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Effect of Rotation on Thermal Convection in an Anisotropic Porous Medium with Temperature-dependent Viscosity

R. K. Vanishree · P. G. Siddheshwar

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Abstract A linear stability analysis is performed for mono-diffusive convection in an anisotropic rotating porous medium with temperature-dependent viscosity. The Galerkin variant of the weighted residual technique is used to obtain the eigen value of the problem. The effect of Taylor–Vadasz number and the other parameters of the problem are considered for stationary convection in the absence or presence of rotation. Oscillatory convection seems highly improbable. Some new results on the parameters' influence on convection in the presence of rotation, for both high and low rotation rates, are presented.

Keywords Anisotropy · Rotation · Porous medium · Variable viscosity · Thermal convection

List of symbols

a	Horizontal wave number
a_c	Critical wave number
Br_D	Brinkman–Darcy number, ΛDa
c	Specific heat
c_p	Specific heat at a constant pressure
Da^{-1}	Inverse Darcy number (porous parameter), $\frac{d^2}{k_v}$
d	Height of the porous layer
$\vec{g}$	Gravitational acceleration $(0, 0, -g)$
k	Permeability
$\boldsymbol{k}$	Permeability tensor, $\frac{1}{k_h}\hat{i}\hat{i} + \frac{1}{k_h}\hat{j}\hat{j} + \frac{1}{k_v}\hat{k}\hat{k}$

R. K. Vanishree (✉)
Maharani's Science college for Women, Bangalore 560 001, India
e-mail: vani_ramana2005@yahoo.co.in

P. G. Siddheshwar
Department of Mathematics, Central College Campus, Bangalore University, Bangalore 560 001, India
e-mail: pgsiddheshwar@bub.ernet.in

$\hat{k}$	Unit vector in the vertical direction
(k_h, k_h, k_v)	Permeability along x, y and z-direction
l, m	Wave numbers
p	Pressure
p^*	Hydrostatic pressure
p_H	Basic state pressure
Pr	Prandtl number, $\frac{\nu\Phi}{\chi_{Tv}}$
$\vec{q}$	(u, v, w), velocity vector
$\vec{q}'$	Velocity of the perturbed state
R	Rayleigh number, $\frac{\alpha\, g\, \Delta T\, d^3}{\nu\, \chi_{Tv}}$
R_D	Darcy–Rayleigh number, RDa
t	Time
T	Temperature field
Ta	Taylor number, $\frac{4\Omega^2 d^4}{\Phi^2 \nu^2}$
T_b	Basic state temperature
T_R	Reference temperature
u, v, w	Dimensional horizontal and vertical velocity components
u^*, v^*, w^*	Dimensionless velocity components
V	Linear variable viscosity parameter, $\Gamma\Delta T$
Va	Vadasz or Prandtl–Darcy number, $PrDa$
Va_D	(Taylor–Vadasz number), $TaDa^2$
x	Horizontal coordinate
x^*	Dimensionless horizontal coordinate
z	Vertical coordinate
z^*	Dimensionless vertical coordinate
(x, y, z)	Cartesian coordinates with z-axis vertically upward

Greek symbols

α	Coefficient of thermal expansion
(χ_{Th}, χ_{Tv})	Thermal conductivities in x- and z-directions
ε	Mechanical anisotropy parameter, $\frac{k_h}{k_v}$
Φ	Porosity of the media
η	Thermal anisotropy parameter, χ_{Th}/χ_{Tv}
γ	Heat capacity ratio, $(\rho_R\, c_p)_m/(\rho_R\, c_p)_f$
τ	Scaled dimensionless time, $t\,Da^{-1}$
ΔT	Temperature gradient
∇	$\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$ (vector differential operator)
∇^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$ (three-dimensional Laplacian operator)
∇_1^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ (two-dimensional Laplacian operator)
Λ	Brinkman number, $\frac{\mu_p}{\mu_f}$
μ	Dynamic viscosity
μ_p	Effective viscosity
μ_f	Viscosity of the fluid
ν	Kinematic viscosity, $\frac{\mu_f}{\rho_R}$

ρ Density
ρ_b Basic state density
ρ_R Density of the liquid at reference temperature $T = T_m$
ψ Stream function
σ Growth rate of perturbation
$\vec{\Omega}$ Angular velocity of rotation
ω Scaled frequency of oscillation
ζ z- component of vorticity, $\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$

Subscripts

b Basic state
c Critical quantity
f Fluid

Superscripts

$'$ Dimensional quantities
$*$ Dimensionless quantities
o Oscillatory
s Stationary

