Transp Porous Med (2007) 67:297–315
DOI 10.1007/s11242-006-9027-x

ORIGINAL PAPER

Fluid convection in a rotating porous layer under modulated temperature on the boundaries

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Received: 8 December 2005 / Revised: 19 May 2006 / Published online: 10 October 2006

Abstract The linear stability of thermal convection in a rotating horizontal layer of fluid-saturated porous medium, confined between two rigid boundaries, is studied for temperature modulation, using Brinkman's model. In addition to a steady temperature difference between the walls of the porous layer, a time-dependent periodic perturbation is applied to the wall temperatures. Only infinitesimal disturbances are considered. The combined effect of rotation, permeability and modulation of walls' temperature on the stability of flow through porous medium has been investigated using Galerkin method and Floquet theory. The critical Rayleigh number is calculated as function of amplitude and frequency of modulation, Taylor number, porous parameter and Prandtl number. It is found that both, rotation and permeability are having stabilizing influence on the onset of thermal instability. Further it is also found that it is possible to advance or delay the onset of convection by proper tuning of the frequency of modulation of the walls' temperature.

Keywords Thermal convection · Modulation · Rayleigh number · Porous medium · Rotation · Galerkin method

Nomenclature

a	Horizontal wave number $\left(a_x^2 + a_y^2\right)^{1/2}$
a_c	Critical wave number
d	Depth of the porous layer
g	Gravitational acceleration
k	Permeability of the porous medium
κ_f	Thermal conductivity of the fluid
κ_s	Thermal conductivity of the solid
κ_m	$\delta\kappa_f + (1-\delta)\,\kappa_s$, effective thermal conductivity of porous media

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p	Pressure
P_l	Porous parameter, k/d^2
P_r	Prandtl number, ν/κ
R	Thermal Rayleigh number, $\frac{\alpha g \Delta T d^3}{\nu\kappa}$
Ω	Angular velocity vector $(0, 0, \Omega)$
T	Taylor number $4\Omega^2 d^4/\nu^2$
R_c	Critical Rayleigh number
T	Temperature
θ	Perturbed temperature
ΔT	Temperature difference between the walls
$\mathbf{V}$	Mean filter velocity, (u, v, w)
x, y, z	Space coordinates
$(\rho c_p)_f$	Heat capacity of the fluid
$(\rho c_p)_s$	Heat capacity of the solid
$(\rho c_p)_m$	$\delta (\rho c_p)_f + (1-\delta)(\rho c_p)_s$ relative heat capacity of the porous medium
$T_S(z)$	Steady temperature field
$T_o(z, t)$	Oscillating temperature field

Greek symbols

ζ	Z-component of vorticity
α	Coefficient of thermal expansion
ε	Amplitude of modulation
δ	Porosity
γ	Heat capacity ratio, $(\rho c_p)_m / (\rho c_p)_f$
κ	Effective thermal diffusivity, $\kappa_m / (\rho c_p)_f$
μ	Coefficient of viscosity
ν	Kinematic viscosity, μ/ρ_R
ρ	Density
ω	Modulation frequency
ϕ	Phase angle

Other symbols

∇_1^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
∇^2	$\nabla_1^2 + \frac{\partial^2}{\partial z^2}$
D	$\frac{\partial}{\partial z}$

1 Introduction

The study of fluid convection in a rotating porous medium is of great practical importance in many branches of modern science such as centrifugal filtration processes, petroleum industry, food engineering, chemical engineering, geophysics and biomechanics. Several studies are available in which phenomena related to the onset of convection in a rotating porous medium have been investigated. A detailed review of most of these findings has been given by Vadasz (1997, 1998), and Nield and Bejan (1999). The problem of thermal instability in a rotating porous medium subject to uniform temperature gradient has been investigated by several authors, Pearlstein

(1981), Chakrabarti and Gupta (1981), Patil and Vaidyanathan (1983), Rudraiah et al. (1986), Prabhamani et al. (1990), Vadasz (1992, 1994), and Qin and Kaloni (1995), for different mathematical models and boundary conditions. Recently, Desaive et al. (2002) have investigated the convective instability in a rotating porous medium, using rigid–rigid boundaries.

There are, however, many practically important situations in which temperature gradient is a function of both space and time. This non-uniform temperature gradient (temperature modulation) can be used as a mechanism to control the convective flow. There can be an appreciable enhancement of heat, mass or momentum if an imposed modulation can destabilize an otherwise stable system. Similarly if it can stabilize an otherwise unstable system, higher efficiency can be achieved in many processing techniques, particularly in solidification processes.

The effect of temperature modulation on thermal stability in a viscous fluid layer was first considered by Venezian (1969), nevertheless a similar problem had been studied earlier by Gershuni and Zhukhovitskii (1963) for a temperature profile, obeying rectangular law. Some other researchers who have investigated temperature modulation of thermal instability in a viscous fluid layer are: Rosenblat and Herbert (1970), Rosenblat and Tanaka (1971), Yih and Li (1972), Roppo et al. (1984), and Bhadauria and Bhatia (2002). Recently, Bhadauria (2005, 2006a) has investigated the effect of temperature modulation on thermal instability in horizontal fluid layer, and studied the effects of rotation and vertical magnetic field. However, the studies related to the effect of temperature modulation on thermal convection in a porous medium has received only limited attention. The effect of temperature modulation on thermal instability in a horizontal porous layer has been studied by Caltagirone (1976), Chhuon and Caltagirone (1979), Rudraiah and Malashetty (1988, 1990), Malashetty and Wadi (1999), and Malashetty and Basavaraja (2002, 2003). Most of these studies are made using free–free boundary conditions, which are less accessible to the experiments. The literature on convection in a porous medium with temperature modulation of rigid–rigid boundaries is scarce. Only very recently Bhadauria (2006b) has investigated this problem and studied the effect of temperature modulation of rigid–rigid boundaries on convection in a sparsely packed porous medium. To the best of author's knowledge, no literature is available in which combined effect of both rotation and temperature modulation has been considered on thermal stability in a porous medium with rigid–rigid boundaries.

Therefore, the objective of the present study is to investigate the combined effect of rotation and temperature modulation of rigid–rigid boundaries on thermal stability of flow through sparsely packed porous medium. Since the porous medium considered is sparsely packed, we use the Brinkman's model that accounts for friction caused by microscopic shear. To modulate the walls' temperature, sinusoidal function has been taken. The results have been obtained for the following three cases: (a) when the plate temperatures are modulated in phase, (b) when the modulation is out of phase, and (c) when only the lower plate temperature is modulated, the upper plate is held at fixed constant temperature. The findings of this study are believed to bridge the gap between the results valid for Darcy model (low permeability) and those valid for classical viscous fluids. The results of the present paper can be used to study the onset of convection in geothermal areas where the ground water flows through a porous medium and is subjected to the earth's rotation.

