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The aggregation of preferences: can we ignore the past?

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Abstract The article shows that a Paretian social welfare function can be history independent and time consistent only if a stringent set of conditions is verified. Individual utilities must be additive. The social welfare function must be a linear combination of these utilities. Social preferences are stationary only if, in addition, all individuals have the same constant discount rate. The results are implemented in two frameworks: deterministic dynamic choice and dynamic choice under uncertainty. The applications highlight that the conditions are unlikely to be met by individual preferences, and that they severely restrict social preferences.

Keywords Social welfare function · Aggregation of preferences · History independence

1 Introduction

When dealing with intertemporal allocation problems, the usual approach consists in treating the aggregate choices as if they were generated by a fictional representative consumer whose utility can be used as a measure of aggregate welfare. For the utility of the representative consumer to have such welfare significance, it should have a normative foundation. The common practice consists in aggregating individuals' utilities into a social welfare function and to impose that the aggregation satisfies Pareto's principle. Social preferences should be the result of a Paretian aggregation of preferences.

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We can therefore build the concept of a benevolent social planner whose preferences, derived from the aggregation of individuals' utilities, are used to provide policy guidance and to choose the intertemporal allocation of resources. The approach permits to deem optimal the course of action prescribed by the social planner. The definition of optimal policies matters for a wide range of issues: economic growth, investment in human capital, environmental preservation, etc.

In models of intertemporal decision making, it is however convenient to assume that preferences have a specific structure. Particular forms of intertemporal utilities have therefore been used to describe the choices of a rational individual decision maker. Quite naturally, we might think that the social planner's preferences should have a structure similar to the individuals' one. The macroeconomic theory has often endorsed this view and taken the form of the social objective from the theory of individual intertemporal decisions.

For instance, the predominant intertemporal social objective, used in the seminal models of optimal growth (Ramsey 1928) and of optimal resource depletion (Dasgupta and Heal 1979), is additively separable and exhibits exponential discounting:[1] $\sum_{t=0}^{+\infty} \beta^t U_t$ (where U_t is the aggregate welfare at time t and β is the social discount factor). This form corresponds to the most standard model of dynamic decision making described by Samuelson (1937).

Subsequent research has postulated more general non additive recursive social objectives (Uzawa 1968; Beals and Koopmans 1969). One interesting feature of the non-additive recursive criteria is that they allow for endogenous discounting. In particular, they can model the fact that impatience decreases with wealth. They may also be used to model preferences for the timing of the resolution of uncertainty (Kreps and Porteus 1978; Epstein and Zin 1989). These features provided new insights in many branches of economics.

For instance, the turnpike nature of optimal growth was shown to be relevant for a larger class of technologies when using non-additive recursive criteria (Epstein and Hynes 1983; Becker et al. 1989; Palivos et al. 1997). Decreasing impatience was also shown to explain why economic development could contribute to an increased demand for environmental preservation (Chavas 2004). Non-additive recursive criteria have been applied to problems in international trade (Obstfeld 1981), to the question of asset pricing (Epstein and Zin 1989) or to the issue of long-run wealth distribution (Lucas and Stokey 1984). These contributions have highlighted new mechanisms and phenomena that may be important to explain dynamic choices.

Both non-additive recursive objectives and the traditional additively separable ones are characterized by the fact that preferences at a given period are independent of what happened in the past. Preferences are history independent. Actually, the mentioned objectives are stationary, a stronger property.

In this article, I investigate the conditions under which a Paretian, time consistent and history independent aggregation of preferences is possible. I find that a major condition bears on admissible individual utilities: they must be additively separable. Another condition bears on the form of social aggregation, which must

[1] Ramsey (1928) actually focuses on the special case $\beta = 1$ and uses a slightly different criterion: $W = \sum_{t=0}^{+\infty}(B - U_t)$, where B represents the "bliss" (the maximal reachable level of welfare).

be additive. I also find necessary and sufficient conditions for obtaining stationarity and Paretian social preferences: the individuals' preferences must display exponential discounting and people must have the same rate of impatience. These conditions are shown to exclude all the non-additive recursive criteria that have been increasingly used in recent years.

Some of these results resemble those obtained separately by Blackorby et al. (2005) in the case of a finite number of overlapping generations. My approach is different, for I work within an infinite horizon framework in order to discuss the criteria used in the macroeconomic literature. Besides, the description of the intertemporal problem in Sect. 2 permits to consider a wider range of intertemporal situations, including situations involving temporal uncertainty, so that I am able to obtain a more general answer.

The framework and some properties satisfied by intertemporal preferences are introduced in the next section. In Sect. 3 individual and social decision problems are presented. In Sect. 4.1, I state the results on possible aggregations in the general framework. Then I derive and discuss their consequences in two specific frameworks: deterministic dynamic choice (Sect. 4.2) and dynamic choice under uncertainty with Kreps and Porteus preferences (Sect. 4.3). The last section contains some concluding remarks. Proofs of the propositions are relegated to an appendix.

2 Dynamic choice

Throughout this article, $\mathbb{N}$ is the set of positive integers and $\mathbb{R}$ the set of real numbers. For $n \in \mathbb{N}$, $\mathbb{R}^n_+$ denotes the non-negative orthand of the Euclidian n-space. Time is discrete and designated by $t \in \mathbb{N}$.

2.1 A general setting

Each period, the set of payoffs is $X = \mathbb{R}^n_+$. A payoff history at time t is $y \in Y_t$, where Y_t is defined recursively by $Y_1 = \{x_0\}$ and $Y_{t+1} = Y_t \times X$.

The set of actions that can be undertaken is A, the set of temporal lotteries over infinite payoff streams. This is the set of temporal lotteries as constructed by Epstein and Zin (1989). They show that it is a separable metric space homeomorphic to $M(X \times A)$, the set of Borel probability measures over $X \times A$ endowed with the weak convergence topology (Epstein and Zin 1989, Theorem 2.1). Hence actions determine prospects of present and future payoffs.

