Transp Porous Med (2008) 74:21–33
DOI 10.1007/s11242-007-9178-4

A New Method for Calculating Two-Phase Relative Permeability from Resistivity Data in Porous Media

Kewen Li

Received: 28 March 2006 / Accepted: 16 October 2007 / Published online: 13 December 2007
© Springer Science+Business Media B.V. 2007

Abstract Many resistivity data from laboratory measurements and well logging are available. Papers on the relationship between resistivity and relative permeability have been few. To this end, a new method was developed to infer two-phase relative permeability from the resistivity data in a consolidated porous medium. It was found that the wetting phase relative permeability is inversely proportional to the resistivity index of a porous medium. The proposed model was verified using the experimental data in different rocks (Berea, Boise sandstone, and limestone) at different temperatures up to 300°F. The results demonstrated that the oil and water relative permeabilities calculated from the experimental resistivity data by using the model proposed in this article were close to those calculated from the capillary pressure data in the rock samples with different porosities and permeabilities. The results demonstrated that the proposed approach to calculating two-phase relative permeability from resistivity data works satisfactorily in the cases studied.

Keywords Relative permeability model · Resistivity index · Capillary pressure · Wetting phase saturation

Nomenclature

A	Across area of rock
E_c	Electric current or flux
G	Conductance of a tube
G_a	Conductance of a porous medium at a water saturation of 100%
G_w	Conductance of a porous medium at a water saturation of S_w
I	Resistivity index
k	Absolute permeability
k_{rnw}	Relative permeability of the nonwetting phase

K. Li (✉)
Energy Resources Engineering, Stanford University, Stanford, USA
e-mail: kewenli@stanford.edu

K. Li
College of Engineering, Peking University, Beijing, China

k_{rw}	Relative permeability of the wetting phase
l	Length of tube or rock
n	Saturation exponent
ΔP	Pressure gradient
q	Volumetric rate of flow
r	Radius of tube
R_0	Resistivity of rock at a water saturation of 100%
R_t	Resistivity of rock at a specific water saturation of S_w
R_w	Resistivity of water
S_w	Wetting phase saturation
S_w^*	Normalized wetting phase saturation
S_{wr}	Residual water saturation
ΔV	Voltage gradient
μ	Viscosity
λ	Pore size distribution index

1 Introduction

One of the important parameters in controlling multiphase fluid flow in porous media is relative permeability. The concept of relative permeability has been adopted in petroleum reservoir engineering, geothermal reservoir engineering, soil science, and many other industries. The direct method to obtain relative permeability is experimental measurement. But it may be difficult to measure in some cases. Such cases include extremely low permeability rocks and special fluid systems in which there are phase transformation and mass transfer between the two phases as pressure changes (Li and Horne 2001, 2004). Relative permeability can also be calculated using other parameters such as capillary pressure. Several mathematical models have been proposed to infer relative permeability from capillary pressure data. Purcell (1949) developed a method to calculate the permeability using capillary pressure curves measured by mercury-injection. This method has been used to calculate multiphase relative permeabilities, as reported by Gates and Leits (1950). Later, Burdine (1953) introduced a tortuosity factor in the model. Corey (1954) and Brooks and Corey (1966) summarized the previous work and modified the method by representing capillary pressure curve as a power law function of the wetting-phase saturation. The modified model was known as the Brooks and Corey relative permeability model. This model has been used in many fields. These include vadose zone studies, subsurface remediation of nonaqueous phase liquids (Parker et al. 1987), oil-water flow in reservoir rocks (Honarpour et al. 1986), and steam-water flow in geothermal reservoirs. It is very difficult to measure steam-water relative permeability because of mass transfer and phase transformation between steam and water as pressure changes. Li and Horne (2005) reported that steam-water relative permeability could be calculated from capillary pressure data. These models provide an easier and more economical approach to obtaining steam-water relative permeability, compared to the experimental technique.

Demond and Roberts (1993) conducted a comparison of experimental measurements with estimates generated with five common methods and their results showed that these methods were limited in their predictive capabilities. Another approach to estimating relative permeability is pore-scale network modeling. For example, Rajaram et al. (1997) used pore-scale network models to investigate the influence of correlations on the capillary pressure-saturation-relative permeability relationships for unconsolidated soils. The predicted relative

permeabilities were compared to the measured values and predictions using the traditional van Genuchten relationships (1980). Rajaram et al. (1997) showed that the pore-scale model could fit the capillary-pressure saturation curves and predict the saturation-relative permeability curves with a degree of accuracy comparable to the van Genuchten relationships (1980).

