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Betting on Machina's reflection example: an experiment on ambiguity

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Abstract In a recent article, Machina (Am Econ Rev forthcoming, 2008) suggested choice problems in the spirit of Ellsberg (Q J Econ 75:643–669, 1961), which challenge tail-separability, an implication of Choquet expected utility (CEU), to a similar extent as the Ellsberg paradox challenged the sure-thing principle implied by subjective expected utility (SEU). We have tested choice behavior for bets on one of Machina's choice problems, the reflection example. Our results indicate that tail-separability is violated by a large majority of subjects (over 70% of the sample). These empirical findings complement the theoretical analysis of Machina (Am Econ Rev forthcoming, 2008) and, together, they confirm the need for new approaches in the analysis of ambiguity for decision making.

Keywords Ambiguity · Choquet expected utility · Experimental economics

JEL Classification C90 · D81

1 Introduction

In the past 20 years, there has been a growing attention in decision theory and decision analysis toward ambiguity (Schmeidler 1989; Gilboa and Schmeidler 1989; Camerer and Weber 1992; Fox and Tversky 1995; Halevy 2007). Simply stated, ambiguity may

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Table 1 Ellsberg acts

Acts	30 Balls	60 Balls	
	Red	Yellow	Black
f_1	50	0	0
g_1	0	0	50
$\hat{f}_1$	50	50	0
$\hat{g}_1$	0	50	50

be defined as uncertainty about unknown probability. The starting point of any study on ambiguity is Ellsberg's well-known two-color example. In this thought experiment, the decision maker has a choice between two bets: betting a sum of money on a red ball drawn from an urn with 50 red balls and 50 black balls or betting the same sum of money on a red ball drawn from a 100-balls urn with unknown numbers of red balls and black balls. Ellsberg predicted that most people would prefer to bet on the first urn and defined this behavior as ambiguity aversion. In such a case, the decision maker prefers the first urn because it provides a clear information—known probabilities—rather than a vague information—unknown probabilities—about the likelihood of receiving the sum of money.

In his 1961 article, Ellsberg also proposed the following choice problem, known as the three-color example. An urn contains 90 balls, 30 of which are red and 60 are either yellow or black in unknown proportion. One ball will be drawn at random. An act pays a particular sum of money depending on the color of the ball drawn. Table 1, above, presents four acts similar to those of Ellsberg (1961). We use different sums of money here in accordance with the design of our experiment. For example, act f_1 has two outcomes: 50 in the event of a red ball, and 0 in the event of a yellow or black ball. Similarly, g_1 pays 50 in the event of a black ball and 0 otherwise.

It has been widely documented (Slovic and Tversky 1974; McCrimmon and Larsson 1979) that people prefer act f_1 to act g_1. As an explanation similar arguments as in the two-color urn problem have been put forward: there is precise information about the likelihood of receiving 50 in act f_1, as opposed to act g_1, where the range of likelihood is between 0 and 2/3. Aversion to this lack of information about the outcome 50 in act g_1 has been identified as a potential cause for the exhibited preferences.

The same aversion to lack of information leads to a preference for act $\hat{g}_1$ over $\hat{f}_1$, because there is a precise $2/3$ chance of getting 50 in $\hat{g}_1$ whereas there is imprecise probability ranging between 1/3 and 1 of getting 50 in $\hat{f}_1$. Choice situations like these, which involve acts over events that have imprecise probabilities (e.g., the events "the drawn ball is black," "the drawn ball is yellow" or "the drawn ball is black or red", and "the drawn ball is yellow or red") have been termed ambiguous.

While most people choose act f_1 over g_1 and also prefer act $\hat{g}_1$ over $\hat{f}_1$, which has been interpreted as ambiguity aversion, there are a few people exhibiting the opposite, ambiguity seeking choice behavior (Slovic and Tversky 1974). Both patterns of choices are in contrast to what subjective expected utility (SEU) would predict. Under SEU, preferences must be consistent in the sense that f_1 is preferred to g_1 if and only if $\hat{f}_1$ is preferred to $\hat{g}_1$. This principle of consistency is a direct implication of the sure-thing principle (Savage 1954), which requires preferences to be independent of common outcomes. Hence, the choice between acts f_1 and g_1 should not depend on the common outcome that obtains if the event "the ball is yellow" occurs. More precisely, the sure-thing principle says that the common outcomes can be replaced by any other common outcomes without influencing the preference; hence, in this example, 0 can be replaced by 50 for the event "the ball is yellow". But notice that this transforms the choice problem f_1 versus g_1 into the problem $\hat{f}_1$ versus $\hat{g}_1$. This way Ellsberg uncovered a major descriptive shortcoming of Savage's SEU.