A temporal lottery can be pictured as an infinite probability tree in which each branch corresponds to a deterministic consumption stream. The specificity of temporal lotteries is that they carefully model the way in which consumption uncertainty is resolved over time.

An important subset of A is $M_\delta(X \times A)$ the set of degenerate lotteries that select a specific current period payoff and a particular temporal lottery for the next period. A degenerate lottery yielding (x, a) with probability 1 can be identified with the consequence $(x, a) \in X \times A$. In the remainder of this article, (x, a) hence denotes with a slight abuse in notation either the degenerate probability or the consequence. But no ambiguity should occur on the specific meaning.

Another interesting subset of A is the subset of deterministic consumption streams $\bar{A} = X^{\infty}$. Any sequence $(x_1, x_2, x_3, \ldots)$ in $\bar{A}$ can be equivalently described as $(x_1, (x_2, x_3, \ldots)) \in X \times \bar{A}$. The set $\bar{A}$ endowed with the product topology is a separable metric space. It can be viewed as a subset of $M_\delta(X \times A)$ since it is homeomorphic to $M_\delta(X \times \bar{A})$.

A dynamic choice problem consists of a sequence of choices on A, given realized past payoffs. I assume that for each $t \in \mathbb{N}$ and each $y \in Y_t$, decision makers choose a course of action by means of a utility function $U_{y,t}$ on A.[2] The utility function $U_{y,t}$ is said *regular* whenever it is continuous with respect to the topology for A and non-constant. I will assume that all decision makers have regular preferences for each $t \in \mathbb{N}$ and each $y \in Y_t$.

2.2 Properties of temporal preferences

It is customary to narrow the scope of intertemporal preferences by imposing particular properties. A widespread assumption is that choices are consistent: the plan selected at time t must also be adopted at time $t + 1$, taking into account the payoff history. This is a key property of rational decision making.

Property 1 (Consistency) *Preferences are (time-)consistent if, for any* $t \in \mathbb{N}$, $y \in Y_t$, $x \in X$, *a and $\hat{a}$ in A:*

$$U_{y,t}(x, a) \geq U_{y,t}(x, \hat{a}) \Leftrightarrow U_{(y,x),t+1}(a) \geq U_{(y,x),t+1}(\hat{a})$$

Property 1 is similar to axiom 3.1 in Kreps and Porteus (1978) and to the strict consistency axiom in Streufert (1998). This first property is usually combined with a second one, called history independence:

Property 2 (History independence) *Preferences are history independent if, for any* $t \in \mathbb{N}$, $y \in Y_t$, *a and $\hat{a}$ in A:*

$$U_{y,t}(a) \geq U_{y,t}(\hat{a}) \Longrightarrow \forall \hat{y} \in Y_t,\ U_{\hat{y},t}(a) \geq U_{\hat{y},t}(\hat{a})$$

It can be shown that when the utility functions are regular, history independent and consistent, there exists a continuous aggregator function V_t which is strictly increasing in its second argument and such that:

$$U_t(x, a) = V_t\left[x, U_{t+1}(a)\right] \quad (1)$$

These recursive preferences are commonplace in the literature. The reason for this success is that they are a simple way to ensure the consistency of temporal choices.[3] This may be too strong an assumption though, as illustrated by the literature on habit

[2] The analysis can be performed with preference relations rather than utility functions, assuming continuity of preferences. To ease the exposition, it is more convenient to work with utility functions.

[3] See Blackorby et al. (1973) and Johnsen and Donaldson (1985) who study the situation of a finite time horizon.

formation, which considers history dependent choices. But the modeling of how past payoffs affect current preferences can be complicated and involves an elaborate mathematical structure. The simplicity and practicality of the recursive specification explain its success.

A related but stronger assumption is that preferences are stationary.

Property 3 (Stationarity) *Preferences are stationary if, for any $x \in X$, a and $\hat{a}$ in A, $t \in \mathbb{N}$, $y \in Y_t$:*

$$U_{y,t}(a) \geq U_{y,t}(\hat{a}) \Longleftrightarrow U_{y,t}(x, a) \geq U_{y,t}(x, \hat{a})$$

This definition of stationarity is consistent with the definition given, for instance, by Koopmans (1960). It is slightly stronger since the condition, usually imposed only at the initial period, is assumed to hold at any period. Under the consistency assumption, this formulation directly implies that preferences exhibit temporal invariance: choices among actions that affect the future are made using the same utility function U in all periods. Stated differently, preferences do not change as time elapses. In that case, the utility must have the following form:

$$U_{y,t}(x, a) = U(x, a) = V\,[x, U(a)] \tag{2}$$

We shall call this form *stationarily recursive* (stationary in short). It is clear from the utility structure that stationarity and consistency imply history independence (and consistency).

Stationarity is a key ingredient of most models of intertemporal planning. It makes it possible for myopic decision makers who use the same objective function in each period to have consistent preferences (Blackorby et al. 1973).

Beside history independence and stationarity, two properties are commonly used in the theory of intertemporal decision making: independence of the future[4] and constant discounting.[5] Combined with Properties 1–3, independence of the future and constant discounting deliver two important families of intertemporal preferences corresponding to particular forms of the aggregator function V_t. First V_t can be additive:

Definition 2 (*Additivity*) Preferences on $X \times A$ are (time-)additive if, for any $t \in \mathbb{N}$, $y \in Y_t$, $x \in X$, $a \in A$, they are represented by:

$$U_{y,t}(x, a) = V_t\left[u_t(x), U_{t+1}(a)\right] = u_t(x) + \beta_t U_{t+1}(a)$$

for some positive real numbers β_t.