However, the capillary pressure technique to compute relative permeability still requires the measurements of capillary pressure, which has the same difficulty as measuring relative permeability in many cases. It would be helpful for engineers and scientists to have a method to infer relative permeability from resistivity data because they are easier to measure and a great amount of resistivity data is available from well-logging. However, such models have been few. Li and Williams (2006) developed a correlation between resistivity and capillary pressure theoretically; but relative permeability was not considered. Pirson et al. (1964) proposed a model to calculate relative permeability from resistivity data; the model reported by Pirson et al. (1964) was empirical. In this study, a new model was derived theoretically to infer relative permeability from resistivity data. Oil and water relative permeability data were calculated from the experimental resistivity data in different rocks using the proposed approach. The results were compared with the oil and water relative permeability data computed from the experimental data of capillary pressure measured in the same core samples.

2 Mathematical Models

The relationship between relative permeability and resistivity index is derived in this section. The main theory behind this is the similarity between fluid flow in a porous medium and electricity flow in a conductive body.

2.1 Calculation of the Wetting-Phase Relative Permeability

The conductance of a porous medium at a water saturation of 100% is:

$$G_a = 1/R_o \tag{1}$$

where R_o is the resistivity at a water saturation of 100%, G_a is the conductance of a porous medium at a water saturation of 100%.

The conductance of a porous medium at a specific water saturation of S_w is:

$$G_w = 1/R_t \tag{2}$$

where R_t is the resistivity and G_w is the conductance at a specific water saturation of S_w.

According to the analogy between fluid flow and electrical flow, the relative permeability of the wetting phase can be calculated using the following equation:

$$k_{rw} = \frac{G_w}{G_a} = \frac{R_o}{R_t} = \frac{1}{I} \tag{3}$$

where I is the resistivity index, k_{rw} is the relative permeability of the wetting phase. The derivation of Eq. 3 is presented in Appendix A.

According to Archie's equation (1942), the following equation applies:

$$I = \frac{R_t}{R_o} = (S_w)^{-n} \tag{4}$$

where n is the Archie's saturation exponent.

At a water saturation of 100%, it is known that $I = 1$, so the value of k_{rw} calculated using Eq. 3 would be equal to 1, which is true. At the residual water saturation, it is known that $k_{rw} = 0$, which implies that I approaches infinity according to Eq. 3. But it is known that the value of I does not approach infinity at the residual water saturation. So the value of k_{rw} calculated using Eq. 3 is greater than zero, which is not consistent with the physical observation.

The relative permeability of the wetting phase calculated using Eq. 3 will be greater than the true value. The reason is that the resistivity counts the average volumetric properties of the pore bodies in a porous medium while permeability counts the properties of pore throats. This may also be why resistivity well-logging can obtain porosity but not permeability.

Considering these problems, Eq. 3 is modified as follows:

$$k_{rw} = \frac{S_w - S_{wr}}{1 - S_{wr}} \frac{1}{I} \tag{5}$$

where S_{wr} is the residual saturation of the wetting phase. According to Eq. 5, $k_{rw} = 1$ at $S_w = 100\%$ and $k_{rw} = 0$ at $S_w = S_{wr}$, which is reasonable.

Equation 5 can also be expressed as follows:

$$k_{rw} = S_w^* \frac{1}{I} \tag{6}$$

S_w^* is the normalized saturation of the wetting phase and is expressed as follows:

$$S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}} \tag{7}$$

Relative permeability of the wetting phase can be calculated using Eq. 6 from resistivity index data once the residual saturation of the wetting phase is available. Note that the residual saturation of the wetting phase can be obtained from the experimental measurement of resistivity in the porous medium.

2.2 Calculation of the Nonwetting-phase Relative Permeability

The wetting-phase relative permeability can be inferred from the resistivity data based on Eq. 6. However, the relationship between nonwetting-phase relative permeability and resistivity has not been established. The computation of nonwetting-phase relative permeability will be described in this section.

