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A Summary of New Predictive High Frequency Thermo-Vibrational Models in Porous Media

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Abstract In this paper, we consider the effect of mechanical vibration on the onset of convection in porous media. The porous medium is saturated either by a pure fluid or by a binary mixture. The importance of a transport model on stability diagrams is presented and discussed. The stability threshold for the Darcy–Brinkman case in the Ra_{Tc}-R and k_c-R diagrams is presented (where Ra_{Tc}, k_c and R are the critical Rayleigh number, the critical wave number and the vibration parameters, respectively). It is shown that there is a significant deviation from the Darcy model. In the thermo-solutal case with the Soret effect, the influence of vibration on the reduction of multi-cellular convection is emphasized. A new analytical relation for obtaining the threshold of mono-cellular convection is derived. This relation shows how the separation factor Ψ is related to the controlling parameters of the problem, $\Psi = f\ (R, \varepsilon^*, Le)$, when the wave number $k \to 0$. The importance of vibrational parameter definition is highlighted and it is shown how, by using a proper definition for vibrational parameter, we may obtain compact relationship. It is also shown how this result may be used to increase component separation.

Keywords Porous media · Darcy–Brinkman model · Scale analysis method · High-frequency vibration · Double diffusive convection · Soret effect · Linear stability · Long-wave mode · Separation

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List of Symbols

Roman Symbols

a^*	Effective thermal diffusivity, $m^2\,s^{-1}$
b	Vibration amplitude, m
C_i	Initial mass fraction
D^*	Mass diffusion coefficient
D_T	Thermodiffusion coefficient
Da	Darcy number
$\mathbf{e}$	The direction of vibration
$\mathbf{g}$	Gravitational acceleration, $m\,s^{-2}$
H	Height, m
$\mathbf{j}$	Unit vector in y direction
k	Wave number
K	Permeability, m^2
Le	Lewis number $(a/D)^*$
P	Pressure, $N\,m^{-2}$
R	Vibration parameter
Ra	Rayleigh number
Ra_v	Vibrational Rayleigh number
T	Temperature, K
t	Dimensional time
$\mathbf{V}$	Velocity, $m\,s^{-1}$
$\mathbf{W}$	Solenoidal vector

Greek Symbols

α	Direction of vibration
β_C	Coefficient of mass expansion
β_T	Coefficient of thermal expansion
γ	The ratio of the Brinkman effective viscosity to the fluid viscosity (μ_e/μ_f)
ε	Porosity
ε^*	Normalized porosity
λ^*	Effective thermal conductivity
ν	Kinematic viscosity, $m^2\,s^{-1}$
ρ	Density, $kg\,m^{-3}$
$(\rho c)^*$	Volumic heat capacity of medium
τ	Vibration period
ψ	Separation factor
ω	Dimensional pulsation

1 Introduction

Microgravity research deals with the effects of reduced gravitational force on physical, chemical, and biological phenomena. Many scientific disciplines are affected by gravity such as fundamental physics, fluid mechanics, transport phenomena, etc. It should be noted that some of these disciplines are laboratory sciences that inherently use controlled and model experiments.

In reduced gravity, the decrease in rates of sedimentation, hydrostatic pressure and buoyancy-driven flows causes other effects to become more important. These effects at the same time can be observed and measured. The acceleration due to vibration can then be treated as an important and interesting experimental parameter. It has been shown that a spacecraft in orbit is subject to many disturbing influences of human as well as equipment origin. These influences result in the appearance of residual accelerations, which are commonly called "g-jitter." As a first approximation, "g-jitter" may be modeled as mono-frequency vibration (see, for example, Alexander 1994 or Nelson 1994).

The exploration of this parameter at normal earth gravity and reduced gravity may provide a better understanding of certain physical process, and possibly may lead to the identification of new phenomena. One idea that may be associated with microgravity is the commercial manufacturing in space environment. We summarize below some of these aspects.

1.1 Material Science and Processing

The microgravity environment provided by an orbiting spacecraft or space station offers new opportunities in the control of the solidification process. Reduction of convective velocities permits, in some cases, more precise control of the temperature and composition of the melt. Likewise, body force effects such as sedimentation will be reduced. In order to accomplish the objectives discussed above, it is necessary to conduct a series of carefully chosen, well conceived experiments. At the same time, these experiments should delineate the advantages and limitations of microgravity research. For example, microgravity experiments may be used to elucidate the essential features of solidification process and suggest better control strategies. These strategies may improve the current technologies for earth-bound experiments; an example is the application of mechanical vibration (shaking) of the container.

Another aspect which is of highest importance is the prediction and control of micro-structures. The region between the advancing solid and dendrite tip is called the "mushy zone." This region is composed of a fine, micrometer length scale mixture of liquid and solid. This closely resembles transport in porous media and this is the reason porous media modeling has received much attention. By carefully controlling the direction in which heat is extracted (directional solidification), interesting controlled solid–liquid micro-structure can be produced.

1.2 Diffusive Transport Processes

Under normal gravitational acceleration, multi-component fluids experience leads to various modes of thermo-solutal convection (depending on the relative orientation of temperature and concentration gradients with each other and with the buoyancy vector). With reduced gravity, there is an attractive opportunity to obtain a better fundamental understanding of temperature–concentration interactions, which may be overshadowed under terrestrial condition.

1.3 Fluid Mechanics and Transport Phenomena

Fluid mechanics and transport phenomena are influenced significantly by gravity. As a consequence, different behavior may be expected for many fluid configurations in a microgravity environment. Also, the reduction of gravitational body forces leads to the dominance by other forces normally obscured in terrestrial environment. Because fluid mechanics and transport processes are involved in many areas of microgravity research, they represent a common theme for fundamental studies.

Fluid mechanics and transport phenomena also play an essential role in many space-based technologies. Space system designers will be constantly challenged to develop new technologies and critical concepts that involve fluid mechanics and transport phenomena in low gravity environment.

Unfortunately, predictive models for the low gravity environments are often inadequate.

Our objective in this article is to highlight some of these predictive models in thermo-vibration problems in porous media.

2 A Brief Summary of Thermal Vibrational Convection in Porous Media

In recent years, effects of mechanical vibration on the stability threshold of thermal systems have been the subject of numerous studies. In broad terms, the subject of thermo-vibration convection concerns the appearance of a mean flow in a confined cavity filled with a fluid (mono or multi-component) subjected to temperature or concentration non-homogeneities. This type of convective motion, in which the buoyancy force may be thought of as time dependent, has attracted the attention of many researchers. Gershuni and Lyubimov (1998) gave a summary of different aspects of the thermo-vibrational problem. Their work mainly covers the Russian researches in this field and focuses on fluid media. One of the most interesting features of this book is a comprehensive treatment of the so-called time-averaged method. In this method, valid under the limiting case of high-frequency and small amplitude vibration, the time dependent acceleration does not appear explicitly in the governing equations. Instead, a vibrational force due to its mean energy appears in the momentum equation. Furthermore, in some cases, the time-averaged method provides us with closed form analytical solutions from which it is possible to study the onset of convection. Given the fact that stability characterization of thermal vibrational convection is generally complicated and depends on many physical parameters, the existence of some closed form relations is quite beneficial in understanding these problems. It should be noted that this method was first proposed by Simonenko and Zenkovskaya (1966). Thermo-vibration problem has received particular attention in porous media too. Generally, these studies can be classified in two groups: porous media saturated by a pure fluid or by a binary-mixture. Here, we report only the researches dealing with the high-frequency and small amplitude vibration; for other cases, the readers can refer to Razi et al. (2008). Zen'kovskaya (1992) studied the effect of vertical vibration (parallel to the temperature gradient) on the onset of convection in a horizontal porous layer. The Darcy model is used in the momentum equation. It is found analytically that vibration has a stabilizing effect (it increases the critical Rayleigh number). In another study, Zen'kovskaya and Rogovenko (1999) completed the previous work by considering arbitrary directions of vibration. The result of their linear stability analysis showed that only the vertical vibration has always a stabilizing effect. In addition, they find that convection under microgravity is possible, provided that the direction of vibrational is not parallel to the temperature gradient. Bardan and Mojtabi (2000) extended the vertical vibration results to the confined cavity geometry. In addition to performing the linear stability analysis, they conducted a weakly nonlinear stability analysis too. They concluded that the primary bifurcations were of symmetry-breaking pitch fork type. Razi et al. (2002) and Charrier-Mojtabi et al. (2003) discussed the validity of the time-averaged formulation. In these papers, through an analysis of a Mathieu equation, the time averaged results were obtained. Bardan et al. (2004) analyzed the importance of vibrational parameter in physical interpretations of the thermal stability results. Finally, Charrier-Mojtabi et al. (2006) revisited the confined cavity and infinite horizontal porous layer problems under the effect of vertical vibration. From a theoretical point of view, they found how we may estimate the stability results of a confined cavity from the results obtained from an infinite horizontal porous layer.