2 Mathematical formulation

Consider a porous medium, which is composed of sparse distribution of particles completely saturated with Boussinesq fluid, and confined between two parallel horizontal walls, at $z = -d/2$ and $z = d/2$, a distance d apart. The walls are infinitely extended in x and y directions, and are rigid. Let the system be rotating uniformly about the z-axis with a constant angular velocity Ω. The effect of rotation is restricted to the Coriolis force, neglecting thus the centrifugal effects; the porous medium is described by the Brinkman's model. The porous medium is regarded as an assemblage of small, identical, spherical particles fixed in the space of porosity close to unity. Then under the Boussinesq approximation the governing equations, for the study of thermal convection in a fluid saturated sparsely packed rotating porous medium, are Rudraiah et al. (1986), Prabhamani et al. (1990),

$$\frac{\partial \mathbf{V}}{\partial t} + \mathbf{V}\cdot\nabla\mathbf{V} + 2\Omega\times\mathbf{V} = -\frac{1}{\rho_{\mathrm{R}}}\nabla p + \frac{\rho}{\rho_{\mathrm{R}}}\mathbf{g} - \frac{\nu}{k}\mathbf{V} + \nu\nabla^2\mathbf{V}, \tag{2.1}$$

$$\left(\rho C_p\right)_m \frac{\partial T}{\partial t} + \left(\rho C_p\right)_f \mathbf{V}\cdot\nabla T = \kappa_m\nabla^2 T, \tag{2.2}$$

$$\nabla\cdot\mathbf{V} = 0, \tag{2.3}$$

$$\rho = \rho_{\mathrm{R}}\left[1 - \alpha\left(T - T_{\mathrm{R}}\right)\right], \tag{2.4}$$

where ρ_{R} and T_{R} are (constants) reference density and temperature, respectively. Walls' temperature is modulated according to the following externally imposed conditions:

$$T(t) = T_{\mathrm{R}} + \Delta T\left[1 + \varepsilon \mathrm{Re}\left\{\mathrm{e}^{\mathrm{i}\omega t}\right\}\right] \qquad \text{at } z = -d/2 \tag{2.5a}$$

$$= T_{\mathrm{R}} + \Delta T\varepsilon \mathrm{Re}\{\mathrm{e}^{\mathrm{i}(\omega t+\phi)}\} \qquad \text{at } z = d/2. \tag{2.5b}$$

Here, ε represents the amplitude of modulation, ΔT is the temperature difference, ϕ is phase angle, and ω is modulation frequency. The applicability of the present theory is seems to be doubtful in the limit $\omega \to 0$ (Venezian 1969, Rosenblat and Herbert 1970, Chhuon and Caltagirone 1979) as in this case non-linear effect becomes important. Therefore the present results would not agree with the results obtained by putting $\omega = 0$ in the above boundary conditions (2.5).

The following three cases are considered: (a) walls' temperature modulation is in phase i.e. $\phi = 0$, (b) temperature modulation is out of phase i.e., $\phi = \pi$, and (c) when only the lower wall's temperature is modulated, the upper wall is held at fixed constant temperature i.e. $\phi = \mathrm{i}\infty$.

2.1 Basic state

An equilibrium solution for the Eqs. (2.1)–(2.4), and (2.5a) can be written as

$$\mathbf{V} = (u, v, w) = 0, \quad T = T_{\mathrm{H}}(z,t), \quad p = p_{\mathrm{H}}(z,t), \quad \rho = \rho_{\mathrm{H}}(z,t). \tag{2.6}$$

The temperature $T_{\mathrm{H}}(z,t)$, pressure p_{H} and density ρ_{H} are given by the equations

$$\gamma\frac{\partial T_{\mathrm{H}}}{\partial t} = \kappa\frac{\partial^2 T_{\mathrm{H}}}{\partial z^2}, \tag{2.7}$$

$$\frac{\partial p_{\mathrm{H}}}{\partial z} = -\rho_{\mathrm{H}} g \tag{2.8}$$

and

$$\rho_{\mathrm{H}} = \rho_{\mathrm{R}} \left[1 - \alpha \left(T_{\mathrm{H}} - T_{\mathrm{R}}\right)\right], \tag{2.9}$$

where $\gamma = \left(\rho C_p\right)_m / \left(\rho C_p\right)_f$ and $\kappa = \kappa_m / \left(\rho C_p\right)_f$. For all the above cases (a), (b) and (c), the solution of the differential Eq. (2.7) subject to the boundary conditions (2.5a) can be written as

$$T_{\mathrm{H}} (z,t) = T_{\mathrm{R}} + T_{\mathrm{S}} (z) + \varepsilon \,\mathrm{Re} \left\{T_{\mathrm{o}} (z,t)\right\}, \tag{2.10}$$

where

$$T_{\mathrm{S}}(z) = \Delta T \left(\frac{1}{2} - \frac{z}{d}\right), \tag{2.11}$$

$$T_{\mathrm{o}}(\mathrm{z,\,t}) = \frac{\Delta T}{\sinh \lambda} \left\{ \mathrm{e}^{i\phi} \sinh \lambda \left(\frac{1}{2} + \frac{z}{d}\right) + \sinh \lambda \left(\frac{1}{2} - \frac{z}{d}\right) \right\} \mathrm{e}^{\mathrm{i}\omega t} \tag{2.12}$$

and

$$\lambda^2 = \mathrm{i}\omega\gamma d^2/\kappa. \tag{2.13}$$

In Eq. (2.10), Re stands for real part.

2.2 Linear stability analysis

Let the system (2.6) be slightly perturbed, then we have

$$\mathbf{V} = \left(u', v', w'\right), \quad T = T_{\mathrm{H}} + \theta', \quad p = p_{\mathrm{H}} + p', \quad \rho = \rho_{\mathrm{H}} + \rho', \tag{2.14}$$

where $\mathbf{V}', \theta', p'$ and ρ' represent the perturbed quantities which are assumed to be small. We substitute (2.14) into (2.1)–(2.4) and linearize with respect to the perturbation quantities $\mathbf{V}', \theta', p'$. Now taking curl twice of the reduced momentum equation (2.1), the system of equations becomes

$$\frac{\partial}{\partial t}\nabla^2 w' = \nu\nabla^4 w' - \frac{\nu}{k}\nabla^2 w' + \alpha g \nabla_1^2 \theta' - 2\Omega \frac{\partial \zeta'}{\partial z} \tag{2.15}$$

$$\frac{\partial \theta'}{\partial t} = -w' \frac{\partial T_{\mathrm{H}}}{\partial z} + \kappa \nabla^2 \theta', \tag{2.16}$$

$$\frac{\partial \zeta'}{\partial t} = 2\Omega \frac{\partial w'}{\partial z} + \nu \nabla^2 \zeta' - \frac{\nu}{k}\zeta', \tag{2.17}$$

where $\zeta' = \frac{\partial v'}{\partial x} - \frac{\partial u'}{\partial y}$ is the vertical component of the vorticity. For convenience, the entire problem has been written in terms of w', θ', ζ'. In the above equations, the value of γ is set equal to one for simplicity. Now using normal mode technique, we seek solutions for the three unknown fields in the form

$$\begin{pmatrix} w' (x,y,z,t) \\ \theta' (x,y,z,t) \\ \zeta' (x,y,z,t) \end{pmatrix} = \begin{pmatrix} w' (z,t) \\ \theta' (z,t) \\ \zeta' (z,t) \end{pmatrix} \exp[\mathrm{i}(a_x x + a_y y)]. \tag{2.18}$$