[4] The property of independence of the future states that for any $t \in \mathbb{N}$, $y \in Y_t$, x and $\hat{x}$ in X, a and $\hat{a}$ in A,

$$U_{y,t}(x, a) \geq U_{y,t}(\hat{x}, a) \Longleftrightarrow U_{y,t}(x, \hat{a}) \geq U_{y,t}(\hat{x}, \hat{a})$$

[5] Assume that the utility functions have the structure exposed in Eq. 1. Whenever the aggregator function V_t is differentiable, the discount factor is defined as $\frac{\partial V_t}{\partial U_{t+1}}$. Preferences exhibit constant discounting if for any $t \in \mathbb{N}$, $\frac{\partial V_t}{\partial U_{t+1}}$ is constant and equal to $\beta_t \in (0, 1)$.

Table 1 Types of intertemporal preferences

Recursive preferences $U_t(x,a) = V_t\left(x, U_{t+1}(a)\right)$ (consistency + history independence)	Additive preferences $U_t(x,a) = u_t(x) + \beta_t U_{t+1}(a)$ (consistency + history independence + independence of the future + constant discounting)
Stationary preferences $U_t(x,a) = V(x, U(a))$ (Consistency + stationarity)	exponential discounting $U_t(x,a) = u(x) + \beta U(a)$ (consistency + stationarity + independence of the future + constant discounting)

If preferences are additive, they are obviously consistent, history independent, independent of the future and they exhibit constant (though time-varying) discounting. The stationary counterpart of additivity is exponential discounting:

Definition 3 (*Exponential discounting*) Preferences on $X \times A$ exhibit exponential discounting if, for any $t \in \mathbb{N}$, $y \in Y_t$, $x \in X$, $a \in A$, they are represented by:

$$U_{y,t}(x,a) = V\left[u(x), U(a)\right] = u(x) + \beta U(a)$$

for some constant positive real numbers β.

In the case of exponential discounting, the discount factor and discount rate are constant through time and respectively equal to β and $\rho = \frac{1-\beta}{\beta}$.

Table 1 summarizes the different types of intertemporal preferences that I have introduced.

3 Aggregating individual preferences

The model of choice laid out in Sect. 2 applies to individuals or to a social planner. Below I discuss the interrelation between individuals' and the planner's preferences. A social aggregation indeed derives the latter from the former.

3.1 Planner's and individuals' preferences

The society is composed of a finite (but possibly large) number $N \in \mathbb{N}$ of infinitely-lived individuals. The set of individuals is denoted $I = \{1, \ldots, N\}$. For the sake of simplicity, I assume that all individuals have the same payoff and action sets for each period: X and A. The individual payoff history set, Y_t, is constructed as indicated in Sect. 2.1. To denote the payoff of a particular individual at a specific date, I use the notation x_t^i (payoff of individual $i \in I$ at period $t \in \mathbb{N}$). Similar notation are used for actions, histories and utility or aggregator functions: subscripts always refer to time, and superscripts to individuals.

All individuals are supposed to make decisions using a process of regular utility functions as described in Sect. 2.1. Denote $U^i_{y^i,t}$ the utility function of individual $i \in I$ at time $t \in \mathbb{N}$ given his payoff history $y^i \in Y_t$. It is assumed that individuals have self-regarding preferences: their utility depends only on their own payoffs and actions and not on the payoffs and actions of other individuals.

Social payoff and action sets are defined as follows. The Cartesian product of individual payoff sets $\mathcal{X} = X^{|I|}$, is the social payoff set at all periods. Similarly, the social payoff history set at time t is $\mathcal{Y}_t = Y_t^{|I|}$ and the social action set is $\mathcal{A} = A^{|I|}$, the same each period. A generic element of $\mathcal{X}$ at time t is x_t, and x_t^i indicates the ith component of x_t. The set $\mathcal{X}^i$ refers to the subset of the space of social payoffs corresponding to individual i's payoffs. Similar notation is used for $\mathcal{Y}_t$ and $\mathcal{A}$.

Social preferences are represented by a process of utility functions $U_{y,t}^S$ on $\mathcal{A}$. The functions $U_{y,t}^S$ are also called social welfare functions. I assume the social planner adopts the model of choice introduced in Sect. 1, so that her utility functions are regular, and thus continuous. There are multiple examples in social choice theory proving that the continuity assumption is not innocuous,[6] but it is required for the society to use the standard model of dynamic choice.

3.2 Social aggregation

The aim of a social aggregation is to derive the social welfare function from individuals' utility functions. Many ethical principles can be invoked to determine how this aggregation should be made. But the most widely accepted principle is that social preferences should be Paretian:

Axiom *(Pareto)* Social preferences are Paretian if for any $t \in \mathbb{N}$, $y \in \mathcal{Y}_t$, a_t and $\hat{a}_t$ in $\mathcal{A}$:

$$U_{y^i,t}^i(a_t^i) \geq U_{y^i,t}^i(\hat{a}_t^i) \quad \forall i \in I \Rightarrow U_{y,t}^S(a_t) \geq U_{y,t}^S(\hat{a}_t)$$

If furthemore there exists $j \in I$ such that $U_{y^j,t}^j(a_t^j) > U_{y^j,t}^j(\hat{a}_t^j)$ then $U_{y,t}^S(a_t) > U_{y,t}^S(\hat{a}_t)$.

Pareto's principle asserts that the unanimous preference of individuals must be respected by the social planner. This principle is the cornerstone of individualistic ethics. Most of the social criteria that have been proposed satisfy a version of this axiom. Besides, the common practice consists in deriving the representative consumer's preferences from a Paretian aggregation of preferences.

Pareto's principle has an important implication concerning the form of the social welfare function when it is continuous. Indeed, it can be proved that if social preferences are Paretian, then at any period t, there exists a social aggregator function $\mathcal{U}_{y,t}$, continuous and increasing in all its arguments such that (see Fleurbaey and Mongin 2005):

$$U_{y,t}^S(a_t) = \mathcal{U}_{y,t}\left[U_{y^1,t}^1(a_t^1), \ldots, U_{y^N,t}^N(a_t^N)\right]$$

for all a_t in $\mathcal{A}$ and for all y in $\mathcal{Y}_t$.

[6] For instance, continuity rules out the leximin criterion.