All the papers cited above deal with porous media saturated by a single component fluid. For the problems involving porous media saturated by binary mixtures, we may cite the work of Sovran et al. (2002). They presented a linear stability analysis of thermo-solutal problem. The Soret effect was also considered in the governing equations. They concluded that vertical vibrations increased the stability threshold. In addition, they presented the results of the Hopf bifurcation for negative separation ratios. Charrier-Mojtabi et al. (2004) investigated the influence of vibration on the Soret-driven convection. The confined cavity and infinite horizontal porous layer saturated by a binary-mixture were considered. Different directions of vibration were considered. From the linear stability analysis, they concluded that vertical vibration had a stabilizing effect on the stationary and Hopf bifurcations. The stability diagrams in $k_c - \Psi$ coordinates illustrated that vibration reduced the critical wave number too. They presented many tables which highlighted the effect of vibration on the Hopf frequency and the Nusselt number. Some analytical relations for the long wave mode instability were proposed too. Elhajjar et al. (2009) revisit the Sovran et al. (2002) problem. They propose a new application for the effect of vibration, namely a better species separation in the case of the long wave mode. Furthermore, for the first time in thermo-vibration problems, they perform a linear stability analysis of the long wave mode. They emphasize that the mono-cellular convection loses its stability via a transient bifurcation. They characterized this transition by its critical Rayleigh, wave number, and oscillatory frequency. In the follow up, they showed that vibration had a stabilizing effect on this kind of instability. Although thermo-solutal convection problems with the Soret effect are important from an applied point of view, thermo-solutal problems without the Soret effect have their own merits. In this kind of thermo-solutal problems, the concentration gradient is imposed and is not induced by a temperature gradient. As we may find analytical relations for the onset of stationary and the Hopf bifurcations, we may have a better opportunity to study the sensitivity of each parameter on the critical values of Rayleigh and wave numbers. For these studies, we may mention Jounet and Bardan (2001) and Mojtabi et al. (2005).

3 The Effect of High-Frequency Vibration on the Onset of Convection in a Horizontal Porous Layer Saturated by a Pure Fluid

The problem of the onset of convection in an infinite horizontal layer is well suited to highlight mathematical and physical features of thermo-vibrational problems. This is why we begin by this problem. Later, we will discuss the effect of different transport models on the convection threshold, namely Darcy and Darcy–Brinkman models.

3.1 Mathematical Formulation

The geometry of the problem consists of two horizontal parallel plates having infinite extension in the Ox direction. These plates are rigid and impermeable; they are kept at constant but different temperatures T_1 and T_2. The distance between the plates is H. The porosity and permeability of the porous material filling the layer are ε and K, respectively. The porous layer and its boundaries are subjected to a harmonic vibration. We suppose that the porous medium is homogenous and isotropic. The fluid is assumed to be Newtonian and to satisfy the Oberbeck–Boussinesq approximation. In the momentum equation, the Darcy–Brinkman model is used. In a coordinate system linked to the porous layer, the gravitational field may be replaced by the sum of the gravitational and vibrational accelerations $\mathbf{g} \rightarrow \mathbf{g} + b\omega^2 \sin(\omega t)\mathbf{j}$, where $\mathbf{j}$ is the unit vector along the axis of vibration, $\mathbf{b}$ is the displacement amplitude, and ω is the angular frequency of vibration. After making standard assumptions (local thermal equilibrium, negligible viscous heating dissipations …), the governing equations for vertical vibration (parallel to the temperature gradient) are written as:

$$\begin{aligned} &\nabla \cdot \mathbf{V} = 0, \\ &\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} + \frac{\mu_f}{K}\mathbf{V} = -\nabla P - \rho_0 \left[\beta_T (T - T_{\mathrm{ref}})\right] \left(\mathbf{g} - b\omega^2 \sin \omega t \mathbf{j}\right) + \gamma \mu_f \nabla^2 \mathbf{V}, \\ &(\rho c)^* \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{V} \cdot \nabla T = \lambda^* \nabla^2 T. \end{aligned} \tag{1}$$

The boundary conditions corresponding to (1) are

$$\begin{aligned} &\mathbf{V}(x, y = 0) = 0, \quad T(x, y = 0) = T_1, \\ &\mathbf{V}(x, y = H) = 0, \quad T(x, y = H) = T_2. \end{aligned} \tag{2}$$

In the system of the Eq. (1), μ_f is the dynamic viscosity of the fluid, $(\rho c)^*$ the effective volumic heat capacity, $(\rho c)_f$ is the volumic heat capacity of fluid, and λ^* represents the effective thermal conductivity, and finally $\gamma = \mu_e/\mu_f$ (μ_e is the Brinkman effective viscosity).

3.2 Time-Averaged Formulation

In order to study the averaged behavior of the system (1)–(2), we use the time-averaged method. This method has been used under the conditions of high frequency and small amplitude of vibration. Under these conditions, it is possible to subdivide the fields into two different parts; the first part varies slowly with time (i.e., the characteristic time is large with respect to the vibration period), while the second part varies rapidly with time and is periodic with a period of $2\pi/\omega$ (this procedure was first used in problems concerning fluid media under vertical vibrational by Simonenko and Zenkovskaya 1966).

On replacing the above-mentioned transformation into Eqs. 1–2, and by performing averaging procedures over a vibration period, we may distinguish the oscillatory fields from mean fields. Two coupled systems of equations are obtained. One governs the mean flow and the other the oscillatory flow. As the problem depends on several time scales and amplitude ratios, special relationships between time scales and amplitude ratios should be found. Before proceeding with a discussion on the time-averaged method, it is informative to describe the nature of momentum and energy equations in the thermo-vibrational problem.

For this reason, we perform an order magnitude analysis in the oscillatory system.

3.3 Scale Analysis Method for the Oscillatory System

The key step in resolving the closure problem lies in establishing relations between oscillatory velocity and temperature fields in terms of the averaged ones. For this purpose, the scale analysis method is used. This method has been successfully employed by Bejan (1994, 2000), Bejan and Nelson (1998), and Nield and Bejan (2006) in predicting boundary layer approximations, the existence of optimal geometries and critical parameters. Later, it became an important tool in Constructal theory (Bejan 2000; Bejan and Lorente 2008). In addition, this procedure in the framework of the averaging method was pioneered in Razi et al. (2002) and was further completed in Razi et al. (2004b). Here, for the first time, we outline this method in the case of Darcy–Brinkman model. The following reference scales are used in the oscillatory system of equations:

$$O(\bar{T} - T_{\mathrm{ref}}) \approx T_1 - T_2 = \Delta T, \quad O\left(\frac{\partial()}{\partial t}\right) \approx \omega(), \quad O\left(\frac{\partial(\,)}{\partial y}\right) \approx \frac{1}{H}. \tag{3}$$

By replacing these scales in the oscillating momentum equation and assuming that for the oscillating temperature scale $T' \ll \Delta T$, the buoyancy terms involving T' may be neglected (the condition for this assumption will be validated later).