Here, $a = (a_x^2 + a_y^2)^{1/2}$ is the horizontal wave number. If we scale length, time, temperature, wave number, frequency, velocity and vorticity according to

$$\mathbf{r} = d\mathbf{r}^*,\ t = t^*/\omega,\ T_{\mathrm{H}} = \Delta T \cdot T_{\mathrm{H}}^*,\ \theta = \Delta T \cdot \theta^*,\ a^2 = d^2 a^{*2},\ \omega = \omega^* \kappa / d^2,$$

$$\mathbf{V}' = (\alpha g\, \Delta T a^2/\nu)\mathbf{V}^*, \zeta' = (\alpha g\, \Delta T a^2/\,(d\nu))\zeta^* \tag{2.19}$$

then the governing equations in non-dimensionalized form are

$$\omega^* \left(D^{*2} - a^{*2}\right) \frac{\partial w^*}{\partial t^*} = P_r \left(D^{*2} - a^{*2}\right) \left\{\left(D^{*2} - a^{*2}\right) - P_l^{-1}\right\} \times w^* - P_r \theta^* - \sqrt{T}\, P_r D^* \zeta^* \tag{2.20}$$

$$\omega^* \frac{\partial \theta^*}{\partial t^*} = -a^2 R \left(\frac{\partial T_{\mathrm{H}}^*}{\partial z^*}\right) w^* + \left(D^{*2} - a^{*2}\right) \theta^* \tag{2.21}$$

$$\omega^* \frac{\partial \zeta^*}{\partial t^*} = \sqrt{T} P_r D^* w^* + P_r \left\{\left(D^{*2} - a^{*2}\right) - P_l^{-1}\right\} \zeta^*, \tag{2.22}$$

where, $P_r = \nu/\kappa$ is the Prandtl number, $P_l = k/d^2$ is the porous parameter, $R = \alpha g\, \Delta T d^3/\nu\kappa$ is the Rayleigh number, $T = 4\Omega^2 d^4/\nu^2$ is the Taylor number. Henceforth the asterisk will be dropped in the above equations. The non-dimensional temperature gradient is given by

$$\frac{\partial T_{\mathrm{H}}}{\partial z} = -1 + \varepsilon\, \mathrm{Re}\left[f\,(z)\, e^{\mathrm{it}}\right], \tag{2.23}$$

where

$$f\,(z) = \frac{\lambda}{\sinh \lambda} \left\{ \mathrm{e}^{\mathrm{i}\phi} \cosh \lambda \left(\frac{1}{2} + z\right) - \cosh \lambda \left(\frac{1}{2} - z\right) \right\} \tag{2.24}$$

and

$$\lambda^2 = \mathrm{i}\omega. \tag{2.25}$$

The boundary conditions for the rigid walls are given by

$$w = Dw = \theta = \zeta = 0 \quad \text{on } z = \pm\frac{1}{2}. \tag{2.26}$$

3 Method

Here we use Galerkin technique, to transform the partial differential equations (2.20)–(2.22) into a system of ordinary differential equations. The latter are then solved numerically. The results have been obtained for moderate values of ε, as we are interested only in the modulating effect of the oscillating temperature gradient. We put

$$w\,(z,t) = \sum_{m=1}^{N} A_m\,(t) \psi_m\,(z)\,, \tag{3.1}$$

$$\theta(z,t) = \sum_{m=1}^{N} B_m(t)\varphi_m(z), \tag{3.2}$$

$$\zeta(z,t) = \sum_{m=1}^{N} C_m(t)\phi_m(z), \tag{3.3}$$

where

$$\psi_m(z) = \begin{cases} \dfrac{\cosh \mu_m z}{\cosh \frac{\mu_m}{2}} - \dfrac{\cos \mu_m z}{\cos \frac{\mu_m}{2}} & \text{if } m \text{ is odd,} \\ \dfrac{\sinh \mu_m z}{\sinh \frac{\mu_m}{2}} - \dfrac{\sin \mu_m z}{\sin \frac{\mu_m}{2}} & \text{if } m \text{ is even,} \end{cases} \tag{3.4}$$

$$\varphi_m(z) = \sqrt{2}\sin m\pi\left(z + \frac{1}{2}\right), \tag{3.5}$$

$$\phi_m(z) = \sqrt{2}\sin\left[(m+1)\pi z + (m-1)\frac{\pi}{2}\right] \quad (m = 1,2,3,\ldots). \tag{3.6}$$

The above functions $\psi_m(z)$, $\varphi_m(z)$ and $\phi_m(z)$ are chosen in such a way that each form an orthonormal set in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$ and vanish at $z = \pm\frac{1}{2}$. For the derivatives of $\psi_m(z)$ to vanish at these boundaries, it is required that μ_m are to be the roots of the characteristic equation(Chandrasekhar 1961, p. 636)

$$\tanh\frac{1}{2}\mu_m - (-1)^m \tan\frac{1}{2}\mu_m = 0. \tag{3.7}$$

We substitute (3.1)–(3.3) into Eqs. (2.20)–(2.22), multiply the equations by $\psi_n(z)$, $\varphi_n(z)$ and $\phi_n(z)$ respectively, $n = 1,2,3,\ldots N$ and then integrate the resulting equations with respect to z, in the interval $\left(-\frac{1}{2},\frac{1}{2}\right)$. We get a system of $3N$ ordinary differential equations for the unknown coefficients $A_n(t)$, $B_n(t)$ and $C_n(t)$,

$$\begin{aligned} \omega\sum_{m=1}^{N}\left[K_{nm} - a^2\delta_{nm}\right]\frac{\mathrm{d}A_m}{\mathrm{d}t} = P_r\sum_{m=1}^{N}\Bigg[&\left\{\left(\mu_m^4 + a^4\right)\delta_{nm} - 2a^2K_{nm}\right\} \\ &-P_l^{-1}\left(K_{nm} - a^2\delta_{nm}\right)\Bigg]A_m \\ &-P_r\sum_{m=1}^{N}P_{nm}B_m - \sqrt{T}P_r\sum_{m=1}^{N}L_{nm}C_m, \end{aligned} \tag{3.8}$$

