Transport in Porous Media (2007) 67:69–91
DOI 10.1007/s11242-006-0021-0 

# Consolidation Statistics Investigation via Thin Layer Method Analysis

M'HAMMED BADAOUI[1], ALI NOUR[2], ABDENNASSER SLIMANI[3] and MOUNIR KHALED BERRAH[4]
[1]*LBE, Faculty of Civil Engineering, USTHB, BP 32 El-Alia, Bab-Ezzaouar, Algiers, Algeria*
[2]*Department of Civil, Geological and Mining Engineering, École Polytechnique de Montréal, P.O. Box 6079, Station "Centre-Ville", Montréal, Quebec, H3C 3A7, Canada.*
[3]*CGS, National Center of Applied Research in Earthquake Engineering, 01 rue Kaddour Rahim BP 252 Hussein-Dey, Algiers, Algeria*
[4]*Ecole Nationale Polytechnique, 10 Avenue Hassan Badi, BP 182, 16200 El-Harrach, Algiers, Algeria*

(Received: 24 August 2005; accepted in final form: 14 February 2006)

**Abstract.** In this paper, the Thin Layer Method (TLM) is adapted for solving one-dimensional primary consolidation problems. It is also combined with a stochastic formulation integrating Monte Carlo simulations to investigate primary consolidation of a random heterogeneous soil profile. This latter is modeled as a set of superposed layers extending horizontally to infinity, and having random properties. Spatial variability of soil properties is considered in the vertical direction only. Soil properties of interest are elastic modulus and soil permeability, modeled herein as spatially random fields. Lognormal distribution is chosen because it is suitable for strictly non-negative random variables, and enables analyzing the large variability of such properties. The statistics regarding final settlement and its corresponding time are investigated by performing a parametric study, which integrates the influence of variation coefficient of both elastic modulus and soil permeability, and vertical correlation length. Obtained results indicate that heterogeneity significantly influences primary consolidation of the soil profile, generating a quite different way of soil grain rearrangement and water pressure dissipation in comparison to the homogeneous case, and causing a delay in the consolidation process.

**Key words:** primary consolidation, heterogeneity, soil profile, elastic modulus, soil permeability, lognormal distribution, Monte Carlo simulation, thin layer method.

## Nomenclature

| | |
|---|---|
| $u_e$ | pore water pressure |
| $\Delta\sigma$ | constrained stress |
| $k$ | coefficient of permeability |
| $\gamma_{\mathrm{w}}$ | unit water weight |
| $e_0$ | void ratio |
| $C_{\mathrm{v}}$ | coefficient of consolidation in the vertical direction |

| | |
|---|---|
| $\upsilon$ | poisson's ratio |
| $N$ | shape function matrix |
| $U_E, \overline{U}_E$ | discrete pore water pressure vector and its Fourier transform |
| $z$ | vertical coordinate |
| $h_i, E_i$ | height and Young modulus of the sublayer $i$ |
| $\xi$ | value corresponding to $\frac{z}{h_i}$ |
| $B_z, A, M$ | matrices in Equation (9) |
| $t$ | time |
| $\omega_R, \phi$ | eigenvalue and eigenvector |
| $\Omega_R, \Phi$ | diagonal matrix of eigenvalues and matrix of eigenvectors |
| $q$ | load's amplitude |
| $H$ | total soil height |
| $U$ | average degree of consolidation |
| $W$ | settlement |
| $I_1$ | unit vector |
| $E_L$ | matrix |
| $\Omega$ | diagonal matrix of $e^{i\omega_R t}$ elements |
| $T_v$ | time factor |
| $H_{dr}$ | length of the longest pore water drainage path |
| $\mu_i, \sigma_i^2$ | mean and Variance values of soil property $i$ |
| $\Delta g(z)$ | a zero mean, unit variance, Gaussian random field |
| $\Psi$ | random phase angle distributed uniformly over the interval $[0, 2\pi]$ |
| $\kappa_z$ | wavenumber in $z$ direction |
| $\Delta\kappa_z$ | wavenumber step in $z$ direction |
| $S(\kappa_z), R(\xi_z)$ | power density and spatial correlation functions |
| $a$ | vertical correlation distance |
| $L_z$ | wavelength |
| $CV_k$, $CV_E$ | permeability and elastic modulus variation coefficients |
| SD, DD | single and double drainage |

## 1. Introduction

When vertical loads are applied on a layer of soil, they cause time-dependent soil deformations. Because air, water and voids are within soil particles structure, vertical loads induce void volume reduction and soil grains rearrangement. These deformations depend basically on the type of soil, drainage conditions, and intensity of applied loads. Such a process is called consolidation and results in soil settlement. The one-dimensional finite non-linear consolidation of thin homogeneous layers is well described for the case of saturated clays by Gibson *et al.* (1967).

The dispersion observed in soil data comes both from the spatial variability and from errors in testing. Geotechnical engineering deals with some of the processes and properties of the geological formations as well as properties of other materials, which must be analyzed from limited observations and few data availability. Due to the prohibitive cost of sampling and measurement errors, deterministic representation of spatial variability of soil properties is not feasible. Hence, reliable final settlement and its corresponding time of a layer of soil cannot be obtained from a deterministic approach. The resort to probabilistic techniques enables modeling uncertainties by analyzing their dispersion effect on the global behavior of the physical system.

The Thin Layer Method (TLM) is suitable for soil profiles modeled as a set of superposed layers extending horizontally to infinity. It was first introduced by Kausel and Roësset (1981) to compute the response of a layered system of finite depth to a concentrated load within (and on) the medium. Kausel and Peek (1982) have presented an explicit closed-form solution for Green's functions corresponding to dynamic loads acting on or within layered strata. This method was applied to obtain the response of multi-layered strata over an elastic half-space, to static and dynamic loads (Seale, 1985; Kausel and Seale, 1987; Seale and Kausel, 1989), with anisotropic materials in both frequency (Kausel, 1986) and time domain (Kausel, 1994). An extension to poroelastic stratum (Bougacha *et al.*, 1993a, b) and to dynamic response of submerged soils (Nogami and Kazama, 1992) was also performed.

In this paper, one prefers to deal with a semi-analytical approach for which the thin layer method (TLM) is adapted for solving one-dimensional primary consolidation problems. It is also combined with a stochastic formulation integrating Monte Carlo simulations to investigate one-dimensional primary consolidation of a random heterogeneous soil profile. The soil profile is modeled as a set of superposed layers extending horizontally to infinity, and having random properties. Spatial variability of soil properties is considered in the vertical direction only. For 1D problem, this technique is more attractive than finite element method (FEM) because the TLM is a semi-analytical one and does not require performing time discretization in the formulation. This is not the case for FEM, where we should be very careful regarding stability and accuracy of the selected time marching algorithm because the integration procedure used for consolidation analysis introduces a relationship between the minimum usable time increment and the element size. Furthermore, if the finite element model is formulated using a compressible element, element matrices should be integrated with a special caution. Any violation of these rules may lead to spurious (non-physical) oscillations and deviation from the real solution of the problem (Desai and Christian, 1977). Hence, adaptation of TLM to consolidation problems serves as a stable numerical deterministic tool for determining consolidation statistics via Monte Carlo simulations, allowing the analysis of moderate to highly heterogeneous media.

In this context, but using different approaches, one mainly notes contributions of Koppula (1988) for the development and dissipation of excess pore pressure in soil under external loads for a random variable representation of the consolidation coefficient, Fenton and Griffiths (2002) for the probabilistic foundation settlement on spatially random soil and Nour *et al.* (2002) for foundation settlement statistics via finite element analysis.

In this study, statistics regarding final settlement and its corresponding time are investigated by performing a parametric study which integrates the influence of variation coefficient of both elastic modulus and soil permeability, and the vertical correlation length.