Studies have shown that the wetting-phase and nonwetting-phase relative permeabilities are coupled and can be calculated using capillary pressure data (Purcell 1949; Brooks and Corey 1966; Li and Horne 2005). Li and Horne (2005) have found that the Purcell model was to be the best fit to the experimental data of the wetting-phase relative permeability in the cases studied, as long as the measured capillary pressure curve had the same residual saturation as the relative permeability curve. Li and Horne (2005) also developed a physical model to explain the insignificance of the effect of tortuosity on the calculation of the wetting-phase relative permeability. As the wetting-phase and nonwetting-phase relative permeabilities are coupled, the nonwetting-phase relative permeability will be computed from the wetting-phase relative permeability which can be inferred from the resistivity data (see Eq. 6). The details are presented in the following.

According to Li and Horne (2005), the wetting-phase relative permeability can be calculated using the Purcell approach (1949):

$$k_{rw} = (S_w^*)^{\frac{2+\lambda}{\lambda}} \tag{8}$$

where λ is the pore size distribution index and can be calculated from capillary pressure data.

After the relative permeability curve of the wetting phase is obtained using Eq. 6, the value of λ can be inferred using Eq. 8.

According to the Brooks–Corey model (1966) and the study by Li and Horne (2005), the relative permeability of the nonwetting phase can be calculated once the value of λ is available. The equation is expressed as follows:

$$k_{\mathrm{rnw}} = (1 - S_{\mathrm{w}}^{*})^{2}\left[1 - (S_{\mathrm{w}}^{*})^{\frac{2+\lambda}{\lambda}}\right] \tag{9}$$

One can see that the entire relative permeability set (both wetting and nonwetting phases) can be inferred from resistivity index data using Eqs. 6 and 9.

3 Results

The experimental data of resistivity and capillary pressure measured by Sanyal (1972) in rocks (Berea, Boise sandstone, and limestone) with different permeability were used to test the models (Eqs. 6 and 9) developed in this study. All of the experimental data used in this study were obtained in drainage process. Firstly, the values of oil/water relative permeability were calculated using Eqs. 6 and 9. Secondly, the oil/water relative permeability data were calculated using the capillary pressure data (Li and Horne 2005). According to the study by Li and Horne (2005), relative permeability could be calculated accurately using the capillary pressure technique. Finally the results of relative permeability inferred from resistivity index and capillary pressure data, respectively, were compared.

3.1 Berea Sandstone

The Berea sandstone sample used by Sanyal (1972) had a porosity of 20.4% and a permeability of 300 md. Fig. 1 shows the relationship between resistivity index and water saturation in Berea sandstone at a temperature of 175°F.

The values of the resistivity index at different water saturation shown in Fig. 1 were measured in oil–water (water displaced by oil) systems. Figure 1 also shows that the relationship between resistivity index and water saturation in Berea sandstone at a temperature of 175°F is

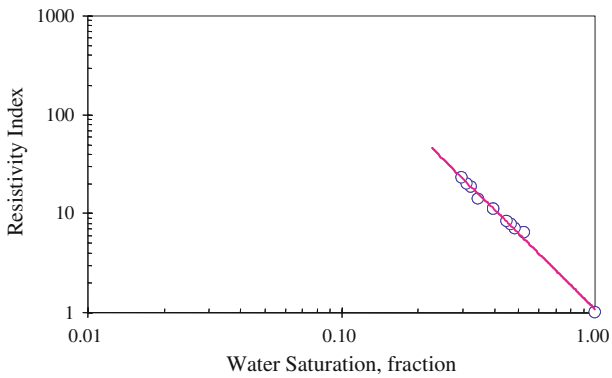

Fig. 1 Relationship between resistivity index and water saturation in Berea sandstone at a temperature of 175°F (Sanyal 1972), goodness of fitting: $R^2 = 0.9941$

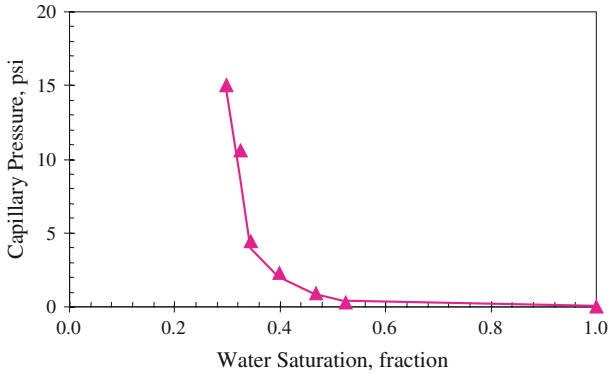

Fig. 2 Capillary pressure data measured in Berea sandstone at a temperature of 175°F (Sanyal 1972)