$$\omega\frac{\mathrm{d}B_n}{\mathrm{d}t} = a^2R\sum_{m=1}^{N}\left[P_{mn} - \varepsilon\mathrm{Re}\left\{F_{nm}\mathrm{e}^{it}\right\}\right]A_m - \left(n^2\pi^2 + a^2\right)B_n, \tag{3.9}$$

$$\omega\frac{\mathrm{d}C_n}{\mathrm{d}t} = \sqrt{T}P_r\sum_{m=1}^{N}R_{nm}A_m \; - P_r\left[(n+1)^2\pi^2 + a^2 + P_l^{-1}\right]C_n \quad (n = 1,2,\ldots,N) \tag{3.10}$$

where δ_{nm} is the Kronecker delta. The other coefficients, which occur in (3.8)–(3.10) are

$$K_{nm} = \int_{-1/2}^{1/2} D^2\psi_m(z) \cdot \psi_n(z)\, dz, \tag{3.11}$$

$$P_{nm} = \int_{-1/2}^{1/2} \varphi_m(z) \cdot \psi_n(z)\, dz, \tag{3.12}$$

$$L_{nm} = \int_{-1/2}^{1/2} D\phi_m(z) \cdot \psi_n(z)\, dz, \tag{3.13}$$

$$R_{nm} = \int_{-1/2}^{1/2} D\psi_m(z) \cdot \phi_n(z)\, dz, \tag{3.14}$$

and

$$F_{nm} = \int_{-1/2}^{1/2} f(z)\,.\psi_m(z)\,.\varphi_n(z)\, dz. \tag{3.15}$$

The coefficients given by (3.11)–(3.15) have been evaluated numerically using Simpson's (1/3)rd rule (Sastry 1993, p. 125). For computational purposes, it is convenient to introduce the notation

$$x_1 = A_1, \quad x_2 = B_1, \quad x_3 = C_1, \quad x_4 = A_2, \quad x_5 = B_2, \quad x_6 = C_2 \text{ etc.} \tag{3.16}$$

and then rearrange Eqs. (3.8)–(3.10) in the form

$$\frac{dx_i}{dt} = G_{ij}(t)\, x_j \quad (i, j = 1, 2, \ldots, 3N), \tag{3.17}$$

where $\left(G_{ij}(t)\right)$ is the matrix of the coefficients in Eqs. (3.8)–(3.10). Since the coefficients $G_{ij}(t)$ are periodic in t with period 2π, therefore the stability of the solution of (3.17) can be discussed on the basis of the classical Floquet theory (Cesari 1963, p. 55). Let

$$x_n(t) = x_{in}(t) = \text{col}\left[x_{1n}(t), x_{2n}(t), \ldots, x_{Ln}(t)\right] \quad (n = 1, 2, 3, \ldots, 3N \;\&\; L = 3N) \tag{3.18}$$

be the solutions of (3.17) which satisfy the initial conditions

$$x_{in}(0) = \delta_{in}. \tag{3.19}$$

The solutions (3.18) with the conditions (3.19) form $3N$ linearly independent solutions of Eq. (3.17). These solutions are obtained by integrating the system (3.17), using Runge-Kutta–Gill Procedure (Sastry 1993, pp. 217, 227). We rearrange the values of $x_{in}(2\pi)$ and get the constant matrix

$$C = [x_{in}(2\pi)]. \tag{3.20}$$

Then using Rutishauser method (Jain et al. 1991, p. 116), eigenvalues $\lambda_1, \lambda_2, \lambda_3, \ldots, \lambda_L$ of the matrix C are found. We define the characteristic exponents χ_r, of the system (3.17) by the relations

$$\lambda_r = \exp\left(2\pi\chi_r\right), \quad r = 1, 2, 3, \ldots, 3N. \tag{3.21}$$

The values of the characteristic exponents determine the stability of the system. We assume that the χ_r are ordered so that

$$\mathrm{Re}\,(\chi_1) \geq \mathrm{Re}\,(\chi_2) \geq \cdots \geq \mathrm{Re}\,(\chi_L)\,. \tag{3.22}$$

Then the system is stable if $\mathrm{Re}(\chi_1) < 0$, while $\mathrm{Re}\,(\chi_1) = 0$ corresponds to one periodic solution and represents a stability boundary. This periodic disturbance is the only disturbance, which will manifest itself at marginal stability.

Thus, the value of the Rayleigh number R for the onset of convection corresponds to $\mathrm{Re}(\chi_1) = 0$. Since in our calculations(in the next section) we find all characteristic exponents as real numbers, therefore the above value of the Rayleigh number corresponds to $\lambda_1 = 1$, the largest eigenvalue of the matrix C. The minimum value of R can be found at some value of the wave number a, for some fixed values of other parameters. This minimum value of the Rayleigh number is known as the critical Rayleigh number (R_c)and the corresponding value of a is known as the critical wave number (a_c).

4 Results and discussion

For the parameter ranges of our interest, it is sufficient to take $N = 4$ (Four Galerkin terms-two even and two odd) in the Galerkin procedure (Fig. 12). Therefore all the following results are related to $N = 4$. The values of the critical Rayleigh number R_C and corresponding values of the critical wave number a_c in the absence of modulation ($\varepsilon = 0$) are found as given below:

$$\varepsilon = 0, \quad T = 0.0, \quad P_l^{-1} = 0.0, \quad a_c = 3.114, \quad R_c = 1709.03. \tag{4.1}$$

Here, result (4.1) corresponds to the non-rotating ($\Omega = 0$), non-porous convection, and are very close to results of Chandrasekhar (1961, p. 43). On comparing (4.1) with the results (1.1)–(1.8) and (2.1)–(2.8) given in Tables 1 and 2, respectively, we find that the effect of rotation and porous medium on the thermal instability is stabilizing as the values of R_c in these cases is higher than 1709.03. From Table 2, when $P_l^{-1} \to 0$ we see that the results correspond to the convection in an ordinary rotating fluid layer and when $P_l \to 0$ we recover the results of Dacry model.