## 2. Objectives and Scope of the Study

It is aimed through this study to analyze primary consolidation of a random heterogeneous soil profile, for which final settlement and its corresponding time are investigated. Soil properties of interest are the elastic modulus and soil permeability, modeled herein as independent spatially random fields, by considering the spatial Gaussian correlation. Their random fields are obtained by adopting the lognormal distribution, which is suitable for strictly non-negative random variables, and enables analyzing the large variability of the medium. Thus, final settlement and its corresponding time are carried out using Monte Carlo simulations combined with deterministic TLM. Thereby the present paper emphasizes the following points:

- Adaptation of TLM for solving one-dimensional primary consolidation and its validation with Terzaghi's solution.
- Combination of TLM with a stochastic formulation integrating Monte Carlo simulations to investigate one-dimensional primary consolidation of a random heterogeneous soil profile.
- A comparative study of two models in relation with two situations often encountered in practice, i.e. (i) model with single drainage (SD) and (ii) model with double drainage (DD).
- A parametric study which develops statistics of final settlement and its corresponding time with regards to the influence of variation coefficient of both elastic modulus and soil permeability, and the vertical correlation length.

## 3. Consolidation Investigation via TLM

### 3.1. CONSOLIDATION THEORY

When a saturated soil is subjected to externally applied loads, its volume decreases. Since both soil particles and water in voids may be considered incompressible, change in volume can only occur if water is forced out of voids. This process is known as consolidation. The one-dimensional soil primary consolidation where the mechanical properties are varying with the vertical coordinate is governed by the well-known following equation (Biot, 1941; Zhuang *et al.*, 2005)

$$\frac{1}{\gamma_\mathrm{w}}\frac{\partial}{\partial z}\left(k_\mathrm{v}\frac{\partial u_e}{\partial z}\right)=\frac{1}{1+e_0}\left(\frac{\partial u_e}{\partial t}-\frac{\partial \Delta\sigma}{\partial t}\right). \tag{1}$$

The boundary conditions are introduced as follows:

$$u_e\,(z=0)=0 \quad u_e\,(z=H)=0 \quad \text{(DD)} \quad \frac{\partial u_e}{\partial z}\,(z=H)=0 \quad \text{(SD)} \tag{2}$$

$u_e$ and $\Delta\sigma$ stand respectively for, the pore water pressure and the constrained stress which induces the pore water pressure dissipation; $k_\mathrm{v}$ is the coefficient of permeability in the vertical direction, $\gamma_\mathrm{w}$ is the unit water weight, $e_0$ is the void ratio and $H$ stands for the total soil height. The soil profile is constituted by a number of layers, each layer is subdivided in sublayers where it is supposed that the mechanical properties are invariant and the pore pressure is linear with respect to the vertical coordinate. Considering Equation (1) for the case of the sublayer $i$, one obtains

$$(C_\mathrm{v})_i\,\frac{\partial^2 u_e}{\partial z^2}=\frac{\partial u_e}{\partial t}-\frac{\partial \Delta\sigma}{\partial t}. \tag{3}$$

The continuity conditions in the interface between the sublayer $i$ and $i+1$ are given by the following equations (at the location $z=H_i$):

$$(u_e)_i=(u_e)_{i+1} \tag{4a}$$

$$\left(k_\mathrm{v}\,\frac{\partial u_e}{\partial z}\right)_i=\left(k_\mathrm{v}\,\frac{\partial u_e}{\partial z}\right)_{i+1} \tag{4b}$$

where $(C_\mathrm{v})_i$ stands for the coefficient of permeability in the vertical direction of the sublayer $i$, given by the following equation:

$$C_\mathrm{v}=\frac{k_\mathrm{v}}{\gamma_\mathrm{w}\left(\frac{(1+\upsilon)\cdot(1-2\upsilon)}{E\,(1-\upsilon)}\right)}. \tag{5}$$

Here, $E$ stands for elastic modulus, $\upsilon$ the Poisson's ratio and $H_i$ designates the location of the interface $i$.

For the case of a multi-layered soil profile, the rigorous solution of Equation (3) involves a significant number of unknowns; making this task rather lengthy and cumbersome. In case of horizontally stratified soil medium, with only vertical spatial variability of soil properties, namely, corresponding to a superposition of several horizontally homogeneous layers of soil; the TLM constitutes an adequate alternative for such problems solution. Indeed, with the TLM, one deals with algebraic equations instead of solving the partial differential equations, leading to a significant reduction of variables of the problem.

### 3.2. ADAPTATION OF THE TLM TO CONSOLIDATION ANALYSIS

This section describes the one-dimensional primary consolidation of a heterogeneous soil profile, modeled as a set of superposed layers which extend horizontally to infinity (Figure 1). It is assumed that the initial distribution of pore water pressure is uniform along the depth. When using the TLM, each layer as illustrated by Figure 2, is subdivided into several sublayers. In each sublayer, it is assumed that the pore water pressure $u_e$ is linearly varying with respect to the vertical coordinate $z$ (depth) (Figures 1 and 2)

$$u_e = N \cdot U_E \tag{6}$$

where

$$U_E = \begin{Bmatrix} U_E^1 \\ U_E^2 \end{Bmatrix} \qquad N = \left[ \xi \ (1-\xi) \right] \tag{7}$$

$N$ is the shape function row vector, for which $\xi = z/h_i$ with $0 \leqslant \xi \leqslant 1$, $U_E$ stands for the discrete pore water pressure vector for each sublayer interface, and $h_i$ designates the height of the sublayer $i$.

To obtain the equations governing the discrete model, Equations (4), (6) and (7) are combined with Equation (3) and the virtual work principle

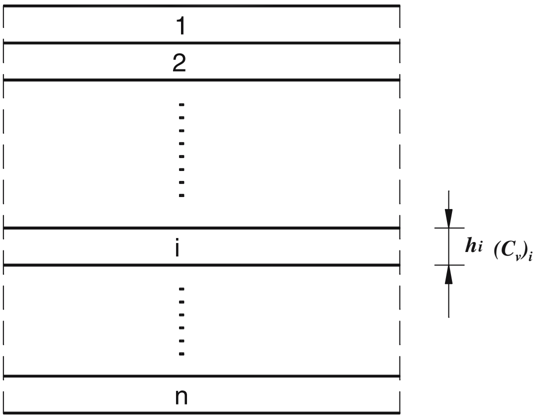



*Figure 1.* Multi-layered soil profile.

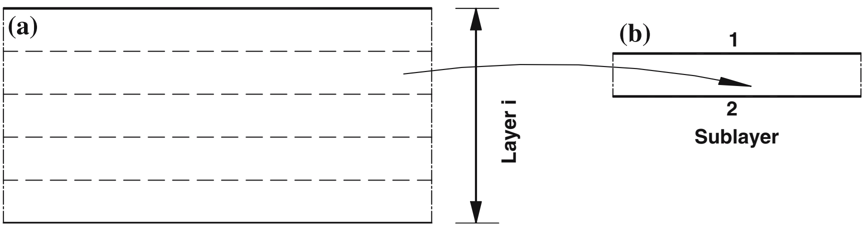



*Figure 2.* Soil layer and soil sublayers definition. (a) Layer. (b) Sublayer.