In order to study the possibility of convective motion in the oscillatory momentum equation, the following expression is considered:

Buoyancy Term (Containing ΔT)$\approx$Inertia (Transient Term in the Momentum Equation)

By replacing the order magnitudes of corresponding terms in this expression, we obtain the oscillating velocity scale

$$v'_{\mathrm{scale}} \approx \varepsilon \beta_T \Delta T b \omega \tag{4}$$

Furthermore, from the inequality *Inertia >>Darcy and Brinkman Friction terms*, we get:

$$\frac{\varepsilon v}{K\omega} \ll 1 \quad \text{or} \quad \tau_{\mathrm{vib}} \ll \tau_{\mathrm{viscD}} \tag{5a}$$

$$\frac{\varepsilon \gamma \nu}{H^2 \omega} \ll 1 \quad \text{or} \quad \tau_{\mathrm{vib}} \ll \tau_{\mathrm{viscB}} \tag{5b}$$

In relation (5a) and (5b), $\tau_{\mathrm{vib}} = 1/\omega$, $\tau_{\mathrm{viscD}} = K/\varepsilon\nu$, and $\tau_{\mathrm{viscB}} = H^2/\varepsilon\gamma\nu$, which represent vibrational, Darcy, and Brinkman viscous time scales, respectively. Assumptions (5a) and (5b) allow us to neglect the viscous terms in the oscillating momentum equation. It should be noted that (5b) is the additional assumption related to applying the Darcy–Brinkman model.

Following the same procedure, the order magnitude of important terms in the oscillatory energy equation is found. Imposing the oscillatory velocity scale in the equality *Convection $\approx$ Transient term* and using the hypothesis $T' \ll \Delta T$ results in:

$$T' \approx \frac{\varepsilon}{\sigma} \beta_T \Delta T^2 \frac{b}{H} \quad \text{or} \quad b \ll \frac{H}{\frac{\varepsilon}{\sigma}\beta_T \Delta T}, \left(\sigma = \frac{(\rho c)^*}{(\rho c)_f}\right) \tag{6}$$

Inequality (6) gives the criterion for small-amplitude vibration. Also, from the following inequality

Transient Term $\gg$ Diffusive (Conductive) Terms

We obtain:

$$\frac{a^*}{\sigma H^2 \omega} \ll 1 \quad \text{or} \quad \tau_{\mathrm{vib}} \ll \tau_{\mathrm{cond}}, \left(a^* = \frac{\lambda^*}{(\rho c)_f}\right) \tag{7}$$

In (7), $\tau_{\mathrm{cond}} = \sigma H^2/a^*$ represents the conductive time scale. Relation (7) allows us to neglect the diffusive terms in the energy equation.

Now that the scale of T' has been found, the final step is to validate our assumptions in the oscillatory momentum equation; in other words, it should be shown under which condition $\rho_0 \beta_T \Delta T b \omega^2$ is the dominant buoyancy force.

Under condition:

$$\omega^2 \gg \frac{g\varepsilon\beta_T \Delta T}{H\sigma} \quad \text{or} \quad \tau^2_{\mathrm{vib}} \ll \tau^2_{\mathrm{buoy}}. \tag{8}$$

$\rho_0 \beta_T \Delta T b\omega^2$ is the dominant buoyancy force in the oscillatory momentum equation. In (8), the gravitational buoyancy time scale is defined as $\tau_{\text{buoy}} = (\varepsilon g \beta_T \Delta T / \sigma H)^{1/2}$.

3.4 Time-averaged System of Equations

By applying assumptions (5a), (5b), (6), (7), and (8) to the oscillatory system of equations and also by applying the Helmholtz decomposition, the oscillatory pressure term may be eliminated. This allows the finding of exact oscillatory velocity and temperature (**W** and $\nabla\phi$ are solenoidal and irrotational parts of the Helmholtz decomposition, respectively):

$$\mathbf{V}' = -(\varepsilon \beta_T b\omega \cos \omega t)\mathbf{W}, \tag{9}$$

$$T' = \left(\frac{\varepsilon}{\sigma} \beta_T b \sin \omega t\right) \mathbf{W} \cdot \nabla \bar{T}. \tag{10}$$

By substituting Eqs. 9 and 10 in the coupling terms of mean fields, the time-averaged equations are found (details can be found elsewhere in Razi et al. 2004b). By introducing reference parameter, T_1–T_2 for temperature, H for height, $\sigma H^2/a^*$ for time, a^*/H for velocity, $\beta_T \Delta T$ for **W**, and $\mu a^*/K$ for pressure, the resulting averaged system in dimensionless form may be written as

$$\begin{aligned}
\nabla \cdot \bar{\mathbf{V}}^* &= 0,\\
B\frac{\partial \bar{\mathbf{V}}^*}{\partial t^*} + \bar{\mathbf{V}}^* &= -\nabla \bar{P}^* + Ra_T \bar{T}^* \mathbf{j} + Ra_v\, \mathbf{W}^* \cdot \nabla \bar{T}^* \mathbf{j} + \gamma\, Da \nabla^2 \bar{\mathbf{V}}^*,\\
\frac{\partial \bar{T}^*}{\partial t^*} + \bar{\mathbf{V}}^* \cdot \nabla \bar{T}^* &= \nabla^2 \bar{T}^*,\\
\nabla \cdot \mathbf{W}^* &= 0,\\
\nabla \times \mathbf{W}^* &= \nabla \bar{T}^* \times \mathbf{j}
\end{aligned} \tag{11}$$

The corresponding boundary conditions for this system are

$$\begin{aligned}
&\forall x^*, \quad \text{for} \quad y^* = 0, \quad \bar{\mathbf{V}}^* = 0, \quad \bar{T}^* = 1, \quad \mathbf{W}_y^* = 0,\\
&\forall x^*, \quad \text{for} \quad y^* = 1, \quad \bar{\mathbf{V}}^* = 0, \quad \bar{T}^* = 0, \quad \mathbf{W}_y^* = 0.
\end{aligned} \tag{12}$$

in which

$$\begin{aligned}
Ra_T &= \frac{K g \beta_T \Delta T H}{\nu a^*}, \quad Ra_v = R^2 Ra_T^2, \quad Da = \frac{K}{H^2}, \gamma = \frac{\mu_e}{\mu_f}\\
&\left(B = \frac{a^* K}{\varepsilon \nu \sigma H^2} = \frac{\tau_{\text{visc}}}{\tau_{\text{cond}}},\ R^2 = \frac{\varepsilon \nu a^*}{2K}\left(\frac{b\omega}{gH}\right)^2\right)
\end{aligned} \tag{12a}$$

In the above relationships, Ra_T is the thermal Rayleigh number, Ra_v is the vibrational Rayleigh number, and B is the ratio of viscous time scale to the conductive time scale. It should be noted that we may define vibrational Rayleigh number as

$$Ra_v = \frac{K \varepsilon (b\omega \beta_T \Delta T)^2}{2\nu a^*} \tag{12b}$$

This definition is more appropriate for weightlessness studies, as it is always positive. In any case, under simultaneous action of vibration and gravitational accelerations, it is better to separate the vibrational parameters from thermal parameters (see Bardan et al. 2004). This remark will be revisited in Sect. 4.4.

3.5 Linear Stability Analysis

3.5.1 Darcy Model

In the presence of vertical vibration (vibration parallel to the temperature gradient), mechanical equilibrium is possible. In order to find the motionless state, the velocity is set equal to zero. The equilibrium state corresponds to a linear distribution of temperature and zero for the solenoidal field.

