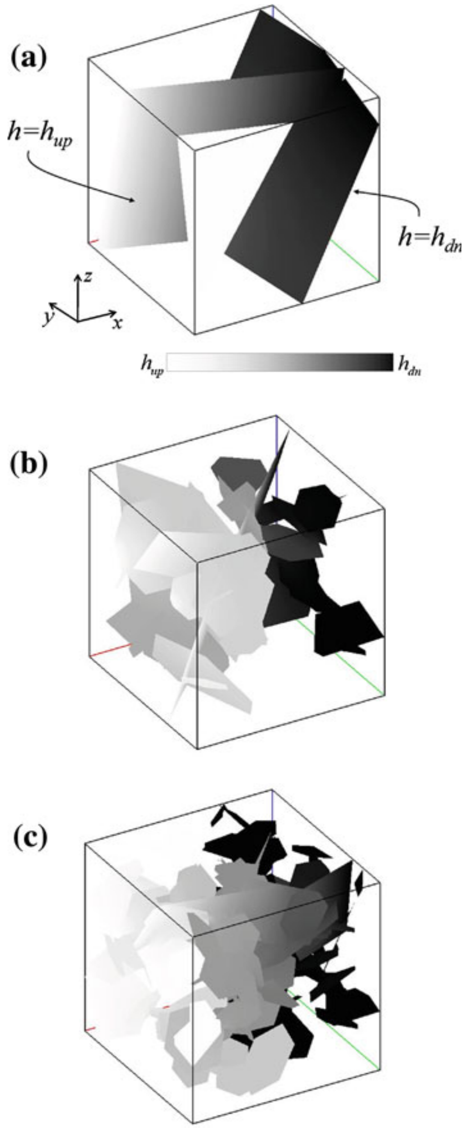

Fig. 1 Examples of generated fracture networks with a power law size distribution with characteristic exponents of **a** 1.0, **b** 3.5, and **c** 4.5, and their hydraulic head distributions

As another system parameter, a percolation parameter is used instead of a fracture density because percolation for a network from the upstream to the downstream depends on the fracture connectivity rather than the density. The percolation parameter (p) indicating the fracture connectivity is defined as $p = \rho v_{ex} \langle R^3 \rangle$ following Mourzenko et al. (2004), where ρ is the total fracture number per unit volume, v_{ex} is the dimensionless shape factor, and $\langle R^3 \rangle$ is the third moment of the fracture size distribution. For each power law network structure, different percolation parameters ranging from 0.3 to 30.5 are applied so that the percolation probability of the networks—a probability for a network to be connected from the upstream to the downstream—varies from under 10 to 100%. The percolation threshold (p_c) is defined as the p where the percolation probability from the upstream to the downstream is 50% (Bour and Davy 1997).

To compute fluid flow in fracture networks, MAFIC, a finite element flow model to simulate flow and transport in a rock mass with a discrete fracture network, is used (Miller et al. 1999). In MAFIC, a fracture network is assumed as a network of interconnecting plates, and each fracture is discretized to several finite elements considering intersections. In MAFIC, the governing equation for incompressible fluid flow in fractures is given by:

$$S\frac{\partial h}{\partial t} - T\overline{\nabla^2}h = q \tag{2}$$

where, S is fracture storavity, h is hydraulic head, t is time, T is fracture transmissivity, and q is source term. To solve Eq. 2, a Galerkin finite element method with triangular elements is utilized with the principle of mass conservation at fracture intersections. For this study, a steady state flow is assumed, and a constant transmissivity is given to individual fractures. For stable calculations, elemental meshes are refined to ensure the relative error to be less than 5% when the number of elements is reduced to a half. Hydraulic head is specified on two opposite planes of the domain and a no-flux condition is applied to the other four plane boundaries. Under these boundary conditions, flow is derived from the upstream to the downstream with the specified head boundaries (Fig. 1). The K_{eqv} of a fracture network is defined by the following relation:

$$K_{eqv} = \frac{Q}{(h_{up} - h_{dn})L} \tag{3}$$

where, Q is the fluid flux from the upstream to the downstream boundaries of the specified head values h_{up} and h_{dn}, respectively.