(Kausel, 1994) is applied, leading thereby to the following result:

$$B_z U_E - A \frac{\partial U_E}{\partial t} = M \frac{\partial \Delta\sigma}{\partial t} \tag{8}$$

where

$$B_z = \frac{(C_\mathrm{v})_i}{h_i} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad A = \frac{h_i}{6} \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad M = \frac{h_i}{2} \begin{Bmatrix} 1 \\ 1 \end{Bmatrix} \tag{9}$$

The system of Equations (8) is provided for a single sublayer. The multilayered soil profile system is obtained by assembling the elementary matrices of each sublayer. The global system obtained exhibits the same form as of the elementary one. Hence, one uses the same notations as for the system of Equations (8). For subsequent analysis, all matrices and vectors correspond to the global system (Kausel, 1994). The set of differential Equations (8) is transformed into a set of algebraic equations in the frequency domain by performing a Fourier transform with respect to time. This leads to the following system of equations:

$$\left(B_z + \mathrm{i}\,\omega\, A\right) \overline{U}_E = -q\, M \tag{10}$$

for which $\overline{U}_E$ is the Fourier transform of $U_E$. Here $\omega$ designates the circular frequency and $i^2 = -1$. Solving Equation (10) requires the resolution of the following eigenproblem:

$$\left(B_z + \mathrm{i}\,\omega_R A\right) \phi = 0 \tag{11}$$

$\omega_R$ is a complex eigenvalue, and $\phi$ represents the corresponding eigenvector, which satisfy the standard normalization conditions, namely

$$\Phi^T A\, \Phi = I \quad \Phi^T B_z\, \Phi = -\mathrm{i}\Omega_R. \tag{12}$$

Here $\Omega_R$ is a diagonal matrix whose elements are the eigenvalues, and $\Phi$ is the corresponding eigenvectors matrix. Solution of the system (10) in frequency domain is given by

$$\overline{U}_E = -\mathrm{i}\, q\, \Phi\ \left(\omega\, I - \Omega_R\right)^{-1}\ \Phi^T\, M \tag{13}$$

The pore water pressure in time domain is given by:

$$U_E = q\, \Phi \Omega\, \Phi^T\, M \tag{14}$$

$\Omega$ is a diagonal matrix whose elements are terms of $\mathrm{e}^{\mathrm{i}\omega_R t}$.

## 4. Consolidation Statistics Investigation via Monte Carlo Simulations Combined with Deterministic TLM

### 4.1. ELASTIC MODULUS AND SOIL PERMEABILITY SPATIAL VARIABILITIES

Soils are geological materials formed by weathering processes and are transported by physical means to their present location. They have been subjected to various stresses, pore fluids, physical, and chemical changes. Thus, it is hardly surprising that mechanical properties of soils vary within resulting deposits. In principle, the spatial variation of mechanical soil properties can be characterized in detail, but only if a large number of tests are made. In reality, the number of tests required exceeds by far what would be practical. Thus, for engineering purposes a simplification is introduced whereby spatial variability of soil properties is decomposed into a deterministic trend, and a random component describing the variability about that trend (Vanmarcke, 1984; Fenton, 1990). In order to analyze the heterogeneous character of soil, soil properties of interest are the elastic modulus and the soil permeability modeled herein as spatially random fields. For the simulation of the random medium, the chosen random variables are defined by their moments of order 1 and 2, which are respectively the mean and the variance, supposed valued from in situ samples.

Elastic modulus and soil permeability are assumed to be lognormally distributed, because this distribution is suitable for strictly non-negative random variables. Furthermore, if one performs simulations considering normal distribution, this leads to negative values for the considered soil properties for large values of variation coefficient. Also, lognormal distribution has a simple relationship to the normal, this latter of which important when simulating via spectral representation, enabling hence analyzing the large variability of the medium.

Because in practice there is no information regarding the correlation between these two soil properties, stochastic independence will be assumed rather than assuming any erroneous correlation. Hence, elastic modulus and soil permeability expressions in terms of the depth $z$, are given by

$$\begin{cases} E(z) = \exp[\mu_{\ln E} + \sigma_{\ln E}\Delta g_E(z)] \\ k(z) = \exp[\mu_{\ln k} + \sigma_{\ln k}\Delta g_k(z)] \end{cases} \tag{15}$$

for which

$$\begin{cases} \sigma^2_{\ln E} = \ln\left(1 + \frac{\sigma_E^2}{\mu_E^2}\right) \\ \mu_{\ln E} = \ln(\mu_E) - \frac{1}{2}\sigma^2_{\ln E} \end{cases} \quad \text{and} \quad \begin{cases} \sigma^2_{\ln k} = \ln\left(1 + \frac{\sigma_k^2}{\mu_k^2}\right) \\ \mu_{\ln k} = \ln(\mu_k) - \frac{1}{2}\sigma^2_{\ln k} \end{cases} \tag{16}$$

where $\mu_E$, $\sigma_E^2$, $\mu_k$ and $\sigma_k^2$ respectively stand for elastic modulus and soil permeability mean and variance. Moreover,

$$\Xi\,(\Delta g_E(z); \Delta g_k(z)) = 0. \tag{17}$$

Here $\Xi$ stands for the mean. The zero mean, unit variance, Gaussian random field $\Delta g(z)$, can be simulated as follows (Shinozuka, 1987; Yamazaki and Shinozuka, 1988):

$$\begin{cases} \Delta g_E(z) = \sqrt{2} \sum_{l=0}^{N_z-1} A_{l,E} \cdot \left[\cos\left(\kappa_{zl} z + \Psi_{l,E}\right)\right] \\ \Delta g_k(z) = \sqrt{2} \sum_{l=0}^{N_z-1} A_{l,k} \cdot \left[\cos\left(\kappa_{zl} z + \Psi_{l,k}\right)\right] \end{cases} \tag{18}$$

with

$$A_{l,E} = \sqrt{2\Delta\kappa_z S_E(\kappa_{zl})} \quad \text{and} \quad A_{l,k} = \sqrt{2\Delta\kappa_z S_k(\kappa_{zl})} \tag{19}$$

$\Psi_l$ is a random phase angle distributed uniformly over the interval $[0, 2\pi]$. $\kappa_z$ is the wavenumber in $z$ direction. It is written as

$$\kappa_{zl} = l\Delta\kappa_z \quad \text{and} \quad \kappa_{zu} = N_z \Delta\kappa_z \tag{20}$$

The wavenumber step $\Delta\kappa_z$ is evaluated from the representation of $S_E(\kappa_z)$ and $S_k(\kappa_z)$ by evaluating the cut-off wavenumber value $\kappa_{zu}$ for $N_z$ increment. Furthermore, the quadrant symmetry of $S_E(\kappa_z)$ and $S_k(\kappa_z)$ with respect to the origin is assumed in Equation (18).

In the purpose of significantly reducing computational time, digital generation of sample functions of Equation (18) is readily performed with the aid of Fast Fourier Transform (FFT) developed by Yamazaki and Shinuzuka (1988) and slightly modified by Zerva (1992). The application of the FFT in Equation (18) provides values for the simulation over half of the wavelength $L_z/2 = \frac{2\pi}{\Delta\kappa_z}$, hence, the simulation should be extended for the entire wavelength, and can be also easily extended to distances longer than the wavelength $L_z$.

### 4.2. COMPUTER IMPLEMENTATION

The above-described procedure is coded in a special-purpose computer program TLMWIN, and simulation strategy is performed at each sublayer center C, whose elastic modulus and soil permeability are taken constant within the sublayer namely, $E(z) = E_C$ and $k(z) = k_C$. Random fields for elastic modulus and soil permeability are generated by using the Monte Carlo simulation method. This method is famous for being the method of statistical experiments and consists of performing a set of probabilistic realizations of the medium used hereunder to predict final settlement and its corresponding time, via deterministic calculation for each realization, and proceeding thereafter to the statistical treatment of the obtained results.

In this study, the power density functions of elastic modulus and soil permeability are set equal i.e. $S_E = S_k = S$. The following Gaussian exponentially decaying form with unit variance is adopted:

$$S(\kappa_z) = \frac{\sigma^2\big|_{\sigma=1} a}{2\sqrt{\pi}} \exp\left(-\left(\frac{\kappa_z a}{2}\right)^2\right) \tag{21}$$

This expression is obtained from the following Wiener–Khinchin expression as follows:

$$S(\kappa_z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R(\xi_z) \exp\left(-\mathrm{i}\kappa_z \xi_z\right) d\xi_z \tag{22}$$

for which

$$R(\xi_z) = \sigma^2\big|_{\sigma=1} \exp\left(-\frac{\xi_z^2}{a^2}\right); \quad \xi_z = z_1 - z_2. \tag{23}$$

Unfortunately, knowledge about variability of the parameters $E$ and $k$ for soils is very poor, and the establishment of a standard expression for the correlation function for soil properties is troublesome as a large number of samples is rarely available. As stated in (Fenton and Griffiths, 1996), since $E$ and $k$ are spatially varying random fields, there will also be a degree of correlation between $E(z_1)$ and $E(z_2)$ ($k(z_1)$ and $k(z_2)$) where $z_1$ and $z_2$ are any two points in the field. So $E$ and $k$ at $z_1$ and $z_2$ will be similar if $z_1$ and $z_2$ are close together. Also, if the two points $z_1$ and $z_2$ are widely separated, less correlation may be expected. Mathematically this concept is captured through the use of a spatial correlation function, which, in this study, is an exponentially decaying function of separating distance $\xi_z = z_1 - z_2$ (Equation (23)), for which the correlation distance $a$ governs the decay rate of correlation between points in the field and can be obtained from the log-data treatment and the non-linear fitting of Equation (23). In this present work, this distance is also supposed to be the same for both soil properties. Also, it is assumed that the mean and the standard deviation of soil properties are constant with depth.