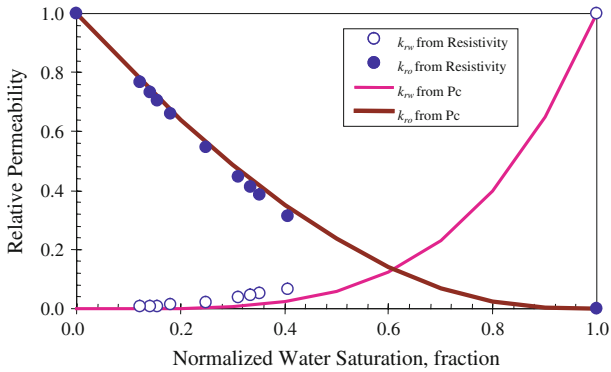

Fig. 3 Relative permeability calculated from resistivity and capillary pressure data in Berea sandstone at a temperature of 175°F

linear on a log–log plot, which is consistent with the Archie's equation (Eq. 4). The regression coefficient $R^2 = 0.9941$.

Oil/water capillary pressure data were measured simultaneously with resistivity in the same core sample. The results are plotted in Fig. 2. Oil/water relative permeability data were calculated from the experimental data of the resistivity index (using Eqs. 6 and 9) and capillary pressure data (see Li and Horne 2005), respectively. The results are compared and shown in Fig. 3. As shown in Fig. 3, the relative permeability data inferred from the resistivity index data are close to those calculated using the capillary pressure data. The oil relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the experimental capillary pressure data.

Sanyal (1972) also conducted the experimental measurements of resistivity index and capillary pressure at different temperatures in the same core sample. The resistivity index data at a temperature of 300°F are plotted in Fig. 4. One can see from Fig. 4 that the resistivity index data also follow the Archie's law (Eq. 4). The regression coefficient $R^2 = 0.9925$.

Figure 5 shows the oil/water capillary pressure data measured simultaneously with resistivity in the same Berea sandstone core sample at a temperature of 300°F. Figure 6 demonstrates the relative permeability data calculated from both the resistivity index data shown in Fig. 4 (using Eqs. 6 and 9) and the capillary pressure data in Fig. 5 (see Li and Horne 2005). One

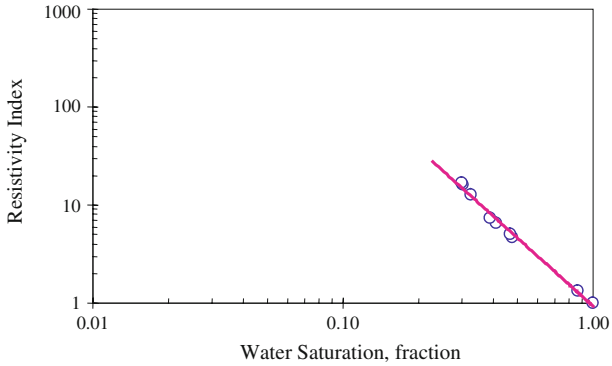

Fig. 4 Relationship between resistivity index and water saturation in Berea sandstone at a temperature of 300°F (Sanyal 1972), goodness of fitting: $R^2 = 0.9925$

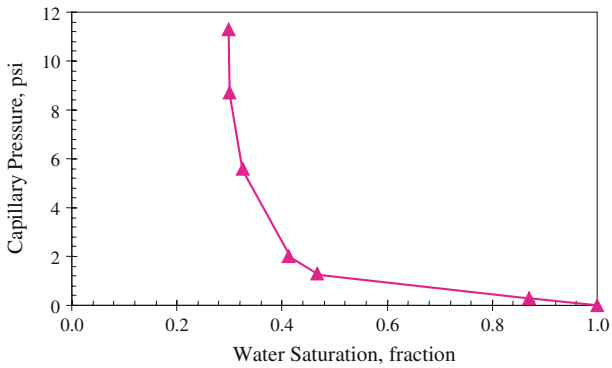

Fig. 5 Capillary pressure data measured in Berea sandstone at a temperature of 300°F (Sanyal 1972)

can see in Fig. 6 that both the oil and water relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the capillary pressure data.