Now when $\varepsilon \neq 0$, we calculate the value of R_c at different values of other parameters. The results have been obtained by solving the Eqs. (3.17) for x_1, x_2, x_3, x_4, x_5,

Table 1 The results correspond to $N = 4$

$P_l = 1.0, \varepsilon = 0$			
S. no.	T	a_c	R_c
1.1	0.0	3.118	1753.6
1.2	1.0	3.119	1754.1
1.3	10.0	3.123	1758.5
1.4	100.0	3.161	1801.4
1.5	200.0	3.201	1848.1
1.6	500.0	3.313	1982.7
1.7	1000.0	3.473	2191.9
1.8	10000.0	4.766	4790.4

Table 2 The results correspond to $N = 4$

$T = 100.0, \varepsilon = 0$			
S. no.	P_l	a_c	R_c
2.1	10^4	3.159	1757.9
2.2	1000.0	3.159	1757.9
2.3	100.0	3.159	1758.3
2.4	10.0	3.159	1762.3
2.5	1.0	3.161	1801.4
2.6	0.1	3.179	2193.0
2.7	0.01	3.236	6089.2
2.8	0.001	3.249	44383.8

$x_6, x_7, x_8, x_9, x_{10}, x_{11}$ and x_{12}. The values of R_c have been calculated for the following three cases: (a) when the walls' temperature are modulated in phase i.e. $\phi = 0$, (b) when the walls' temperature are modulated out of phase i.e. $\phi = \pi$, and (c) when only the bottom plate temperature in modulated, the upper plate is held at a fixed constant temperature i.e. $\phi = i\infty$.

The variation of R_c with ω, for cases (a), (b) and (c), have been depicted in the Figs. 1, 2 and 3, respectively, at different values of the Taylor number T, the other parameters are $P_l = 1.0, \varepsilon = 0.4, P_r = 1.0$. From the Fig. 1, we observe that an increase in the value of T increases the value of R_c, thus the effect of large values of the Taylor number T is to stabilize the system, as convection starts at higher Rayleigh number. Also this shows that rotation delays the onset of convection, thus stabilizing the system in presence of modulation. Now to discuss the effect of modulation, we consider a graph in Fig. 1 corresponding to a particular value of T (say $T = 1.0$). We find that initially (for small ω) the effect of modulation is destabilizing, as the value of R_c is smaller than the corresponding value 1754.1, in unmodulated case (Table 1). Modulation effect is small for small values of ω, and becomes maximum (destabilizing) near $\omega = 17$. Modulation stabilizes the system at around $\omega = 60$, and then its effect disappears altogether when $\omega \to \infty$, which is clear from the graph, since R_c approached the same value 1754.1, which is obtained in the unmodulated case (Table 1). In Figs. 2 and 3, we find the same qualitative effect of Taylor number T as obtained in Fig. 1. However, here the effect of modulation is greatest (stabilizing) near $\omega = 0$ and disappears altogether when the frequency ω becomes sufficiently large. For intermediate values of ω, the effect of modulation is found to be less stabilizing. To compare the values of R_c for different cases, we have depicted the variation of R_c with ω at $T = 500.0$ and 1000.0, in Figs. 4 and 5, respectively. From these figures, we find that for some particular value of ω, the value of R_c in case (c) is smaller than the value of R_c in case (b) but higher than the value of R_c in case (a), thus maximum stabilization occurs in case (b) i.e. for out of phase modulation.

In Figs. 6–10, we have shown the variation of Rc with ω, for all the three cases, at $P_l = 1000.0$, 10.0, 1.0, 0.1 and 0.01, respectively, the values of other parameters are $T = 100.0$, $\varepsilon = 0.4$, $P_r = 1.0$. From Figs. 6–8, we observe that an increase in the value of P_l decreases the value of Rc, thus the effect of large values of the porous parameter P_l is to advance the onset of convection, as convection occurs at an early point. However as the value of P_l becomes smaller, R_c increases, thus showing the stabilizing effect on the system (Figs. 9, 10). In Fig. 6, for a particular graph (say for $P_l = 1000.0$), we find that for small values of ω the effect of modulation is destabi-

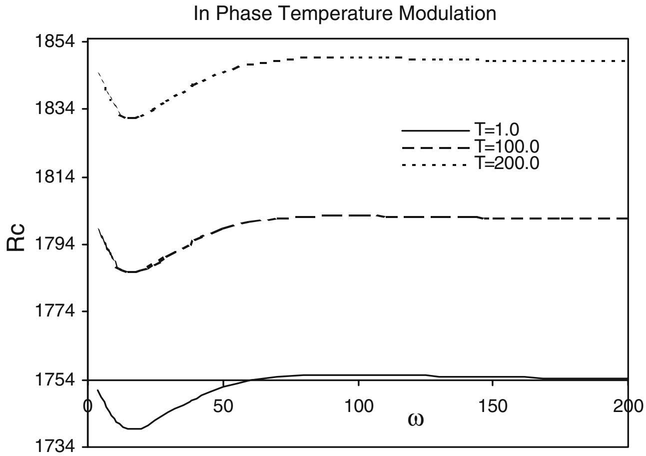

Fig. 1 Variation of R_c with $\omega.\varepsilon = 0.4$, $P_r = 1.0$, $P_l = 1.0$

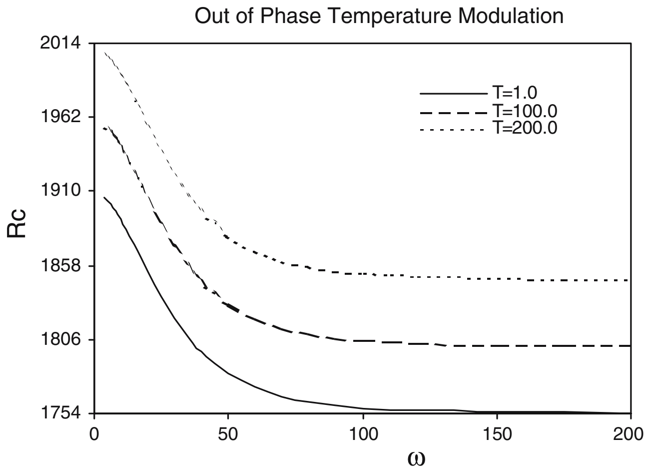

Fig. 2 Variation of R_c with $\omega.\varepsilon = 0.4$, $P_r = 1.0$, $P_l = 1.0$

lizing, as convection occurs at an earlier point than in the unmodulated case (see the Table 2). For intermediate values of ω, R_c is minimum at $\omega = 17$, thus the effect of modulation is most destabilizing, the effect becomes stabilizing at around $\omega = 60$, and then reduces to zero when ω becomes very large. Figs. 7 and 8 show that for small and intermediate values of ω, the effect of modulation is stabilizing and falls of to zero as ω goes to infinity. However, the stabilization is found to be greatest near $\omega = 0$, as the value of R_c is highest here. In figures 9, 10, we compare the values of R_c for different cases, at $P_l = 0.1$ and 0.01, respectively. For some particular value of ω, it is found that the value of R_c in case (c) is smaller than the value of R_c in case (b) but higher than the value of R_c in case (a), thus maximum stabilization occurs in case (b) i.e. for out of phase modulation.