1 Introduction

Thermal convection in a rotating porous medium is a phenomenon relevant to many fields. It has various applications in geophysics, food processing, engineering, and nuclear reactors. Many authors have investigated the effect of external constraint such as rotation on convection in a porous medium. Cellular convection in a rotating, fluid-saturated porous medium was studied by Rudraiah and Rohini (1975) and Rudraiah and Srimani (1976). Stability of finite amplitude and overstable convections of a conducting fluid through a fixed porous bed were studied by Rudraiah and Vortmeyer (1978). Palm and Tyvand (1984) investigated thermal convection in a rotating porous layer. The effect of Coriolis force and non-uniform temperature gradient on Rayleigh–Bénard convection was established by Rudraiah and Chandna (1985). Jou and Liaw (1987) studied the thermal convection in a porous medium subject to transient heating and rotation. Vadasz (1993, 1994, 1997, 1998a,b) extensively studied the flow through a porous medium with rotational effects such as three-dimensional free convection in a long rotating porous box, stability of free convection in a narrow porous layer subject to rotation, stability of free convection in a rotating porous layer distant from the axis of rotation, flow in a rotating porous medium, Coriolis effect on gravity-driven convection in a rotating porous layer heated from below, and free convection in a porous medium. Transition and chaos in free convection in a rotating porous layer were studied by Vadasz and Olek (1998). Straughan (2000) established a sharp non-linear stability threshold in rotating porous convection. Govender and Vadasz (2002) made a moderate time linear study of moderate Stephan number convection in rotating mushy layers. Riahi (2003, 2006) studied stationary and oscillatory modes of flow instability in a rotating porous layer during alloy solidification, and non-linear convection in a rotating mushy layer. Govender (2006) studied the effect of anisotropy on stability of convection in a rotating porous layer distant from the center of rotation. Riahi (2007a,b) analyzed the inertial effects on the rotating fluid flow in a porous layer, and inertial and Coriolis effects on oscillatory flow in a horizontal dendrite layer. Combined effect of thermal modulation and rotation on the onset of stationary convection in a porous layer was studied by Malashetty and Mahantesh Swamy (2007) . Govender and Vadasz (2007) studied the effect of mechanical and thermal anisotropy on the stability of gravity-driven convection in a rotating porous medium in the presence of thermal non-equilibrium. Effect of temperature modulation on the onset of Darcy convection in a rotating porous medium was studied by Bhadauria (2008). Linear stability of solutal convection in rotating, solidifying mushy layers with permeable mush–melt interface was established by Govender (2008).

Most of the above investigators have studied convection in a low-porosity, rotating, and isotropic porous medium with constant viscosity. Temperature dependence of viscosity gives rise to temperature-dependent Darcy and Brinkman frictions. Patil and Vaidyanathan (1983) analyzed setting up of convection currents in a rotating porous medium under the influence of variable viscosity. Richardson and Straughan (1993) studied the non-linear stability and the Brinkman effect on convection with temperature-dependent viscosity (linear dependence) in a porous medium. Effect of radiation on non-Darcy free convection from a vertical cylinder embedded in a fluid-saturated porous medium with a temperature-dependent viscosity was studied by El-Hakiem and Rashad (2007). A good account of convection problems in a porous medium is given in Vafai and Hadim (2000); Ingham and Pop (2002), and Nield and Bejan (2006).

The object of this article is to study the effect of rotation on mono-diffusive convection in a high-porosity, anisotropic porous medium with temperature-dependent viscosity.

2 Mathematical Formulation

Consider a rotating porous layer of infinite horizontal extent occupied by a Boussinesquian fluid with temperature-dependent viscosity, confined between rigid isothermal boundaries at $z = 0$ and $z = d$ at which the temperatures are T_0 and T_1, respectively. Let $\vec{\Omega}$ denote the angular velocity of rotation of the medium. The porous medium is assumed to have high porosity and hence the fluid flow is governed by the Brinkman model with effects of Coriolis force and centrifugal acceleration included. An appropriate single-phase heat transport equation is chosen with effective heat capacity ratio and effective thermal diffusivity. Thus, the governing equations for the Rayleigh–Bénard situation in a fluid, with the non-Boussinesq effect of temperature-dependent viscosity, occupying a rotating porous layer are

Conservation of mass

$$\nabla \cdot \vec{q} = 0, \tag{2.1}$$

Conservation of linear momentum

$$\begin{aligned} \frac{\rho_R}{\Phi}\frac{\partial \vec{q}}{\partial t} + \frac{\rho_R}{\Phi^2}(\vec{q}\cdot\nabla)\vec{q} &= -\nabla p + \rho\vec{g} - \mu_f \boldsymbol{k}\cdot\vec{q} \\ &+ 2\frac{\rho_R}{\Phi}\left(\vec{q}\times\vec{\Omega}\right) + \nabla\cdot\left[\mu_p\left(\nabla\vec{q} + \nabla\vec{q}^{\,Tr}\right)\right], \end{aligned} \tag{2.2}$$

Conservation of energy

$$\gamma\frac{\partial T}{\partial t} + \vec{q}\cdot\nabla T = \chi_{Tv}\left[\eta\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\partial^2 T}{\partial z^2}\right], \tag{2.3}$$

Equation of state

$$\rho = \rho_R \left[1 - \alpha(T - T_0)\right], \tag{2.4}$$

Thermorheological equations

$$\mu_f(T) = \frac{\mu_1}{1 - \Gamma(T - T_0)}, \tag{2.5}$$

$$\mu_p(T) = \frac{\mu_2}{1 - \Gamma(T - T_0)}, \tag{2.6}$$

where

$$p = p^* - \frac{\rho_R}{2\Phi} \nabla \left(\left| \vec{\Omega} \times \vec{r} \right|^2 \right).$$

The thermorheological Eqs. 2.5 and 2.6 have been used, following Nield (1996).