An infinite number of equally probable fracture networks can be generated for a given power law exponent with the same structural parameters. Although the average behavior is defined for given network structures, the network properties such as K_{eqv} can vary significantly among realizations, as observed in field experiments (Hsieh et al. 1993). In order to quantify a system's variability for given network structures, we analyze the coefficient of variation (CV) of K_{eqv} for the realizations of fracture networks with various power law exponents and with different percolation parameters. CV is determined by dividing the standard deviation with the mean, and thus can evaluate the uncertainty for the typical behavior of K_{eqv}: larger CV values mean that the system varies more among realizations and thus it is more difficult to estimate the system properties with given statistical properties. Note that if CV, that is the system variability, becomes smaller among realizations, then the same statistics can guarantee similar system properties and thus, they are considered to be sufficient information to characterize a system. 200–1,200 network realizations are generated for each p and a, and the equivalent permeabilities are averaged over the number of realizations for each case. To make sure that the number of realizations is statistically significant, it is ensured in each Monte Carlo simulation that the average and standard deviation of K_{eqv} of the first and second halves of the total number of realizations are similar to those of the whole realizations within a 10% error range following Ji et al. (2004). Note that we include the fracture networks that are not connected from the upstream to the downstream in the Monte Carlo analyses by regarding their K_{eqv} are zero.

When a part of a fracture system is characterized deterministically and the statistical properties of the system are known, the deterministic information can be incorporated in the statistical conceptualization. We define the characterization level as the ratio of the number of deterministically conceptualized fractures to the total number of fractures in the domain, and use it as one of the system parameters in this study. For each p and a, a subset of a fracture population is generated to represent deterministic fractures considering the given characterization level, and is incorporated in the whole realization. Then, considering the given p, the rest of a population is generated stochastically for each realization. We apply different characterization levels for fracture networks with the same structural properties resulting in the characterization levels ranging from 0 to 90%. Networks with power law exponents of 1.0 and 4.5 are used to observe the reduction of the ensemble variability from the characterization level. Clemo and Smith (1997) indicated that some fractures may play more (or less) important roles for transport in a network by analyzing the geometrical properties of the fractures acting as main transport paths, but this speculation is beyond the scope of this article and it is not considered.

3 Results and Discussions

The percolation thresholds are estimated for various power law fracture networks. Then, the ensemble mean of the equivalent permeability is calculated for different p and power law fracture networks (Fig. 2). When p is high and the fractures are well-connected, a power law governs the relation between the mean K_{eqv} and $p - p_c$ for the whole a as shown by Mourzenko et al. (2004). However, as the percolation parameter decreases, the relation between the mean K_{eqv} and $p - p_c$ is deviated from the power law. This deviation is related to a domain size (L) and the correlation length (ξ) of a fracture network, where ξ is defined as an average distance between two fractures that belong to the percolation cluster in the percolation theory. According to the percolation theory, a power law governs the relation between the hydraulic properties of a system and the percolation parameter. However, Ji et al. (2004) observed the equivalent permeability of a 2-D lattice network diverts from a power law below a specific percolation parameter (p_s) where the domain size equals to the percolation correlation length, and explained the reason: when L is larger than ξ of a given fracture network the percolation cluster shows a homogeneous behavior. In the opposite case, however, it becomes a fractal material because ξ is a measure of the largest hole in the percolation cluster. To check if those results can be applied to a 3-D discrete fracture network, we calculate the p_s, where L equals to ξ, in our 3-D DFNs as follows, and compare them to the p where the mean K_{eqv} for each a deviates from a power law.

According to the percolation theory, the relation between ξ and $p - p_c$ obeys a power law relation (Stauffer and Aharony 1991):

$$\xi \propto (p - p_c)^{-\nu}, \tag{4}$$

where ν is a characteristic power law exponent. It is related with the domain size L as follows. Assume that the mass of the cluster equals the size of the cluster. Let $M(L)$ be the mass of the percolation cluster, then, $M(L)$ is given by (Stauffer and Aharony 1991):