Generalizations of SEU have been developed in order to tackle the issues raised by Ellsberg and more generally to take into account sensitive behavior towards ambiguity. Among the most influential of these theories, Multiple Priors (Gilboa and Schmeidler 1989), Choquet Expected Utility (CEU: Gilboa 1987; Schmeidler 1989; Sarin and Wakker 1992), and Cumulative Prospect Theory (CPT: Tversky and Kahneman 1992; Wakker and Tversky 1993). Although we present our results in relation to a fundamental principle underlying CEU, our findings also apply to CPT because, if consequences are all gains (or all losses), CPT agrees with CEU.

The main derivations of CEU build up the idea of rank-dependence introduced for risk by Quiggin (1982). Paraphrasing Diecidue and Wakker (2001), the intuition of rank-dependence may be expressed as the fact that "the attention paid to an event depends not only on the event but also on how good the outcome yielded by the event is in comparison to the outcomes yielded by the other events". This has two main consequences. First, attitudes toward risk are no longer modeled solely through the utility function but also through the perception of risk and uncertainty. Second, not only the likelihood of an event matters but also its ranking compared to others possible events. More accurately, non-expected utility with rank-dependence (including CEU) restricts the sure-thing principle to comonotonic acts and this can explain the pattern of preference derived from the Ellsberg paradox (Chew and Wakker 1996). Comonotonicity may be defined as follows: if two acts have the same ranking of events, then any change of their common outcomes that does not change the ranking of events should leave the preference between these acts unaffected. Under the comonotonic sure-thing principle, preferences must be independent of common outcomes only for comonotonic acts. In Ellsberg's three-color example, the event "the ball is yellow" is rank-ordered differently in the two proposed choices: it is of rank 0 (associated with the worst consequence) in choice between f_1 and g_1 and of rank 1 (associated with the best consequence) in choice between $\hat{f}_1$ and $\hat{g}_1$. As a consequence, acts are not comonotonic in the Ellsberg's three-color example, and independence does not need to hold.

The comonotonic sure-thing principle is, on its own, more general than tail-separability, the condition we test experimentally. According to tail-separability if two acts have the same tail, on best or worst outcomes, then any change of their common tail should leave the preference between these acts unaffected. CEU implies both tail-separability and the comonotonic sure-thing principle. Tail-separability may be defined in two distinct ways depending on whether indifference between adjacent outcomes is allowed or not. In the former case, we have weak tail-separability, and in the latter case, strong tail-separability.

Two reasons may explain CEU's success. First, CEU keeps the main structure of SEU but introduces more realistic, but still measurable, features of individual behavior. The idea of rank-dependence inherent to CEU has proven to be capable of explaining the observed deviations from expected utility (the Allais paradox and the Ellsberg paradox). Second, CEU provides generalizations of classic results in various areas of economics such as insurance demand, portfolio choice and asset pricing, or inequality measurement (see Mukerji and Tallon 2004 for a survey). Throughout this article, we choose to concentrate on CEU as the main and most popular non-expected utility theory but our results may be extended not only to any model that implies tail-separability but also to the main Multiple Priors models (Baillon et al. 2008).

In his *reflection example* Machina (2008) modified the original three-color example of Ellsberg by adding a further imprecise probability event, a fourth color event. This modified Ellsberg urn contains 50 red or yellow balls in unknown proportion and 50 black or green balls in unknown proportion. An important aspect of the structure of the reflection example is *informational symmetry*: Notably, there are two symmetric events with precise probabilities (events "the drawn ball is red or yellow" and "the drawn ball is black or green" are equally likely) and further, within the two events "the drawn ball is red or yellow" and "the drawn ball is black or green" the ambiguity about the distribution of colors is similar. Machina illustrated that having two informationally symmetric sources of ambiguity poses serious difficulties for tail-separability under CEU. More precisely, Machina showed that a specific replacement of common outcomes at the tails of acts with other common outcomes leads to a reflected pair of acts that were informationally symmetric to the original acts, so that a preference in the former pair of acts would be reflected in the latter pair of acts, contradicting the consistency requirement under CEU.