2.1 Basic State

The quiescent basic state of the liquid is described by

$$\begin{aligned} \frac{\partial()}{\partial t} &= 0, \vec{q}_b = (0, 0, 0), T = T_b(z), \\ \rho &= \rho_b(z), \mu_f = \mu_{f_b}(z), \mu_p = \mu_{p_b}(z). \end{aligned} \tag{2.7}$$

The pressure p_b, temperature T_b, density ρ_b, and the viscosities satisfy

$$\frac{dp_b}{dz} = -\rho_b g, \tag{2.8}$$

$$\frac{d^2 T_b}{dz^2} = 0 \tag{2.9}$$

$$\rho_b = \rho_R \left[1 - \alpha(T_b - T_0)\right], \tag{2.10}$$

$$\mu_{f_b}(T) = \frac{\mu_1}{1 - \Gamma\left(T_b - T_0\right)}, \tag{2.11}$$

and

$$\mu_{p_b}(T) = \frac{\mu_2}{1 - \Gamma(T_b - T_0)}. \tag{2.12}$$

Solving Eq. 2.9 for T_b using the boundary conditions

$$\begin{aligned} T_b &= T_0 + \Delta T \text{ at } z = 0, \\ T_b &= T_0 \text{ at } z = 1, \end{aligned}$$

we get

$$T_b - T_0 = \Delta T(1 - z), \tag{2.13}$$

$$\rho_b = \rho_R \left[1 + \alpha \Delta T(1 - z)\right], \tag{2.14}$$

$$\mu_{f_b}(T) = \frac{\mu_1}{1 + \Gamma \Delta T\left(1 - z\right)}, \tag{2.15}$$

and

$$\mu_{p_b}(T) = \frac{\mu_2}{1 + \Gamma \Delta T(1 - z)}. \tag{2.16}$$

2.2 Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have

$$\begin{aligned}\vec{q} &= \vec{q}_b + \vec{q}\,', T_b = T_b(z) + T', p_b = p_b(z) + p', \rho_b = \rho_b(z) + \rho',\\ \mu_f &= \mu_{f_b}(z) + \mu'_f, \mu_p = \mu_{p_b}(z) + \mu'_p.\end{aligned} \tag{2.17}$$

The prime indicates that the quantities are infinitesimal perturbations. Substituting Eq. 2.17 into Eqs. 2.1–2.3, and using the basic-state solution, we get the linearized equations governing the infinitesimal perturbations in the form:

$$\nabla \cdot \vec{q}\,' = 0, \tag{2.18}$$

$$\begin{aligned}\frac{\rho_R}{\Phi}\left[\frac{\partial \vec{q}\,'}{\partial t}\right] &= -\nabla p' + \alpha \rho_R g T' \hat{k} + 2\frac{\rho_R}{k_v}\left(\vec{q}\,' \times \vec{\Omega}\right)\\ &\quad -\mu_{f_b} \boldsymbol{k} \cdot \vec{q}\,' + \nabla \mu_{p_b} \cdot \left(\nabla \vec{q}\,' + \nabla \vec{q}\,'^{Tr}\right) + \mu_{p_b} \nabla^2 \vec{q}\,',\end{aligned} \tag{2.19}$$

$$\gamma \frac{\partial T'}{\partial t} = -\frac{\Delta T}{d} w' + \chi_{Tv}\left[\eta\left(\frac{\partial^2 T'}{\partial x^2} + \frac{\partial^2 T'}{\partial y^2}\right) + \frac{\partial^2 T'}{\partial z^2}\right]. \tag{2.20}$$

Operating curl twice on Eq. 2.19, to eliminate the pressure, we get

$$\begin{aligned}-\frac{\rho_R}{\Phi}\frac{\partial}{\partial t}\left(\nabla_1^2 w'\right) &= -\alpha \rho_R g \nabla_1^2 T' - \mu_{p_b} \nabla^4 w' - 2\frac{\partial \mu_{p_b}}{\partial z}\nabla^2\left(\frac{\partial w'}{\partial z}\right)\\ &\quad + \frac{\mu_{f_b}}{k_v}\nabla_1^2 w' + \frac{\mu_{f_b}}{k_v}\frac{1}{\varepsilon}\frac{\partial^2 w'}{\partial z^2} + \frac{1}{k_v}\frac{1}{\varepsilon}\frac{\partial \mu_{f_b}}{\partial z}\frac{\partial w'}{\partial z}\\ &\quad + \frac{\partial^2 \mu_{p_b}}{\partial z^2}\left[\nabla_1^2 w' - \frac{\partial^2 w'}{\partial z^2}\right] + 2\frac{\rho_R}{\Phi}\Omega\frac{\partial \zeta}{\partial z},\end{aligned} \tag{2.21}$$

where
$\zeta = \left(\frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}\right)$ is the z-component of the vorticity, $\vec{\omega}\,' = \nabla \times \vec{q}\,'$.