3.2 Boise Sandstone

The Boise sandstone core sample had a porosity of 32% and a permeability of 960 md. The values of porosity and permeability are greater than those of the Berea sandstone used by Sanyal (1972). Figures 7 and 8 show the resistivity index data and capillary pressure versus water saturation in Boise sandstone at a temperature of 175°F.

The relative permeability data calculated from both the resistivity index data shown in Fig. 7 (using Eqs. 6 and 9) and the capillary pressure data in Fig. 8 are plotted in Fig. 9. Figure 9 shows that the oil and water relative permeabilities calculated from the resistivity index data are also close to those calculated from the capillary pressure data.

The results at a temperature of 300°F are demonstrated in Figs. 10–12. Figure 10 shows the resistivity data and Fig. 11 shows the capillary pressure data. Figure 12 plots the relative permeability calculated from the resistivity index shown in Fig. 10 (using Eqs. 6 and 9) and the capillary pressure in Fig. 11, respectively. One can see from Fig. 12 that the values of the oil relative permeability calculated from resistivity index are close to those inferred

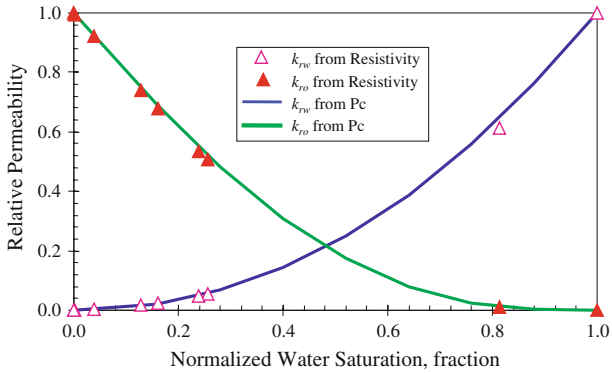

Fig. 6 Relative permeability calculated from resistivity and capillary pressure data in Berea sandstone at a temperature of 300°F

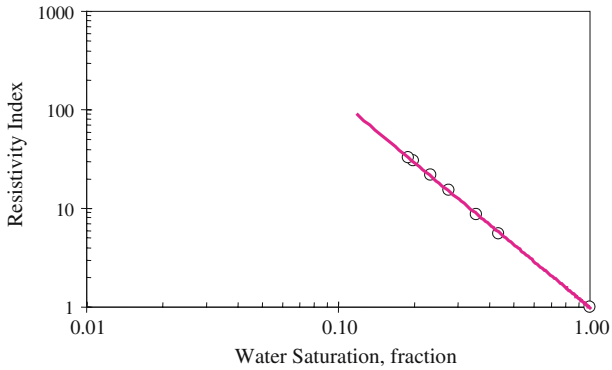

Fig. 7 Relationship between resistivity index and water saturation in Boise sandstone at a temperature of 175°F (Sanyal 1972) , goodness of fitting: $R^2 = 0.9996$

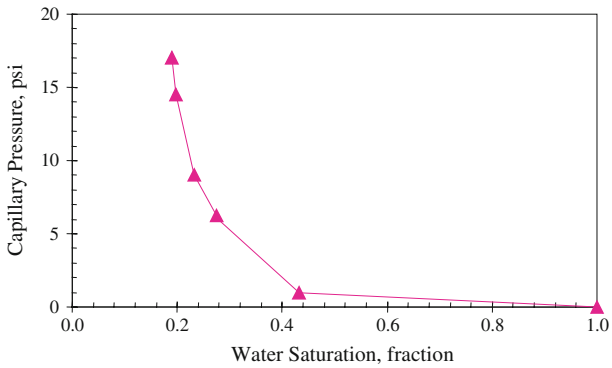

Fig. 8 Capillary pressure data measured in Boise sandstone at a temperature of 175°F (Sanyal 1972)