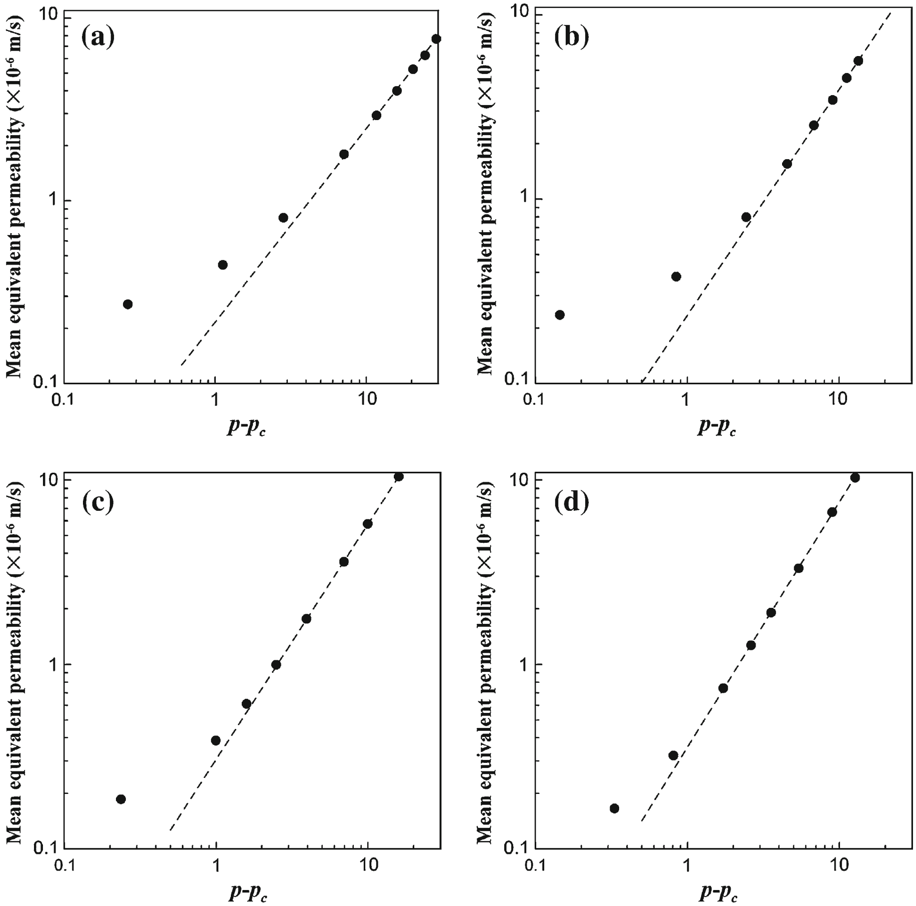

Fig. 2 Mean equivalent permeabilities and the normalized differences between the estimated mean K_{eqv} from a power law and the observed mean K_{eqv} for various percolation parameters at **a** $a = 1.0$, **b** $a = 2.5$, **c** $a = 3.5$, and **d** $a = 4.5$. The dotted lines indicate the best fitted power law equations for each a

$$M(L) \propto \begin{cases} L^D & L < \xi \\ \xi^D (L/\xi)^d & L > \xi \end{cases}, \tag{5}$$

where D is the fractal dimension of the percolation cluster, and d is the dimension of the system. When the corresponding density is defined as $\rho_{\text{pc}} = M(L)/L^d$, the density of the percolation cluster is given by:

$$\rho_{\text{pc}} \propto \begin{cases} L^{D-d} & L < \xi \\ \xi^{D-d} & L > \xi \end{cases}, \tag{6}$$

From Eq. 4, Eq. 6 can be modified as follows:

$$\rho_{\text{pc}} \propto \begin{cases} L^{D-d} & L < \xi \\ (p - p_{\text{c}})^{\nu(d-D)} & L > \xi \end{cases}, \tag{7}$$

and we can determine the p_{s} with Eq. 7 because ρ_{pc} deviates from the power law when p is larger than p_{s} thus ξ is smaller than L. Figure 3 shows the relations between the mean