Now the equation for ζ can be obtained by differentiating x- and y-components of Eq. 2.17 partially w.r.t. y and x, respectively, and then subtracting the resulting equations from one another. This gives us the vorticity transport equation:

$$\frac{\rho_R}{\Phi}\frac{\partial \zeta}{\partial t} = 2\frac{\rho_R}{\Phi}\Omega\frac{\partial w'}{\partial z} - \frac{\mu_{f_b}}{k_v}\frac{1}{\varepsilon}\zeta + \frac{\partial \mu_{p_b}}{\partial z}\frac{\partial \zeta}{\partial z} + \mu_{p_b}\nabla^2 \zeta. \tag{2.22}$$

We now non-dimensionalize Eqs. 2.20–2.22 using the following definitions:

$$\begin{aligned}(x^*, y^*, z^*) &= \left(\frac{x}{d}, \frac{y}{d}, \frac{z}{d}\right), w^* = \frac{w'}{(\chi_v/d)}, T^* = \frac{T'}{\Delta T},\\ \zeta^* &= \frac{\zeta}{\left(\chi_v/d^2\right)}, t^* = \frac{t}{\left(d^2/\nu\right)}.\end{aligned} \tag{2.23}$$

Substituting Eq. 2.23, along with Eqs. 2.11 and 2.12, in Eqs. 2.20–2.22, we get the following equation on dropping the asterisks

$$\frac{\partial}{\partial \tau}(\nabla^2 w) = R_D \nabla_1^2 T + g(z)\left[Br_D \nabla^4 w - \left(\nabla_1^2 w + \frac{1}{\varepsilon}\frac{\partial^2 w}{\partial z^2}\right)\right] + V\,[g(z)]^2 \left[2Br_D \nabla^2 \left(\frac{\partial w}{\partial z}\right) - \frac{1}{\varepsilon}\frac{\partial w}{\partial z}\right] - \sqrt{Va_D}\frac{\partial \zeta}{\partial z} - 2Br_D V^2\,[g(z)]^3 \left(\nabla_1^2 w - \frac{\partial^2 w}{\partial z^2}\right), \tag{2.24}$$

$$\frac{\partial \zeta}{\partial \tau} = \sqrt{Va_D}\frac{\partial w}{\partial z} + g(z)\left(Br_D \nabla^2 - \frac{1}{\varepsilon}\right)\zeta + V Br_D\,[g(z)]^2 \frac{\partial \zeta}{\partial z} \tag{2.25}$$

$$Va\frac{\partial T}{\partial \tau} = w + \left[\eta\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \frac{\partial^2 T}{\partial z^2}\right], \tag{2.26}$$

where $g(z) = \{1 + V(1-z)\}^{-1}$. In the above equations, the dimensionless quantities τ, Va, Va_D, R_D, Br_D, ε, η and Λ are introduced as done by Vadasz (1998a,b). Equations 2.24–2.26 are three equations in the three unknowns w, ζ, and T. Equations 2.24–2.26 are solved subject to the conditions

$$w = Dw = D\zeta = T = 0 \text{ at } z = 0, 1. \tag{2.27}$$

The choice of rigid boundaries is to bring in the boundary effect modeled by the Brinkman term.

The infinitesimal perturbations w, T, and ζ are assumed to be periodic waves, and hence these permit normal mode solutions in the form (see Chandrasekhar 1961)

$$\begin{bmatrix} w \\ \zeta \\ T \end{bmatrix} = e^{\sigma\tau}\begin{bmatrix} w(z) \\ \zeta(z) \\ T(z) \end{bmatrix} e^{i(lx+my)} \tag{2.28}$$

where the imaginary part of σ is the scaled frequency, $w(z)$, $\zeta(z)$, and $T(z)$ are the amplitudes, and l and m are the horizontal components of the wave number such that $a^2 = l^2 + m^2$. The amplitudes must satisfy the boundary conditions of Eqs. 2.27. Substituting Eq. 2.28 into Eqs. 2.24–2.26, we get

$$g(z)\left[Br_D\left(D^2 - a^2\right)^2 + a^2 - \frac{1}{\varepsilon}D^2\right]w - \sigma\left(D^2 - a^2\right)w - R_D a^2 T + V\,[g(z)]^2\left(2Br_D\left(D^2 - a^2\right) - \frac{1}{\varepsilon}\right)Dw - \sqrt{Va_D}D\zeta + 2Br_D V^2\,[g(z)]^3\left(D^2 + a^2\right)w = 0, \tag{2.29}$$

$$g(z)\left[Br_D\left(D^2 - a^2\right) - \frac{1}{\varepsilon}\right]\zeta - \sigma\zeta + \sqrt{Va_D}Dw + Br_D V\,[g(z)]^2 D\zeta = 0, \tag{2.30}$$

$$\left(D^2 - \eta a^2\right)T - Va\sigma T + w = 0, \tag{2.31}$$

where σ is, in general, complex and $D = \frac{d}{dz}$. We discuss marginal stability considering both stationary and oscillatory convections. The boundary conditions for solving Eqs. 2.29–2.31 are obtained from Eq. 2.27, on using Eq. 2.28, in the form

$$w = Dw = D\zeta = T = 0 \text{ at } z = 0, 1. \tag{2.32}$$

3 Application of Galerkin Variant of Weighted-Residuals Technique

Equations 2.29–2.31 are solved using the Galerkin variant of weighted-residuals technique (Finlayson 1972). This method gives quite general results on the eigen value of the problem using the trial functions for the lowest eigen value. We obtain an approximate solution of the differential equations with the given boundary conditions by choosing trial functions for the velocity and temperature perturbations that may satisfy the boundary conditions but may not exactly satisfy the differential equations. This leads to residuals when the trial functions are substituted into the differential equations. The method requires the residual to be orthogonal to each individual trial function. In the Galerkin variant of the weighted-residuals procedure, we expand the velocity and temperature in the form