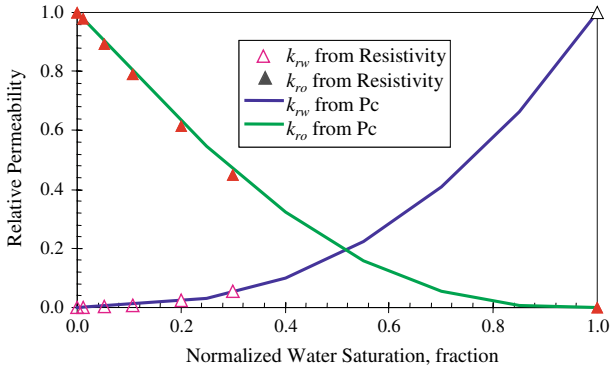

Fig. 9 Relative permeability calculated in Boise sandstone at a temperature of 175°F

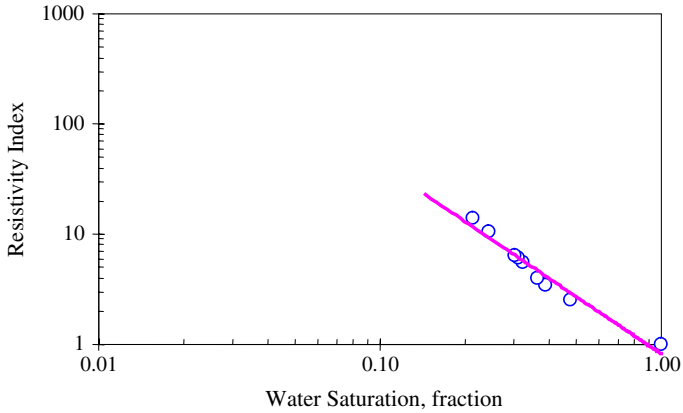

Fig. 10 Resistivity index in Boise sandstone at a temperature of 300°F (Sanyal 1972), goodness of fitting: R^2=0.9934

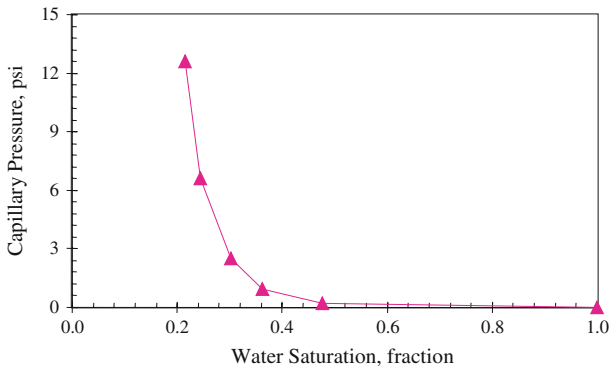

Fig. 11 Capillary pressure data measured in Boise sandstone at a temperature of 300°F (Sanyal 1972)

from the capillary pressure data. However, the water relative permeability calculated from resistivity index data is smaller than those inferred from capillary pressure. This may be the experimental error because of the high test temperature.

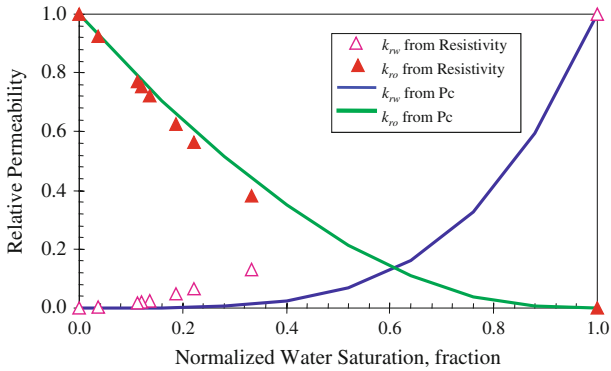

Fig. 12 Relative permeability calculated from resistivity and capillary pressure in Boise sandstone at a temperature of 300°F

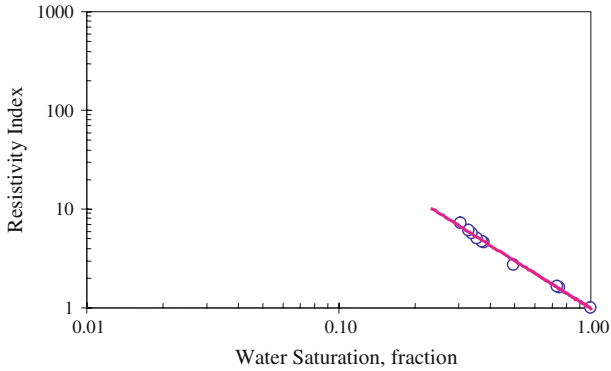

Fig. 13 Resistivity index verus water saturation in limestone at a temperature of 300°F (Sanyal 1972), goodness of fitting: $R^2 = 0.9943$