One notes, because the discretization is finer in the vicinity of the drains, the used FTT algorithm for calculating sample functions of Equation (18), produces the values $\Delta g$ at the space coordinates $(z_r)$ which generally does not coincide with the desired layer centroid coordinates of the mesh. Thus, the desired values at the coordinates $z_C$ are obtained using the neighbor Spline interpolation technique in the $\Delta g(z_r)$ field as stated in (Nour *et al.*, 2002).

The use of the FFT to produce realizations must be carried out with caution. The FFT approach gives spurious large lag correlations because it assumes the signal to be periodic. One is well aware of this point. Indeed, in the simulation used in the paper, the number of increments used in Equation (18) is determined in such a manner that all field points could be represented by less than one wavelength.

## 5. Application to Primary Consolidation of Random Heterogeneous Soil Profile

### 5.1. PRELIMINARIES

In this section, we analyze by using the above procedure, primary consolidation variability of an heterogeneous soil profile. The one-dimensional aspect of the problem is assumed, therefore, spatial variability of the mechanical soil properties is considered in the $z$ direction only. With respect to $(x - y)$ plane, it is assumed that soil properties are invariant, interpreted as average over the plane or as having an infinite length of correlation. So, one is dealing with the probabilistic investigation of 1D primary consolidation using the technique of application of stochastic input soil parameters into deterministic numerical analysis (Elkateb *et al.*, 2003). In this framework, the TLM is chosen as the deterministic numerical tool, and is adapted herein for solving 1D primary consolidation problems. This technique is more attractive for 1D problems than FEM, because the TLM is a semi-analytical one and does not require performing time discretization in the formulation. This is not the case for FEM, where one has to be very careful regarding stability and accuracy of the selected time marching algorithm. Hence, adaptation of TLM to consolidation problems serves as a numerical deterministic tool for determining consolidation statistics via Monte Carlo simulations, allowing hence the analysis of moderate to highly heterogeneous media.

For the sake of illustration, let us consider a 8 m saturated soil profile, loaded with an uniformly distributed pressure $q = 0.1$ MPa applied on the top. The following data are used (Nasri and Magnan, 1997): mean soil elastic modulus $\mu_E = 20$ MPa, Poisson's ratio $\upsilon = 0$, mean soil permeability $\mu_k = 10^{-9}$ m/s, unit water weight $\gamma_w = 0.01$ MN/m$^3$ and vertical correlation distance $a = 0.5$ m. These data correspond to a coefficient of consolidation $C_v = 2 \times 10^{-6}$ m$^2$/s.

The TLM model used in the analysis, boundary conditions and position of the applied loads are shown in Figure 3. For the problem under consideration, two situations often encountered in practice are investigated: model with single drainage (SD) and model with double drainage (DD). It is obvious that time corresponding to 100% of the degree of consolidation is equal to infinity; however, there is no standard formulation in the literature allowing for the determination of that time. In this study, the time cor-

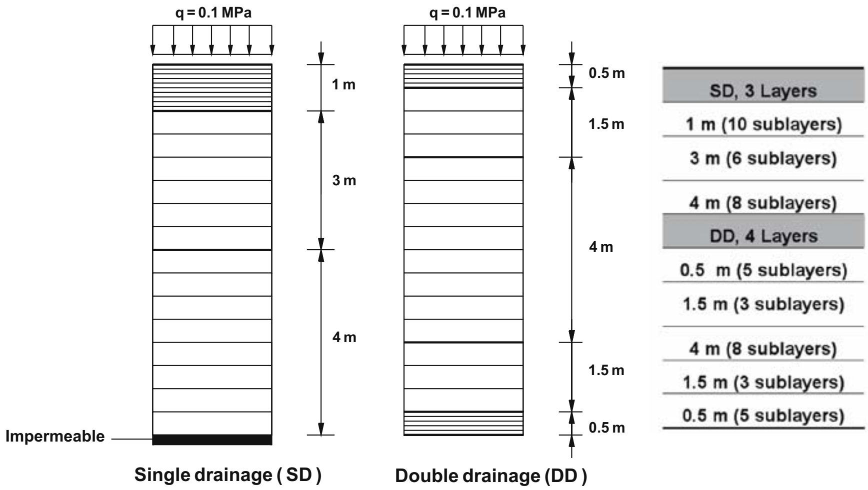



*Figure 3.* Thin layer Model.

responding to final settlement is determined in the process of time marching, $t_i = t_{i-1} + \Delta t$; in such a manner that $|u_i - u_{i-1}| < \varepsilon$ with $\varepsilon = 10^{-16}$, for which $u_i$ stands for settlement at the time station $i$.

One notes that different layers and sublayers are used in the analysis for SD and DD models. It is important to ensure that the vertical correlation length should be captured by at least one sublayer of the TLM mesh. Furthermore, in order to get an accurate solution with the TLM model, each layer is subdivided into several sublayers. The number of layers and sublayers used in the analyses are well depicted by Figure 3. For 1D consolidation problem at time $t = 0$, the pore water pressure $u_e$ is equal to the applied load at all points except at the permeable boundaries. This phenomenon could be captured by well refining the mesh near these boundaries. For this reason, as shown by Figure 3 the size of the sublayers for both models (SD and DD) is equal to 0.1 m near the boundaries and 0.5 m elsewhere.

### 5.2. VALIDATION OF TLM SOLUTION WITH TERZAGHI'S THEORY

It is aimed throughout this section, to demonstrate the validity of the TLM in solving one-dimensional primary consolidation problem. This technique is well-suited for a multi-layered soil profile with layers extending horizontally over a length several times greater than the soil profile thickness. The results obtained from TLM are compared to the analytical solution given by Terzaghi's theory (Kézdi, 1974). When using the TLM, one finds that the average degree of consolidation is governed by Equation (24)

$$U = 1 - \frac{M^T \phi\, \Omega \phi^T M}{H}. \tag{24}$$

The settlement at ground surface is given as

$$W = M^T E_L (2q\, I_1 - U_E) \tag{25}$$

for which $I_1$ stands for unit vector of the same dimension as $M$, and the submatrice $E_L$ is given by Equation (26)

$$E_L = \frac{1}{E_i} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \tag{26}$$

Figure 4 shows, homogeneous (mean values) single layer results regarding consolidation degree, final settlement and interstitial pressure for $T_v = 0.050$, $T_v = 0.195$ and $T_v = 0.848$, where $T_v$ is the time factor (Biot, 1941; Kézdi, 1974).

Thus, results obtained from TLM, are in good agreement with Terzaghi's solutions for both cases of double and single drainage. This demonstrates the validity of TLM in solving one dimensional primary consolidation problems.

### 5.3. RESULTS AND ANALYSIS

In this section, the statistics regarding final settlement and its corresponding time will be analyzed by performing a parametric study. Although it is reported in the literature that the range of variability of the elastic modulus is around $CV_E \approx 0.4 - 0.5$, it may reach greater values (Becker, 1996). It is intended in this paper to investigate the variability of the elastic modulus from 0 to 2. Regarding soil permeability, to the authors knowledge, there is practically no information about its variability range. One notes that Fenton and Griffiths (1995, 1996) varied $CV_k$ from 0.1 to 8. In this paper, soil permeability variation coefficient $CV_k$ is also varied from 0 to 2. It is often considered in soil mechanics that in the same soil the permeability and the compressibility are strongly correlated. Thus a small permeability corresponds to a small compressibility.