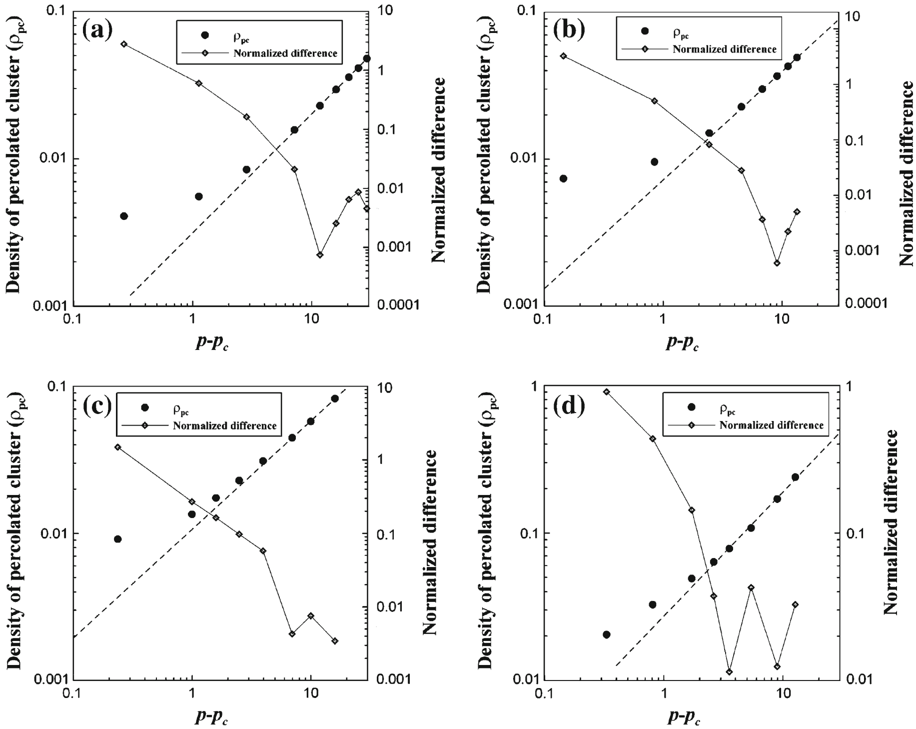

Fig. 3 Mean density of the percolation cluster for various $p - p_c$ at **a** $a = 1.0$, **b** $a = 2.5$, **c** $a = 3.5$, and **d** $a = 4.5$

ρ_{pc} and $p - p_c$ at various a. When p is high, a power law governs the relation between the mean ρ_{pc} and $p - p_c$ as Eq. 7 says. However, as p decreases the mean ρ_{pc} deviates from the power law and tends to converge to a constant. To determine p_s, we calculate the normalized difference between the estimated mean ρ_{pc} from a power law ($\rho_{pc,est}$) and the observed mean ρ_{pc} ($\rho_{pc,obs}$) defined as $\frac{|\rho_{pc,est} - \rho_{pc,obs}|}{\rho_{pc,est}}$, and assume that the deviation begins when more than 10% normalized difference is observed. The opened diamonds in Fig. 3 show the calculated normalized differences between $\rho_{pc,est}$ and $\rho_{pc,obs}$, and the p_s for each a is determined using them. The determined p_s for each a is almost same to the p where the mean K_{eqv} starts to deviate from a power law, and these results show that the K_{eqv} of a 3-D fracture network also diverts from a power law below p_s.

Figure 4a shows the change of CV with p for each a, and the dotted lines in Fig. 4 indicate the p_s of each power law fracture network. CV decreases from ~10 to ~0.1 as p increases. In poorly connected networks where p is smaller than p_s, CV is larger than unity, which means that it is unlikely to predict the order of magnitude of the equivalent permeability with the given statistical properties. In well-connected networks where p is larger than p_s, however, CV becomes smaller than unity, which indicates that the uncertainty in the order of the equivalent permeability (as estimated from the statistical properties of fractures) is low.

Figure 4b–c shows the changes of CV with different characterization levels and the dotted lines indicate the p_s for power law fracture networks with the exponents of 1.0 and 4.5. The changes of CV for a given characterization level depends on percolation parameter. When p is smaller than p_s, no significant reduction of CV is observed although 90% of a