3.3 Limestone

The limestone core sample had a porosity of 19% and a permeability of 410 md. The resistivity index and capillary pressure data at a temperature of 300°F are depicted in Figs. 13 and 14, respectively. The results of relative permeability calculated from the resistivity index and the capillary pressure data are plotted in Fig. 15. One can see in Fig. 15 that the oil relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the capillary pressure data in the limestone core sample. The difference between the relative permeability inferred from the resistivity index and those calculated from the experimental data of capillary pressure is acceptable in terms of reservoir engineering applications.

4 Conclusion

On the basis of the present study, the following conclusions may be drawn in the cases studied. A new method was developed to infer two-phase relative permeability from resistivity

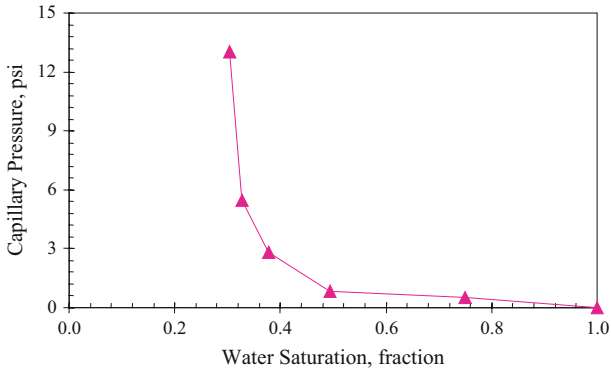

Fig. 14 Capillary pressure data measured in limestone at a temperature of 300°F (Sanyal 1972)

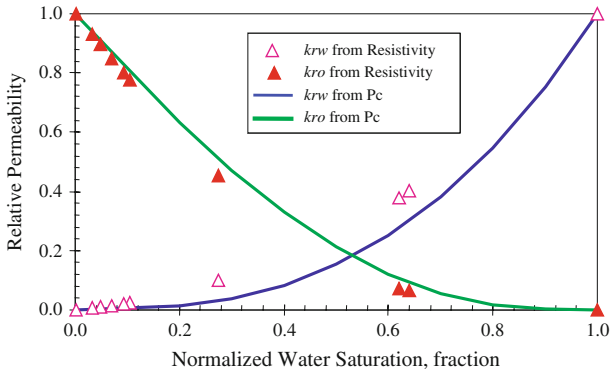

Fig. 15 Relative permeability calculated from resistivity and capillary pressure data in limestone at a temperature of 300°F

index data. Theoretically, the wetting phase relative permeability is inversely proportional to the resistivity index of a porous medium. The proposed model works satisfactorily for the experimental data presented. The values of the nonwetting phase relative permeability inferred from the resistivity index data are almost equal to those calculated from the experimental capillary pressure data. For the wetting-phase relative permeability, the values inferred from the resistivity index are close to those calculated from capillary pressure in most of the cases studied.

Acknowledgements This research was conducted with financial support to the Stanford Geothermal Program from the US Department of Energy under grant DE-FG07-02ID14418, the contribution of which is gratefully acknowledged.

Appendix A: Derivation of Relationship Between Relative Permeability and Resistivity Index

Fatt (1956) reported a method to obtain dynamic properties of networks with tubes by means of a network of resisters used as an analog model. The analogy between Poiseuille's law and Ohm's law was used by Fatt (1956). The analogy between fluid flow and electric flow has

been used by many researchers (King 1989; Li and Firoozabadi 2000, etc.). The Poiseuille's law for a cylinder is:

$$q = \frac{\pi r^4}{8\mu l}\Delta P \tag{A-1}$$

where q is the volumetric rate of flow, μ is the viscosity, ΔP is the pressure gradient, and r is the tube radius, and l is the tube length.

