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A note on a recent paper by Dagsvik on IIA and random utilities

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Abstract In a recent paper in this journal, Dagsvik derives the class of independent random utility representations that are "equivalent" to the independence-from-irrelevant-alternatives (IIA) assumption by Luce (Individual choice behavior: a theoretical analysis. Wiley, New York, 1959). In this short note, we clarify the relations between this paper by Dagsvik, and a paper in Lindberg's 2012 thesis.

Keywords Random utility · IIA (Independence from irrelevant alternatives) · Choice probabilities

1 Introduction

In a recent paper in this journal, Dagsvik (2016) addresses the question of what independent random utility (RU) representations are equivalent[1] to the famous independence-from-irrelevant-alternatives (IIA) assumption of Luce (1959). There

[1] The term "equivalent" in the title of Dagsvik's paper is somewhat misleading since additional conditions are required, including infinitely expandable choice sets. Indeed, it is shown by counterexample in Lindberg (2012b) that the stated equivalence does not hold without this latter assumption.

Tony E. Smith was the Opponent to P. O. Lindberg's thesis.

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are clear connections between Dagsvik's paper and the paper Lindberg (2012b), in Lindberg's thesis (Lindberg 2012a).[2] As we shall clarify below, the key contribution of Dagsvik's paper is to establish a simpler proof of the main result in Lindberg (2012b) (though under stronger assumptions).

2 Subject matter

To shorten the presentation, and to simplify the comparison, we will utilize the notation of Dagsvik (D hereafter). We give a short outline of Lindberg's (L hereafter) approach, commenting in square brackets on the ways in which D deviates from L's approach.

Like D, L assumes independent random utilities U_j, $j \in S$, where S can be taken to be a (possibly infinite) set of integers. L further assumes that each U_j has support on an arbitrary (closed) interval D_j in R, with cdf F_{w_j} generated by a continuous, positive density. [D assumes that all supports are $[0, \infty)$ and that each F_{w_j} is strictly increasing and continuously differentiable.]

The question both authors ask is for what utility distributions, F_{w_j}, are the associated choice probabilities of the "Luce form"

$$P_C(j) =_{df} \Pr\left\{U_j = \max_{k \in C} U_k\right\} = \frac{w_j}{\sum_{k \in C} w_k} \tag{1}$$

for arbitrary $j \in C \subseteq S$, and for appropriate positive parameters w_j (i.e., the ones following from the IIA assumption).

[Here D assumes that the parameters w_j in (1) are same as those indexing the F_{w_j}.]

Inspired by the concept of "uniform expansion" of choice sets in Yellott (1977), L introduces non-uniform expansions of choice sets, where one is allowed to let arbitrary numbers of independent copies of the U_j compete for the maximum utility. By the use of such non-uniform expansions, one may choose competing utilities consisting of a copy of one utility, say U_1, together with $m > 0$ independent copies of another utility, say U_k, and then let m tend to infinity. [D does the same. But he lets m take all the possible values to draw his conclusion.]

Further, to establish his result, L must invoke a regularity condition which is somewhat complex to explain. In essence, it says that certain densities connected to U_1 and U_k cannot both agree and disagree in arbitrarily small punctuated neighborhoods of the supremum of the set of points where they disagree [D avoids this assumption by invoking Hausdorff's powerful theorem on the moment problem, which greatly simplifies the argument.]

The result both authors arrive at is that under the assumptions made the cdfs of the U_j must of the form

$$F_{w_j} = (F_1)^{\alpha_j}, \tag{2}$$

for appropriate positive scalars α_j. [D arrives at this result (page 4, first paragraph, last line); but he assumes (without loss of generality) that $\alpha_j = w_j$, and $w_1 = 1$]. Along the way, L proves that the supports are equal which D assumes.

[2] Professor Dagsvik was a specially invited reviewer of this thesis work.

Up to here, the papers follow parallel lines. L has more general assumptions on the supports, but at the same time needs to put stronger conditions on the behavior of the F_{w_j} (his regularity condition). Here, the use of Hausdorff's result (as suggested by D) permits shorter proofs.

In the end, D makes a rather standard monotone transformation of utilities, U_j (necessarily leaving all choice probabilities unaltered), which shows that (2) can also be characterized in terms of utilities of the form, $U_j = w_j \varepsilon_j$, with ε_j having cdf, $\exp\left(-\frac{1}{u}\right)$, which is a special case of the Fréchet distribution.[3] This distribution is in turn seen to be a simple transformation of the double exponential distribution originally shown by Yellott (1977) to characterize choice probabilities of the Luce form for the case of uniformly expandable choice sets. In summary, D's transformation together with the above application of Hausdorff's result, constitute the main differences between the two papers.

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[3] While this cdf is termed by D as a type I extreme value distribution (following Resnick 1987), it is most commonly designated as the type II extreme value distribution (following the original paper of Fisher and Tippett 1928).