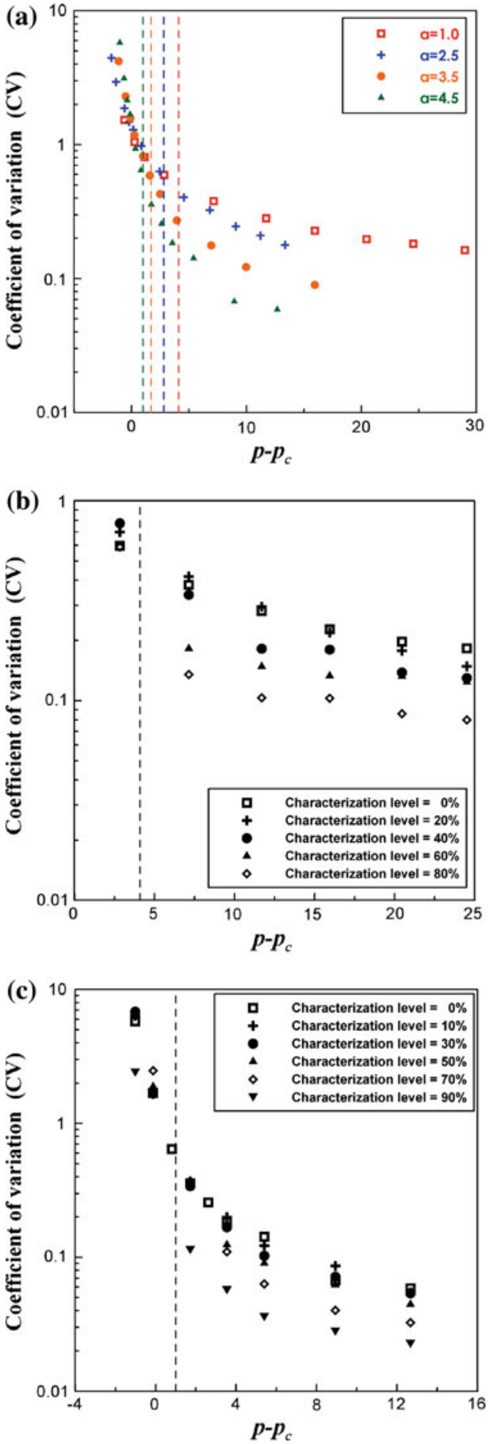

Fig. 4 **a** Coefficients of variation for various percolation parameters at each a; and changes of the coefficients of variation with different characterization levels at **b** $a = 1.0$, and **c** $a = 4.5$. The dotted lines show the specific percolation parameter for each power law fracture network of a

fracture population is from the deterministic conceptualization, and 10% from the statistical conceptualization. However, when p is larger than p_s, CV decreases by more than 30% if the given characterization level is above 40%. These results imply that even a small portion of statistically generated fractures in a poorly connected network may cause significant uncertainty in the estimated system. On the other hand, in well-connected networks, where p is larger than p_s, the system property such as K_{eqv} estimated from the statistical properties has a low variability and additional deterministic properties can further reduce its variability.

4 Summary and Conclusions

The influences of the fracture connectivity and characterization level on the uncertainty of the estimated system properties from statistically conceptualized DFNs are examined by analyzing K_{eqv} of the stochasitically generated fracture networks with various fracture connectivities, structures, and characterization levels. The results show that the uncertainty of the statistical conceptualization of a fracture network is acceptable in a well-connected system, where p is greater than p_s, because estimates of system properties such as an equivalent permeability are similar among realizations. However, in a poorly connected network, where p is below p_s, the uncertainty of system properties induced from statistically conceptualized fractures is large, and most fractures need to be explicitly identified. Deterministic information for a portion of a fracture network can reduce the uncertainty of the system properties estimated from the statistical information only when p is larger than p_s. These results suggest that the specific percolation parameter can be a criterion for evaluating the applicability of the statistical conceptualization in a DFN construction for modeling groundwater flow in a fractured rock. These conclusions can be expanded to the real-world applications: because the p_s of a given fracture network is related to the correlation length the statistical conceptualization of fractures for modeling can be applied by calculating the correlation length with the observed fracture properties and using the larger model domain than the correlation length. Note that the correlation length of a given fracture network can be calculated following Kapitulnik et al. (1983).

Natural fracture systems can have more complexity than the fracture networks used in this study: the orientations and locations of fractures may not be statistically uniform and homogeneous, and individual fractures will have variable apertures and roughness. Nevertheless, our results show that the specific percolation parameter and/or correlation length of a fracture network can be determined based on Monte Carlo analyses of the equivalent permeability and used to evaluate the applicability of the stochastically constructed DFN for modeling groundwater flow in the network, which provides a stepping-stone for the expansion of our understanding of flow and transport in a fracture system and a proper approach to describe it.

Acknowledgment This work was supported by the Korean Nuclear Energy R&D program of the Ministry of Science and Technology, Korea.

References

Bear, J.: Modeling flow and contaminant transport in fractured rocks. In: Bear, J., Tsang, C.-F. de Marsily, G. (eds.) Flow and Contaminant Transport in Fractured Rock. Academy Press, Inc., San Diego (1993)

Bogdanov, I.I., Mourzenko, V.V., Thovert, J.-F., Adler, P.M.: Effective permeability of fractured porous media with power-law distribution of fracture sizes. Phys. Rev. E **76**, 036309 (2007)

Bonnet, E., Bour, O., Odling, N.E., Davy, P., Main, I., Cowie, P., Berkowitz, B.: Scaling of fracture systems in geological media. Rev. Geophys. **39**, 347–383 (2001)

Bour, O., Davy, P.: Connectivity of random fault networks following a power law fault length distribution. Water Resour. Res. **33**, 1567–1583 (1997)

