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Rational choices: an ecological approach

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Abstract

We address the oft-repeated criticism that the demands which the rational choice approach makes on the knowledge and cognition of a decision-maker (DM) are way beyond the capabilities of typical human intelligence. Our key finding is that it may be possible to arrive at this ideal of rationality by means of cognitively less demanding, heuristic-based ecological reasoning that draws on information about others' choices in the DM's environment. Formally, we propose a choice procedure under which, in any choice problem, the DM, first, uses this information to shortlist a set of alternatives. The DM does this shortlisting by a mental process of categorization, whereby she draws similarities with certain societal members—the ingroup—and distinctions from others—the outgroup—and considers those alternatives that are similar (dissimilar) to ingroup (outgroup) members' choices. Then, she chooses from this shortlisted set by applying her preferences, which may be incomplete owing to limitations of knowledge. We show that, if a certain homophily condition connecting the DM's preferences with her ingroup–outgroup categorization holds, then the procedure never leads the DM to making bad choices. If, in addition, a certain shortlisting consistency condition holds vis-a-vis non-comparable alternatives under the DM's preferences, then the procedure results in rational choices.

Keywords Rational choice · Ecological rationality · Ingroup–outgroup categorization · Fast and frugal heuristics · Homophily

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1 Introduction

In neo-classical economics, to say that a decision-maker (DM) is rational entails that this DM has complete and transitive preferences over the set of relevant alternatives and, in any choice problem, chooses that alternative that is best according to this preference relation.[1] Since preferences are not directly observable, in terms of its empirical content, this perspective of rationality takes the DM's choices as the primitive concept. Within the revealed preference tradition, the key theoretical step is to ask whether it is possible to back out a DM's rational preferences from her observed choices. That is, to ask if it is possible to think of this DM's choices as if they were the result of maximizing some such underlying preferences. A classic result in choice theory establishes that this is essentially the case when the DM's choices are internally consistent. In other words, rationality in economics boils down to internal consistency of choice. A related observation that follows is that, within this paradigm, the focus is on what the DM chooses rather than on how she arrives at such choices in terms of the reasoning involved.

As is well known, this standard model of rationality in economics has received its fair share of criticism, and here, we would like to re-visit certain aspects of that criticism. One of the earliest and most influential criticisms of this worldview came from Herbert Simon (Simon 1955, 1956). One of the key ways in which Simon's bounded rationality perspective differed from the standard model was in terms of its focus on not just the outcome of choice—substantive rationality—but also on the decision-making procedure by which such choices are arrived at—procedural rationality. What Simon pointed out was that, when viewed from this procedural perspective, the standard model of rational choice makes way too stringent demands on a DM's knowledge and cognitive-computational capacities—demands that typical human intelligence would find overwhelming. In this paper, we want to take this criticism of the standard model that it is blind to the decision-making procedure and the cognitive constraints faced therein seriously. Our broad goal is to see if it is possible for a DM to arrive at the stringent ideal of rationality set by the standard model by means of simple and cognitively less demanding heuristic-based reasoning that draws on information regarding choices that may be available in her environment.

Specifically, we identify and engage with two types of challenges that the decision-making process may involve. First, we consider the possibility that, owing to limitations of knowledge, a DM's preferences may not be complete and she may be unable to rank every pair of alternatives. Specifically, we consider a DM who has asymmetric and transitive preferences that are not necessarily complete. If that is so, then it may not always be possible for the DM to determine the best alternative in a given choice problem. That such incompleteness of preferences is not merely a theoretical curiosity, but, rather, a real possibility has been acknowledged going at least

[1] In this paper, we restrict attention to strict preferences and unique choices (choice functions). As such, when we say that preferences are complete, we mean that, for any two distinct alternatives, x and y, the DM either strictly prefers x to y or y to x.

as far back as the seminal paper by Robert Aumann (Aumann 1962).[2] Second, we engage with the possibility that even if a DM's preferences are complete and transitive in a particular choice problem and, therefore, (in the context of a finite set of alternatives), a best alternative theoretically exists, it may not always be possible for the DM to figure out what this best alternative is owing to cognitive limitations. This may be particularly so if the choice problem under consideration involves a large set of alternatives.

In this paper, we take these two challenges that real world decision-making may involve as our motivation and propose a choice procedure that embodies the spirit of how a DM may go about meeting such challenges. Our procedure appeals to the idea of ecological rationality, which refers to decision-making processes that exploit the structure of information in a given environment to arrive at efficacious choices (Todd and Gigerenzer 2012). Specifically, the choice procedure that we propose captures the behavior of a DM who uses information about the choices that others in society make in similar situations as she faces to simplify her decision-making process. To fix ideas, think of the DM as a new entrant to a society whose entrenched individuals are all rational in the traditional sense. We assume that the choice problems that this DM faces have been previously faced by the entrenched individuals of this society and information about their choices on these problems is available to her. Under our procedure, in any choice problem, the DM uses others' choice data on that problem to create a "small" shortlist of alternatives to consider. From a cognitive and decision-making perspective, this set of shortlisted alternatives is a much simpler and more effective object for her to consider because it is a smaller set and this smallness is produced by, presumably, drawing on useful information from others' choices. It is to this shortlisted set that she applies her preferences to choose an alternative. Arguably, this is a much simpler task cognitively than choosing an alternative from the original set. In other words, the choice procedure that we are proposing is a two-stage procedure under which, in the first stage, the DM uses information about others' choices to create a shortlist of alternatives and, in the second stage, chooses from this set based on her preferences. We call this choice procedure the ecological shortlist heuristic (ESH).

The key ingredient of the ESH, of course, is in the way it uses the choice data of others to create a set of shortlisted alternatives in any choice problem. The question that bears answering, therefore, is about the psychological and cognitive underpinnings that determine the way in which the DM does this shortlisting. Our answer to this question draws on a very robust idea from social psychology—that groups exert a constitutive psychological influence on individual attitudes and behavior. Specifically, in any given social situation, it appears that the human mind is hardwired to organize the others of her social world within an ingroup–outgroup division. The ingroup consists of those individuals that, for instance, she likes, relates to and identifies with, thinks of as her kin or friends, etc. On the other hand, the outgroup consists of those that are outside this circle of identification and connection. When it comes to behavior and attitudes, the outcome of such hardwiring is to imitate that of those in the ingroup and differentiate from that of those in the outgroup. It is as if the DM's inner cognition tells her, "You

[2] In this paper, Aumann writes: "Of all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others of the axioms, it is inaccurate as a description of real life; but unlike them, we find it hard to accept even from the normative viewpoint" (Aumann 1962, p.446).

are (not) like them and you will (not) like things that they like." The shortlisting that the DM does under the ESH choice procedure imbibes this social psychology. Specifically, in any choice problem, she shortlists those alternatives that she thinks of as being "similar" to the choices of members of her ingroup and "different" from those of members of her outgroup, with the procedure making precise the way in which this similarity and difference work.

Much work in social psychology has talked about such hardwiring of group attitudes. For instance, one prominent line of research is the self-categorization theory (SCT) of Turner (1985). Here, self-categorization refers to an individual's cognitive representation of herself with respect to salient social categories, i.e., as similar to certain categories—her ingroup—and as distinct from others—her outgroup. The central hypothesis of SCT is how the salience of such ingroup–outgroup categorization produces a certain depersonalization of the self, whereby an individual stereotypes herself as a representative exemplar or prototype of a social category and views the world from the perspective of such a categorization.[3] Indeed, our formulation of the ecological shortlisting process draws inspiration from the meta contrast principle of social psychology which argues that cognitively salient categories form in a way that maximizes intragroup similarities and intergroup differences (Hornsey 2008).