For linear stability analysis, the temperature, velocity, and solenoidal fields are perturbed around the equilibrium state. By performing the standard linearizing procedure and developing disturbances in normal modes, we obtain:

$$(-\lambda B+1)\left(\frac{d^2\phi(y^*)}{dy^{*2}}-k^2\phi(y^*)\right)=-IkRa_T\theta(y^*)+k^2Ra_v f(y^*),$$
$$-\lambda\theta(y^*)+Ik\phi(y^*)=\frac{d^2\theta(y^*)}{dz^{*2}}-k^2\theta(y^*),$$
$$-k^2 f(y^*)+\frac{d^2 f(y^*)}{dy^{*2}}=-Ik\theta(y^*). \tag{13}$$

In (13), k is the wave number and ϕ, θ, and f represent amplitude of velocity, solenoidal, and temperature disturbances, respectively. Also, λ characterizes the eigenvalue of the system, which is generally a complex number ($\lambda = \lambda_r + I\lambda_i$). It should be noted that velocity boundary conditions in (12) should be modified and the slip condition should be imposed. There exist exact solutions of sinusoidal form for this system, which upon replacing in (13), results in the following relation for the marginal stability ($\lambda = 0$):

$$Ra_T=\frac{(\pi^2+k^2)^2}{k^2}+Ra_v\frac{k^2}{\pi^2+k^2}. \tag{14}$$

For all values of control parameters, it has been verified numerically that $\lambda_i = 0$. It can be understood from the above equation that, under micro-gravity ($Ra_T = 0$), the system remains thermally stable. Under the condition of vibration in the presence of gravity, Ra_v can be replaced by $R^2 Ra_T^2$. From Eq. 14, we get:

$$Ra_T=\frac{k^2+\pi^2}{2k^2R^2}\left[1-\sqrt{1-4R^2(k^2+\pi^2)}\right]. \tag{15}$$

Figure 1 shows the critical Rayleigh number as a function of R for the layer heated from below. We conclude that vibration has a stabilizing effect, and the critical Rayleigh number increases. At the same time, vibration decreases the critical wave number. Another interesting feature of Eq. 15 is that it gives additional information for complete stabilization:

$$R=\frac{1}{2\pi},\quad (k_c\to 0) \tag{16}$$

we may observe the existence of this asymptote in Fig. 2.

3.5.2 Darcy–Brinkman Model

In this section, we consider two different sets of boundary conditions, namely free surface and rigid boundaries.

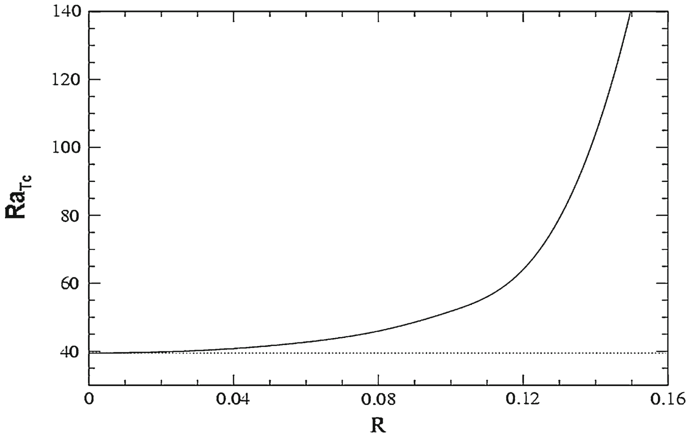

Fig. 1 Influence of vibration parameter R on the onset of convection in a horizontal porous layer saturated by a pure fluid (Darcy model) for the layer heated from below

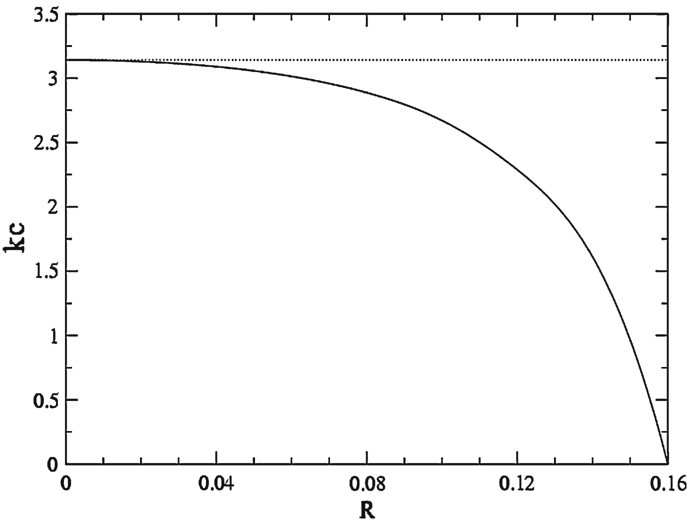

Fig. 2 Influence of vibration parameter R on the critical wave number k_c (Darcy model) for the layer heated from below

Free Surface Boundary Conditions (Unrealistic Boundary Condition): For this case, there is an exact solution for the system of equations and the procedure is the same as previous section. We find an analytical relation which resembles (14):

$$Ra_T = \frac{(\pi^2 + k^2)^2}{k^2}\left[1 + \gamma Da(\pi^2 + k^2)\right] + R^2 Ra_T^2 \frac{k^2}{k^2 + \pi^2} \tag{17}$$

From a comparison of Eqs. 14 and 17, we conclude that the Brinkman term only modifies the first part, and it has no effect on the second part (vibrational effect). This is due to the fact that vibrational effect under high frequency and small amplitude is obtained by neglecting the viscous terms. The critical Rayleigh and wave numbers can be found from the following system:

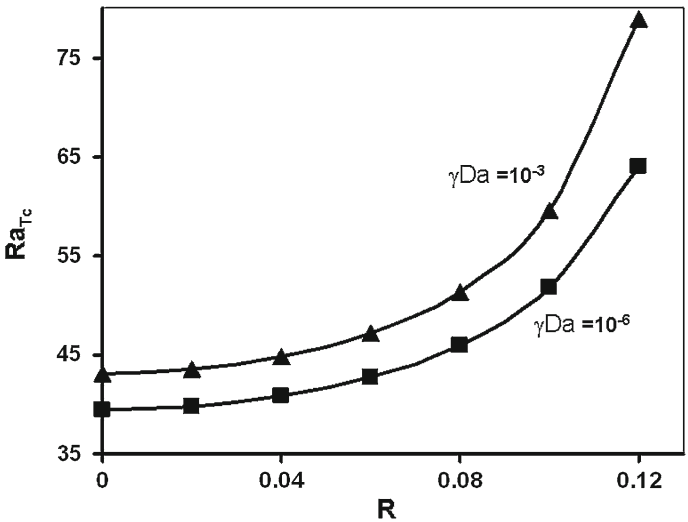

Fig. 3 Influence of vertical vibration on the critical Rayleigh number for different values of γDa for the layer heated from below

$$\begin{cases} Ra_{Tc} = \dfrac{\left[(2\pi^2 - k_c^2) + 2\gamma Da(\pi^4 - k_c^4)\right](\pi^2 + k_c^2)^2}{\pi^2 k_c^2} \\ R^2 = \dfrac{\pi^2\left[(\pi^2 - k_c^2) + \gamma Da(\pi^2 - 2k_c^2)(\pi^2 + k_c^2)\right]}{(\pi^2 + k_c^2)\left[(2\pi^2 - k_c^2) + 2\gamma Da(\pi^4 - k_c^4)\right]^2} \end{cases} \tag{18}$$

In this situation, the critical vibrational parameter for absolute stabilization is modified:

$$R_{\max}^2 = \frac{1}{4\pi^2(1 + \gamma\pi^2 Da)}, \quad k_c \to 0 \tag{19}$$

Equation 19 shows that the introduction of the Brinkman term leads to a reduction of the critical vibrational parameter. This is due to the additional frictional effect in the time-averaged momentum equation.

Rigid Boundary Conditions: Figure 3 shows the effect of γDa parameter on the onset of convection. For the layer heated from below, we see that the Brinkman model modifies the critical Ra_{Tc} values significantly. This means that by increasing γDa, there is a deviation from the Darcy model. This deviation depends largely on vibrational parameter (Razi et al. 2005). For the layer heated from above, the conductive solution is always stable.

For an alternative mathematical description (see Maliwan 2004).

3.6 Some Key Results

In this section, we studied the effect of transport models on the stability threshold under the effect of vibration. The direction of vibration is vertical (parallel to the temperature gradient). The time-averaged method is adopted. For the first time, the time-averaged governing equations for the Darcy–Brinkman model are obtained. The additional assumption in obtaining this system of equations has been explained. We showed that there is a significant deviation (20%) from the Darcy model in determining the critical Rayleigh number. This effect is not as significant in critical wave numbers.