It is worth noting that consolidation statistics are evaluated from the study of 1000 simulated samples. After completing 1000 realizations, 1000 values for final settlement and its corresponding time are computed. In order to determine the statistics, as shown by Figure 5, histograms are plotted corresponding to single and double drainage cases. The shape of the histograms suggests a lognormal distribution which is adopted in this study. Superimposed on the histogram, is the fitted lognormal distribution with parameters given by $m_{\ln W}$, $S_{\ln W}$.

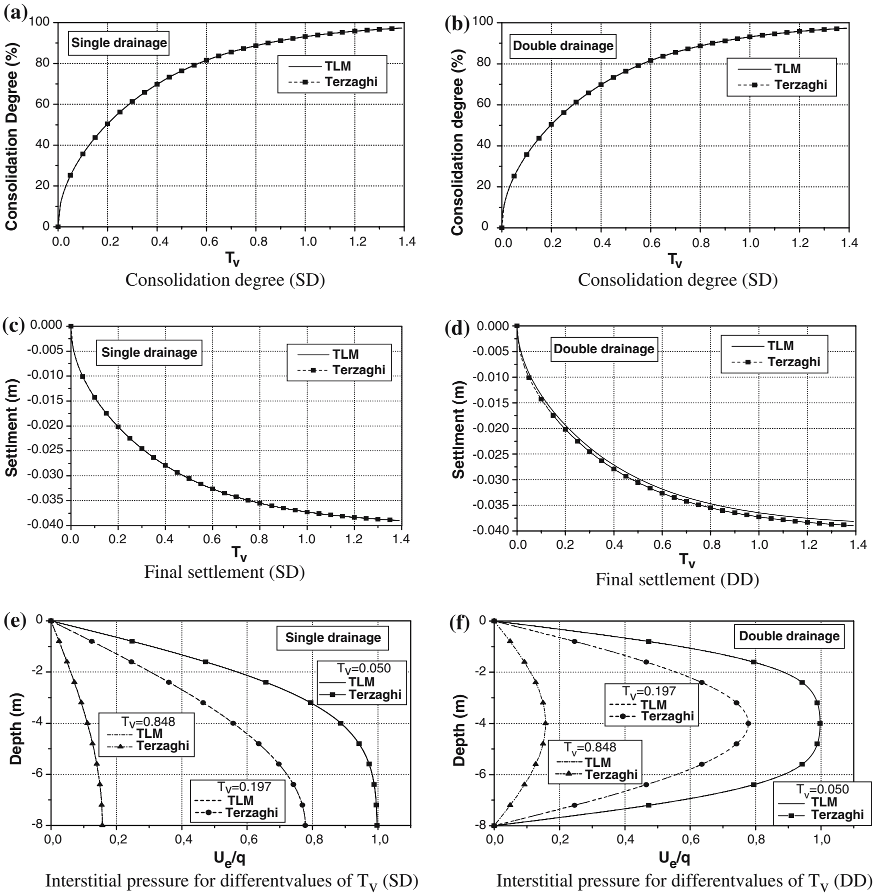



*Figure 4.* (a) Consolidation degree (SD). (b) Consolidation degree (DD). (c) Final settlement (SD). (d) Final settlement (DD). (e) Interstitial pressure for different values of $T_v$ (SD). (f) Interstitial pressure for different values of $T_v$ (DD). Comparison of TLM solution with Terzaghi's theory.

However, the parameters of the fitted distribution are estimated by the method of moments from the ensemble of realizations. Figure 5 shows that the fit appears reasonable. The Chi-Square goodness (Ang and Tang, 1975) of fit test is used to evaluate the fit of the assumed lognormal distribution. Therefore, Figure 6 is plotted based on the entire simulations done in this paper, and illustrates the probability of rejecting the lognormal distribution versus the number of simulations in (%). Figures 6(a) and 6(b) indicate that the hypothesis of lognormal distribution is a reasonable one for the settlement (SD and DD). Regarding the final settlement time (SD and DD), as shown by Figures 6(c) and 6(d), and despite the fact that the test is not successful for all histograms, the lognormal distribution is adopted as it captures the major trends in the histogram.

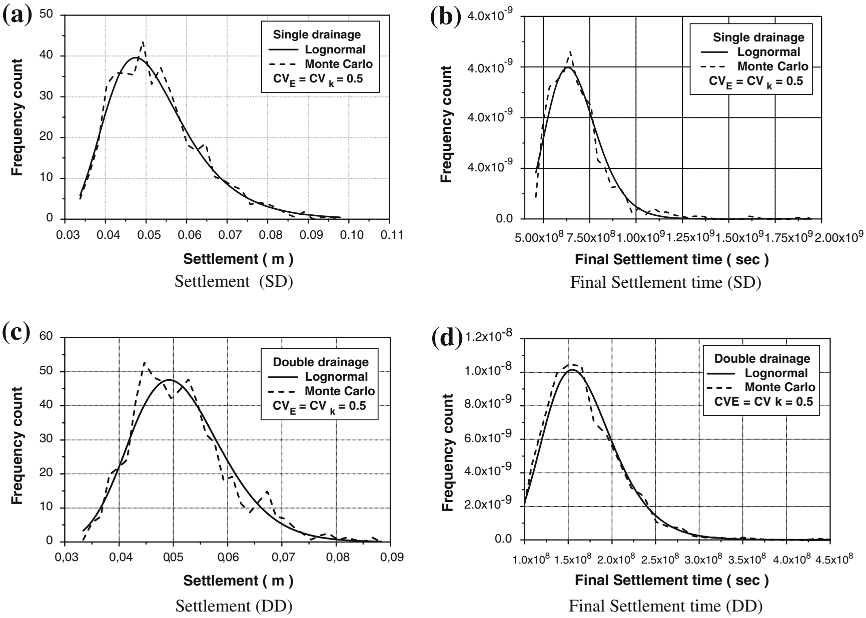



*Figure 5.* (a) Settlement (SD). (b) Final Settlement time (SD). (c) Settlement (DD). (d) Final Settlement time (DD). Typical frequency histogram and fitted lognormal distribution.

Once the parameters $m_{\ln \mathrm{W}}$ and $S_{\ln \mathrm{W}}$ are determined from the fitted log-normal distribution, the statistics are obtained by using the following formulas:

$$m_{\mathrm{W}} = \exp\left(m_{\ln \mathrm{W}} + \frac{1}{2} S^2_{\ln(\mathrm{W})}\right); \quad S_{\mathrm{W}} = \sqrt{m^2_{\mathrm{W}} \left\{\exp\left(S^2_{\ln(\mathrm{W})}\right) - 1\right\}} \tag{27}$$

### 5.3.1. *Influence of Variation Coefficient*

In this section, the influence of the variation coefficient of both, elastic modulus and soil permeability on consolidation statistics is investigated. To do so, $\mathrm{CV}_E$ and $\mathrm{CV}_k$ are varied from 0 to 2.

Figure 7 shows typical realization curves of consolidation degree, final settlement and interstitial pressure corresponding to $T_{\mathrm{v}} = 0.03125$ for single and $T_{\mathrm{v}} = 0.12500$ for double drainage cases (both cases refer to the same time of $10^6$ s). One observes that the heterogeneity generates a different way of soil grain rearrangement and water pressure dissipation in comparison to the homogeneous case. Moreover, another realization gives a quite different curve of degree of consolidation, final settlement and interstitial pressure. One notes that the curve based on Terzaghi equation is plotted with the mean values ($E(z) = \mu_E = 20$ MPa and $k(z) = \mu_k = 10^{-9}$ m/s).