Our analysis of the ESH focusses on two key questions. First, we delve into the question of how efficacious is the ESH in terms of leading the DM towards good choices. After all, the cognitive simplicity afforded by the ESH will not be worth much if it consistently leads the DM towards inferior choices. Specifically, think of a choice problem in which by the DM's preferences, a best alternative actually does exist. How effective is the ESH in picking out this alternative? We show that the ESH can be very efficacious in this regard if a certain homophily-type condition that connects the DM's preferences with her ingroup–outgroup categorization holds. If this condition holds, then the ESH is guaranteed to pick the best alternative in a choice problem, provided such a best alternative exists as per the DM's preferences. At the same time, we point out that if the DM's ingroup–outgroup categorization does not respect this homophily condition, then the ESH can lead her to make biased choices that contradict her preferences. Second, we look at the question of when it is the case that the ESH results in rational choices (in the traditional economic sense) on the part of the DM. We show that if, along with the homophily condition, a certain consistency condition applies to the ecological shortlisting process, then the DM's choices following the ESH can, indeed, be rational. Therefore, when both these conditions hold, the DM's choices under the ESH can be rationalized by a complete and transitive preference ranking, which, in

[3] In making this case, social psychologists have assembled an impressive body of evidence. For instance, the minimal group paradigm studies by Tajfel et al. (1971) clearly show that even the meaningless categorization of lab subjects on the basis of trivial and random criteria such as, for instance, the result of a coin-flip led to ingroup favoritism and discrimination against the outgroup. In many of these experiments, subjects were asked to anonymously divide a fixed sum of money between a member from their ingroup and one from their outgroup, who were, in turn, anonymous except for their group membership. In such settings, subjects chose to allocate as much as 70% of the money to the ingroup member. These findings have been replicated in several other settings, as well. In the context of the experimental economics literature, Chen and Li (2009), for instance, show that a randomly assigned group identity amplifies social preferences by inducing "ingroup altruism" and "outgroup envy".

turn, is a completion of the incomplete primitive preferences that the DM starts off with.

Our paper relates to the fast and frugal heuristics program of Gigerenzer et al. (1999). Their central hypothesis there is that, from an operational point of view, rationality is about the use of fast and frugal heuristics through which smart inferences can be made. Furthermore, these heuristics are successful to the degree that they are ecologically rational. The ESH that we develop here is a type of fast and frugal heuristic. Where our work differs from their's is in the contention they make that there is a radical disconnect between this and the rational choice approach. The results that we derive for the ESH in this paper seem to suggest that this need not necessarily be true as the ecological rationality embedded in the ESH can be a fast and frugal way to lead the DM towards the rational choice benchmark. The spirit of this observation connects our work to that of Mandler et al. (2012). They show how the fast and frugal heuristic of making choices by proceeding sequentially using a checklist of desirable properties is essentially equivalent to the utility maximization paradigm.

Our paper also relates to the literature on the theories of behavioral choice. Like many of the papers in this area, ours too explicitly spells out a psychologically motivated sequential choice procedure under which a cognitive phenomenon—that of ecological shortlisting—constrains the available set of alternatives and it is from this constrained set that the DM chooses. In terms of this structure, our model bears resemblance to Manzini and Mariotti (2007), Manzini and Mariotti (2012), Masatlioglu et al. (2012), Cherepanov et al. (2013), and Lleras et al. (2017), amongst others. Of particular interest to us in this context is the paper by Cuhadaroglu (2017), who models social influence within the behavioral choice paradigm.[4] One way in which our ESH procedure differs from the choice procedure she introduces is in that, under her procedure, the DM first applies her preferences to the set of available alternatives and only consults other individuals' preferences if her preferences are not decisive. In the ESH, on the other hand, reference to others' preferences comes at the first stage, and the DM's preferences enter the picture in the second stage and acts on the shortlisted alternatives from the first stage. We should point out one major difference between our paper and this literature when it comes to its substantive behavioral content. Whereas most of this literature has looked at heuristic-based, psychologically motivated choice procedures as a way of explaining departures from the rational choice benchmark, our focus here goes precisely in the opposite direction. One of the keys questions that we address is how the ESH may allow the DM to attain the rational benchmark.

The rest of the paper is organized as follows. Section 2 introduces the setup. Section 3 formally introduces and defines the ESH. Section 4 contains our substantive analysis of the ESH and the formal results addressing the main questions of the paper. Proofs of the results are presented in the Appendix.

[4] Another recent paper which develops the theme of social influence within a slightly different theoretical framework is Fershtman and Segal (2018).

2 Primitives

Let X be a finite set of alternatives with typical elements denoted by x, y, z, etc. $\mathcal{P}(X)$ will denote the set of non-empty subsets of X with typical elements S, T etc. We refer to any such $S \in \mathcal{P}(X)$ as a choice problem. A choice function on X is a mapping, $c : \mathcal{P}(X) \rightarrow X$, that for any $S \in \mathcal{P}(X)$ picks an element $c(S) \in S$. Let $\mathcal{I} = \{1, \ldots, n-1\}$ be a set of individuals in society, with typical elements denoted by i, j, etc. Each individual $i \in \mathcal{I}$ has a choice function c_i on X. We assume that each such individual is rational in the traditional economic sense, i.e., each c_i satisfies the weak axiom of revealed preferences (WARP). WARP demands that if an alternative x is chosen from some set where y is available, then y is never chosen from any set where x is available. Formally:

Definition 2.1 A choice function $c : \mathcal{P}(X) \rightarrow X$ satisfies WARP if, for all $S, T \in \mathcal{P}(X)$:

$$[c(S) = x, y \in S, x \in T] \Rightarrow [c(T) \neq y].$$

It is well known that if an individual's choice function satisfies WARP, then her choices can be rationalized by a strict preference ranking, formally, a complete, asymmetric and transitive binary relation.[5]

Proposition 2.1 *A choice function $c : \mathcal{P}(X) \rightarrow X$ satisfies WARP if and only if there exists a strict preference ranking $\succ \subseteq X \times X$, such that, for any $S \in \mathcal{P}(X)$:*

$$c(S) = \{x \in S | x \succ y, \forall y \in S \backslash \{x\}\}.$$

Therefore, we can identify for each $i \in \mathcal{I}$, a strict preference ranking $\succ_i$ on X that rationalizes the choice function c_i. Finally, in the way of notation, note that, in the subsequent presentation, for any finite set A, $\#A$ will denote the number of elements in A. Furthermore, for any binary relation $\succ$ on X, we will denote its restriction to any set $S \in \mathcal{P}(X)$ by $\succ_S$.

3 The ecological shortlist heuristic

Consider a society with these $n-1$ rational individuals and, in the way of motivation, think of individual n as a new entrant to this society. In what follows, we often refer to n as the decision-maker (DM). Our starting point is the observation that this DM may not be able to rank every pair of alternatives in X, which is our domain of interest. As suggested in the Introduction, such incompleteness in preferences reflects the fact that anyone who like this DM is exposed to, say, a new environment may not have the requisite knowledge to form complete preference judgments over all alternatives involved. Formally, we assume that the DM's primitive preference judgments are captured by an asymmetric and transitive binary relation $\succ^* \subseteq X \times X$. In keeping

5 We refer to a binary relation $B \subseteq X \times X$ as (1) complete if for all $x, y \in X$, $x \neq y$, either xBy or yBx; (2) asymmetric if for all $x, y \in X$, $[xBy] \Rightarrow \neg[yBx]$ and (3) transitive if for all $x, y, z \in X$, $[xBy \wedge yBz] \Rightarrow xBz$.

with the spirit of this work, we interpret this binary relation as capturing preference judgments that the DM can in principle arrive at if she were to critically think and invest cognitive resources.

What about choices? Her primitive preferences are no doubt incomplete, but it does not necessarily imply that, in any choice problem $S \in \mathcal{P}(X)$, she is not able to choose a desired alternative. For any such set S, denote the $\succ^*$-maximal element of S by $x^*(S)$, whenever it exists. That is, $x^*(S) \in S$ is such that $x^*(S) \succ^* y$, for all $y \in S \backslash \{x^*(S)\}$. Clearly, if a $\succ^*$-maximal element exists, then it is unique. Given that $\succ^*$ is not complete, for many such sets $S \in \mathcal{P}(X)$, a maximal element may not exist. On the other hand, for certain sets, the maximal element may exist and the DM can in principle choose the maximal element. Even in those cases, though, the process of figuring out the maximal element may be a cognitively demanding task for the DM, especially if the set under consideration is a large one, involving lots of alternatives.