4 Influence of High-Frequency Mechanical Vibration on a Porous Medium Saturated by a Binary Mixture

In this section, we study the effect of vibration on a porous medium saturated by a multi-component fluid. This problem is more interesting than a single-component fluid for in addition to buoyancy forces due to the gravity and vibration we may also have the effect of dissipative mechanism. This mechanism is cause by diffusion (or thermo-diffusion). As put forward in Van Vaerenberg and Legros (1990), the Soret effect must be taken into account as it can modify the concentration gradient in the liquid–solid interface. Under the Soret effect, the temperature gradient invokes a concentration gradient in a binary mixture (see De Groot and Mazur 1984). From a physical point of view, we may encounter oscillatory instability (Hopf bifurcation), which is normally absent in a single component fluid (pure fluid) situation under the effect of vibration (see Sect. 3). Many instability modes such as stationary multi-cellular, mono-cellular, and oscillatory multi-cellular may be observed too.

This problem in the context of the high-frequency vibration in a binary mixture was pioneered by Gershuni et al. (1997) for horizontal vibration and Gershuni et al. (1999) for vertical vibration. Razi et al. (2004a) completed this study for arbitrary directions of vibration. The geometry was an infinite horizontal fluid layer. They found that vibration modifies the stability diagram and can be effectively used to control the onset of convection. Generally, the vertical vibration (parallel to the temperature gradient) increases the stability threshold. For the same problem, under finite frequency situations, Smorodin et al. (2002) showed that vertical vibration had a stabilizing effect in the synchronous mode.

Due to the application and importance of solidification control and the existing analogies with porous media, which was previously explained (Sect. 1), many researchers studied this problem in recent years. In this context, we may mention Sovran et al. (2002), Maliwan (2001), Maliwan et al. (2002), and Charrier-Mojtabi et al. (2004). Elhajjar et al. (2009) proposed a new application of vibration in increasing separation. They also pioneered the stability analysis of the long-wave mode, too.

For the case in which the concentration gradient is imposed independently and is not generated by the temperature gradient, we may cite Jounet and Bardan (2001) and Mojtabi et al. (2005).

4.1 Problem Description

The geometry of the problem consists of a rectangular cavity filled with a porous medium and saturated by a binary mixture. The aspect ratio is defined as $A = L/H$, where H represents the height and L the length of the cavity. The cavity boundaries are rigid and impermeable; the horizontal boundaries can be heated from below or above. The governing equations are written in a reference frame linked to the cavity. For the high frequency and small amplitude vibration, the time-averaged equations in dimensionless form are written as Elhajjar et al. (2009) for vertical vibration and Charrier-Mojtabi et al. (2004) for arbitrary directions of vibration):

$$\nabla \cdot \bar{\mathbf{V}}^* = 0$$

$$B\frac{\partial \bar{\mathbf{V}}^*}{\partial t^*} + \bar{\mathbf{V}}^* = -\nabla \bar{P}^* + Ra\left(\bar{T}^* + \psi \bar{C}^*\right)\mathbf{j}$$
$$+ Ra_v(\mathbf{W}_T^* + \psi \mathbf{W}_c^*) \cdot \nabla\left(\bar{T}^* + \frac{\psi}{\varepsilon^*}\bar{C}^*\right)\mathbf{e}$$

$$\frac{\partial \bar{T}^*}{\partial t^*} + \bar{\mathbf{V}}^* \cdot \nabla \bar{T}^* = \nabla^2 \bar{T}^*$$
$$\varepsilon^* \frac{\partial \bar{C}^*}{\partial t^*} + \bar{\mathbf{V}}^* \cdot \nabla \bar{C}^* = \frac{1}{Le}\left(\nabla^2 \bar{C}^* - \nabla^2 \bar{T}^*\right)$$
$$\nabla \cdot \mathbf{W}_T^* = 0, \quad \nabla \cdot \mathbf{W}_c^* = 0$$
$$\nabla \bar{T}^* \times \mathbf{e} = \nabla \times \mathbf{W}_T^*, \quad \nabla \bar{C}^* \times \mathbf{e} = \nabla \times \mathbf{W}_c^* \quad \text{where } \mathbf{e} = \cos(\alpha)\mathbf{i} + \sin(\alpha)\mathbf{j} \tag{20a}$$

where $\mathbf{W}_T^*$ and $\mathbf{W}_c^*$ are the solenoidal vectors corresponding to the temperature and concentration, respectively. For finding the assumptions leading to this system, see Appendix. The corresponding boundary conditions are given by

$$\begin{aligned} &\bar{V}^* \cdot \mathbf{n} = \mathbf{W}_T^* \cdot \mathbf{n} = \mathbf{W}_c^* \cdot \mathbf{n} \quad \forall M \in \partial\Omega \\ &y^* = 0 : \bar{T}^* = 1, \quad \mathbf{J}_{\mathrm{m}}^* \cdot \mathbf{n} = 0 \\ &y^* = 1 : \bar{T}^* = 0, \quad \mathbf{J}_{\mathrm{m}}^* \cdot \mathbf{n} = 0 \\ &x^* = 0, \ A : \frac{\partial \bar{T}^*}{\partial x^*} = \frac{\partial \bar{C}^*}{\partial x^*} = 0 \end{aligned} \tag{20b}$$

In obtaining the system of Eq. (20), we assumed that Dufour effect is negligible (as we are interested in liquid mixture). $\mathbf{J}_{\mathrm{m}}^*$ represents the non-dimensional mass flux ($\mathbf{J}_{\mathrm{m}}^* = \nabla C^* - \nabla T^*$).

System (20a) depends on the following parameters: the thermal Rayleigh number $Ra = Kg\beta \Delta T H/\nu a^*$, the vibrational Rayleigh number $Ra_v = R^2 Ra_T^2$, the separation factor $\psi = -C_i(1-C_i)(\beta_c/\beta_T)D_T/D^*$, the normalized porosity ε^* ($\varepsilon^* = \varepsilon/\sigma$) where $\sigma = (\rho c)^*/(\rho c)_f$, the Lewis number Le ($Le = a^*/D^*$ in which a^* is the effective thermal diffusivity and D^* is the effective mass diffusivity), the coefficient of the unsteady Darcy term in the momentum equation B (in porous media B is normally very small $\approx 10^{-5}$), and finally α which is the direction of vibration with respect to the heated boundary.

4.2 Mechanical Equilibrium (or Quasi-Equilibrium)

When the direction of vibration is parallel to the temperature gradient, i.e., $\alpha = \pi/2$, there exists a mechanical equilibrium, for both an infinite horizontal layer and a confined cavity. This solution is characterized by

$$\mathbf{V}_0 = 0, \quad T_0 = 1 - y, \quad C_0 = c_1 - y, \quad \mathbf{W}_{T0} = 0, \quad \mathbf{W}_{C0} = 0 \tag{21}$$

However, for other directions of vibration, the situation is quite different. For an infinite horizontal layer, there exists a mechanical quasi-equilibrium solution, which is represented by zero mean velocity and generally non-zero oscillatory part. This motionless state for an infinite horizontal layer is characterized by

$$\mathbf{V}_0 = 0, \quad T_0 = 1 - y, \quad C_0 = c_1 - y, \quad \mathrm{W}_{T0_x} = c_2 - y\cos\alpha; \quad \mathrm{W}_{T0_y} = 0, \tag{22}$$
$$\mathrm{W}_{C0_x} = c_3 - y\cos\alpha, \quad \mathrm{W}_{C0_y} = 0$$

4.3 Formulation of the Stability Problem in an Infinite Horizontal Porous Layer

In order to investigate the stability of the conductive solution, the fields are perturbed around the equilibrium solution. Then, after performing linearization, the disturbances are developed in the form of normal modes:

$$(\xi', \theta', \eta', F_T', F_C') = (\hat{\xi}, \hat{\theta}, \hat{\eta}, \hat{F}_T, \hat{F}_\eta)\exp(-\lambda t + Ikx) \tag{22a}$$

In the above equation, the disturbances $\hat{\xi}, \hat{\theta}, \hat{\eta}, \hat{F}_T, \hat{F}_\eta$ represent the amplitude of velocity stream function, temperature, the transformation $(c' - \theta')$, solenoidal stream function related to the temperature, and finally solenoidal stream function related to η.