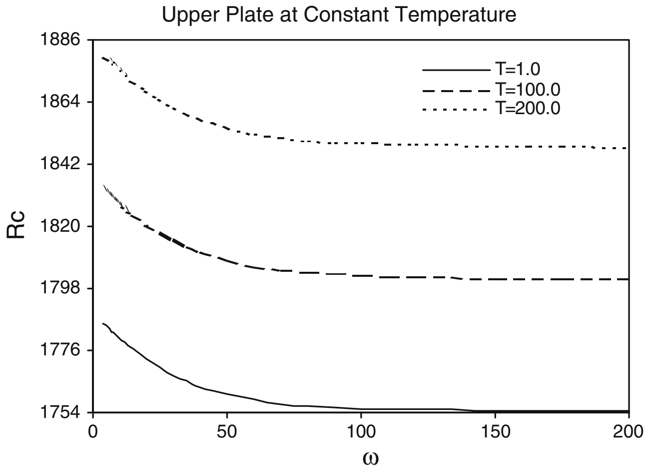

Fig. 3 Variation of R_c with $\omega.\varepsilon = 0.4$, $P_r = 1.0$, $P_l = 1.0$

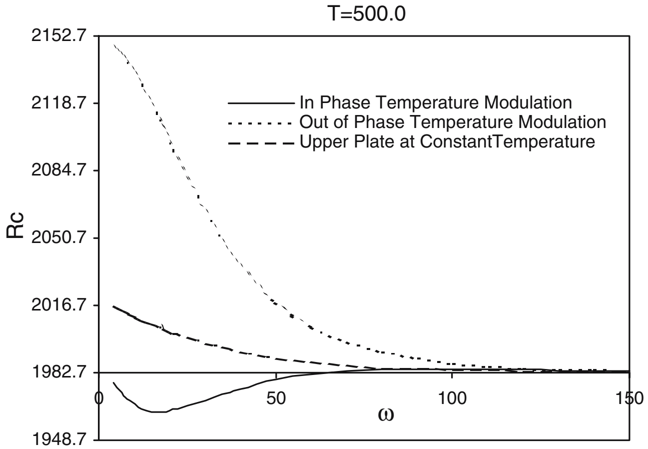

Fig. 4 Variation of R_c with $\omega.\varepsilon = 0.4$, $P_r = 1.0$, $P_l = 1.0$, $a_c = 3.313$

As we know that at high frequency modulation becomes very fast, therefore temperature in the fluid layer is unaffected by the modulation except for a thin layer (Venezian 1969), so that we find almost the same value of R_c as in the unmodulated case (see the tables), for large value of ω. The convective waves propagation across the porous layer is higher when the modulation is out of phase[case (b)], while it is lower when only the lower wall temperature is modulated [case (c)] or when the temperature modulation is in phase [case (a)], therefore convection occurs at higher Rayleigh number in case (b) than in other two cases.

In Fig. 11 the variation of R_c with ω has been depicted, for out of phase modulation at $T = 100.0$, $P_l = 1.0$, $P_r = 1.0$ and $\varepsilon = 0.4$, and the results are compared corresponding to $N = 4$ and $N = 6$. It is found here that in all cases the error in the

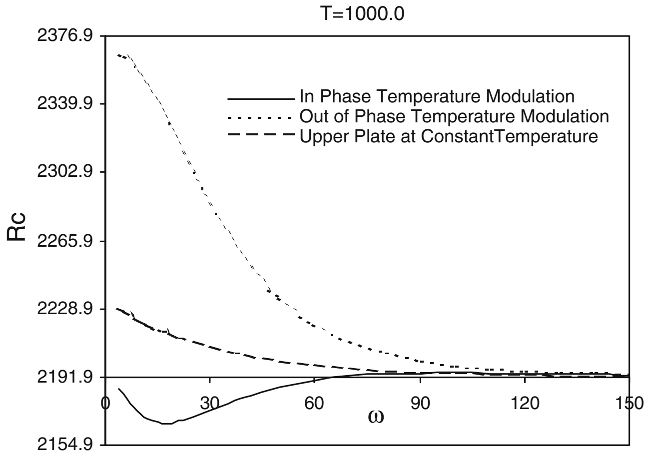

Fig. 5 Variation of R_c with $\omega.\varepsilon = 0.4$, $P_r = 1.0$, $P_l = 1.0$, $a_c = 3.473$

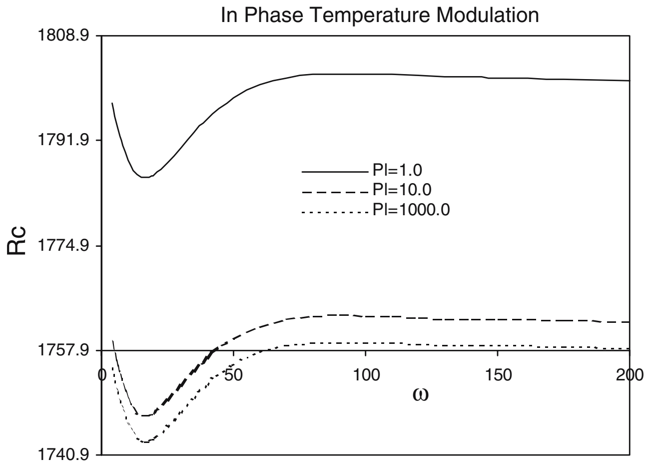

Fig. 6 Variation of R_c with $\omega.\varepsilon = 0.4$, $P_r = 1.0$, $T = 100.0$

results corresponding to $N = 4$ and $N = 6$ is less than 0.09%. This justifies the reason of taking $N = 4$ in our calculations.

Now we calculate the value of R_{YL} [which is R_c of Yih and Li (1972) in their Figs. 1, 2] and compare it with Yih–Li's results. The value of R_{YL} is the minimum value of R as a function of the wave number a for fixed values of the other parameters. Here in Fig. 12, we plot R_{YL} with respect to ε for $T = 100.0$, $P_l = 1.0$, $\omega = 17$, $P_r = 1.0$. Since our thermal boundary conditions for case (b) are similar (but not exactly same) to that of Yih and Li (1972) therefore the results have been obtained here only for case (b) i.e. for out of phase modulation. In the graph, we find two types of curves; one corresponding to synchronous and other corresponds to the subharmonic solution (half-frequency). In the Fig. 12, S is for synchronous solutions and H is for subharmonic (half-frequency) solutions. Each of these two curves represents the minimum