With this as background, we proceed to propose a simple heuristic that allows the DM to determine her choices in a cognitively less demanding fashion. The key feature of the heuristic is that it enables the DM to integrate available information in her environment, specifically, the choice information of the other $n-1$ individuals to simplify her decision-making process. The heuristic captures a two-stage decision-making process. Faced with any choice problem $S \in \mathcal{P}(X)$, in the first stage, the DM uses the choices of the other individuals on S to shortlist a (presumably small) set of alternatives that she considers. In the second stage, from this shortlisted or consideration set, she chooses the best alternative according to $\succ^*$. Observe that this two-stage process can drastically cut down on the number of preference judgments and inferences that the DM potentially needs to make to arrive at her choice, especially when the choice problem at hand involves lots of alternatives.

To set up the heuristic formally, assume that individual n, our DM, knows the choices made by the other $n-1$ entrenched individuals in society in any choice problem, i.e., she knows the family of choice functions $(c_i)_{i=1}^{n-1}$. Furthermore, the DM works within an ingroup–outgroup mindset as far as relating to the others in society. Specifically, at the level of her perception, there exists a partition of the other $n-1$ individuals in $\mathcal{I}$ into an ingroup, I_n, and an outgroup, I_n^c. The ingroup represents those members of society with whom she identifies and the outgroup those ones from whom she wants to differentiate herself. Therefore, which alternatives in a choice problem will the DM shortlist or consider in the first stage? For reasons discussed in the Introduction, presumably, those ones that are similar to the choices of the members of her ingroup and different from the choices of the members of her outgroup. For any choice problem S, we denote this set of shortlisted alternatives, referred to as the consideration set, by $\Gamma(S)$, and define it formally by the following:

$$\Gamma(S) = \text{argmax}_{x \in S} \Big\{ \alpha \big[\#\{i \in I_n : c_i(S) = x\} \big] - (1-\alpha) \big[\#\{j \in I_n^c : c_j(S) = x\} \big] \Big\},$$

where $\alpha \in (0, 1)$. In other words, for any choice problem $S \in \mathcal{P}(X)$, the DM observes the choices from S of the rest of the individuals in $\mathcal{I}$ and uses these to form her consideration set in the following manner. As suggested above, the heuristic judgment involved in forming the consideration set is to shortlist alternatives that are similar to the choices of the members of her ingroup and different from those of the members of her outgroup. As such, for any alternative $x \in S$, whenever someone from her ingroup chooses this alternative, it serves as a normative approval of x. At the same time, whenever someone from her outgroup chooses it, it serves as a normative disapproval of this alternative. Hence, $\#\{i \in I_n : c_i(S) = x\}$ captures the total number of approvals and $\#\{j \in I_n^c : c_j(S) = x\}$ captures the total number of disapprovals corresponding to this alternative. Furthermore, the parameter α captures the relative importance of the ingroup in relation to the outgroup in her perception. Hence, approvals are weighted by α and disapprovals by $1 - \alpha$. The difference between the total weighted approvals and disapprovals reflects the ecological support in favor of an alternative. As such, the DM shortlists or considers those alternatives from S that have maximum ecological support.

We can now formally define our choice heuristic. As noted earlier, in the second stage, in any choice problem S, the DM chooses the $\succ^*$-maximal element from $\Gamma(S)$. In the way of notation, $c_n : \mathcal{P}(X) \to X$ denotes n's choice function.

Definition 3.1 $c_n : \mathcal{P}(X) \to X$ is an ecological shortlist heuristic (ESH) if, for all $S \in \mathcal{P}(X)$: $c_n(S) = x^*(\Gamma(S))$.

A couple of comments are due. First, observe that, in any choice problem S, if $\Gamma(S)$ is a singleton, then the second stage of the choice procedure is redundant. Second, if $\Gamma(S)$ is not a singleton, then the heuristic requires that the $\succ^*$-maximal element exists in $\Gamma(S)$.

Before concluding this section, we present a couple of examples that illustrate the working of the ESH. The examples also set the stage for a formal analysis of the key properties of the ESH that follows in the next section. The first example illustrates the efficacy of the ESH.

Example 3.1 (Vacation) Isabel, our DM (individual 5) is a fun loving 20 year old girl. She is planning her next vacation and the possible destinations that she has in mind are Barcelona (x), Paris (y), and Istanbul (z). Isabel has been to Barcelona and Paris before, and knows these cities well and strictly prefers Barcelona to Paris ($x \succ^* y$). However, Istanbul is a bit of an unknown for her having never been there before. Therefore, she is unable to compare it with the other two cities. That is, her primitive preferences are $\succ^* = \{(x, y)\}$, which are clearly incomplete. Her two best friends, Liz (individual 1) and Andy (individual 2), as well as her parents (individuals 3 and 4) have been to all three cities and have a complete preference ranking over them, which are captured by their choice functions in Table 1.[6] For this problem at hand, Isabel thinks of her friends as her ingroup and her parents as her outgroup, i.e., $I_5 = \{1, 2\}$ and $I_5^c = \{3, 4\}$. Furthermore, the decision weights on the ingroup and outgroup are equal, i.e., $\alpha = \frac{1}{2}$. Then, Isabel's choices given in Table 1 are an ESH. Note that they satisfy WARP and can be rationalized by the strict preference ranking, $x \succ_5 y \succ_5 z$.

The example above shows how, despite having incomplete preferences, the ESH allows Isabel to make choices over all choice problems. Not just that, the choices that

[6] Specifically, $x \succ_1 y \succ_1 z$, $y \succ_2 x \succ_2 z$, $z \succ_3 y \succ_3 x$, and $z \succ_4 y \succ_4 x$.

Table 1 Choices of Individual 1, 2, 3, 4, and 5

	$\{x, y\}$	$\{y, z\}$	$\{x, z\}$	$\{x, y, z\}$
$c_1(.)$	x	y	x	x
$c_2(.)$	y	y	x	y
$c_3(.)$	y	z	z	z
$c_4(.)$	y	z	z	z
$\Gamma_5(.)$	$\{x\}$	$\{y\}$	$\{x\}$	$\{x, y\}$
$c_5(.)$	x	y	x	x

she ends up making are rational in the traditional sense of the term. Essentially, choosing according to the ESH allows her to complete her incomplete primitive preferences $\succ^*$. Furthermore, the choices that she ends up making are good ones in the sense that none of them go against these preferences.

However, it is not the case that pursuing the logic of the ESH always leads to rational choices or even good ones. We have seen above a situation where the ESH guides the DM towards good choices. As the reader may have inferred, the reason behind this has something to do with the fact that the ingroup–outgroup categorization of Isabel in this situation has a connection to her preferences. Specifically, the only comparison that she can make is between Barcelona and Paris, and over this pair, the preferences of her parents run completely counter to her's, whereas that of at least one of her friends does not. As such, the shortlisting that results from taking her parents as her outgroup and her friends as her ingroup is potentially informative for her. However, there is no guarantee that such a connection between the DM's preferences and ingroup–outgroup categorization always exists. One of the great dangers of working through the prism of such categorizations is that they may get caught up in things like ethnocentric biases and end up having no connection to DM's preferences. In those cases, choices resulting from the ESH may not have desirable properties as the following example illustrates.

Example 3.2 (Ethnocentricism) Ravi, our DM (individual 4), is a new occupant of an apartment building occupied by Shiv (individual 1), Rashid (individual 2), and Firdose (individual 3). Ravi and Shiv are Hindus, whereas Rashid and Firdose are Muslim. This example delves into the dietary choices which they make while living in this apartment building as neighbors. The alternatives are a diet that includes beef (x); a diet that includes fish but no meat (y); a vegetarian diet (z). As it turns out, all 4 of them have identical rational preferences, with the diet that includes beef being the top alternative, followed by the diet that includes fish but no meat, followed by the vegetarian diet. That is, $x \succ^* y \succ^* z$ for Ravi and the other three's choices produced by these preferences are as specified in Table 2. Suppose Ravi is ethnocentric and thinks of the two Muslims in this situation as his outgroup and the lone Hindu as his ingroup. Furthermore, the decision weights on the ingroup and outgroup are equal, i.e., $\alpha = \frac{1}{2}$. Then, the choices specified for him in Table 2 constitutes an ESH. Clearly, these choices violate WARP.