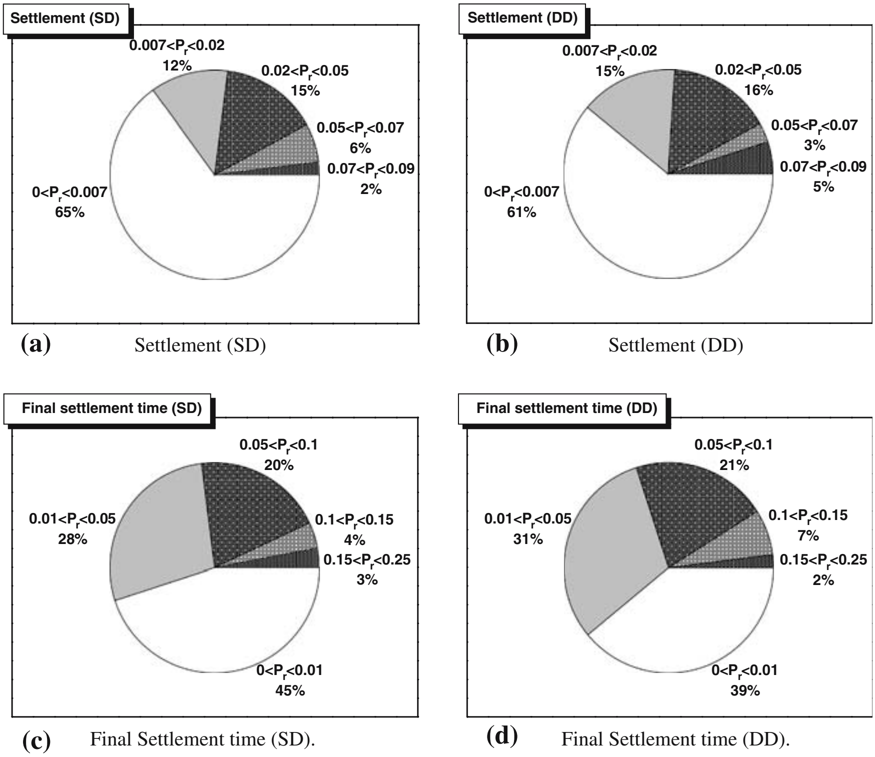



*Figure 6.* (a) Settlement (SD). (b) Settlement (DD). (c) Final Settlement time (SD). (d) Final Settlement time (DD). Diagram showing the probability of rejecting the lognormal distribution.

Figure 8 depicts final settlement statistics versus elastic modulus and soil permeability variabilities for both, single and double drainage cases. One notes that final settlement statistics are estimated when the interstitial pressure tends to zero. One observes that whatever the variability of soil permeability is, as the coefficient of variation of elastic modulus $\mathrm{CV}_E$ increases, the induced settlement increases too. That is an indication that the simulated soil medium becomes softer. Because final settlement statistics depend on long term mechanical soil properties, the statistics are independent from the variation coefficient of soil permeability $\mathrm{CV}_k$.

Also, one sees that statistics are slightly greater for the case of single drainage than they are for the case of double drainage. This result is in accordance with Equation (25) (with $U_E = 0$), as the difference in final settlement between these two cases depends only on the lower layer (lower position) characteristics, and is equal to

$$W_{\mathrm{SD}} - W_{\mathrm{DD}} = \frac{q\,h_n}{E_n} \tag{28}$$

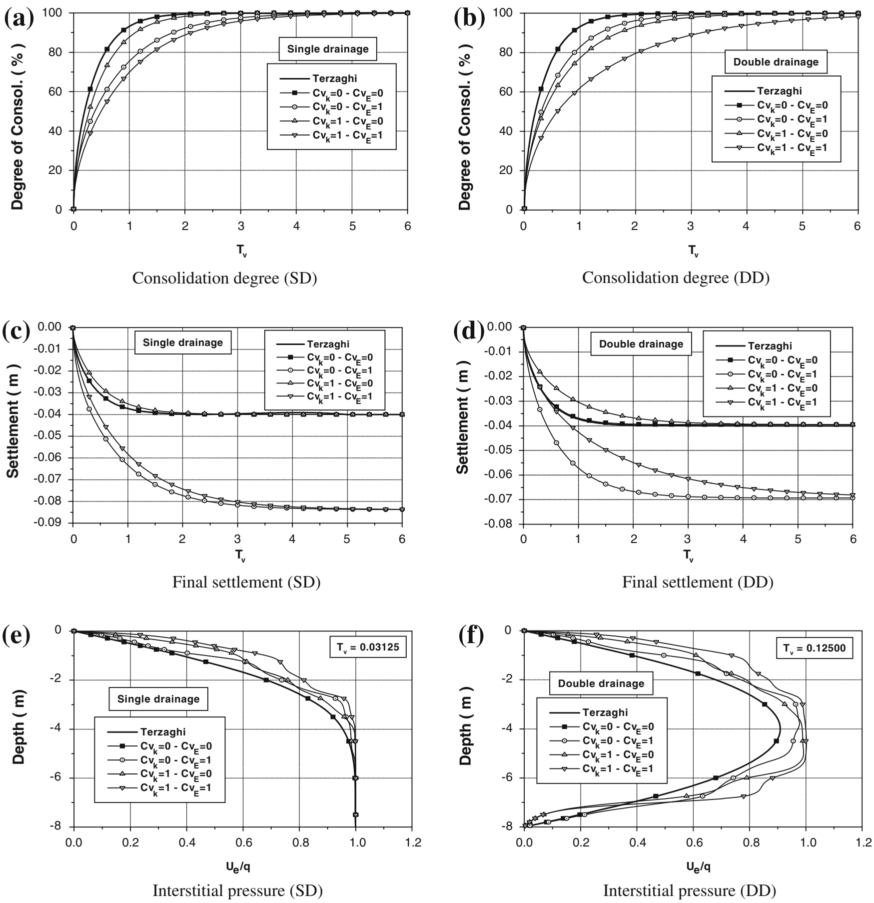



*Figure 7.* (a) Consolidation degree (SD). (b) Consolidation degree (DD). (c) Final settlement (SD). (d) Final settlement (DD). (e) Interstitial pressure (SD). (f) Interstitial pressure (DD). Typical realizations curves.

for which $h_n$ and $E_n$ stand for the lower layer height and its corresponding elastic modulus, respectively.

Figures 9 and 10 show statistics of time corresponding to the final settlement, interpreted as time of achievement of primary consolidation, for both single and double drainage cases. One observes that as the coefficients of variation of both elastic modulus $\mathrm{CV}_E$ and soil permeability $\mathrm{CV}_k$ increase, final settlement time increases too. This indicates that heterogeneity causes a delay in consolidation process. This can be interpreted also by the fact that in 1D seepage analysis, the effective permeability is given by the harmonic average, which is always governed by the lowest permeability zones. Furthermore, results obtained show that for large values of $\mathrm{CV}_E$ and $\mathrm{CV}_k$, corresponding to highly heterogeneous medium, the consolidation