It is fruitful to consider a more specific, additively separable, form of the social aggregator function. The formula is widespread in the literature on the aggregation of preferences and it is obtained as a result in the main propositions of this article.

Definition 4 Social preferences at time t constitute an additive aggregation of preferences if there exists a profile of individual utility functions $\left[U^S_{y,t}, U_{y^1,t}, \ldots, U_{y^N,t}\right]$ such that for all $y \in \mathcal{Y}_t,\ a_t \in \mathcal{A}$,

$$U^S_{y,t}(a_t) = U^1_{y^1,t}(a^1_t) + \cdots + U^N_{y^N,t}(a^N_t)$$

represents the planner's preferences.

Remark that the symmetric linear form of the social aggregation is contingent on the choice of a particular utility profile. For different utility representations $\tilde{U}^i_{y^i,t} = \phi_i^{-1}\left(U^i_{y^i,t}\right)$, the aggregation would be

$$\tilde{U}^S_{y,t}(a_t) = \phi_S\left(\phi_1\left(\tilde{U}^1_{y^1,t}(a^1_t)\right) + \cdots + \phi_N\left(\tilde{U}^N_{y^N,t}(a^N_t)\right)\right)$$

In addition to the fact that social utility functions can be obtained as a sum of individual utility functions, an important result of the paper will be that they are obtained as the linear combination of very specific individual utility functions, namely additively separable ones.

4 Possible aggregations

It is now possible to address the core issue of this article: under what conditions can a consistent Paretian aggregation of preferences be history independent?

4.1 Possibility results for a history independent planner

Proposition 1 enunciates the conditions under which it is possible to aggregate individual preferences into a Paretian social objective satisfying history independence and consistency.

Proposition 1 *Suppose that social preferences are Paretian. His preferences are consistent and history independent if and only if*

1. *individuals have additive preferences and*
2. *social preferences are an additive aggregation of preferences which is the sum of additive individual utility functions.*

Proof The detailed proof is in the appendix. It makes use of separability theorems by Debreu (1959) and Gorman (1968). Indeed Pareto's principle is a separability condition: each individual's action set is separable from others' action sets. Similarly, history independence implies that the future is separable from the past each period.

There is sufficient overlap between these sets so that Gorman's theorem on overlapping separable sets applies. We obtain a strong separability condition that leads to Debreu's theorem on additive representations of preferences. □

Proposition 1 is the main result of this article. It indicates that Pareto's principle, history independence and consistency reduce the possibilities of social choice. Restrictions are imposed both on individuals' utilities and on the form of the social aggregation:

1. Restrictions on individuals' preferences mean that it is not always possible to find social choice rules satisfying Pareto's principle and having a recursive form. Whenever individuals' utilities do not meet the conditions stated in Proposition 1, it is impossible to aggregate preferences into a history independent and consistent social objective.
2. The restriction on the form of the social aggregation means that the possibilities for the social planner to trade-off individuals' welfare are also limited. The fact that the aggregation is linear in the specific additive individual utility functions also limits the model of intertemporal choice a social planner can adopt. Social preferences must be additive.

The economic literature frequently makes the stronger assumption that the social objective is stationary. The most common form of stationary utility is exponential discounting. But the class of stationary preferences is pretty wide-ranging. In particular, it is possible to find stationary objectives with non-constant discount rates (see Uzawa 1968; Beals and Koopmans 1969). The following proposition indicates that such objectives cannot be the result of a Paretian aggregation:

Proposition 2 *Suppose that social preferences are Paretian. The social objective is consistent and stationary if and only if individuals and the planner have preferences exhibiting exponential discounting and have the same rate of time preference.*

Proof See the appendix. □

Proposition 2 makes it clear that a specific form of stationarity is required for individual and social utilities, namely exponential discounting. In addition people must have identical time preferences. Although some people seem to be more patient than others, any departure from the homogeneous patience case would introduce non-stationarity in the planner's objective.

To sum up, additivity and exponential discounting are the major conditions imposed by Propositions 1 and 2. Their definitions in Sect. 2.2 indicate that they only apply to preferences on $M_\delta(X \times A)$. This makes sense given that the properties of consistency, history independence and stationarity were also defined on this restricted domain. But we can show that the propositions also have significant implications for choices in several frameworks of particular interest. This also permits to better assess the restrictiveness of the conditions imposed by Propositions 1 and 2.

4.2 Deterministic dynamic choice

The simplest and most natural framework dealing with intertemporal preferences considers deterministic dynamic choices over (consumption) programs X^{∞}. As indicated in Sect. 2.2, this set is homeomorphic to $M_{\delta}(X \times \bar{A}) \subset M_{\delta}(X \times A)$ so that we can directly apply the results of Propositions 1 and 2.

In Proposition 1, it was shown that individual i's preferences for any sequence ${}_t\chi^i$ in $\bar{A}$ must be represented by a utility function

$$U_t^i(x_t^i, a_{t+1}^i) = u_t^i(x_t^i) + \beta_t U_{t+1}^i(a_{t+1}^i)$$

Proceeding by induction yields the additively separable representation of preferences over programs:

$$U_t^i(x_t^i, \ldots, x_T^i, \ldots) = \sum_{\tau=t}^{\infty} \beta_{t,\tau} u_\tau^i(x_\tau^i) \tag{3}$$

with $\beta_{t,T} = \prod_{\tau=t+1}^{T} \beta_\tau$. Proposition 2 further imposes that the functions u_t^i in Equation (3) are the same in all periods and that all individuals have the same constant discount factor β.

Additively separable forms have the noticeable consequence that trade-offs between goods at each period are independent of what happens in other periods. Preferences are said to be period independent. This property precludes any complementarities between consumptions at different periods.

One can wonder whether the restrictions imposed by Propositions 1 and 2 allow for preferences that can sufficiently portray the observed intertemporal choice patterns. The answer is globally negative. There is evidence that intertemporal non-complementarities are absent from empirical data (see Deaton 1971). There is also evidence that people are not equally patient and that a higher wealth may increase patience (such evidence is reviewed and discussed in Becker and Mulligan (1997)). As noted in Sect. 4.1, these findings suggest that Propositions 1 and 2 can be viewed as impossibility results.