In the example above, Shiv, Rashid, and Firdose have identical preferences. As such, the categorization of Shiv as the ingroup and Rashid and Firdose as the outgroup does

Table 2 Choices of Individual 1, 2, 3, and 4

	$\{x, y\}$	$\{y, z\}$	$\{x, z\}$	$\{x, y, z\}$
$c_1(.)$	x	y	x	x
$c_2(.)$	x	y	x	x
$c_3(.)$	x	y	x	x
$\Gamma_4(.)$	$\{y\}$	$\{z\}$	$\{z\}$	$\{y, z\}$
$c_4(.)$	y	z	z	y

not have a basis in preferences. Rather, they reflect Ravi's ethnocentric bias. Because of that, even though Ravi's primitive preferences are rational, he ends up making choices that are not. Specifically, under the ESH, he ends up making choices that run counter to his preferences. For instance, the diet that includes beef is Ravi's top alternative according to his preferences, but, under the ESH, he never ends up choosing it in any choice problem.

With these two examples as background, we now proceed to a formal analysis that provides us with a language with which to understand the difference in outcomes between the two.

4 Efficacy and rationality under the ESH

We are now in a position to address the two substantive questions of the paper. First, how efficacious is the ESH in terms of helping the DM arrive at good choices, where the notion of good is a subjective one guided by $\succ^*$. For instance, if in a given choice problem $S \in \mathcal{P}(X)$, $\succ^*$ does happen to have a maximal element, then does the ESH manage to pick this alternative. Second, is it possible for the ESH choices to be fully rational in the traditional economic sense. That is, can a choice function c_n that is an ESH satisfy WARP and, if so, under what conditions. We introduce two conditions that connect the DM's primitive preferences $\succ^*$ with her ingroup–outgroup categorization and the underlying ecological shortlisting to provide sharp answers to these questions.

Our first condition is in the nature of a preference-based homophily condition that establishes a connection between the DM's ingroup–outgroup categorization and her preferences $\succ^*$. As we have noted, the DM's ingroup consists of those individuals with whom she identifies, whereas the outgroup consists of those from whom she wants to differentiate herself. If such a categorization is to be connected to the DM's preferences, then it stands to reason that the DM will show a greater propensity to identify with those individuals whose preferences are "closer" to her's and differentiate from those whose preferences are "farther." That is what preference-based homophily—the tendency to identify and relate more closely with others sharing similar characteristics, in this case, preferences—would entail. To formalize this idea, we construct a homophily index that captures the degree of association between the DM's ingroup–outgroup categorization and her preferences in the following way. Consider any choice problem S and strict preference rankings $\succ$ and $\succ'$ on S.[7] Suppose that $\succ$ is "closer" to $\succ^*_S$

[7] For the index, we consider rankings that have a specific relationship with each other.

than $\succ'$ is in a well-defined sense that we elaborate below.[8] Furthermore, let $N(\succ)$ and $N(\succ')$ denote the set of individuals in $\mathcal{I}$ whose preferences on S are given by $\succ$ and $\succ'$, respectively. The homophily index that we construct will attain a higher score the more we are able to associate sets like $N(\succ)$ with the ingroup and sets like $N(\succ')$ with the outgroup. This is the case when more are the number of individuals in $N(\succ)$ from the ingroup; and more are the ones in $N(\succ')$ from the outgroup.

To construct the index and formalize this condition, we first need to define a notion of the difference between two asymmetric binary relations, so that we can meaningfully talk about the degree of closeness between two sets of preferences. To that end, note that for any set of alternatives $S \in \mathcal{P}(X)$ and any two asymmetric binary relations P and P' on S, the two may differ on anywhere between zero and $\frac{\#S \cdot (\#S-1)}{2}$ binary comparisons in S. For example, take $S = \{x, y, z\}$ and suppose that $P = \{(x, y), (x, z), (y, z)\}$ and $P' = \{(y, x), (y, z)\}$. Then, P and P' differ on two binary comparison, i.e., that between x and y, and x and z. Denote by $|P - P'|_S$ the number of binary comparisons in S that any two asymmetric binary relations, P and P', differ on. It turns out that:

$$|P - P'|_S = \#\left((P \backslash P') \cup (P' \backslash P) \backslash P^{-1}\right),$$

where the binary relation P^{-1} is defined as follows: $(x, y) \in P^{-1}$ if $(y, x) \in P$. Note that if both P and P' are strict preference rankings, then $(P' \backslash P) \backslash P^{-1} = \emptyset$ and $|P - P'|_S = \#(P \backslash P')$. Clearly, $|P - P'|_S = |P' - P|_S$. In what follows, we think of $|P - P'|_S$ as representing the difference between two asymmetric binary relations P and P' on any such set S.

Next, consider the following definition that connects two strict preference rankings on a given set. It is on a certain class of preference rankings of this type that we define the homophily index.

Definition 4.1 A pair of strict preference rankings $\succ$ and $\succ'$ on $S \in \mathcal{P}(X)$, $\#S \geq 2$, is tops only permutation (TOP) if there exists $x, y \in S$, $x \neq y$, and a bijection $f : S \to S$ with $f(x) = y$, $f(y) = x$ and $f(z) = z$ for all $z \in S \backslash \{x, y\}$, such that (i) $v \succ' w \iff f(v) \succ f(w)$, for any $v, w \in S$ and (ii) $x \succ z$ or $x \succ' z$ for all $z \in S \backslash \{x\}$. We refer to such a pair of rankings as (xy)-TOP.

That is, a pair of strict preference rankings, $\succ$ and $\succ'$, on a given set is TOP if the position of the respective top elements under the two rankings is interchanged while maintaining the position of all the other elements.

Finally, for any strict preference ranking $\succ$ on a set $S \in \mathcal{P}(X)$, let $N(\succ) \subseteq \mathcal{I}$ be the set of individuals in $\mathcal{I}$ whose preferences on S are given by $\succ$:

$$N(\succ) = \{i \in \mathcal{I} : \succ_{iS} = \succ\} = \{i \in \mathcal{I} : c_i(T) = x^*(T, \succ), \forall T \subseteq S, T \neq \emptyset\},$$

where, for any set $\emptyset \neq T \subseteq S$, $x^*(T, \succ)$ denotes the $\succ$-maximal element of T.

We will refer to a TOP pair, $\succ$ and $\succ'$, on S as relevant if: (i) there exists $i \in \mathcal{I}$, such that either $\succ_{iS} = \succ$ or $\succ_{iS} = \succ'$, and (ii) $| \succ - \succ^*_S |_S \neq | \succ' - \succ^*_S |_S$. That

[8] $\succ^*_S$ denotes the restriction of $\succ^*$ to S.

is, a TOP pair on a choice problem S is relevant if there exists some individual in $\mathcal{I}$ whose preferences on S coincide with one of these rankings, and one of the rankings is closer to the DM's preferences than the other. For any $S \in \mathcal{P}(X)$ and $x, y \in S$, we will say that S is (xy)-relevant if there exists a relevant (xy)-TOP pair on S. We will now define a homophily index for any such pair of relevant alternatives in a choice problem.