We present details of the reflection example in Sect. 2, where we also illustrate that CEU requires consistent choice behavior between acts and their reflected, informationally symmetric, acts. We tested these predictions in our experiment and found that more than 70% of subjects violate models implying weak tail-separability suggesting sensitivity to the informationally symmetric structure in Machina's choice problem.

The remainder of the article is organized as follows: Section 2 presents the theoretical and conceptual framework. Section 3 details the experimental studies. We report the findings in Sect. 4. In Sect. 5 we confront the results in respect to CEU and attitudes toward ambiguity. Section 6 concludes.

2 Framework

In this section, we recall the reflection example of Machina (2008). To have a clear picture of the challenge for the non-expected utility theories, including the rank-dependent theories, it is important to introduce some notation. We recall briefly the classical SEU model of Savage (1954) and present the sure-thing principle that underlies this theory before we look at variations of this property that underpin rank-dependent theories.

As in the framework of Savage, we assume a state space S, subsets of which we call events. An act f assigns to each event a consequence. For simplicity of exposition, the set of consequences is $\mathbb{R}$, designating money. For our purposes it will be sufficient to look at simple acts, that is, acts that have only finitely many consequences. An act, therefore, can be represented as $f = (E_1, x_1;\ldots;E_n, x_n)$ for a natural number n, with the understanding that x_i is obtained if event E_i is true. With this notation it is implicitly assumed that the collection of events $\{E_1;\ldots;E_n\}$ forms an (ordered) partition of the state space S, that is, they are mutually exclusive and exhaustive.

We assume a preference $\succsim$ over acts, denoting weak preference, and we adopt the usual notation $\succ$ and $\sim$ for strict preference and indifference, respectively ($\precsim$ and $\prec$ denote corresponding reversed preferences). Next we look at different models to evaluate acts such that the assigned values allow for a comparison of acts in agreement with the preference $\succsim$. That is, we consider functions V that assign to each act a real value such that $V(f) \geq V(g)$ whenever $f \succsim g$, for any acts f, g.

2.1 Subjective expected utility

Subjective expected utility holds if each act $f = (E_1, x_1;\ldots;E_n, x_n)$ is evaluated by $\sum_{i=1}^{n} p(E_i)u(x_i)$. Here p is a uniquely defined probability measure and the continuous and strictly increasing utility function u, which assigns to each consequence a real number, is cardinal.

We write $h_E f$ for the act that agrees with h if event E obtains and otherwise agrees with the act f. A necessary condition for SEU is that the preference satisfies the sure-thing principle:

$$h_E f \succsim h_E g \Leftrightarrow h'_E f \succsim h'_E g,$$

for all acts $h_E f$,$h_E g$, $h'_E f$, and $h'_E g$. Thus, under SEU, the preference between any two acts is independent of consequence-event pairs that are common.

2.2 Choquet expected utility

Choquet expected utility holds if each act $f = (E_1, x_1;\ldots;E_n, x_n)$ is evaluated by $\sum_{i=1}^{n} \pi(E_j : x_j \geq x_i)u(x_i)$. The utility under CEU is also cardinal, like under SEU. The difference between the two models consists in the weights that precede utility when evaluating an act. The weights $p(E_i)$ under SEU are generated by a probability measure, thus, an additive measure on the state space S, while the weights $\pi(E_j : x_j \geq x_i)$ under CEU are generated by a (possibly) non-additive measure. This capacity v, assigns weight 0 to the empty set and weight 1 for the entire state space S and is monotonic (i.e., $v(E_j \cup E_i) \geq v(E_i)$ for all events $E_j, E_i \in S$). The decision weights $\pi(E_j : x_j \geq x_i)$, $i = 1, \ldots, n$, are defined as follows: Take any permutation ρ of $\{1, \ldots, n\}$ such that $x_{\rho(1)} \geq \cdots \geq x_{\rho(n)}$. Then, $\pi(E_j : x_j \geq x_i) = v(\cup_{x_{\rho(k)} \geq x_i} E_{\rho(k)}) - v(\cup_{x_{\rho(k)} \geq x_i, \rho(k) \neq i} E_{\rho(k)})$, $i = 1, \ldots, n$.
Like the probability measure p under SEU, the capacity v is uniquely determined under CEU.