Ohm's law is:

$$E_c = G\Delta V \tag{A-2}$$

Where E_c is the electric current or flux and is equivalent to the volumetric rate of flow in Poiseuille's law, G is the conductance, and ΔV is the voltage gradient. Both laws give the flux as a function of the potential drop and the resistance of the medium. Fatt (1956) demonstrated the interchangeability of tubes and electrical resistors.

By analogy then (Fatt 1956),

$$G = \frac{\pi r^4}{8\mu l} \tag{A-3}$$

Darcy's law is:

$$q = \frac{Ak}{\mu l}\Delta P \tag{A-4}$$

Here A is the cross area of the rock and l is the length of the rock. The analogy between the Darcy's law and the Ohm's law is obvious.

Using the same procedure, one can obtain:

$$G_a = \frac{Ak}{\mu l} \tag{A-5}$$

For a rock saturated with water partially, Eq. A-5 can be written:

$$G_w = \frac{Akk_{rw}}{\mu l} \tag{A-6}$$

Equation 3 can be obtained by combining Eqs. A-5 and A-6.

References

Archie, G.E.: The electrical resistivity log as an aid in determining some reservoir characteristics. AIME Petroleum Tech. 1–8 (1942)

Brooks, R.H., Corey, A.T.: Properties of porous media affecting fluid flow. J. Irrig. Drain. Div. **6**, 61 (1966)

Burdine, N.T.: Relative permeability calculations from pore size distribution data. Trans. AIME **198**, 71 (1953)

Corey, A.T.: The interrelation between gas and oil relative permeabilities. Prod. Mon. **19**, 38 (1954)

Demond, A.H., Roberts, P.V.: Estimation of 2-phase relative permeability relationships for organic liquid contaminants. Water Resour. Res. **29**(4), 1081–1090 (1993)

Fatt, I.: The network model of porous media, III Dynamic properties of networks with tube radius distribution. AIME Trans. **207**, 164–177 (1956)

Gates, J.I., Leitz, W.J.: Relative permeabilities of California cores by the capillary pressure method. Paper presented at the API meeting, Los Angeles, California, p 286 May 11 1950

Honarpour, M.M., Koederitz, L., Harvey, A.H.: Relative Permeability of Petroleum Reservoirs. CRC press, Boca Raton, Florida, USA (1986) ISBN 0-8493-5739-X, 19

King, P.R.: The use of renormalization for calculating effective permeability. Transport Porous Media **4**, 237 (1989)

Li, K., Horne, R.N.: An experimental and theoretical study of steam-water capillary pressure. SPEREE 477–482 (2001)

Li, K., Horne, R.N.: Steam-water and air-water capillary pressures: Measurement and comparison. J. Can. Petrol.Technol. **43**(7), 24–30 (2004)

Li, K., Firoozabadi, A.: Phenomenological modeling of critical condensate saturation and relative permeabilities in gas condensate systems. SPEJ 138–147 (2000)

Li, K., Horne, R.N.: Comparison of methods to calculate relative permeability from capillary pressure in consolidated water-wet porous media. Water Resour. Res., 42, W06405, doi:10.1029/2005WR004482

Li, K., Williams, W.: Determination of capillary pressure function from resistivity data. Transport Porous Media (accepted, 2006)

Parker, J.C., Lenhard, R.J., Kuppusamy, T.: A parametric model for constitutive properties governing multiphase flow in porous media. Water Resour. Res. **23**(4), 618–624 (1987)

Pirson, S.J., Boatman, E.M., Nettle, R.L.: Prediction of relative permeability characteristics of intergranular reservoir rocks from electrical resistivity measurements. J. Petrol. Technol. 561–570 (1964)

Purcell, W.R.: Capillary pressures-their measurement using mercury and the calculation of permeability. Trans. AIME **186**, 39 (1949)

Rajaram, H., Ferrand, L.A., Celia, M.A.: Prediction of relative permeabilities for unconsolidated soils using pore-scale network models. Water Resour. Res. **33**(1), 43–52 (1997)

Sanyal, S.K.: The effect of Temperature on electrical resistivity and capillary pressure behavior of porous media, Ph.D. report, Stanford University, Stanford, California (1972)

Van Genuchten, M.T.: A closed form equation for predicting the hydraulic conductivity of unsaturated soils. Soil Sci. Soc. Am. J. **44**, 892–898 (1980)