For any choice problem S that is (xy)-relevant, $x, y \in S$, let $(\succ^k, \succ'^k)_{k=1}^K$ be the collection of all relevant (xy)-TOP pairs on S. Specifically, for any such pair $(\succ^k, \succ'^k)$, we order the two rankings, such that $| \succ^k - \succ^*_S |_S < | \succ'^k - \succ^*_S |_S$. As mentioned earlier, this inequality captures the idea that, in choice problem S, the preferences of individuals in $N(\succ^k)$ are closer to the DM's preferences than those of individuals in $N(\succ'^k)$. If the DM's ingroup–outgroup categorization follows the spirit of preference-based homophily, then it should be possible to identify $N(\succ^k)$ with the ingroup and $N(\succ'^k)$ with the outgroup. What stands in the way of such an identification are the individuals in the ingroup who are part of $N(\succ'^k)$ (the set $I_n \cap N(\succ'^k)$) and the individuals in the outgroup who are part of $N(\succ^k)$ (the set $I_n^c \cap N(\succ^k)$). Call this set, $(I_n \cap N(\succ'^k)) \cup (I_n^c \cap N(\succ^k))$, the non-homophilous group and contrast it with the homophilous group, given by the set $(I_n \cap N(\succ^k)) \cup (I_n^c \cap N(\succ'^k))$. A sensible way to measure the degree of homophily in this situation, therefore, is by the fraction of individuals in the homophilous group. However, when doing so, we need to take into account the relative importance of the ingroup and the outgroup in the DM's perception as captured by the parameter α. As such, define a homophily score for the pair $(\succ^k, \succ'^k)$ by the following:

$$\sigma_{xy}^S(\succ^k, \succ'^k) = \frac{\alpha\#[I_n \cap N(\succ^k)] + (1-\alpha)\#[I_n^c \cap N(\succ'^k)]}{\alpha\#[I_n \cap (N(\succ^k) \cup N(\succ'^k))] + (1-\alpha)\#[I_n^c \cap (N(\succ^k) \cup N(\succ'^k))]}.$$

The numerator of the fraction in the RHS is the α-weighted average across in/outgroups of the number of individuals in the homophilous group; the denominator is the α-weighted average across in/outgroups of the number of individuals whose preferences are either $\succ^k$ or $\succ'^k$ in this choice problem.

We can now determine a homophily index with respect to the collection $(\succ^k, \succ'^k)_{k=1}^K$ by taking a weighted average of the individual homophily scores, $\sigma_{xy}^S(\succ^k, \succ'^k)$, $k = 1, \ldots, K$, for each of the relevant (xy)-TOP pairs. Define the weight associated with any such (xy)-TOP pair $(\succ^k, \succ'^k)$ by the following:

$$w_{xy}^S(\succ^k, \succ'^k) = \frac{\alpha\#[I_n \cap (N(\succ^k) \cup N(\succ'^k))] + (1-\alpha)\#[I_n^c \cap (N(\succ^k) \cup N(\succ'^k))]}{\sum_{\hat{k}=1}^K \{\alpha\#[I_n \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))] + (1-\alpha)\#[I_n^c \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))]\}}.$$

We can then define the homophily index with respect to the collection $(\succ^k, \succ'^k)_{k=1}^K$ by the following:

$$\sigma_{xy}^S = \sum_{k=1}^{K} w_{xy}^S(\succ^k, \succ'^k) \cdot \sigma_{xy}^S(\succ^k, \succ'^k).$$

We can now state the homophily condition that establishes a connection between the DM's ingroup–outgroup categorization and her preferences. In the way of terminology, we say that $x, y \in X$ are $\succ^*$-comparable if either $x \succ^* y$ or $y \succ^* x$. On the other hand, if neither $x \succ^* y$ nor $y \succ^* x$, then we say that x and y are not $\succ^*$-comparable.

Condition 4.1 ***(Homophily)*** *For any $x, y \in X$ that are $\succ^*$-comparable and $S \in \mathcal{P}(X)$ that is (xy)-relevant, $\sigma_{xy}^S \geq \frac{1}{2}$.*

That is, for any alternatives $x, y \in S$ if the collection of relevant (xy)-TOP pairs in S is non-empty and, hence, the homophily index, σ_{xy}^S, is well defined, we consider $\frac{1}{2}$ as the cut-off score that this index has to exceed to think of the DM's ingroup–outgroup categorization as preference-homophilous with respect to this collection. The homophily condition requires that for any x, y that are $\succ^*$-comparable and for which the homophily index σ_{xy}^S is well defined in a choice problem S, this index be no smaller than the homophily cut-off of $\frac{1}{2}$.

The reader should be able to verify that the ingroup–outgroup categorization of Ravi in Example 3.2 does not satisfy the homophily condition. To see this, consider the set $S = \{x, y\}$. For this set, of course, there is just one (xy)-TOP pair on S: $\succ = \{(x, y)\}$ and $\succ' = \{(y, x)\}$. The preferences of the three other individuals coincide with $\succ$ on this set. Furthermore, Ravi's preferences on this set is $\succ_S^* = \{(x, y)\}$ and, hence, $|\succ - \succ_S^*|_S < |\succ' - \succ_S^*|_S$. Hence, this TOP pair is relevant and the set S is (xy)-relevant, which implies that the homophily index σ_{xy}^S is well defined. However:

$$\begin{aligned}\sigma_{xy}^S &= \frac{\alpha\#[I_n \cap N(\succ)] + (1-\alpha)\#[I_n^c \cap N(\succ')]}{\alpha\#[I_n \cap (N(\succ) \cup N(\succ'))] + (1-\alpha)\#[I_n^c \cap (N(\succ) \cup N(\succ'))]} \\ &= \frac{0.5 \times 1 + 0.5 \times 0}{0.5 \times 1 + 0.5 \times 2} = \frac{1}{3} < \frac{1}{2}.\end{aligned}$$

On the other hand, Isabel's ingroup–outgroup categorization in Example 3.1 does satisfy the Homophily condition. Since Isabel's preferences are given by $\succ^* = \{(x, y)\}$, to verify this, we need to consider the sets $S = \{x, y, z\}$ and $S' = \{x, y\}$. On the set S, verify that there is one relevant (xy)-TOP pair: $\succ = \{(x, y), (y, z), (x, z)\}$ and $\succ' = \{(y, x), (x, z), (y, z)\}$. Specifically, note that $|\succ - \succ_S^*|_S = 2 < 3 = |\succ' - \succ_S^*|_S$. The homophily index with respect to this TOP pair is as follows:

$$\begin{aligned}\sigma_{xy}^S &= \frac{\alpha\#[I_n \cap N(\succ)] + (1-\alpha)\#[I_n^c \cap N(\succ')]}{\alpha\#[I_n \cap (N(\succ) \cup N(\succ'))] + (1-\alpha)\#[I_n^c \cap (N(\succ) \cup N(\succ'))]} \\ &= \frac{0.5 \times 1 + 0.5 \times 0}{0.5 \times 2 + 0.5 \times 0} = \frac{1}{2}.\end{aligned}$$

Next, consider the set $S' = \{x, y\}$. The only (xy)-TOP pair on S' is $\succ = \{(x, y)\}$ and $\succ' = \{(y, x)\}$. It is straightforward to verify that this pair is also relevant with

$| \succ - \succ_{S'}^{*} |_{S'} < | \succ' - \succ_{S'}^{*} |_{S'}$. The homophily index defined with respect to this pair is as follows:

$$\sigma_{xy}^{S'} = \frac{\alpha \#[I_n \cap N(\succ)] + (1-\alpha)\#[I_n^c \cap N(\succ')]}{\alpha \#[I_n \cap (N(\succ) \cup N(\succ'))] + (1-\alpha)\#[I_n^c \cap (N(\succ) \cup N(\succ'))]}$$
$$= \frac{0.5 \times 1 + 0.5 \times 2}{0.5 \times 2 + 0.5 \times 2} = \frac{3}{4}.$$

Hence, Isabel's ingroup–outgroup categorization satisfies the Homophily condition.

We can now state our first main result which establishes the efficacy of the ESH when the DM's ingroup–outgroup categorization satisfies homophily. In the way of notation, define the base relation $\succ_n \subseteq X \times X$ as: $x \succ_n y$ if $x = c_n(\{x, y\}), x, y \in X$, $x \neq y$.