A necessary property of rank-dependent utility models is *tail-separability*. It states that, if two acts share a common tail, then this tail can be modified without altering the preference between the acts. Let us formally introduce *weak* tail-separability:

$$h_E f \succsim h_E g \Leftrightarrow h'_E f \succsim h'_E g,$$

for all acts $h_E f$,$h_E g$, $h'_E f$, and $h'_E g$, such that either all outcomes that obtain under event E are ranked weakly above those of f and of g or all outcomes that obtain under event E are ranked weakly below those of f and of g. *Strong* tail-separability holds if we require the previous equivalence to hold whenever all outcomes that obtain under event E are ranked strictly above those of f and of g or all outcomes that obtain under event E are ranked strictly below those of f and of g.

The two variants of tail-separability, both being implications of the sure-thing principle, are equivalent if other standard assumptions are invoked. These assumptions are required under SEU and CEU, under CPT and under the outcome-dependent capacity model of Chew and Wakker (1996). All these models imply both forms of tail-separability.

At this stage, it is important to clarify the extent to which our experimental results apply. While we are providing experimental evidence against weak tail-separability our tests do not say anything about strong tail-separability. This point will be further illustrated in Sect. 2.3 when we review Machina (2008) refection example. Moreover, we like to note here that the specific test of weak separability that we focus on is not exclusively a test of CEU and the other rank-dependent theories. Because we test weak tail-separability only by looking at extreme consequences, the results we obtain also provide a test, and as we show a challenge, for other "separable" models that have been put forward in the literature on ambiguity (for a more detailed discussion see Baillon et al. 2008). However, given the popularity of CEU for the analysis of ambiguity, we present our results in relation to (weak) tail-separability. As an illustration of the general property that we are testing, consider the following four acts (assuming that outcomes are ordered from best to worst and that $x_{j-1} \geq y_j$ and $z_{j-1} \geq x_j$):

$$\begin{aligned}
h_1 &= (G_1, z_1; \ldots; G_{j-1}, z_{j-1}; E_j, x_j; \ldots; E_n, x_n)\\
h_2 &= (G_1, z_1; \ldots; G_{j-1}, z_{j-1}; F_j, y_j; \ldots; F_n, y_n)\\
h_3 &= (G_1, x_1; \ldots; G_{j-1}, x_{j-1}; E_j, x_j; \ldots; E_n, x_n)\\
h_4 &= (G_1, x_1; \ldots; G_{j-1}, x_{j-1}; F_j, y_j; \ldots; F_n, y_n)
\end{aligned}$$

The two acts h_1 and h_2 share common outcomes on the same events that give outcomes strictly better than x_j (ie: $z_1, \ldots, z_{j-1}$); they have a *common upper tail* (G_1, $z_1; \ldots; G_{j-1}, z_{j-1}$). Similarly, acts h_3 and h_4 have a common upper tail (G_1, $x_1; \ldots; G_{j-1}, x_{j-1}$). Due to tail-separability, a preference for h_1 over h_2 implies the preference for h_3 over h_4 (see Appendix 1 for a proof). The two acts h_2 and h_4 share common outcomes on the same events that give outcomes strictly lower than z_{j-1} and x_{j-1}; they have a *common lower tail* ($F_j, y_j; \ldots; F_n, y_n$). Similarly, acts h_1 and h_3 have a common lower tail ($E_j, x_j; \ldots; E_n, x_n$). The same reasoning as before applies and the tail-separability property implies $h_1 \succsim h_2 \Leftrightarrow h_3 \succsim h_4$. In other words, in both cases the preference is determined by the tail on which acts differ.

One should remark here that the common tail may not need to be maximal i.e., must not contain the whole sequence. (F_n, y_n) is a common lower tail for acts h_2 and h_4, but also any tail formed by the partition $\{F_n \backslash H, H\}$ of event F_n. Then the common tail (F_n, y_n) can be replaced by the common tail $(F_n \backslash H, w_n, H, w_{n+1})$ if $w_{n+1} \leq w_n \leq z_{j-1}$. Such a replacement of a common tail is possible only if outcomes are not required to be strictly rank-ordered: $x_1 \geq \cdots \geq x_n$. If outcomes are strictly rank-ordered: $x_1 > \cdots > x_n$, then it is not possible to split an event further. The former defines *weak tail-separability* which is the matter of this article; the latter, *strong tail-separability* is beyond our scope.