$$\begin{aligned} w(z,\tau) &= \sum A_i(\tau)w_i(z), \\ \zeta(z,\tau) &= \sum B_i(\tau)\zeta_i(z), \\ T(z,\tau) &= \sum C_i(\tau)T_i(z), \end{aligned} \tag{3.1}$$

where $w_i(z)$, $\zeta_i(z)$ and $T_i(z)$ are trial functions that have to satisfy the boundary conditions (2.32). For the purpose of illustration, we present below the single-term version of the technique.

Multiplying Eqs. 2.29–2.31 by w, ζ, and T, respectively, and integrating the resulting equations by parts with respect to z between 0 and 1, and taking $w = Aw_1$, $T = BT_1$, and $\zeta = C\zeta_1$, in which A, B, and C are constants, and w_1, T_1, and ζ_1 are trial functions that satisfy the boundary conditions, yield the following equations for the Darcy–Rayleigh number, R_D:

$$R_D = \frac{(G_3 - Va\sigma E_2)\,[Va_D F_1 D_{11} - (G_1 - \sigma G_2)(G_4 - \sigma F_5)]}{a^2 D_{10}^2 (G_4 - \sigma F_5)}, \tag{3.2}$$

where

$$\begin{aligned} G_1 &= Br_D\left(D_1 + a^4 D_2 - 2a^2 D_3\right) + 2Br_D V\left(D_4 - a^2 D_5\right) + 2Br_D V^2\left(D_6 + a^2 D_7\right) \\ &\quad + a^2 D_2 - \frac{1}{\varepsilon}(D_3 + V D_5), \end{aligned}$$

$$G_2 = (D_8 + a^2 D_9),\ G_3 = E_1 - \eta a^2 E_2,\ G_4 = Br_D\left(F_2 - a^2 F_3\right) + Br_D V F_4 - \frac{1}{\varepsilon}F_3,$$

$$D_1 = \left\langle w_1 D^4 w_1 g(z)\right\rangle,\ D_2 = \left\langle w_1^2 g(z)\right\rangle,\ D_3 = \left\langle w_1 D^2 w_1 g(z)\right\rangle,\ D_4 = \left\langle w_1 D^3 w_1\,[g(z)]^2\right\rangle,$$

$$D_5 = \left\langle w_1 D w_1\,[g(z)]^2\right\rangle,\ D_6 = \left\langle w_1 D^2 w_1\,[g(z)]^3\right\rangle,\ D_7 = \left\langle w_1^2\,[g(z)]^3\right\rangle,\ D_8 = \left\langle w_1 D^2 w_1\right\rangle,$$

$$D_9 = \left\langle w_1^2\right\rangle,\ D_{10} = \langle w_1 T_1\rangle,\ D_{11} = \langle w_1 D\zeta_1\rangle,\ E_1 = \left\langle T_1 D^2 T_1\right\rangle,\ E_2 = \left\langle T_1^2\right\rangle,\ F_1 = \langle \zeta_1 D w_1\rangle,$$

$$F_2 = \left\langle \zeta_1 D^2 \zeta_1 g(z)\right\rangle,\ F_3 = \left\langle \zeta_1^2 g(z)\right\rangle,\ F_4 = \left\langle \zeta_1 D\zeta_1\,[g(z)]^2\right\rangle,\ F_5 = \left\langle \zeta_1^2\right\rangle,$$

$\langle \cdots \rangle$ denotes integration with respect to z between $z=0$ and $z=1$.We note here that R_D in Eq. 3.2 is a functional and the Euler–Lagrange equations for the extremization of R_D are Eqs. 2.29–2.31.

For stationary convection, we set $\sigma = 0$ and then Eq. 3.2 becomes

$$R_D^s = \frac{G_3(Va_D F_1 D_{11} - G_1 G_4)}{a^2 G_4 D_{10}^2}. \tag{3.3}$$

For oscillatory instability, we set $\sigma = i\omega$ in Eq. 3.2, which gives

$$R_D^o = \frac{G_3G_4(Va_DF_1D_{11}-G_1G_4)+\omega^2\left(VaE_2\begin{Bmatrix}F_1F_5D_{11}Va_D+G_2G_4^2\\+\omega^2F_5^2G_2\end{Bmatrix}-G_1G_3F_5^2\right)+i\omega N}{a^2D_{10}^2\left(G_4^2+\omega^2F_5^2\right)}, \tag{3.4}$$

where

$$N = \left(G_3G_4 + \omega^2VaE_2F_5\right)(G_2G_4 + G_1F_5) + (G_3F_5 - VaE_2G_4)\left(Va_DF_1D_{11} + \omega^2G_2F_5 - G_1G_4\right). \tag{3.5}$$