It is also important to study the consequence of the propositions on the model of intertemporal choice a social planner can adopt. Proposition 1 implies that social welfare function is a symmetric linear combination of additive separable individual utility functions. Social welfare at period t is therefore ordinally equivalent to

$$\sum_{i \in I} U_t^i(x_t^i, \ldots, x_T^i, \ldots) = \sum_{\tau=t}^{\infty} \beta_{t,\tau} u_\tau^S(x_\tau) \tag{4}$$

with $u_t^S(x_t) = \sum_{i \in I} u^i(x_t^i)$.

Social welfare must therefore be period independent. In such a case, the distribution of advantages at each point of time is independent of people's prospects. Each individual life's period is treated separately, which precludes any idea of compensation for intertemporally correlated bad outcomes. Proposition 2 constrains the social planner to have preferences exhibiting a constant discount rate, which is the same as the individuals' one. The main consequence is that the whole class of non-additive recursive criteria is excluded.

4.3 Dynamic choice under uncertainty

Kreps and Porteus (1978) have provided axiomatics of preferences that takes into account the timing of the resolution of uncertainty in dynamic problems. These preferences also make it possible to disentangle risk aversion and intertemporal elasticity of substitution. The axiomatic construction was extended to the case of an infinite horizon by Epstein and Zin (1989). It involves defining temporal lotteries describing when the uncertainty is resolved.

The Kreps and Porteus (K–P) model also encompasses the more standard expected utility model of von Neumann and Morgenstern (vNM). vNM preferences correspond to the special case in which the timing of the resolution of uncertainty does not matter. In this section, I consider the more general K–P model, but the conclusions will apply to the vNM model as well: the vNM model will be proved to be a requirement for the aggregation of preferences to be possible.

Under Kreps and Porteus axioms, history independent preferences have the following structure:

Definition 5 A decision maker has history independent K–P preferences on temporal lotteries if there exist functions $v_t : X \times A \to \mathbb{R}$ for each $t \in \mathbb{N}$, $y \in Y_t$, $x_t \in X$ and $a_{t+1} \in A$ such that the process of preferences is given by the utility functions U_t defined recursively by:

$$U_t(a_t) = E_{a_t}\left[v_t\left(x_t, U_{t+1}(a_{t+1})\right)\right] \tag{5}$$

In the above expression, $E_a v = \int_{X \times A} v \, \mathrm{d}a$ is the expectation with respect to the Bernoulli probability measure a. Remark that the function v_t represents preferences over the set $M_\delta(X \times A)$: the form of the function clearly implies that K–P preferences are consistent.

Kreps and Porteus (1978) prove that if the aggregator function $v_t(.)$ in Eq. 5 is linear with respect to its second argument, the decision maker is indifferent to the timing of the resolution of uncertainty. If it is so, we are back to the vNM construction: the decision maker is an expected utility maximizer. In that case, v_t is called a Bernoulli utility function.

Proposition 1 has the following implication:

Proposition 3 *Assume that the social planner has history independent K–P preferences. A Paretian aggregation of individual preferences over temporal lotteries can be history independent if and only if all individuals and the planner have vNM preferences with additively separable Bernoulli utility functions, as described in Equation (3), and the social Bernoulli function is a sum of individual ones, as described in Eq. 4.*

Proof See the appendix. □

All individuals and the planner should be indifferent to the timing of the resolution of uncertainty if we want a history independent social objective. This conflicts with the empirical findings by Chew and Ho (1994), which suggests that the timing indifference assumption cannot appropriately represent the individuals' actual choices.

The result also excludes the use of the C.E.S. aggregator function proposed by Epstein and Zin (1989) to define the social planner's welfare. Although this work and others have provided many illuminating insights on individual intertemporal choices, the form of preferences they suggest cannot be the result of a Paretian aggregation, unless it corresponds to the standard additively separable vNM case.

Proposition 3 has also important consequences within the vNM expected utility model of choice under uncertainty. The proposition bears some similarities with Harsanyi (1955) theorem, which states that a Paretian aggregation of individual vNM utilities can only be represented by a sum of individual Bernoulli utility functions. The additional insight is that when the planner's objective is history independent, the Bernoulli utility functions of all decision makers must be additively separable.

This condition of additive separability has implications similar to those found in the deterministic case. Indeed, individuals' and the planner's preferences should then not only be independent of the past but also risk independent of the future.

But the additive separability of Bernoulli utility functions also has implications in terms of "correlation aversion".[7] Indeed, it implies that the decision makers must be indifferent to the correlation of their intertemporal outcomes. On the contrary, many papers have underlined that correlation aversion seems to have realistic empirical implications (see Epstein and Tanny 1980; or Eeckhoudt et al. 2007). Once again, this can be perceived as supporting the view that Proposition 1 is an impossibility result. This also limits the planner's possible preferences.

5 Conclusion

I have found stringent conditions for a Paretian aggregation of intertemporal utilities to be history independent. They bear both on individual preferences and on the permitted aggregations.

Individual utilities must be additively separable. We know that this has strong implications in terms of time independence, risk independence, etc. In the context of uncertainty, it also means indifference to the intertemporal correlation of outcomes. When considering temporal lotteries, this results in indifference to the timing of the resolution of uncertainty. If we want a stationary and Paretian social objective, the only admissible individual utilities are those exhibiting exponential discounting. In addition, people must have the same rate of time discounting, canceling out an important dimension of the heterogeneity in preferences.

The form of the social welfare function is also constrained. The social objective must be an additive aggregation of preferences which is linear in additively separable individual utility functions. As a consequence, the social welfare function must also

[7] The concept was brought to the economic literature by Richard (1975). It was further discussed in Epstein and Tanny (1980).

be additive. This severely restrains the models of dynamic choice the social planner can adopt.