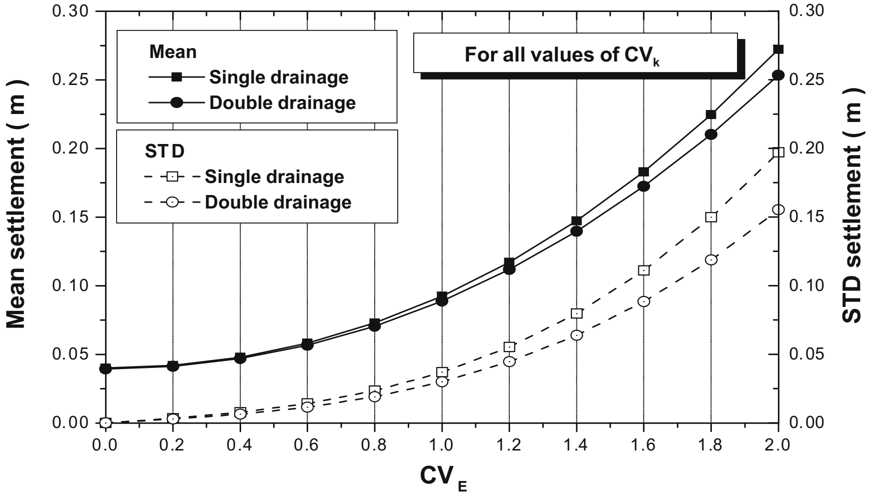
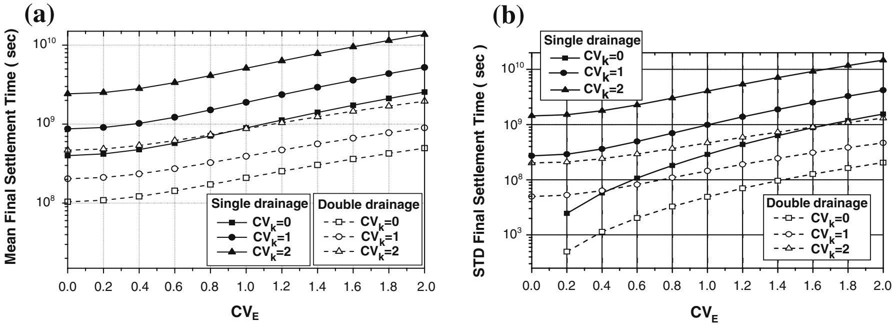



*Figure 8.* Final settlement variability versus $CV_E$.




Mean Standard deviation (STD)

*Figure 9.* (a) Mean. (b) Standard deviation (S.D). Final settlement time variability versus $CV_E$.

process takes a very long time before its achievement. Also, it is found that consolidation process is more accelerated for the double drainage case, because water is evacuated from the top and the bottom boundaries which takes less time than if water is evacuated from one boundary. Furthermore, it is observed that shape of curves of Figures 9 and 10 is similar, because time corresponding to final settlement depends on the coefficient of consolidation ($C_v$) which is in bilinear relation with $E$ and $k$ (Equation (4)).

### 5.3.2. *Vertical Correlation Length Influence*

This section deals with correlation length influence on consolidation statistics. When $a \to 0$ as shown by Figure 11, the power spectral density

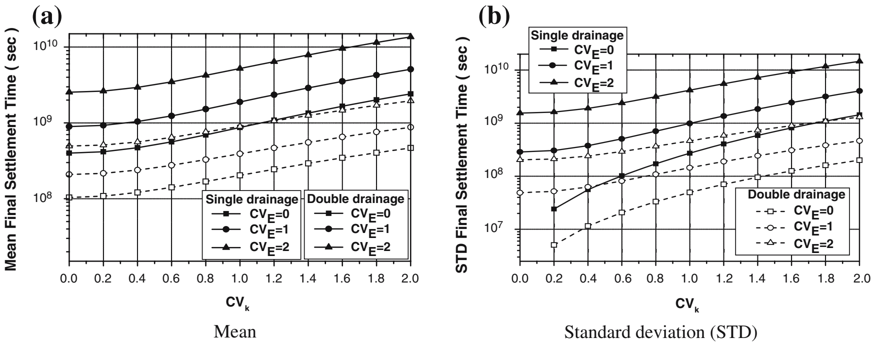



*Figure 10.* (a) Mean. (b) Standard deviation (S.D). Final settlement time variability versus $CV_k$.

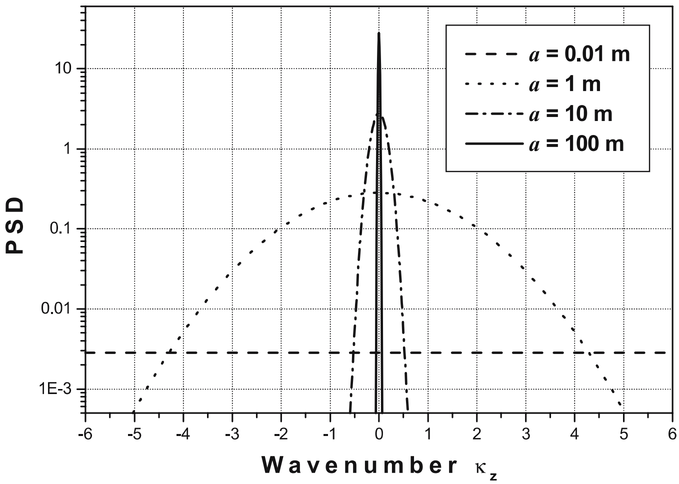



*Figure 11.* Power spectral density function versus $\kappa_z$.

function is defined in a broad band wavenumber range, the wavenumber step becomes very important and approaches infinity, hence wavelength of the simulated field tends to zero according to vertical direction, whereas elastic modulus and soil permeability become white noise fields, with $E$ and $k$ values at any two distinct independent points, a physically unrealizable situation. In fact, correlation lengths less than size of laboratory samples used to estimate elastic modulus and soil permeability have little meaning.

They are interpreted either as variability which is smaller than measurement scale, or as related to measurement errors. In this situation one deals with vanishing consolidation variance and consolidation is expected to be as obtained in the deterministic case, with elastic modulus and soil permeability equal to $\mu_E$ and $\mu_k$ everywhere. When $a \to \infty$ as shown by Figure 11, the power spectral density function is defined in a very

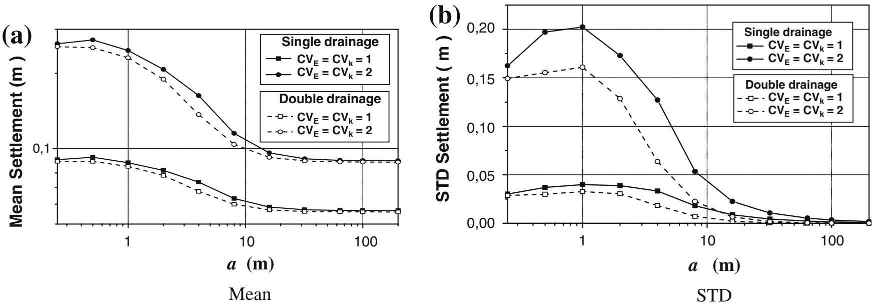



*Figure 12.* (a) Mean. (b) S.D. Final settlement versus vertical correlation length.

narrow band wavenumber range (Dirac), the wavenumber step becomes very small and approaches zero, hence wavelength of the simulated field tends to infinity according to vertical direction, and all points on the field are completely correlated, which corresponds to a homogenization of the simulated medium. In this situation, elastic modulus and soil permeability tend respectively to $\mu_E/\sqrt{1+\mathrm{CV}_E^2}$ and $\mu_k/\sqrt{1+\mathrm{CV}_k^2}$ everywhere (Nour *et al.*, 2002) but each realization will be different.

As shown in Figure 12, the vertical correlation length exerts a great influence on final settlement statistics, for both cases single and double drainage. One observes for relatively small correlation lengths, important final settlement values in comparison to the large ones, whereas, for large correlation length, the mean final settlement is independent from the vertical correlation length, and the standard deviation tends to zero. This is also valid for the difference in settlement between single and double drainage cases.

Figure 13 illustrates final settlement time statistics versus vertical correlation length, for both cases, single and double drainage. Results obtained indicate that consolidation process is very slow for relatively small values of the vertical correlation length; whereas for large correlation lengths, this process is faster and the mean of final settlement time becomes independent from the vertical correlation length. Furthermore, the corresponding standard deviation decreases significantly and tends to zero when $a \to \infty$.