Since R_D is a real quantity, either $\omega = 0$ (stationary) or $N = 0$ ($\omega \neq 0$, oscillatory). The latter condition, on simplification, yields the frequency of oscillations and the oscillatory Rayleigh number in the form:

$$\omega^2 = -\frac{G_2G_3G_4^2 + G_3F_1F_5D_{11}Va_D + VaE_2G_4(G_1G_4 - Va_DF_1D_{11})}{F_5^2(VaE_2G_1 + G_2G_3)}, \tag{3.6}$$

$$R_D^o = \frac{G_3G_4(Va_DF_1D_{11}-G_1G_4)+\omega^2\left(VaE_2\begin{Bmatrix}F_1F_5D_{11}Va_D+G_2G_4^2\\+\omega^2F_5^2G_2\end{Bmatrix}-G_1G_3F_5^2\right)}{a^2D_{10}^2\left(G_4^2+\omega^2F_5^2\right)}. \tag{3.7}$$

In evaluating R_D^s and R_D^o, we have assumed $w_1 = z^2(z-1)^2$, $T_1 = Sin\pi z$ and $\zeta_1 = Cos\pi z$.

4 Results and Discussions

In this article, a study is made of the effects on temperature-dependent viscosity and rigid-body rotation on the onset of convection in a fluid-saturated anisotropic porous medium. It is important to note that the viscosity μ decreases with increase in temperature T and that the μ–T curve is concave upward. This aspect is well covered by the thermorheological Eqs. 2.5 and 2.6. With control of convection as the motivation for the problem, the following effects on the classical Rayleigh–Bénard problem are considered:

(i) porous medium inhibition of convection,
(ii) anisotropy of the medium,
(iii) variable viscosity, and
(iv) Coriolis force.

These four effects are, respectively, represented by the inverse Darcy number Da^{-1}, the anisotropy parameters (ε, η), the variable viscosity parameter (also called thermorheological parameter) V, and the Taylor–Vadasz number Va_D. The formulation of the problem involves several assumptions (Knobloch 1998)—the lateral boundaries are far enough not to influence rotating convection and that the Froude number is quite small. The latter assumption facilitates the restoration of the conduction state as an equilibrium solution. Experimentally, the lateral boundary effect and the centrifugal effect have been shown by Ecke et al. (1992)

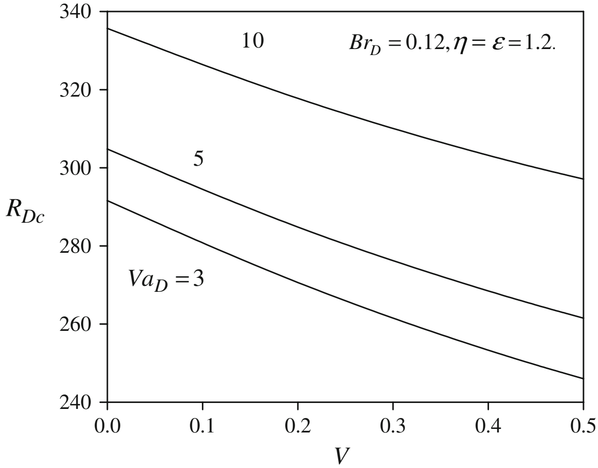


Fig. 1 Plot of critical Darcy–Rayleigh number R_{Dc} (stationary) versus variable viscosity parameter V for different values of Taylor–Vadasz number Va_D

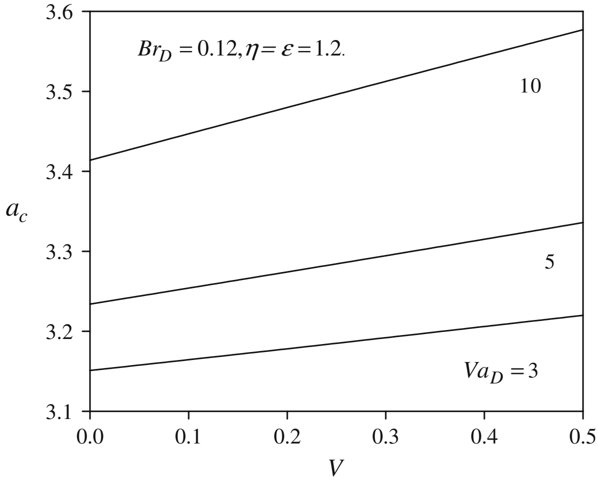


Fig. 2 Plot of critical wave number a_c (stationary) versus V for different values of Va_D.

to be quite important, but in a theoretical study to keep the problem manageable and focus on Bénard-like situations, it is common practice to exclude these effects. In order to conform to standard practices in a porous medium, we have used the Darcy–Rayleigh number rather than the viscous Rayleigh number that is used in the Chandrasekhar (1961) formulation of the problem in a clear fluid. The main emphasis of this study is to consider the effect of temperature-dependent viscosity on the onset of convection via the stationary mode, as oscillatory convection is found to be highly improbable. Before embarking on a discussion of the results depicted by the Figs. 1, 2, 3, 4, 5, 6, 7 and 8, we note that, unlike the case of a clear fluid, critical convection is always stationary for all considered values of the Vadasz and Taylor–Vadasz numbers.

Figure 1 reveals that the effect of increasing thermorheological parameter V is to destabilize the system. Figure 2 reveals that the effect of increasing Va_D is to decrease the cell size at the onset of convection. This result can be seen from the fact that the wave length is inversely proportional to the wave number.