The results can be read in either of two ways. One might first be left with the impression that the restrictions uncovered in this article should be accepted, despite their lack of realism. Indeed, linear aggregations of additively separable utilities are commonplace in economic theory. However, the applications to specific setting have highlighted that such a construction rules out significant phenomena such as endogenous discounting, correlation aversion, and so forth. Many welfare functions considered in the macroeconomic literature, in particular non-additive recursive preferences, would be excluded, for they cannot embody the decisions made by a history independent Paretian social planner, except for their additive special case. The exclusion seems unfortunate, for these social welfare functions have been theoretically fruitful and have provided many new insights concerning dynamic social choices.

A second reading of the results is that we should abandon the assumption of history independence. The property has indeed no particular normative appeal. But then the dependence must be carefully modeled. For instance, we can try to aggregate the recursive preferences of heterogeneous individuals. Lucas and Stokey (1984) have shown that Paretian equilibria can be computed in that case. Their results also suggest that when considering an additive aggregation, the dependence of the past takes the form of changing weights in the social welfare function. Some research could be undertaken in that promising direction. Hopefully, we could obtain social objectives indicating how past inequalities should modify current choices.

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Appendix

Definitions and preliminary results

To prove Proposition 1, I use results by Debreu (1959) and Gorman (1968). In order to make the article self-contained, I shall first introduce some definitions and present these results.

Consider a continuous preference ordering $\succeq$ on a product space $S = S_1 \times S_2 \times \cdots \times S_M$. The set of factors' indexes is $J = \{1, \ldots, M\}$. For a subset K of J, $S_K = \prod_{k \in K} S_k$. For $x \in S$, I denote by x_j the component in S_j.

Definition *(Separability)* The factors S_K are said to be separable for the preference ordering $\succeq$ if, for any $x,\ x',\ y$ and y' in S such that $x_k = x'_k$ and $y_k = y'_k$ for all $k \in K$ and $x_j = y_j$ and $x'_j = y'_j$ for all $j \in J \setminus K$:

$$x \succeq y \Longleftrightarrow x' \succeq y'$$

We also need a minimum sensitivity requirement called essentiality:

Definition *(Essentiality)* The factor S_j is essential for preference ordering $\succeq$ if there exist x and y such that $x_j \neq y_j$ and $x_k = y_k$ for all $k \in J \setminus \{j\}$ and $x \succ y$.

Two important results can now be stated.

Proposition (Debreu 1959) *Assume that for all subsets K of J the factors S_K are separable for the preference ordering $\succeq$. If there exist at least three factors which are essential, then there exist continuous functions $u_j(.)$ on each S_j such that for any x and y in S:*

$$x \succeq y \Longleftrightarrow U(x) = \sum_{j \in J} u_j(x_j) \geq \sum_{j \in J} u_j(y_j) = U(y)$$

To obtain separable sets, a second result can be used:

Proposition (Gorman 1968) *Let (I, K, L, M) be a partition of the set of indexes J. If $S_{I \cup K}$ and $S_{L \cup K}$ are separable and essential for the preference ordering $\succeq$, then S_I, S_K, S_L, $S_{I \cup L}$ and $S_{I \cup K \cup L}$ are separable and essential for the preference ordering $\succeq$.*

Preliminary lemma

The following lemma will be used in the proofs:

Lemma 1 *If social preferences are Paretian and consistent (resp. history independent, stationary, Kreps and Porteus), all individuals must have consistent (resp. history independent, stationary, Kreps and Porteus) preferences.*

Proof Take a and $\hat{a}$ in $\mathcal{A}$ such that $a^i \neq \hat{a}^i$ and $a^j = \hat{a}^j$, for $j \in I \setminus \{i\}$. By Pareto axiom, $U^S_{y,t}(a) \geq U^S_{y,t}(\hat{a}) \Leftrightarrow U^i_{y^i,t}(a^i) \geq U^i_{y^i,t}(\hat{a}^i)$. Combining Pareto's Axiom and the consistency of social preferences, we obtain that for any $i \in I$, $y^i \in Y^i_t$, $x^i \in X$, a^i and $\hat{a}^i$ in A:

$$U^i_{y^i,t}(x^i, a^i) \geq U^i_{y^i,t}(x^i, \hat{a}^i) \Leftrightarrow U^i_{(y^i,x^i),t+1}(a^i) \geq U^i_{(y^i,x^i),t+1}(\hat{a}^i)$$

This is the definition of individual preferences being consistent.

The same line of argument can be used to prove the other claims:

- For history independence, consider $a, \hat{a} \in \mathcal{A}$ and $y, \hat{y} \in \mathcal{Y}$ such that $a^i \neq \hat{a}^i$ and $y^i \neq \hat{y}^i$ but $a^j = \hat{a}^j$ and $y^j = \hat{y}^j$, for $j \in I \setminus \{i\}$.
- For stationarity, consider $x, \hat{x} \in \mathcal{X}$ and $a, \hat{a} \in \mathcal{A}$ such that $x^i \neq \hat{x}^i$ and $a^i \neq \hat{a}^i$ but $x^j = \hat{x}^j$ and $a^j = \hat{a}^j$, for $j \in I \setminus \{i\}$.
- For Kreps and Porteus preferences, the K–P independence axiom is applied to one individual component only while all other individuals take the same action.

Proof of Proposition 1 An additive aggregation of preferences which is linear in additive representations of individuals' preferences is clearly consistent and history independent.

To prove the necessity part of Proposition 1, consider social preferences $\succeq^S_{y,t}$ on $\mathcal{X} \times \mathcal{A}$ defined for any $a_t, \hat{a}_t \in \mathcal{A}$ by $a_t \succeq^S_{y,t} \hat{a}_t \Longleftrightarrow U^S_{y,t}(a_t) \geq U^S_{y,t}(\hat{a}_t)$. By history independence they can be expressed as $\succeq_t^S$ and we know that the factors $\mathcal{A}$ are separable for $\succeq_t^S$. Pareto's axiom implies that for any $i \in I$,the set $\mathcal{X}^i \times \mathcal{A}^i$ is separable for $\succeq_t^S$. Since I assume that individuals' utility functions are not constant, each $\mathcal{X}^i \times \mathcal{A}^i$ and $\mathcal{A}$ are essential.