2.3 The reflection example

Machina (2008) presented two new choice problems, namely the 50:51 example and the reflection example. In our experiment we investigate the latter through an urn containing 20 identical balls except for color. Ten of these balls are red or yellow in unknown proportion, and the remaining 10 are black or green in unknown proportion. One ball is drawn at random from the urn. Acts that give different outcomes depending on the color drawn are described in Tables 2 and 3. The choice pattern $f_t \succ g_t$ and $\hat{f}_t \succ \hat{g}_t$ is shortly designated by $f_t \hat{f}_t$. Index t refers to choices between bets of a table t.

$\hat{f}_2(\hat{g}_2)$ is obtained from $f_2(g_2)$ as follows. Suppose that $g_2 \succsim f_2$ is observed (the arguments below also apply if $f_2 \succsim g_2$ is assumed). We rewrite this to highlight the common lower tails $(G; 25; R; 0)$ and $(R; 0)$ of g_2 and f_2.

Table 2 The reflection example with lower tail shifts

Acts	10 Balls			
	R	Y	B	G
f_2	0	50	25	25
g_2	0	25	50	25
$\hat{f}_2$	25	50	25	0
$\hat{g}_2$	25	25	50	0

Table 3 The reflection example with Upper Tail Shifts

Acts	10 Balls			
	R	Y	B	G
f_3	50	50	25	75
g_3	50	25	50	75
$\hat{f}_3$	75	50	25	50
$\hat{g}_3$	75	25	50	50

$$g_2 \succsim f_2 \Leftrightarrow (B, 50;\ Y, 25;\ G, 25;\ R, 0) \succsim (Y, 50;\ B, 25;\ G, 25;\ R, 0).$$

Now we replace only the common tail $(R, 0)$ with the common tail $(R, 25)$ and obtain

$$\begin{aligned}&(B, 50;\ Y, 25;\ G, 25;\ R, 0) \succsim (Y, 50;\ B, 25;\ G, 25;\ R, 0)\\ &\quad\Leftrightarrow (B, 50;\ Y, 25;\ G, 25;\ R, 25) \succsim (Y, 50;\ B, 25;\ G, 25;\ R, 25).\end{aligned}$$

As remarked in the previous section, this latter replacement is allowed under the weak tail-separability. Under the strong tail-separability the common tail $(R, 0)$ can only be replaced by a common tail (R, z) with $z < 25$, hence, to get the above equivalence, one would need to invoke additional preference conditions like outcome-continuity. However, notice something typical for rank-dependence with weakly rank-ordered outcomes: we can rewrite the acts in the last indifference by interchanging the order of the events G and R without affecting the preference between those acts. This would not be possible when outcomes are required to be strictly rank-ordered. Hence, we obtain the equivalence:

$$\begin{aligned}&(B, 50;\ Y, 25;\ G, 25;\ R, 25) \succsim (Y, 50;\ B, 25;\ G, 25;\ R, 25)\\ &\quad\Leftrightarrow (B, 50;\ Y, 25;\ R, 25;\ G, 25) \succsim (Y, 50;\ B, 25;\ R, 25;\ G, 25).\end{aligned}$$

where the last two acts have common lower tails $(R, 25;\ G, 25)$ and $(G, 25)$. We, finally, replace the common tail $(G, 25)$ with the common tail $(G, 0)$ and obtain

$$\begin{aligned}&(B, 50;\ Y, 25;\ R, 25;\ G, 25) \succsim (Y, 50;\ B, 25;\ R, 25;\ G, 25)\\ &\quad\Leftrightarrow (B, 50;\ Y, 25;\ R, 25;\ G, 0) \succsim (Y, 50;\ B, 25;\ R, 25;\ G, 0).\end{aligned}$$

where the last preference is equivalent to $\hat{g}_2 \succsim \hat{f}_2$.

Notice, that the exercise of replacing common lower tails with other common tails, which transforms the choice problem "f_2 vs. g_2" into "$\hat{f}_2$ vs. $\hat{g}_2$" has also led to a replacement of known probability events with unknown probability events when going from f_2 to $\hat{f}_2$. It has also led to the opposite reflected replacement of unknown probability events with known probability events when going from g_2 to $\hat{g}_2$. That is, the precise information that the likelihood of obtaining 25 in the event "the drawn ball is black or green" in act f_2 has now changed into the ambiguous information that the likelihood of obtaining 25 ranges between 0 and 1 in act $\hat{f}_2$. Similarly, the imprecise information that the likelihood of obtaining 25 ranges between 0 and 1 in act g_2 has now been changed into the precise information that the likelihood of obtaining 25 in the event "the drawn ball is red or yellow" in act $\hat{g}_2$.