It is assumed that the perturbation quantities are sufficiently small so that the second-order terms may be neglected. The system of equations for amplitudes can be written as

$$\begin{aligned}
&-(\lambda B + 1)(D^2 - k^2)\hat{\xi} = IkRa\left[(1+\psi)\hat{\theta} + \psi\hat{\eta}\right] \\
&+ Ra_v\left[-k^2(W_{T0x} + \psi W_{C0x})\left(\left(1 + \frac{\psi}{\varepsilon^*}\right)\hat{\theta} + \frac{\psi}{\varepsilon^*}\hat{\eta}\right)\sin\alpha\right. \\
&-IkD\,(W_{T0x} + \psi W_{C0x})\left(\left(1 + \frac{\psi}{\varepsilon^*}\right)\hat{\theta} + \frac{\psi}{\varepsilon^*}\hat{\eta}\right)\cos\alpha \\
&-Ik\,(W_{T0x} + \psi W_{C0x})D\left(\left(1 + \frac{\psi}{\varepsilon^*}\right)\hat{\theta} + \frac{\psi}{\varepsilon^*}\hat{\eta}\right)\cos\alpha \\
&-k^2\left(1 + \frac{\psi}{\varepsilon^*}\right)((1+\psi)\hat{F}_T + \psi\hat{F}_\eta)\sin\alpha \\
&\left.-Ik\left(1 + \frac{\psi}{\varepsilon^*}\right)D((1+\psi)\hat{F}_T + \psi\hat{F}_\eta)\cos\alpha\right] \\
&\lambda\hat{\theta} + Ik\hat{\xi} = (D^2 - k^2)\hat{\theta},\ \varepsilon^*\lambda(\hat{\theta} + \hat{\eta}) + Ik\hat{\xi} = \frac{1}{Le}(D^2 - k^2)\hat{\eta} \\
&(D^2 - k^2)\hat{F}_T = D\hat{\theta}\cos\alpha - Ik\sin\alpha,\ (D^2 - k^2)\hat{F}_\eta = D\hat{\eta}\cos\alpha - Ik\hat{\eta}\sin\alpha
\end{aligned} \tag{23}$$

The corresponding boundary conditions are given as

$$\begin{aligned}
&\hat{\xi}(x,0) = \hat{\theta}(x,0) = D\hat{\eta}(x,0) = \hat{F}_T(x,0) = \hat{F}_\eta(x,0) = 0 \\
&\hat{\xi}(x,1) = \hat{\theta}(x,1) = D\hat{\eta}(x,1) = \hat{F}_T(x,1) = \hat{F}_\eta(x,1) = 0
\end{aligned} \tag{24}$$

The solution of system (23)–(24) leads to a spectral amplitude problem in which λ is related to the important thermo-physical parameters of the problem, namely $\lambda = \lambda\ (Ra, Ra_v, \Psi, \alpha, \varepsilon^*, k, Le)$. Generally, the decay rate λ is a complex number, i.e., $\lambda = \lambda_r + I\lambda_i$. For a stationary bifurcation, the stability domain is determined by setting $\lambda = 0$. In the case of an oscillatory bifurcation, the stability domain is determined by setting $\lambda_r = 0$ (λ_i represents the frequency of the oscillating instability).

4.4 Limiting Case of the Long-Wave Mode Instability ($\alpha = \pi/2$)

The results of the previous studies indicate that the long wave mode ($k = 0$) is the dominant mode of the Soret-driven convection under the effect of mechanical vibration in binary liquids. For this reason, we study the special case of the long wave mode theoretically. In some related studies in fluid media, Gershuni et al. (1997) and Gershuni et al. (1999), and Razi et al. (2004a) showed that asymptotic analysis results in a closed form relation for the stability threshold. In order to obtain such a relation, a regular perturbation method with the wave number as the small parameter is performed (for simplifying the procedure, we drop the hat symbol in (23) and (24)):

$$\xi = \sum_{n=0}^{\infty} k^n \xi_n;\ \theta = \sum_{n=0}^{\infty} k^n \theta_n;\ \eta = \sum_{n=0}^{\infty} k^n \eta_n;\ F_T = \sum_{n=0}^{\infty} k^n F_{T_n};\ F_\eta = \sum_{n=0}^{\infty} k^n F_{\eta_n};$$
$$\lambda = \sum_{n=0}^{\infty} k^n \lambda_n \qquad (25)$$

By substituting expressions (25) in the amplitude equations resulting from the linear stability analysis and factoring the same order of k, we find a sequential system of equations:
For the zeroth order (k^0):

$$\xi_0 = 0;\ \theta_0 = 0;\ \ \eta_0 = cst;\ F_{\eta 0} = 0;\ F_{T_0} = 0;\ \lambda_0 = 0 \qquad (26)$$

For the first-order (k^1):

$$\theta_1 = 0,\ \xi_1 = -\frac{I Ra_T \psi \eta_0}{2}(y^2 - y),\ \eta_1 = cst;\quad F_{T_1} = 0;\ F_{\eta 1} = -\frac{I\eta_0}{2}(y^2 - y);$$
$$\lambda_1 = 0 \qquad (27)$$

For the second-order(k^2):

For this order of k, we obtain the following system of equations:

$$\begin{cases} -\dfrac{d^2\xi_2}{dy^2} = I Ra_T \psi \eta_1;\ \dfrac{d^2\theta_2}{dy^2} = I\xi_1 \\ \varepsilon^*\lambda_2\eta_0 + I\xi_1 = \dfrac{1}{Le}\left(\dfrac{d^2\eta_2}{dy^2} - \eta_0\right);\ \dfrac{d^2 F_{T_2}}{dy^2} = 0;\ \dfrac{d^2 F_{\eta 2}}{dy^2} = -I\eta_1 \end{cases} \qquad (28)$$

subjected to the corresponding boundary conditions

$$At\ y = 0 \ \text{ and } \ y = 1 : \xi_2 = \theta_2 = \frac{d\eta_2}{dy} = F_{T_2} = F_{\eta 2} = 0$$

After invoking the solvability condition, we find:

$$\varepsilon^*\lambda_2 = \frac{1}{Le} - \frac{\psi\ Ra_T}{12} \qquad (29)$$

We note that λ_2 is a real number ($\lambda_2 \in R$), which means that instability is of stationary type. For the marginal stability λ_2 is set equal to zero and we obtain:

$$Ra_{Tcs} = \frac{12}{\psi\, Le} \qquad (30)$$

As mentioned in Razi et al. (2004a) in thermo-vibration problems in fluid-media, we may continue this procedure up to the fourth order of k. By setting $\lambda_4 = 0$, we find the following relation:

$$204\frac{1+\psi}{Le\psi} - 14 Le Ra_v \psi\left(1 + \frac{\psi}{\varepsilon^*}\right) = 160 \qquad (31)$$

The solution of this nonlinear equation shows from which value of ψ the long wave-mode may appear. The results also emphasize the fact that vibration increases the long wave mode domain. In other words, this means that the long wave mode settles in at the lower value of ψ under the effect of vertical vibration.

In addition, relation (31) can be written in a more compact form if we use the alternative definition of vibrational Rayleigh number ($Ra_v = R^2\ Ra_T^2$):

$$\psi = \frac{204 - 2016R^2}{160Le - 204 + \frac{2016R^2}{\varepsilon^*}} \quad \text{for} \quad k \to 0 \tag{32}$$

Relation (32) is the new result, which provides us with a theoretical basis for the previous publications, Sovran et al. (2002), Charrier-Mojtabi et al. (2004), and Elhajjar et al. (2009). In addition, the importance of vibrational Rayleigh definition is highlighted one more time in the Soret driven convection.