Fig. 3 Plot of R_{Dc} versus V for different values of mechanical anisotropy parameter ε

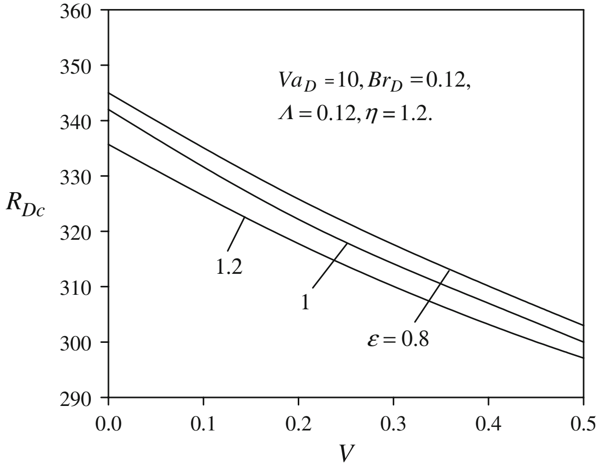


Fig. 4 Plot of a_c versus V for different values of ε

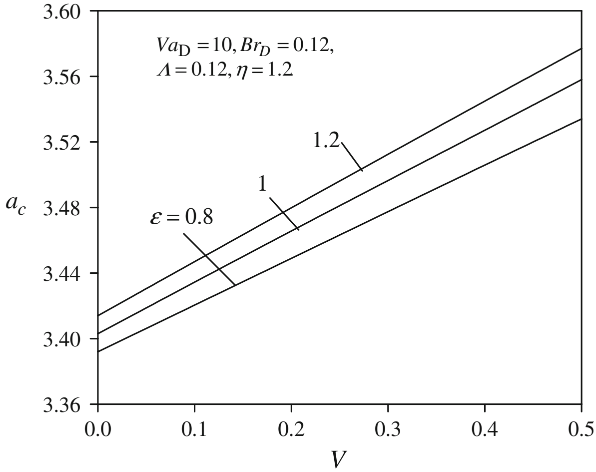


Fig. 5 Plot of R_{Dc} versus V for different values of thermal anisotropy parameter η

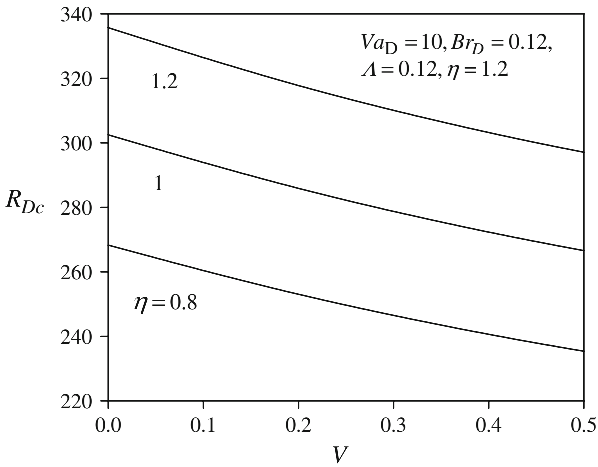


Fig. 6 Plot of a_c versus V for different values of η

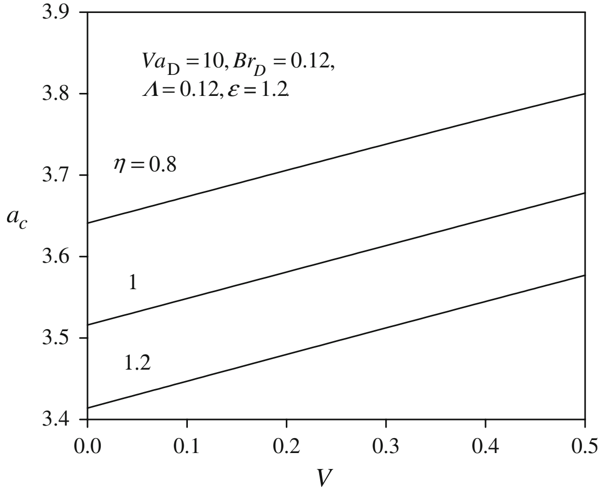
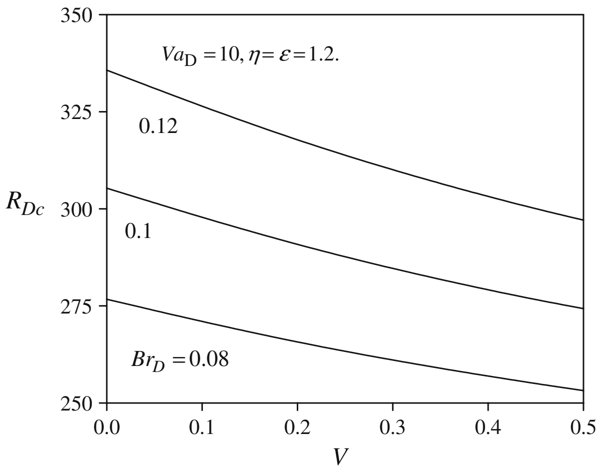


Fig. 7 Plot of R_{Dc} versus V for different values of Brinkmann–Darcy number Br_D



Epherre (1975) concluded that the effects of mechanical anisotropy and thermal anisotropy parameters are opposite in their influences on the critical Darcy–Rayleigh number. In so far as the critical wave number is concerned, the two anisotropy effects superpose on each other while influencing the cell size. This observation by Epherre (1975) in the absence of rigid-body rotation continues to be observed in the case when rotation is present. We see that increase in the value of ε is to decrease R_{Dc} and the opposite is seen with η. We also find that the observations made by Epherre (1975) on R_{Dc} and a_c hold good in the case of variable-viscosity fluids also. The above results can be seen in Figs. 3, 4, 5 and 6.