By Gorman's (1968) theorem, all of the following are separable and essential: $\mathcal{X}^i$, $\mathcal{A}^i$ and $\mathcal{X}^i \times (\mathcal{A} \setminus \mathcal{A}^i)$.[8] For $i \neq j$, $\mathcal{X}^i \times (\mathcal{A} \setminus \mathcal{A}^i)$ and $\mathcal{X}^j \times (\mathcal{A} \setminus \mathcal{A}^j)$ are hence separable and essential so that $\mathcal{X}^j \times \mathcal{A}^i$ and $\mathcal{X}^i \times \mathcal{A}^j$ are separable and essential.[9] By appropriate unions and intersections of these sets, it can be showed that any subset of factors in $\mathcal{X} \times \mathcal{A}$ is separable and essential for $\succeq_t^S$. By Debreu's (1959) theorem, there must exist real-valued functions u_t^i on X and W_t^i on A such that for any (x_t, a_{t+1}) and $(\hat{x}_t, \hat{a}_{t+1})$ in $\mathcal{X} \times \mathcal{A}$:

$$(x_t, a_{t+1}) \succeq_t^S (\hat{x}_t, \hat{a}_{t+1}) \Longleftrightarrow \sum_{i \in I} u_t^i(x_t^i) + \sum_{i \in I} W_t^i(a_{t+1}^i)$$
$$\geq \sum_{i \in I} u_t^i(\hat{x}_t^i) + \sum_{i \in I} W_t^i(\hat{a}_{t+1}^i)$$

Consider (x_t, a_{t+1}) and $(\hat{x}_t, \hat{a}_{t+1})$ such that $(x_t^i, a_t^i) \neq (\hat{x}_t^i, \hat{a}_t^i)$ while, for all $j \neq i$, $(x_t^j, a_t^j) = (\hat{x}_t^j, \hat{a}_t^j)$. By Pareto's Axiom, $U_t^i(x_t^i, a_{t+1}^i) \geq U_t^i(x_t^i, a_{t+1}^i) \Longleftrightarrow (x_t, a_{t+1}) \succeq_t^S (\hat{x}_t, \hat{a}_{t+1}) \Longleftrightarrow u_t^i(x_t^i) + W_t^i(a_{t+1}^i) \geq u_t^i(\hat{x}_t^i) + W_t^i(\hat{a}_{t+1}^i)$. Hence $\tilde{U}_t^i(x_t^i, a_t^i) = u_t^i(x_t^i) + W_t^i(a_{t+1}^i)$ must represent individual i's preferences on $M_\delta(X \times A)$ at time t. Moreover the sum of these utilities, $\tilde{U}_t^S = \sum_{i \in I} \tilde{U}_t^i$, represents social preferences.

Individual preferences are represented by $\tilde{U}_t^i$: they must be history independent. They must also be consistent in view of the Lemma 1. We hence know that $W_t^i(a_{t+1}^i)$ must represent individual i's preferences on A at time $t+1$.

In principle, we could have $W_t^i = \phi_t^i \left[\tilde{U}_{t+1}^i\right]$, with ϕ_t^i a strictly increasing continuous function. But, for social preferences to be consistent, we also need $\sum_{i \in I} W_t^i$ to represent the same preferences as $\tilde{U}_{t+1}^S$ so that for any $a_{t+1} \in \mathcal{A}$, $\sum_{i \in I} \phi_t^i \left[\tilde{U}_{t+1}^i(a_{t+1}^i)\right] = \psi_t \left[\tilde{U}_{t+1}^S(a_{t+1}^i)\right] = \psi_t \left[\sum_{i \in I} \tilde{U}_{t+1}^i(a_{t+1}^i)\right]$. Denoting $z_{t+1}^i \equiv \tilde{U}_{t+1}^i(a_{t+1}^i)$, this yields the Pexider functional equation $\sum_{i \in I} \phi_t^i \left[z_{t+1}^i\right] = \psi_t \left[\sum_{i \in I} z_{t+1}^i)\right]$ whose solution is (Aczél 1966 Theorem 1 and Corollary p. 142): $\psi_t(z) = \beta_t z + \gamma_{t+1}$ and, for all $i \in I$, $\phi_t^i(z_t^i) = \beta_t z_{t+1}^i + \gamma_{t+1}^i$, with $\gamma_{t+1} = \sum_{i \in I} \gamma_{t+1}^i$ and $\beta_t \in \mathbb{R}_+$.

To sum up: $\bar{U}_t^i(x_t^i, a_{t+1}^i) = u(x_t^i) + \beta_t \bar{U}_{t+1}^i(a_{t+1}^i)$, where $\bar{U}_t^i = \tilde{U}_t^i + \frac{\gamma_t^i}{\beta_{t-1}}$, represents individual i's preferences. Individuals' preferences are additive. And $\bar{U}_t^S = \sum_{i \in I} \bar{U}_t^i$ represents social preferences so that the planner also has additive preferences on $\mathcal{X} \times \mathcal{A}$.

[8] To obtain this result, denote $I = \mathcal{X}^i$, $K = \mathcal{A}^i$, $L = \mathcal{A} \setminus \mathcal{A}^i$ so that $I \cup K = \mathcal{X}^i \times \mathcal{A}^i$ and $L \cup K = \mathcal{A}$ and apply Gorman's theorem.

[9] This result is obtained by taking $I = \mathcal{X}^i \times \mathcal{A}^j$, $K = \mathcal{A} \setminus (\mathcal{A}^i \times \mathcal{A}^j)$, $L = \mathcal{X}^j \times \mathcal{A}^i$ and applying Gorman's theorem.