Bour, O., Davy, P.: On the connectivity of three-dimensional fault networks. Water Resour. Res. **34**, 2611–2622 (1998)

Clemo, T., Smith, L.: A hierarchical model for solute transport in fractured media. Water Resour. Res. **33**, 1763–1784 (1997)

de Dreuzy, J.-R., Davy, P., Bour, O.: Percolation parameter and percolation-threshold estimates for three-dimensional random ellipses with widely scattered distributions of eccentricity and size. Phys. Rev. E **62**, 5948–5952 (2000)

de Dreuzy, J.-R., Davy, P., Bour, O.: Hydraulic properties of two-dimensional random fracture networks following a power law length distribution: 2. Permeability of networks based on lognormal distribution of apertures.. Water Resour. Res. **37**, 2065–2078 (2001)

de Dreuzy, J.-R., Darcel, C., Davy, P., Bour, O.: Influence of spatial correlation of fracture centers on the permeability of two-dimensional fracture networks following a power law length distribution. Water Resour. Res. **40**, W01502 (2004). doi:10.1029/2003WR002260

Dershowitz, W.S., Wallman, P., Kinrod, S.: Discrete fracture modeling for the Stripa site characterization and validation drift inflow predictions, Stripa Project Tech. Rep. 91-16, SKB, Stockholm (1991)

Dershowitz, W., Lee, G., Geier, J., Foxford, T., LaPointe, P., Thomas, A.: FracMan Interactive Discrete Feature Data Analysis, Geometric Modeling, and Exploration Simulation, User Documentation. Golder Associates Inc., Seattle (1998)

Hartley, L., Hoch, A., Jackson, P., Joyce, S., McCarthy, R., Rodwell, W., Swift, B., Marsic, N.: Groundwater flow and transport modeling during the temperate period for the SR-Can assessment—Forsmark area, version 1.2, R-06-98. SKB, Stockholm (2006)

Hsieh, P.A., Shapiro, A.M., Barton, C.C., Haeni, F.P., Johnson, C.D., Martin, C.W., Paillet, F.L., Winter, T.C., Wright, D.L. : Methods of characterizing fluid movement and chemical transport in fractured rock. In: Chaney, J.T., Hepburn, J.C. (eds.) Field Trip Guide Book for Northeastern United States, USGS, Boulder (1993)

Ji, S.-H., Lee, K.K., Park, Y.C.: Effects of the correlation length on the hydraulic parameters of a fracture network. Transp. Porous Media **55**, 153–168 (2004)

Kapitulnik, A., Aharony, A., Deutscher, G., Stauffer, D.: Self similarity and correlations in percolation. J. Phys. A **16**, L269–L274 (1983)

Kirkpatrick, S.: Percolation and conduction. Rev. Mod. Phys. **45**, 574–588 (1973)

Long, J.C.S., Remer, J.S., Wilson, C.R., Witherspoon, P.A.: Porous media equivalents for networks of discontinuous fractures. Water Resour. Res. **18**, 645–658 (1982)

Miller, I., Lee, G., Dershowitz, W.: MAFIC Matrix/Fracture Interaction Code with Heat and Solute Transport, User Documentation, v 1.6.. Golder Associate Inc., Seattle (1999)

Moench, A.F.: Double porosity models for a fissured groundwater reservoir with fracture skin. Water Resour. Res. **20**, 831–846 (1984)

Mourzenko, V.V., Thovert, J.-F., Adler, P.M.: Macroscopic permeability of three-dimensional fracture networks with power-law size distribution. Phys. Rev. E **69**, 066307 (2004)

Stauffer, D., Aharony, A.: Introduction to Percolation Theory, 2nd edn. Taylor & Francis Ltd., London (1991)

Surrette, M., Allen, D.: Quantifying heterogeneity in variably fractured sedimentary rock using a hydrostructural domain. GSA Bull. **120**, 225–237 (2008)

Thury, M., Gautschi, A., Mazurek, M., Muller, W., Naef, H., Pearson, F., Vomvoris, S, Wilson, W.: Geology and hydrogeology of the crystalline basement of northern Switzerland—synthesis of regional investigations 1981–1993 within the Nagra radioactive waste disposal programme, NTB93-01. Nagra, Wettingen (1994)

Tsang, Y.W., Tsang, C.F., Hale, F.V., Dverstorp, B.: Tracer transport in a stochastic continuum model of fractured media. Water Resour. Res. **32**, 3077–3092 (1996)