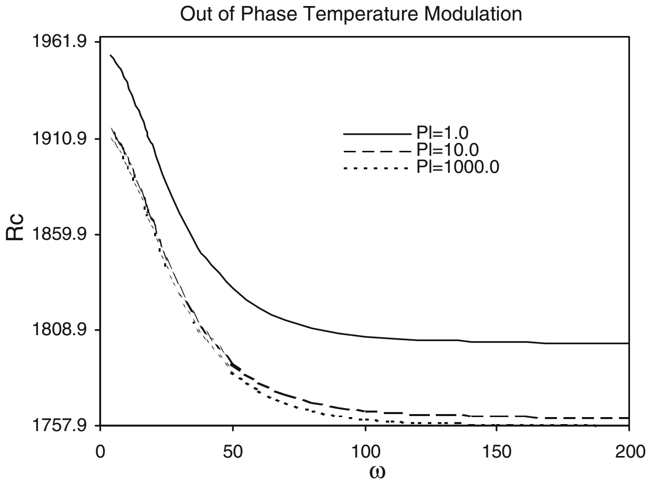

Fig. 7 Variation of R_c with ω. $\varepsilon = 0.4$, $P_r = 1.0$, $T = 100.0$

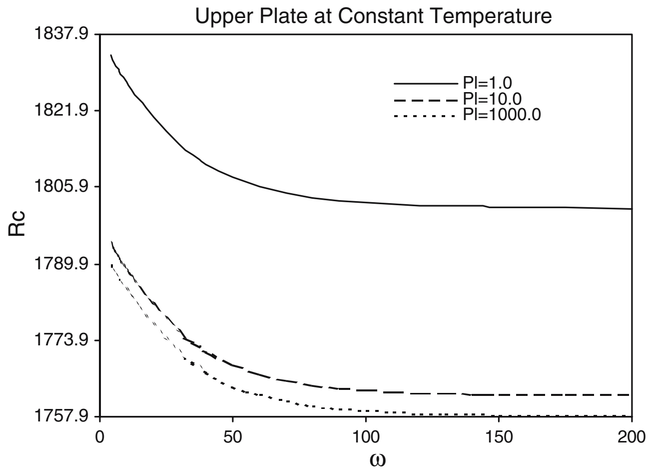

Fig. 8 Variation of R_c with ω. $\varepsilon = 0.4$, $P_r = 1.0$, $T = 100.0$

of the modes of solutions in terms of the values of R_{YL}. Also in this figure, we have shown the variation of R_c (Rayleigh number at neutral stability) with ε. For both R_{YL} and R_c, we get a combination of synchronous and subharmonic solutions. On comparing the values of R_{YL} and Rc, we find that R_{YL} is smaller than Rc. Initially, we find that the system becomes more and more stabilized as ε increases upto 0.758 and then less stabilized as ε increased further. On further increasing the value of ε, we see that at around $\varepsilon = 2.2$ the system becomes destabilized. This destabilizing effect of modulation may be due to the finite amplitude convection at higher ε. On comparing the present values of R_{YL} and R_c respectively with the Figs. 2 and 3 of Yih and Li (1972), we find a very good agreement between them. Only difference is that in the present results the values of the Rayleigh numbers are slightly greater

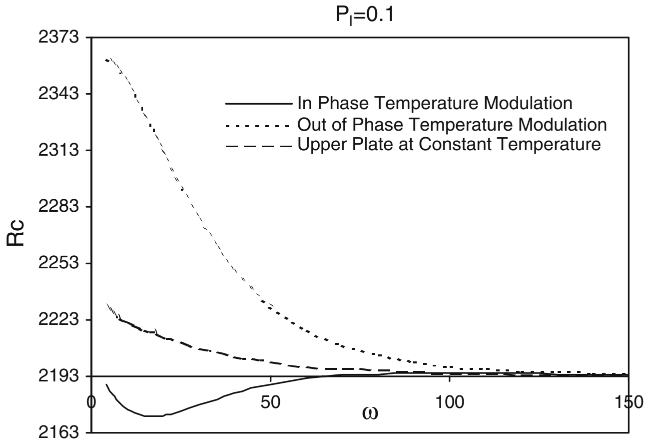

Fig. 9 Variation of R_c with ω. $\varepsilon = 0.4$, $P_r = 1.0$, $T = 100.0$, $a_c = 3.179$

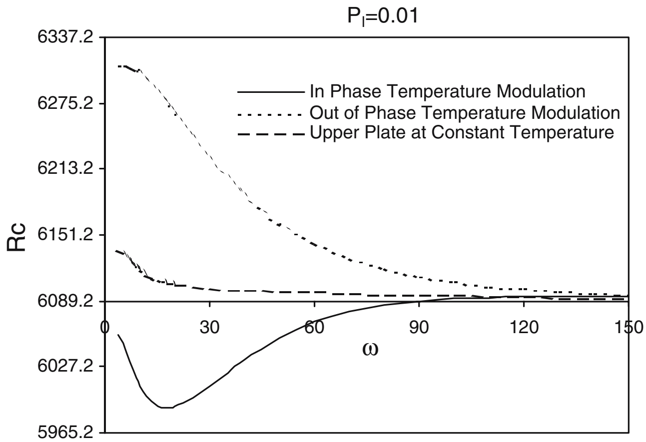

Fig. 10 Variation of R_c with ω. $\varepsilon = 0.4$, $P_r = 1.0$, $T = 100.0$, $a_c = 3.236$

than those of Yih and Li (1972), which is obvious due to the effects of rotation and porous medium. We expect qualitatively same behaviour of the Rayleigh numbers in other two cases(a) and (c) also, with slight difference that in case (a) the effect of modulation on the system would be destabilizing from the very beginning.

In Fig. 13, we consider the variation of the corresponding value of a_{YL} with ε. We observe that initially for synchronous solutions the value of a_{YL} decreases upto $\varepsilon = 0.758$, and then there is a jump for subharmonic solutions as the value of ε becomes slightly greater 0.758. Same jump behaviour is found every time whenever there is a change from one type of solution to other type.

Also we have checked the variation of R_{YL} with ω and found that for all the cases (a), (b) and (c), there exists only synchronous solution, while Yih and Li (1972) in their Fig. 1 find a combination of both synchronous and subharmonic solutions. This difference may be due to two possible reasons: one, since during calculations we find that the variation in the value of a_{YL} is not much different from a_c (wave number at