## 6. Conclusions

The present paper emphasizes the following points: (i) adaptation of TLM for solving 1D primary consolidation and its validation with Terzaghi's solution (ii) combination of TLM with a stochastic formulation integrating Monte Carlo simulations to investigate one-dimensional primary consolidation of a random heterogeneous soil profile (iii) a parametric study which develops statistics of final settlement and its corresponding time through

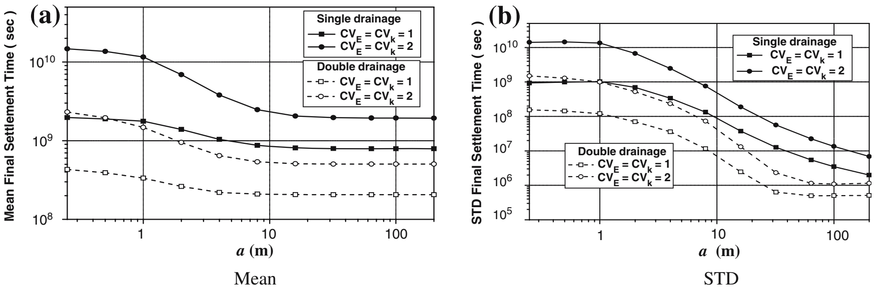



*Figure 13.* (a) Mean. (b) S.D. Final settlement time variability versus correlation length.

a comparative study of two models in relation with two situations often encountered in practice, model with single drainage (SD) and model with double drainage (DD).

Regarding the first point, the TLM seems to be more attractive than FEM for solving 1D consolidation problems. In our case, the TLM serves as a stable numerical deterministic tool for determining consolidation statistics via Monte Carlo simulations. It is found that TLM results are in good agreement with Terzaghi's solutions for both cases of double and single drainage.

Regarding the second point, primary consolidation of a random heterogeneous soil profile is analyzed. Soil properties of interest are elastic modulus and soil permeability, modeled herein as independent spatially random fields. These properties are obtained by adopting a lognormal distribution, which enables analyzing their large variability. Thus, the statistics regarding final settlement and the corresponding time are evaluated using Monte Carlo simulations combined with deterministic Thin Layer Method. So, consolidation statistics are reasonably well represented by the proposed simulation technique. Obtained results for single and double drainage models indicate that heterogeneity significantly influences consolidation of soil profile, generating a quite different way of soil grain rearrangement and water pressure dissipation in comparison to the homogeneous case, and causing a delay in consolidation process.

Regarding the third point, the performed parametric study shows that whatever the variability of soil permeability is, as the coefficient of variation of elastic modulus increases, the induced settlement increases too, which means that the simulated soil medium becomes softer. Also, as the coefficients of variation of both elastic modulus and soil permeability increase, final settlement time increases too. Furthermore, results obtained show that for highly heterogeneous medium, the consolidation process takes a very long time before its achievement. On the other hand, the vertical correlation length has a great influence on the final settlement statistics, for both cases single and double drainage. So, one notes for relatively small correlation lengths, important final settlement values, whereas, for large correlation length, mean final settlement is independent from the vertical correlation length, and the standard deviation tends to zero. Also, results obtained indicate that the consolidation process is very slow for relatively small values of the vertical correlation length; whereas for large correlation lengths, this process is faster and the mean final settlement time becomes independent of the vertical correlation length. Furthermore, the corresponding standard deviation tends to zero.

Despite the model used is this paper is simple, as 1D problem is adopted, but it provides guidance for sophisticated models to achieve more realistic modeling of soil media.

## References

Ang, A. H. S. and Tang, W. H.: 1975, *Probability Concepts in Engineering Planning and Design*, John Wiley & Sons.

Becker, E. D.: 1996, An overview of the foundation design process (Part I). Development for the national building code of Canada (Part II), *Can. Geotech. J.* **33**, 956–1007.

Biot, M. A.: 1941, General theory of three-dimensional consolidation, *J. Appl. Phys.* **12**(2), 155–164.

Bougacha, S., Tassoulas, J. L. and Roësset, J. M.: 1993, Analysis of foundations on fluid-filled poroelastic stratum, *J. Eng. Mech. ASCE* **119**(8), 1632–1648.

Bougacha, S., Roësset, J. M. and Tassoulas, J. L.: 1993, Dynamic stiffness of foundations fluid-filled poroelastic stratum, *J. Eng. Mech. ASCE* **119**(8), 1649–1662.

Desai, C. S. and Christian, J. T.: 1977, *Numerical Methods in Geotechnical Engineering*, McGraw-Hill, New York, N.Y.

Elkated, T., Chalaturnyk, R. and Robertson, P. K.: 2003, An overview of soil heterogeneity: quantification and implication on geotechnical filed problems, *Can. Geotech. J.* **40**, 1–15.

Fenton, G. A.: 1990, Simulation and analysis of random fields, PhD thesis, Princeton University.

Fenton, G. A. and Griffiths, D. V.: 1995, Flow through earthdams with spatially random permeability, in: *Proceedings of the 10th ASCE Engineering Mechanics Conference*, Boulder, Colorado.

Fenton, G. A. and Griffiths, D. V.: 1996, Statistics of free surface flow through stochastic earth dam, *J. Geotech. Eng. ASCE* **122**(6), 427–436.

Fenton, G. A. and Griffiths, D. V.: 2002, Probabilistic foundation settlement on spatially random soil, *J. Geotech. Geoenviron. Eng. ASCE* **128**(5), 381–390.

Gibson, R. E., England, G. L. and Hussey, M. J. L.: 1967, The theory of one dimensional consolidation of saturated clays, *Géotechnique* **17**, 261–273.

Kausel, E. and Roësset, J. M.: 1981, Stiffness matrices for layered soils, *Bull. Seism. Soc. Am.* **71**(6), 1743–1761.

Kausel, E. and Peek, R.: 1982, Dynamic loads in the interior of a layered stratum: An explicit solution, *Bull. Seism. Soc. Am.* **72**(5), 1459–1481.

Kausel, E.: 1986, Wave propagation in anisotropic layered media, *Int. J. Numer. Meth. Eng.* **23**, 1567–1578.

Kausel, E. and Seale, S. H.: 1987, Static loads in layered halfspaces, *J. Appl. Mech. ASME* **54**(2), 403–408.

Kausel, E.: 1994, Thin layer method: Formulation in the time domain, *Int. J. Numer. Meth. Eng*. **37**, 927–941.

Kézdi, A.: 1974, *Soil physics, Handbook of Soil Mechanics*, Vol. 1, Elsevier.

Koppula, S. D.: 1988, Pore pressure development: Probability analysis, in: P. D. Spanos, (ed.), *Probabilistic Methods in Civil Engineering*, ASCE Publication, New York, pp. 57–60.

Nasri, V. and Magnan, J. P.: 1997, Comportement numérique des éléments finis dans l'analyse des problèmes de consolidation des sols, *Bull. Liaison Ponts et Chaussées*, Paris, France, **210**, 41–53.

Nogami, T. and Kazama, M.: 1992, Dynamic response analysis of submerged soil by Thin Layer Element Method, *Soil Dyn. Earthquake Eng*. **11**, 17–26.

Nour, A., Slimani, A. and Laouami, N.: 2002, Foundation settlement statistics via finite element analysis, *J. Comp. Geotechnics* **29**(8), 641–672.

Seale, S. H.: 1985, Dynamic loads in layered halfspaces, PhD thesis, Department of Civil Engineering, MIT, Cambridge, Mass.

Seale, S. H. and Kausel, E.: 1989, Point loads in cross-anisotropic layered halfspaces, *J. Eng. Mech. ASCE* **115**(31), 509–524.

Shinozuka, M.: 1987, Stochastic fields and their digital simulation, in: G. I. Schueller and M. Shinozuka, (eds.), *Stochastic Methods in Structural Dynamics*, Martinus Nijhoff Publishers, Boston, Mass., pp. 93–133.

Vanmarcke, E. H.: 1984, *Random Fields: Analysis and Synthesis*, The MIT Press, Cambridge, Mass.

Yamazaki, F. and Shinozuka, M.: 1988, Digital generation of non-Gaussian stochastic field, *J. Eng. Mech. ASCE* **114**(7), 1183–1197.

Zerva, A.: 1992, Seismic ground motion simulations from a class of spatial variability model, *Earthquake Eng. Struct. Dyn*. **21**, 351–361.

Zhuang, Y. C., Xie, K. H. and Li, X. B.: 2005, Nonlinear analysis of consolidation with variable compressibility and permeability, *J. Zhejiang Univ. Sc*. **6A**(3), 181–187.