These theoretical relations for the long wave mode for different orientation of vibration are compared with the relations previously obtained in fluid media (Razi et al. 2004a; Mojtabi et al. 2005); see Table 1. The comparison of the results reveals that the Darcy model can capture the essential physical features of binary mixture under the effect of vibration in fluid media.

Relation (32) also shows how by selecting the vibrational parameter, we may increase the mono-cellular region significantly.

4.5 Stability Analysis Results for Arbitrary Values of Wave Number

The aim of this section is to present the effect of vibration on the critical Rayleigh number for arbitrary values of critical wave number.

The results are presented in the stability diagram Ra_{Tc}-ψ and k_c-ψ. Only $\psi > 0$ region is considered. For a complete analysis, the readers are referred to Charrier-Mojtabi et al. (2004) or Ehajjar et al. (2008). Figure 4 illustrates the effect of vibration on the onset of convection for the layer heated from below. For this case, the numerical values of physical parameters are chosen as $Le = 100$, $B = 10^{-6}$, and $\varepsilon^* = 0.5$. The vibrational Rayleigh number is varied in the interval $0 < Ra_v < 50$. We can see that all the curves in this region fall between two limiting ones: 1. ($Ra_v = 0$, classical Soret driven convection under static gravity) and 2. ($k = 0$, or the long wave mode instability). Therefore, we conclude that $Ra_T > 12/(Le\Psi)$ is a sufficient condition for the onset of convection. Figure 5 shows the effect of vibration on the critical wave numbers. It is evident that vibration decreases the multi-cellular instability; in other words, vibration reduces the critical wave number. Another interesting result is that vibration reduces the mono-cellular threshold values of Ψ for convection (cf. Fig. 6). This conclusion is in perfect agreement with relations (31) and (32). Table 2 provides a comparison of the critical values of thermal Rayleigh and wave numbers of classical situation (absence of vibration) with the situation under different values of vibrational Rayleigh number. For a detailed analysis of the classical case of the Soret driven convection (in the absence of vibration field), the readers are referred to Charrier-Mojtabi et al. (2007).

4.6 Separation Management Under High Frequency Vibration

As mentioned elsewhere (Bonneville 1990), finding experiments that emphasize the benefits of residual acceleration is of prominent importance. Recently, Elhajjar et al. (2009) proposed such an application. They showed that in the case of mono-cellular convection, there is a possibility of increasing separation (cf. Fig. 7). Their study is theoretical as well as numerical. It should be added that the separation is defined as $S = mA$, in which m is the slope of the concentration field in the horizontal direction, and A is the aspect ratio. They show that there

Table 1 Comparison of results of the long-wave mode for the onset of convection in the presence of vibration

Physical situation	Fluid layer	Porous layer
Vertical vibration	$Ra = \frac{720}{\psi\, Le}, \psi_{k\to 0} = \frac{131 - 71280R^2}{71280R^2 - 131 + 34Le}$	$Ra = \frac{12}{\psi\, Le}, \psi_{k\to 0} = \frac{204 - 2016R^2}{\frac{2016R^2}{\varepsilon^*} - 204 + 160Le}$
Microgravity conditions	$Ra_v = \frac{720}{\psi(1+\psi)Le\cos^2\alpha}$	$Ra_v = \frac{12\varepsilon^*}{\psi(1+\psi)Le\cos^2\alpha}$
Simultaneous action of vibration and gravitation	$Ra = \frac{-\psi\, Le \pm \sqrt{(\psi\, Le)^2 + 2880\psi(1+\psi)Le\cos^2\alpha\ R^2}}{2\psi(1+\psi)Le\ \cos^2\alpha\ R^2}$	$Ra = \frac{-\varepsilon^*\psi\, Le \pm \sqrt{(\varepsilon^*\psi\, Le)^2 + 48\varepsilon^*\psi(1+\psi)Le\cos^2\alpha\ R^2}}{2\psi(1+\psi)Le\ \cos^2\alpha\ R^2}$

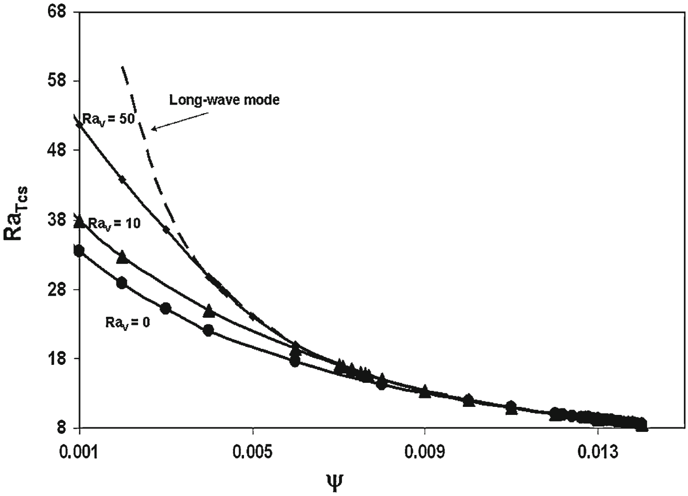

Fig. 4 Effect of vertical vibration on the onset of Soret-driven convection, $Le = 100$, $\varepsilon^* = 0.5$, and $B = 10^{-6}$ for the layer heated from below

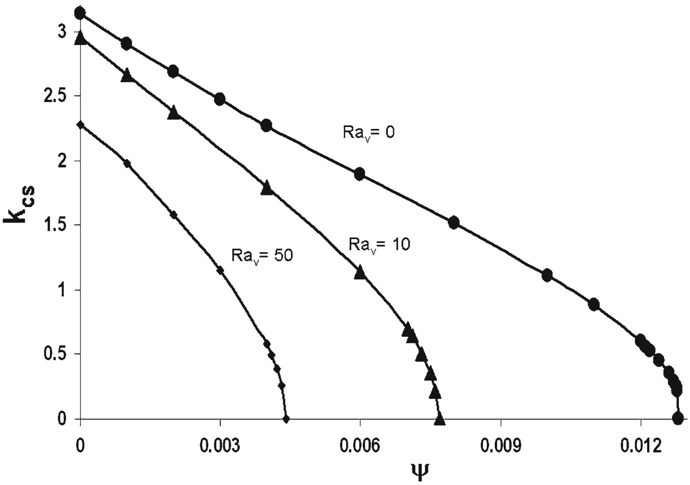

Fig. 5 Effect of vertical vibration on the critical wave number in Soret-driven convection $Le = 100$, $\varepsilon^* = 0.5$, and $B = 10^{-6}$ for the layer heated from below

Table 2 Comparison of the critical Rayleigh (Ra_{Tc}) and wave numbers (k_c) for $Le = 100$, $B = 10^{-6}$, and $\varepsilon^* = 0.5$ for different physical situations

ψ	$Ra_v = 0$ (No vibration)		$Ra_v = 10$		$Ra_v = 50$	
	Ra_{Tc}	k_c	Ra_{Tc}	k_c	Ra_{Tc}	k_c
0	39.48	3.14	44.32	2.95	60.38	2.38
0.001	33.55	2.90	37.94	2.66	51.76	1.98
0.002	28.88	2.68	32.82	2.37	43.76	1.58
0.004	22.12	2.27	25.07	1.79	29.87	0.58

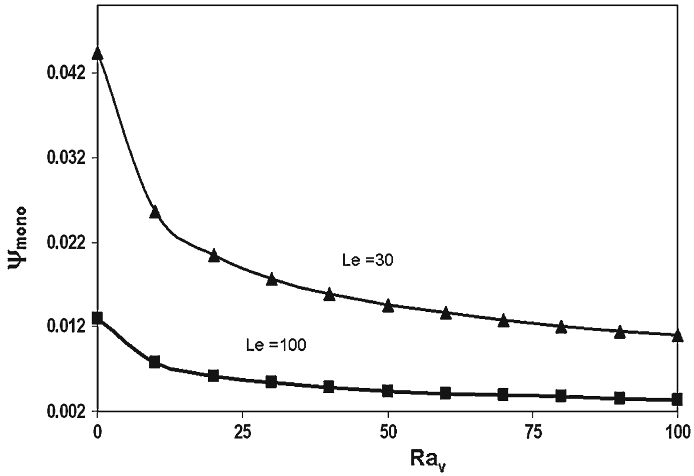

Fig. 6 Effect of vertical vibration on the onset of mono-cellular separation ratios, $\varepsilon^* = 0.5$ and $B = 10^{-6}$ for the layer heated from below