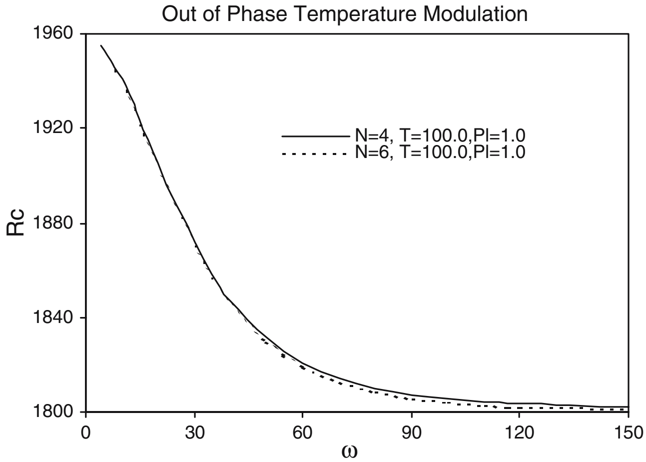

Fig. 11 Variation of R_c with ω. $\varepsilon = 0.4$, $P_r = 1.0$, $a_c = 3.161$

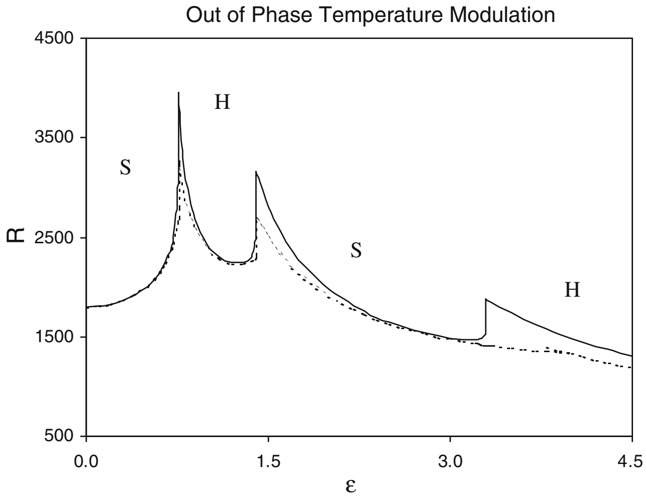

Fig. 12 Variation of R_{YL} and R_c with ε. $\omega = 17.0$, $P_r = 1.0$, $P_l = 1.0$, $T = 100.0$. Solid lines represent to R_c, dotted lines represent to R_{YL}

neutral stability), therefore the value of R_{YL} is very close to R_c and so only one solution (as in Figs. 1–11); second, the present thermal boundary conditions are different from theirs. Since the variation in R_{YL} with ω is found to be very close to that of R_c (Figs. 1–11) therefore we do not find appropriate to depict it here again.

Figure 14 shows the variation of R_c with the Prandtl number P_r, for case (c) at $T = 100.0$, 150.0 and 200.0, respectively, the values of other parameters are $P_l = 1.0$, $\varepsilon = 0.4$, $\omega = 50.0$. From the figure one can see that the maximum stabilization of the system occurs at around $P_r = 1.0$, and the same behaviour is expected in other two cases (a) and (b) also, therefore we have considered $P_r = 1.0$ in all our above calculations.

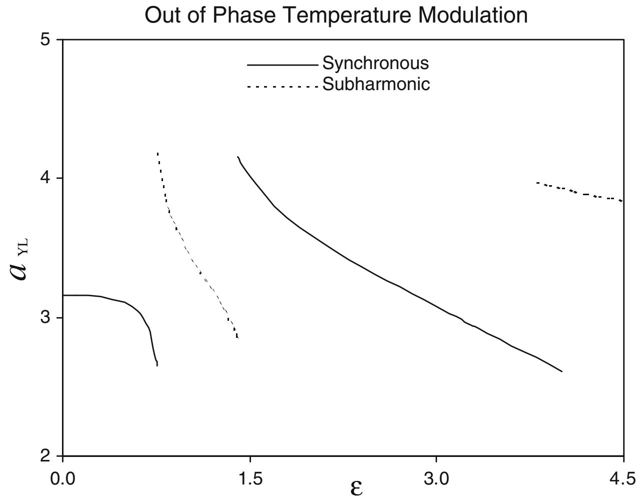

Fig. 13 Variation of a_{YL} with ε. $\omega = 17.0$, $P_r = 1.0$, $P_l = 1.0$, $T = 100.0$

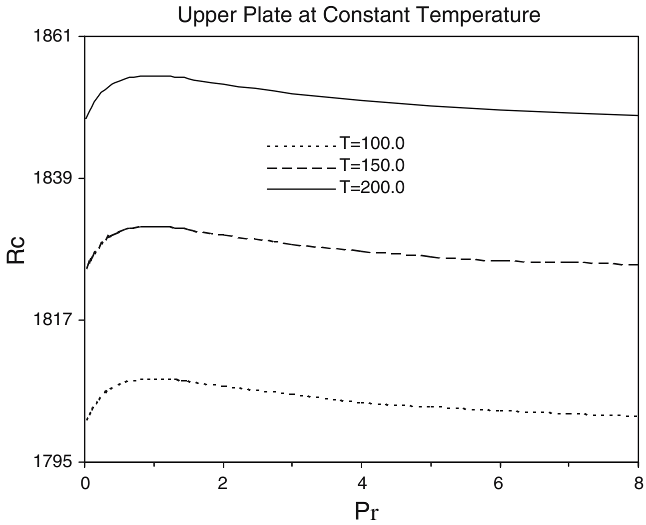

Fig. 14 Variation of R_c with P_r. $\varepsilon = 0.4$, $P_l = 1.0$, $\omega = 50.0$, $T = 100.0$

5 Conclusion

The problem of linear stability of fluid convection has been investigated in a sparsely packed rotating porous medium subject to time periodic heating of the rigid boundaries, and three types of modulation effects have been considered. The solution is obtained under the assumptions that disturbances are infinitesimal, and the amplitude of the applied temperature field is small. The following conclusions are drawn:

1. In case of in phase modulation, initially for small ω, the effect of modulation is destabilizing, it becomes most destabilizing at around $\omega = 17$, the effect decreases

for intermediate values of ω, becomes stabilizing on further increasing the value of ω, and finally disappears as ω becomes very large.
2. In case of out of modulation or when only the lower wall temperature is modulated, we find that the effect of modulation is most stabilizing near $\omega = 0$, becomes less stabilizing for intermediate values of ω, and finally disappears as ω goes to infinity.
3. The effect of increasing porous parameter P_l is to decrease the value of the critical Rayleigh number R_c. Thus, the effect of increasing permeability is to advance the onset of convection in the presence of thermal modulation.
4. The Brinkman's model serves to bridge the gap between the viscous fluid limit and the Darcy limit in the sense that when $P_l^{-1} \to 0$ (high permeability) we get the results of viscous fluid layer, and when $P_l \to 0$ (low permeability) we find Darcy limit results.
5. It is found that the effect of increasing the value of Taylor number is to delay the onset of convection, thus making the system more stabilizing. This confirms the well-established fact that the effect of rotation is to stabilize the system. This is because, in the presence of the Coriolis force, the disturbance in the fluid will not be able to move up or down as easily as without rotation. When $T = 0.0$, we obtain $R_c = 1753.6$ (Table 1), which is exactly the same as obtained by Bhadauria (2006b) in his study of non-rotating porous convection.
6. On calculating the variation of R_{YL} and R_c with respect to ε, the amplitude of modulation, it was found that the solution consists of two regions; one corresponding to synchronous and the other one corresponding to the subharmonic.

The author is thankful to the referees for their useful comments and valuable suggestions, which have helped in improving the presentation and quality of the paper.

Acknowledgements This work was carried out as part of a major research project [No.F.8-10/2003(SR)] awarded by University Grants Commission (UGC), India. The financial assistance from UGC is gratefully acknowledged.

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