Proof of Proposition 2 The sufficiency part of the proposition is obvious.

For the necessity part, note first that by Lemma 1, individuals must have stationary preferences. Note also that social preferences are consistent and stationary, hence consistent and history independent. Thus, Proposition 1 applies and individual preferences are additive. The only additive and stationary preferences are those exhibiting exponential discounting: there exist functions u^i and U^i and a constant $\beta_i \in \mathbb{R}_+$ such that for any $i \in I$, $x^i \in X$ and $a^i \in A$, $U^i(x^i, a^i) = u^i(x^i) + \beta_i U^i(a^i)$ represents individual i's preferences. $U^S(x, a) = \sum_{i \in I} U^i(x^i, a^i)$ represents social preferences on $\mathcal{X} \times \mathcal{A}$. But for this aggregation to be stationary, it is necessarily that $\beta_i = \beta$, for all $i \in I$.

Proof of Proposition 3 By Lemma 1, all individuals must have history independent K–P preferences. Their utility functions are thus defined on A by Eq. 5. Denote v_t^i an aggregator function for individual i at time t. $U_t^i(a_t^i) = E_{a_t^i} v_t^i$ represents individual i's preferences on A. Similar notation are used to describe the planner's preferences on $\mathcal{A}$.

Restricting attention to the subset $M_\delta(\mathcal{X} \times \mathcal{A})$, we can apply Proposition 1. So there exists utility functions $\tilde{v}_t^i$ representing individuals' preferences such that $\tilde{v}_t^i(x_t^i, a_{t+1}^i) = u_t^i(x_t^i) + \beta_t \Phi_t^i \left(U_{t+1}^i(a_{t+1}^i)\right)$ for some increasing functions Φ_t^i and $\tilde{v}_t^S = \sum_{i \in I} \tilde{v}_t^i$ represents social preferences. The functions $\tilde{v}_t^i$ are ordinally equivalent to the aggregator function v_t^i so there exist increasing functions Ψ_t^i such that $v_t^i \equiv \Psi_t^i \circ \tilde{v}_t^i$. There also exists an increasing functions Ψ_t^S such that $v_t^S \equiv \Psi_t^S \circ \tilde{v}_t^S$.

Now the time consistency of social preferences implies that $\sum_{i \in I} \Phi_t^i \left(U_{t+1}^i(a_{t+1}^i)\right)$ must be ordinally equivalent to $U_{t+1}^S(a_{t+1}^S) = E_{a_{t+1}} \left[v_{t+1}^S(x_{t+1}^S, a_{t+2}^S)\right]$. Hence there exists an increasing function F_{t+1}^S such that:

$$
\begin{aligned}
F_{t+1}^S & \left(\sum_{i \in I} \Phi_t^i \left(E_{a_{t+1}^i} \left[v_{t+1}^i(x_{t+1}^i, a_{t+2}^i)\right]\right)\right) \\
&= F_{t+1}^S \left(\sum_{i \in I} \Phi_t^i \left(U_{t+1}^i(a_{t+1}^i)\right)\right) \\
&= E_{a_{t+1}} \left[v_{t+1}^S(x_{t+1}^S, a_{t+2}^S)\right] \\
&= E_{a_{t+1}} \left[\Psi_{t+1}^S \left(\sum_{i \in I} (\Psi_{t+1}^i)^{-1} \circ v_{t+1}^i(x_{t+1}^i, a_{t+2}^i)\right)\right]
\end{aligned}
$$

Consider simple Bernoulli distributions yielding $(\bar{x}_{t+1}, \bar{a}_{t+2})$ with probability p and $(\underline{x}_{t+1}, \underline{a}_{t+2})$ with probability $1 - p$. Denote $w^i = p\ v_{t+1}^i(\bar{x}_{t+1}^i, \bar{a}_{t+2}^i)$, $z^i = (1 - p)\ v_{t+1}^i(\underline{x}_{t+1}^i, \underline{a}_{t+2}^i)$, $\bar{\Psi}_{t+1}^i = p\ \Psi_{t+1}^i$, $\underline{\Psi}_{t+1}^i = (1-p)\ \Psi_{t+1}^i$, $\bar{\Psi}_{t+1}^S = p\ \Psi_{t+1}^S$ and $\underline{\Psi}_{t+1}^S = (1-p)\ \Psi_{t+1}^S$, the above equalities yield the following general Pexider–Sincov functional equation:

$$
F_{t+1}^S \left(\sum_{i \in I} \Phi_t^i \left(w^i + z^i\right)\right) = \bar{\Psi}_{t+1}^S \left(\sum_{i \in I} (\bar{\Psi}_{t+1}^i)^{-1}(w^i)\right) + \underline{\Psi}_{t+1}^S \left(\sum_{i \in I} (\underline{\Psi}_{t+1}^i)^{-1}(z^i)\right)
$$

The solution of this equation (Aczél 1966, Theorem 1 and Corollary p. 302) implies that for any $z \in \mathbb{R}^I$, $F^S_{t+1}\left(\sum_{i\in I}\Phi^i_t(z^i)\right) = \sum_{i\in I} b^i_t z^i + c$ for some positive real numbers a^i_t and constant c. Hence, functions Φ^i_t are linear. It must also be the case that functions Ψ^i_t are linear so that functions v^i_t can be written $v^i_t(x^i_t, a^i_{t+1}) = u^i_t(x^i_t) + \beta_t U^i_{t+1}(a^i_{t+1})$. By Kreps and Porteus (1978) result, we know that all individuals have vNM preferences. Besides the Bernoulli utility functions v^i_t are time additive. Finally, functions $\tilde{v}^i_t$ (resp. $\tilde{v}^S_t$) are also Bernoulli utility functions because they are equal to v^i_t (resp. v^S_t) up to a positive affine transformation. Hence, there exists a Bernoulli utility function for the social planner, namely $\tilde{v}^i_t$, which is a sum of individual Bernoulli utility functions.

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