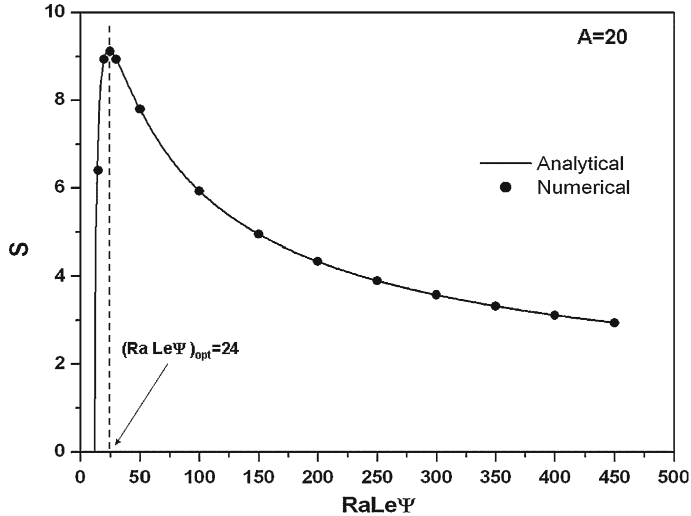

Fig. 7 Separation (S) versus ($RaLe\Psi$)

is an optimum value for ($LeRa\Psi$), for which S possesses a maximum value:

$$(LeRa\Psi)_{\text{opt}} = 24 \tag{33}$$

Equations 33 and 32 may be considered simultaneously to provide us with the set of controlling parameter to achieve maximum separation. Figure 8 gives a qualitative representation of flow and concentration fields under different physical situations. For the case studied, $A = 10$, $Le = 2$, $\Psi = 0.4$, $Ra = 15.7$, and $\varepsilon^* = 0.5$. The vibrational Rayleigh numbers considered are set to 0 and 20 ($Ra_v = 0$ and 20). In 8(a), the convective flow has multi-cellular nature and it is not possible to achieve the separation of the components. However in 8(b), the component separation in the binary mixture is achieved under the effect of vibration (all

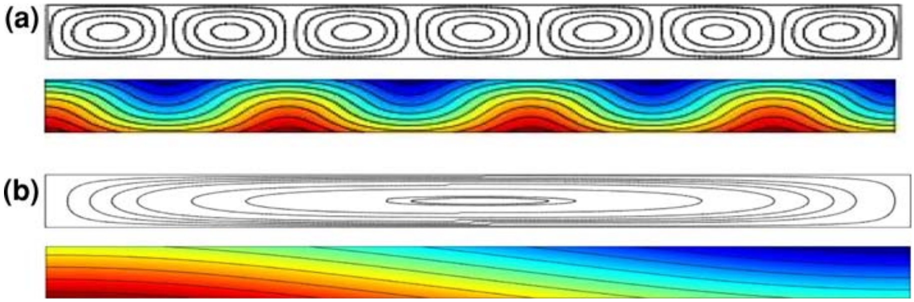

Fig. 8 Streamlines and iso-concentrations. (**a**) Static gravity ($Ra_v = 0$). (**b**) Simultaneous effects of vibration and gravitation ($Ra_v = 20$), $Ra_T = 15.7$, $Le = 2$, $A = 10$, and $\varepsilon^* = 0.5$

parameters are kept constant except Ra_v, which is set equal to 20). It is noteworthy that vibration has drastically changed the flow patterns too.

4.7 Summary of Key Results

In this section, we present new results for the Soret-driven convection under the effect of vibration. The vibration is in the range of high-frequency and small amplitude and its direction is taken parallel to the temperature gradient. The focus was placed on the long wave mode instability. An exact analytical relation for characterizing the mono-cellular domain is presented. From this equation, we may find the value of the vibrational parameter, for which the mono-cellular flow becomes the dominant flow pattern. This conclusion along with better separation results of component under the influence of vibration proposed by Elhajjar et al. (2009) may be used to achieve maximum components separation.

Appendix

Under the Boussinesq approximation, the dimensional governing equations for the thermo-solutal convection with Soret effect can be written as

$$\begin{aligned}
&\nabla \cdot \mathbf{V} = 0,\\
&\frac{\rho_0}{\varepsilon}\frac{\partial \mathbf{V}}{\partial t} + \frac{\mu_f}{K}\mathbf{V} = -\nabla P - \rho_0[\beta_T(T - T_{\text{ref}})\\
&\qquad - \beta_C(C - C_{\text{ref}})]\,(\mathbf{g} - b\omega^2 \sin\omega t\,\mathbf{j}),\\
&(\rho c)^* \frac{\partial T}{\partial t} + (\rho c)_f \mathbf{V}\cdot\nabla T = \lambda^* \nabla^2 T,\\
&\varepsilon\frac{\partial C}{\partial t} + \mathbf{V}\cdot\nabla \mathbf{C} = D^*\nabla^2 C + D_T C_i(1 - C_i)\nabla^2 T
\end{aligned} \tag{34}$$

The corresponding boundary conditions are

$$\begin{aligned}
&\mathbf{V}\cdot\mathbf{n} = 0 \quad \forall M \in \partial\Omega,\\
&y = 0 : T = T_1, \quad \mathbf{J}_{\text{m}}\cdot\mathbf{n} = 0,\\
&y = H : T = T_2, \quad \mathbf{J}_{\text{m}}\cdot\mathbf{n} = 0,\\
&x = 0, L : \frac{\partial T}{\partial x} = \frac{\partial C}{\partial x} = 0
\end{aligned} \tag{35}$$

In (A2), the mass flux vector is $\mathbf{J}_\mathrm{m} = D^*\nabla C + D_T C_i\ (1 - C_i)\ \nabla T$, where C_i represents initial mass fraction of the denser component.

Following the time-averaged procedure explained in Sects. 3.2 and 3.3, we transform the velocity, pressure, temperature, and mass fraction fields as the superposition of the time-averaged values (mean values calculated over a vibration period) plus oscillating ones:

$$\mathbf{V} = \bar{\mathbf{V}}(t) + \mathbf{V}^*(\omega \mathrm{t}), \quad P = \bar{P}(t) + P'(\omega t),$$
$$T = \bar{T}(t) + T'(\omega t), \quad C = \bar{C}(t) + C'(\omega t) \tag{36}$$

By replacing (36) in (34) and (35), and performing the time averaging over a vibrational period, we may obtain two coupled systems of equations. One governs the time-averaged system of equations, the other governs the oscillating one. By making the following assumptions in the oscillating system

$$\omega \gg \max\left(\frac{\varepsilon\nu_f}{K}, \frac{a^*}{\sigma H^2}, \frac{D^*}{\varepsilon H^2}\right), \quad \omega^2 \gg \max\left(\frac{\varepsilon\beta_T \Delta T g}{\sigma H}, \frac{\beta_C \Delta C g}{H}\right)$$
$$\max\left(\frac{\varepsilon b\beta_T \Delta T}{\sigma H}, \frac{\varepsilon b\beta_C \Delta C}{\sigma H}\right) \ll 1 \tag{37}$$

we may simplify this system of equations significantly. The conditions on frequency in 37 represent the high frequency assumptions. The second inequality in 37 is the small amplitude assumption (it comes from the fact that the temperature and concentration oscillatory fields are much less than their averaged counterparts). By applying (37) in the oscillating system of equations, we may simplify it drastically. By applying the Helmholtz decompositions to the simplified oscillating momentum equation, we may eliminate the pressure term and find the oscillating fields. On substituting these oscillating fields in the coupling terms in the momentum, energy, and concentration equations, we obtain the system of Eq. (20a).

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