Proposition 4.1 *Suppose c_i, $i = 1, \ldots, n-1$, satisfies WARP, c_n is an ESH and Homophily holds. Then, for any $S \in \mathcal{P}(X)$:*

1. *if $x \in \Gamma(S)$, then there does not exist $y \in S \backslash \Gamma(S)$, such that $y \succ^{*} x$*
2. *if there exists $x^{*} \in S$, s.t. $x^{*} \succ^{*} y$, for all $y \in S \backslash \{x^{*}\}$, then $c_n(S) = x^{*}$.*

Furthermore, $\succ^{*} \subseteq \succ_n$.

Proof: Please refer to Sects. A.1 and A.2

The result establishes that whenever Homophily holds, the ESH is efficacious both in terms of shortlisting a good set of alternatives and in leading the DM towards a good choice. First, it establishes that, in any choice problem, the set of alternatives that are shortlisted in the first stage and form the consideration set is a good one in the sense that there exists no alternative that is not shortlisted that is strictly preferred under the DM's preferences to an alternative that is shortlisted. Second, it establishes that if in any choice problem, a $\succ^{*}$-maximal alternative exists, the heuristic picks it up. As such, the heuristic never leads the DM to choices that go against her primitive preferences. This is one sense in which the heuristic eases the cognitive load that the DM faces when trying to figure out what her most preferred alternative is. If, after carefully contemplating through her preferences, the DM is indeed able to arrive at a best alternative, then the ESH is guaranteed to find this alternative in a fast and frugal way. It is also worth pointing out that one of the criticisms that is often directed towards heuristic-based decision-making is that it leads to biases and sub-optimal choices (Tversky and Kahneman 1974). The conclusions of the Proposition together provide a defense for the ESH from such criticism whenever the DM's ingroup–outgroup categorization satisfies the homophily condition.

Observe that in Example 3.2 where Ravi's ingroup–outgroup categorization violates Homophily, the conclusions of the Proposition fail to hold. For instance, $\Gamma(\{x, y, z\}) = \{y, z\}$, but $x \succ^{*} y$. Furthermore, in that set, x is the $\succ^{*}$-maximal element, but $c_4(\{x, y, z\}) = y$. On the other hand, in Example 3.1 where Isabel's ingroup–outgroup categorization satisfies Homophily, it is straightforward to verify that the conclusions, indeed, hold.

Having established the efficacy of the ESH, we now proceed to address the other main question of this paper, which is to investigate the possibility of providing an ecological basis for the traditional view of rationality in economics. Specifically, we look at when is it true that ESH choices satisfy WARP and, hence, can be rationalized by a strict preference ordering.

We introduce a second condition that along with Homophily ensures that if the DM's choice function is an ESH, then it, indeed, satisfies WARP. This condition requires the ecological shortlisting to follow a certain consistency when dealing with any pair of alternatives that are not comparable under the DM's primitive preferences.

Condition 4.2 ***(Ecological Shortlisting Consistency)*** *For any $x, y \in X$ that are not $\succ^*$-comparable:*

$$\Gamma(\{x, y\}) = \{x\} \Longrightarrow [\forall S \in \mathcal{P}(X) \text{ and } x, y \in S, y \in \Gamma(S) \Rightarrow x \in \Gamma(S)].$$

This condition requires that, for any pair of alternatives that the DM cannot compare, ecological shortlisting should not result in conflicting inferences for her. Specifically, for two such alternatives x and y, if x is the unique element shortlisted in the set $\{x, y\}$, then there should not be a choice problem in which y is shortlisted but x is not.

Proposition 4.2 *Suppose c_i, $i = 1, \ldots, n-1$, satisfies WARP, c_n is an ESH, and Homophily and Ecological Shortlisting Consistency holds. Then, c_n satisfies WARP. Furthermore, $\succ^* \subseteq \succ_n$, with $\succ_n$ rationalizing c_n.*

Proof: Please refer to Sects. A.1 and A.2

This result provides sufficient conditions that identify when ESH choices are rational. In so doing, it shows how it may be possible to arrive at the stringent ideal of rationality in economics by means of a simple heuristic-based reasoning that builds on information about others' choices that may be available in the DM's environment. Referring back to Example 3.1, we have already noted that Isabel's ingroup–outgroup categorization satisfies Homophily. It is also straightforward to verify that Ecological Shortlisting Consistency holds. As such, Isabel's choices satisfy WARP.

A Appendix

A.1 Preliminaries

We first prove two lemmas that we use to prove our results.

Lemma A.1 *If $(\succ, \succ')$ is an (xy)-TOP pair on $S \in \mathcal{P}(X)$ with $x \succ z$, for all $z \in S \backslash \{x\}$, and $\succ^*$ is an asymmetric, transitive binary relation on S with $x \succ^* y$, then*

$$| \succ - \succ^* |_S < | \succ' - \succ^* |.$$

Proof Since $(\succ, \succ')$ is an (xy)-TOP pair, $w \succ z$ if and only if $w \succ' z$ for all $w, z \in S\backslash\{x, y\}$. Hence:

$$| \succ - \succ^* |_{S\backslash\{x,y\}} - | \succ' - \succ^* |_{S\backslash\{x,y\}} = 0,$$

and it follows that:

$$\begin{aligned}|\succ - \succ^*|_S - |\succ' - \succ^*|_S =& |\succ - \succ^*|_{\{x,y\}} - |\succ' - \succ^*|_{\{x,y\}}\\ &+ \sum_{z\in S\setminus\{x,y\}} \left[|\succ - \succ^*|_{\{x,z\}} - |\succ' - \succ^*|_{\{x,z\}}\right]\\ &+ \sum_{z\in S\setminus\{x,y\}} \left[|\succ - \succ^*|_{\{y,z\}} - |\succ' - \succ^*|_{\{y,z\}}\right].\end{aligned}$$

Define the sets $S_1 = \{z \in S : x \succ z \succ y\}$ and $S_2 = \{z \in S : y \succ z\}$. Clearly, $S_1 \cup S_2 \cup \{x, y\} = S$. Furthermore, since $x \succ z$, $x \succ' z$, $y \succ z$ and $y \succ' z$, for all $z \in S_2$, it follows that:

$$\begin{aligned}\sum_{z\in S_2} |\succ - \succ^*|_{\{x,z\}} &= \sum_{z\in S_2} |\succ' - \succ^*|_{\{x,z\}}, \text{ and}\\ \sum_{z\in S_2} |\succ - \succ^*|_{\{y,z\}} &= \sum_{z\in S_2} |\succ' - \succ^*|_{\{y,z\}}.\end{aligned}$$

Therefore:

$$\begin{aligned}\sum_{z\in S\setminus\{x,y\}} \left[|\succ - \succ^*|_{\{x,z\}} - |\succ' - \succ^*|_{\{x,z\}}\right] &= \sum_{z\in S_1} \left[|\succ - \succ^*|_{\{x,z\}} - |\succ' - \succ^*|_{\{x,z\}}\right]\\ \sum_{z\in S\setminus\{x,y\}} \left[|\succ - \succ^*|_{\{y,z\}} - |\succ' - \succ^*|_{\{y,z\}}\right] &= \sum_{z\in S_1} \left[|\succ - \succ^*|_{\{y,z\}} - |\succ' - \succ^*|_{\{y,z\}}\right].\end{aligned}$$

Next, define the sets

$$\begin{aligned}V_1 &= \{z \in S_1 : z \succ^* x\}, \quad V_2 = \{z \in S_1 : x \succ^* z\}\\ \hat{V}_1 &= \{z \in S_1 : z \succ^* y\}, \quad \hat{V}_2 = \{z \in S_1 : y \succ^* z\}\end{aligned}$$

and verify that

- $|\succ - \succ^*|_{\{x,y\}} - |\succ' - \succ^*|_{\{x,y\}} = -1$
- $\sum_{z\in S_1} \left[|\succ - \succ^*|_{\{x,z\}} - |\succ' - \succ^*|_{\{x,z\}}\right] = \#V_1 - \#V_2$
- $\sum_{z\in S_1} \left[|\succ - \succ^*|_{\{y,z\}} - |\succ' - \succ^*|_{\{y,z\}}\right] = \#\hat{V}_2 - \#\hat{V}_1$.