Givler and Altobelli (1994) in their pioneering article have discussed the need to have actual viscosity and effective viscosity in modeling a porous medium using the Brinkman–Darcy model. In view of the fact that boundary effects can be seen only when we have a rigid boundary, appropriate boundary conditions have been used to bring in this effect. In a medium in which the Brinkman term is important, ratio Λ of actual and effective viscosities appears in the governing equations. When scaled with the Darcy number, this can be called

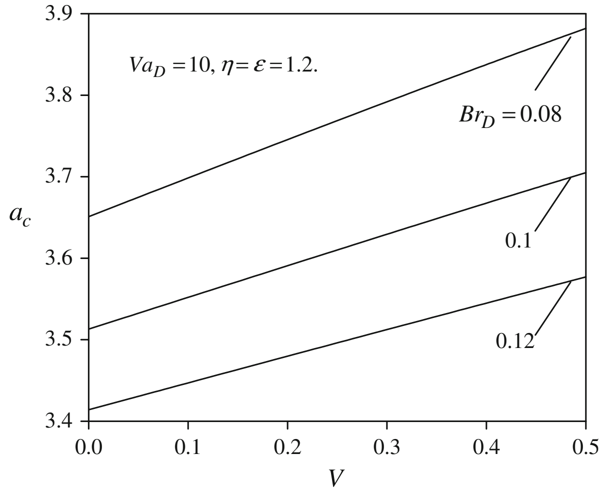


Fig. 8 Plot of a_c versus V for different values of Br_D

the Brinkman–Darcy number, Br_D. In the light of the observation made by Givler and Altobelli (1994) that the viscosity ratio can take values less than as well as greater than unity, in the article we have taken values of Br_D to be slightly lesser than or greater than 0.1. The effect of Br_D is to stabilize the system. It is also seen that the effect of increasing Br_D is to increase the cell size. These results on the effects of Br_D on R_{Dc} and a_c as discussed above can be seen in Figs. 7 and 8, respectively.

We now focus attention on Eq. 3.6 to conclude that oscillatory convection is improbable in a rotating porous medium. Notice that by inspecting Eq. 3.6 we find that the only way by which ω^2 can take a positive value is when $G_1G_4 - Va_D F_1 D_{11} > 0$. We also note that the E_i's, the F_i's, the G_i's, and D_{11} are all positive. The above line of thought suggests that by merely evaluating the ratio $r = \frac{Va_D F_1 D_{11}}{G_1 G_4}$ we can comment on the possibility or otherwise of oscillatory convection. For all parameter combinations, we found on computation that r is less than unity which thus precludes the possibility of oscillatory motions in the present problem. Computations further reveal that the effect of increasing V is to diminish the magnitude of r.

5 Conclusions

Stationary convection is preferred to oscillatory convection in the case of a rotating high-porosity medium occupied by a variable viscosity liquid. It is found that for all rotation rates temperature-dependent viscosity destabilizes the system. The effect of anisotropy parameters on the onset of convection in a rotating medium is qualitatively similar to that in the non-rotating case. Our results for a constant viscosity liquid occupying a low-porosity isotropic medium coincide with those of Vadasz (1998a,b) due to the fact that when $Br_D = 0$, $V = 0$ and $\varepsilon = \eta = 1$ the equations reduce to those reported by the author. One can easily verify that in this case the stationary and oscillatory Rayleigh numbers are given by

$$R_D^s = \frac{\delta^2 \left[\delta^2 + Va_D \pi^2\right]}{a^2}, \tag{5.1}$$

$$R_D^o = \frac{\delta^4\left(\omega^2+1\right) - Va\omega^2\left(\delta^2(\omega^2+1) - Va_D\right) Va_D\delta^2}{a^2(\omega^2+1)}, \tag{5.2}$$

where the frequency of oscillations is given by

$$\omega^2 = \frac{(\delta^2 - Va)Va_D}{\left(\delta^4 + Va\delta^2\right)} - 1, \tag{5.3}$$

where $\delta^2 = \pi^2 + a^2$.

At this point we note that our results and discussions, and conclusions are based on the choice of T_0 (temperature of the upper boundary) as the reference temperature. As pointed out by Nield (1996), the study can be carried out with $T_m = \frac{(T_0+\Delta T)+T_0}{2}$ (average temperature of the two boundaries) as the reference temperature. With T_0 replaced by T_m in the above analysis we conclude the following:

(a) R_{Dc} increases as V increases, but the variation is very weak as pointed out by Nield (1996).
(b) a_c decreases as V increases, but changes are observed only in the third decimal digit.
(c) The effect of Br_D and Va_D on R_{Dc} and a_c is the same as that when T_0 is used.

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