Now, since $\succ^*$ is transitive and $x \succ^* y$, we have that $z \succ^* x \Rightarrow z \succ^* y$. Hence, $V_1 \subseteq \hat{V}_1$ and $\#V_1 \leq \#\hat{V}_1$. Furthermore, $y \succ^* z \Rightarrow x \succ^* z$. Hence, $\hat{V}_2 \subseteq V_2$ and $\#\hat{V}_2 \leq \#V_2$. Accordingly, $[\#V_1 - \#\hat{V}_1] + [\#\hat{V}_2 - \#V_2] \leq 0$ and, therefore:

$$\begin{aligned}0 &\geq [\#V_1 - \#V_2] + [\#\hat{V}_2 - \#\hat{V}_1]\\ &= \sum_{z\in S_1} \left[|\succ - \succ^*|_{\{x,z\}} - |\succ' - \succ^*|_{\{x,z\}}\right]\\ &\quad + \sum_{z\in S_1} \left[|\succ - \succ^*|_{\{y,z\}} - |\succ' - \succ^*|_{\{y,z\}}\right]\end{aligned}$$

Hence, $|\succ - \succ^*|_S - |\succ' - \succ^*|_S < 0$. □

Before stating the next lemma, we introduce some notation. For any choice problem $S \in \mathcal{P}(X)$ and any alternative $x \in S$ define the ecological support for x in this choice problem by the following:

$$\gamma_S(x) = \alpha\#\{i \in I_n : c_i(S) = x\} - (1-\alpha)\#\{j \in I_n^c : c_j(S) = x\}.$$

That is:

$$\Gamma(S) = \operatorname*{argmax}_{x \in S} \gamma_S(x).$$

Lemma A.2 *Suppose that c_i, $i = 1, \dots, n-1$, satisfies WARP, c_n is an ESH, and Homophily holds. If $x \succ^* y$, $x, y \in X$, then for any $S \in \mathcal{P}(X)$ with $x, y \in S$, $\gamma_S(x) \geq \gamma_S(y)$.*

Proof Let $x, y \in S$ and $x \succ^* y$. First, note that if there exists no $i \in \mathcal{I}$, such that $c_i(S) = x$ or $c_i(S) = y$, then $\gamma_S(x) = \gamma_S(y) = 0$ and our desired conclusion follows immediately. Therefore, consider the case where there exists $i \in \mathcal{I}$ with $c_i(S) = x$ or $c_i(S) = y$. This means that there exists at least one (xy)-TOP pair, $\succ$ and $\succ'$, on S, such that $\succ_{iS} = \succ$ or $\succ_{iS} = \succ'$. Furthermore, since $x \succ^* y$, from the last Lemma, we know that for any (xy)-TOP pair, $\succ$ and $\succ'$, on S with, say, x the top element of $\succ$ and y of $\succ'$, we have $| \succ - \succ_S^* |_S < | \succ' - \succ_S^* |_S$. Therefore, under this case, the collection of relevant (xy)-TOP pairs on S is non-empty. Let $(\succ^k, \succ'^k)_{k=1}^K$ be the set of all relevant (xy)-TOP pairs on S, where $x \succ^k z$ for all $z \in S\backslash\{x\}$ and $y \succ'^k z$, for all $z \in S\backslash\{y\}$, for all $k = 1, \dots, K$. As such, the Homophily condition implies that σ_{xy}^S, the homophily index defined with respect to this collection of relevant (xy)-TOP pairs, is such that

$$\sigma_{xy}^S = \sum_{k=1}^K w_{xy}^S(\succ^k, \succ'^k) \cdot \sigma_{xy}^S(\succ^k, \succ'^k) \geq \frac{1}{2}.$$

That is:

$$\sum_{k=1}^K \left[\frac{\alpha\#[I_n \cap (N(\succ^k) \cup N(\succ'^k))] + (1-\alpha)\#[I_n^c \cap (N(\succ^k) \cup N(\succ'^k))]}{\sum_{\hat{k}=1}^K \{\alpha\#[I_n \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))] + (1-\alpha)\#[I_n^c \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))]\}} \right] \times \left[\frac{\alpha\#[I_n \cap N(\succ^k)] + (1-\alpha)\#[I_n^c \cap N(\succ'^k)]}{\alpha\#[I_n \cap (N(\succ^k) \cup N(\succ'^k))] + (1-\alpha)\#[I_n^c \cap (N(\succ^k) \cup N(\succ'^k))]} \right] \geq \frac{1}{2}.$$

That is:

$$\sum_{k=1}^K \left[\frac{\alpha\#[I_n \cap (N(\succ^k) \cup N(\succ'^k))] + (1-\alpha)\#[I_n^c \cap (N(\succ^k) \cup N(\succ'^k))]}{\sum_{\hat{k}=1}^K \{\alpha\#[I_n \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))] + (1-\alpha)\#[I_n^c \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))]\}} \right] \times \left[\frac{\alpha\#[I_n \cap N(\succ^k)] + (1-\alpha)\#[I_n^c \cap N(\succ'^k)]}{\alpha\#[I_n \cap (N(\succ^k) \cup N(\succ'^k))] + (1-\alpha)\#[I_n^c \cap (N(\succ^k) \cup N(\succ'^k))]} - \frac{1}{2} \right] \geq 0.$$

Denote:

$$\theta = \sum_{\hat{k}=1}^{K}\{\alpha\#[I_n \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))] + (1-\alpha)\#[I_n^c \cap (N(\succ^{\hat{k}}) \cup N(\succ'^{\hat{k}}))]\},$$

and we have that

$$\begin{aligned}
&\frac{1}{2\theta}\sum_{k=1}^{K}\{2(\alpha\#[I_n \cap N(\succ^{k})] + (1-\alpha)\#[I_n^c \cap N(\succ'^{k})]) - \alpha\#[I_n \cap (N(\succ^{k}) \cup N(\succ'^{k}))] \\
&\quad - (1-\alpha)\#[I_n^c \cap (N(\succ^{k}) \cup N(\succ'^{k}))]\} \geq 0 \\
&\Rightarrow \sum_{k=1}^{K}\{2\alpha\#[I_n \cap N(\succ^{k})] + 2(1-\alpha)\#[I_n^c \cap N(\succ'^{k})] - \alpha\#[I_n \cap N(\succ^{k})] \\
&\quad - \alpha\#[I_n \cap N(\succ'^{k})] - (1-\alpha)\#[I_n^c \cap N(\succ^{k})] - (1-\alpha)\#[I_n^c \cap N(\succ'^{k})]\} \geq 0.
\end{aligned}$$

That is:

$$\begin{aligned}
&\sum_{k=1}^{K}\{\alpha\#[I_n \cap N(\succ^{k})] + (1-\alpha)\#[I_n^c \cap N(\succ'^{k})] \\
&\qquad - \alpha\#[I_n \cap N(\succ'^{k})] - (1-\alpha)\#[I_n^c \cap N(\succ^{k})]\} \geq 0 \\
&\quad \Rightarrow \alpha\sum_{k=1}^{K}\#[I_n \cap N(\succ^{k})] - (1-\alpha)\sum_{k=1}^{K}\#[I_n^c \cap N(\succ^{k})] \\
&\quad \geq \alpha\sum_{k=1}^{K}\#[I_n \cap N(\succ'^{k})] - (1-\alpha)\sum_{k=1}^{K}\#[I_n^c \cap N(\succ'^{k})] \\
&\quad \Rightarrow \alpha\sum_{k=1}^{K}\#\{i \in I_n : \succ_{iS} = \succ^{k}\} - (1-\alpha)\sum_{k=1}^{K}\#\{j \in I_n^c : \succ_{jS} = \succ^{k}\} \\
&\quad \geq \alpha\sum_{k=1}^{K}\#\{i \in I_n : \succ_{iS} = \succ'^{k}\} - (1-\alpha)\sum_{k=1}^{K}\#\{j \in I_n^c : \succ_{jS} = \succ'^{k}\} \\
&\quad \Rightarrow \alpha\#\{i \in I_n : x \succ_i z, \forall z \in S\backslash\{x\}\} - (1-\alpha)\#\{j \in I_n^c : x \succ_j z, \forall z \in S\backslash\{x\}\} \\
&\quad \geq \alpha\#\{i \in I_n : y \succ_i z, \forall z \in S\backslash\{y\}\} - (1-\alpha)\#\{j \in I_n^c : y \succ_j z, \forall z \in S\backslash\{y\}\}.
\end{aligned}$$

That is:

$$\begin{aligned}
&\alpha\#\{i \in I_n : c_i(S) = x\} - (1-\alpha)\#\{j \in I_n^c : c_j(S) = x\} \\
&\quad \geq \alpha\#\{i \in I_n : c_i(S) = y\} - (1-\alpha)\#\{j \in I_n^c : c_j(S) = y\}
\end{aligned}$$

Or, $\gamma_S(x) \geq \gamma_S(y)$. □

A.2 Proof of Propositions

Proof of Proposition 4.1

Consider $x \in \Gamma(S)$ and $y \in S \backslash \Gamma(S)$. That is, $\gamma_S(x) > \gamma_S(y)$. Clearly, it cannot be the case that $y \succ^* x$, for then, by Lemma A.2, $\gamma_S(y) \geq \gamma_S(x)$.

Let $x^* \in S$ be such that $x^* \succ^* y$, for all $y \in S \backslash \{x^*\}$. By Lemma A.2, $\gamma_S(x^*) \geq \gamma_S(y)$, for all $y \in S \backslash \{x^*\}$. Hence, $x^* \in \Gamma(S)$ and $c_n(S) = x^*$.

Let $x \succ^* y$. By Lemma A.2, $\gamma_{\{x,y\}}(x) \geq \gamma_{\{x,y\}}(y)$. Therefore, either $\Gamma(\{x, y\}) = \{x, y\}$ or $\Gamma(\{x, y\}) = \{x\}$. In either case, $c_n(\{x, y\}) = x$, which implies $x \succ_n y$. Hence, $\succ^* \subseteq \succ_n$.

Proof of Proposition 4.2

Consider any sets $S, T \in \mathcal{P}(X)$, such that $c_n(S) = x$, $y \in S$, $x \in T$. To establish that c_n satisfies WARP, we need to show that $c_n(T) \neq y$. If $y \notin T$, the conclusion is immediate. Therefore, assume that $y \in T$.

Our desired conclusion follows if we can show that $c_n(S \cap T) = x$. We first show that $x \in \Gamma(S \cap T)$. Clearly, $\Gamma(S \cap T) \neq \emptyset$. Pick $w \in \Gamma(S \cap T)$. If $w = x$, we have reached the desired conclusion. Therefore, consider $w \neq x$. First, note that it cannot be the case that $w \succ^* x$. Clearly, this cannot be the case if $w \in \Gamma(S)$ since $c_n(S) = x \succ^* y$, for all $y \in \Gamma(S) \backslash \{x\}$. On the other hand, if $w \in S \backslash \Gamma(S)$, then $\gamma_S(x) > \gamma_S(w)$, by the definition of $\Gamma(S)$. However, if $w \succ^* x$, then by Lemma A.2, $\gamma_S(w) \geq \gamma_S(x)$! Second, if $x \succ^* w$ then, by Lemma A.2, we have $\gamma_{S\cap T}(x) \geq \gamma_{S\cap T}(w)$. Hence, $x \in \Gamma(S \cap T)$. Finally, consider the case that neither $x \succ^* w$ nor $w \succ^* x$. In this case, first, it has to be that $w \in S \backslash \Gamma(S)$ for if $w \in \Gamma(S)$, then $x \succ^* w$. Second, $\Gamma(\{x, w\}) \neq \{x, w\}$ for if this were so, the ESH cannot determine what $c_n(\{x, w\})$ is. Therefore, $\Gamma(\{x, w\}) = \{x\}$ or $\Gamma(\{x, w\}) = \{w\}$. However, $\Gamma(\{x, w\}) \neq \{w\}$, since $\Gamma(\{x, w\}) = \{w\}$, $x \in \Gamma(S)$ and $w \in S \backslash \Gamma(S)$ violates Ecological Shortlisting Consistency. Hence, $\Gamma(\{x, w\}) = \{x\}$. Therefore, by the same condition, $w \in \Gamma(S \cap T)$ implies that $x \in \Gamma(S \cap T)$. That is, we have shown that $x \in \Gamma(S \cap T)$ under all possible cases.

Finally, note that since c_n is an ESH, the set $\Gamma(S \cap T)$ has a $\succ^*$-maximal element. Arguments made above establish that this element has to be x and not some $w \neq x$. To reiterate, if it was such a w, then clearly $w \in S \backslash \Gamma(S)$, i.e., $\gamma_S(x) > \gamma_S(w)$. However, $w \succ^* x$ would imply by Lemma A.2 that $\gamma_S(w) \geq \gamma_S(x)$! Hence, $x \succ^* y$, for all $y \in \Gamma(S \cap T) \backslash \{x\}$ and $c_n(S \cap T) = x$.

We have already shown above in the proof of Proposition 4.1 that if Homophily holds, then $\succ^* \subseteq \succ_n$. Furthermore, it can be shown that if c_n satisfies WARP, then the base relation $\succ_n$ is complete and transitive and, indeed, rationalizes c_n. We omit those details here as they are quite standard.

References

Aumann, R. J. (1962). Utility theory without the completeness axiom. *Econometrica*, *30*, 445–462.

Chen, Y., & Li, S. X. (2009). Group identity and social preferences. *The American Economic Review*, *99*, 431–457.

Cherepanov, V., Feddersen, T., & Sandroni, A. (2013). Rationalization. *Theoretical Economics*, *8*, 775–800.

Cuhadaroglu, T. (2017). Choosing on influence. *Theoretical Economics*, *12*, 477–492.

Fershtman, C., & Segal, U. (2018). Preferences and social influence. *American Economic Journal: Microeconomics*, *10*, 124–142.

Gigerenzer, G., Todd, P. M., & ABC Research Group (1999). *Simple Heuristics That Make Us Smart*. Oxford: Oxford University Press.

Hornsey, M. J. (2008). Social identity theory and self-categorization theory: A historical review. *Social and Personality Psychology Compass*, *2*, 204–222.

Lleras, J. S., Masatlioglu, Y., Nakajima, D., & Ozbay, E. Y. (2017). When more is less: limited consideration. *Journal of Economic Theory*, *170*, 70–85.

Mandler, M., Manzini, P., & Mariotti, M. (2012). A million answers to twenty questions: choosing by checklist. *Journal of Economic Theory*, *147*, 71–92.

Manzini, P., & Mariotti, M. (2007). Sequentially rationalizable choice. *The American Economic Review*, *97*, 1824–1839.

Manzini, P., & Mariotti, M. (2012). Categorize then choose: boundedly rational choice and welfare. *Journal of the European Economic Association*, *10*, 1141–1165.

Masatlioglu, Y., Nakajima, D., & Ozbay, E. Y. (2012). Revealed attention. *American Economic Review*, *102*, 2183–2205.

Simon, H. A. (1955). A behavioral model of rational choice. *The Quarterly Journal of Economics*, *69*, 99–118.

Simon, H. A. (1956). Rational choice and the structure of the environment. *Psychological Review*, *63*, 129–138.

Tajfel, H., Billig, M. G., Bundy, R. P., & Flament, C. (1971). Social categorization and intergroup behaviour. *European Journal of Social Psychology*, *1*, 149–178.

Todd, P. M., & Gigerenzer, G. (2012). *Ecological rationality: Intelligence in the world*. Oxford: Oxford University Press.

Turner, J. C. (1985). Social categorization and the self-concept: A social cognitive theory of group behavior. In E. J. Lawler & C. T. Greenwich (Eds.), *Advances in Group Processes: Theory and Research* (Vol. 2, pp. 77–122). Greenwich: JAI Press.

Tversky, A., & Kahneman, D. (1974). Judgment under uncertainty: Heuristics and biases. *Science*, *185*, 1124–1131.