Observe that there is informational symmetry when comparing acts f_2 and $\hat{g}_2$: There is a 50% chance of getting 25 in each act and an imprecise probability of getting 50 or 0. Except for the names of the corresponding events there is no informational asymmetry about the outcomes of the respective acts. Likewise, there is informational symmetry between acts $\hat{f}_2$ and g_2: There is an imprecise probability p ranging between 0 and 1/2 of getting 0, an imprecise probability q ranging between 0 and 1/2 of getting 50, and an imprecise probability, $1 - p - q$, of getting 25. So, weak tail-separability at the lower tail has reflected the ambiguity that may have influenced a preference for

f_2 over g_2 into a similar situation of ambiguity that may influence a choice of $\hat{g}_2$ over $\hat{f}_2$.

To see why this reflection poses a problem for CEU, assume that utility of 0 is 0, and consider the choice pattern $f_2\hat{g}_2$ (note that a similar argument applies for $g_2\hat{f}_2$). Substitution of CEU gives

$$\begin{aligned} f_2 \succ g_2 \Rightarrow v(Y)u(50) &+ [v(Y \cup B \cup G) - v(Y)]u(25) \\ &> v(B)u(50) + [v(Y \cup B \cup G) - v(B)]u(25) \end{aligned}$$

and

$$\begin{aligned} \hat{g}_2 \succ \hat{f}_2 \Rightarrow v(B)u(50) &+ [v(R \cup Y \cup B) - v(B)]u(25) \\ &> v(Y)u(50) + [v(R \cup Y \cup B) - v(Y)]u(25) \end{aligned}$$

Consequently,

$$f_2 \succ g_2 \Rightarrow v(Y) > v(B) \tag{1}$$

$$\hat{g}_2 \succ \hat{f}_2 \Rightarrow v(B) > v(Y) \tag{2}$$

Because the revealed beliefs (1) and (2) are contradictory, informational symmetry leads to preferences $f_2\hat{g}_2$ or $g_2\hat{f}_2$ that are not compatible with weak tail-separability. A CEU decision maker should exhibit either $f_2\hat{f}_2$ or $g_2\hat{g}_2$. A CEU decision maker who furthermore follows informational symmetry should exhibit $f_2 \sim g_2$ and $\hat{f}_2 \sim \hat{g}_2$.

Table 3 shows the reflection example with upper tail shifts. Acts $\hat{f}_3$ and $\hat{g}_3$ are obtained from events f_3 and g_3 by an ordered sequence of upper tail shifts. $\hat{f}_3$ is obtained from f_3 by two successive shifts. First, a shift of the payoffs in event G from €75 down to €50. Second, a shift of the payoffs in event R from €50 up to €75. The same applies for the way $\hat{g}_3$ is obtained from g_3. As previously noted, these shifts also create a mirror-image effect by making f_3 (g_3) symmetric with $\hat{g}_3$ ($\hat{f}_3$). As a consequence, choice patterns which correspond to informational symmetry are $f_3\hat{g}_3$ or $g_3\hat{f}_3$ while strict choice patterns which correspond to CEU are $f_3\hat{f}_3$ or $g_3\hat{g}_3$. One may note that $f_3 \sim g_3$ and $\hat{f}_3 \sim \hat{g}_3$ are the only choice patterns consistent both with informational symmetry and CEU.

Choice situations presented above enable us to test for preference conditions that allow discriminating between behaviors which are consistent with CEU with weak tail-separability and behaviors which follow informational symmetry. More accurately, any strict choice pattern consistent with informational symmetry violates weak tail-separability and is therefore a preference reversal under a CEU representation based on such hypothesis.

2.4 Proper criteria to analyze ambiguity

In an earlier draft of his article, Machina proposed three criteria to analyze ambiguity for the reflection example. Depending on the criteria retained, a decision maker's behavior may or may not be compatible with weak tail-separability. In what follows, we refer to acts of Table 2 to highlight these aspects.

- *Individual payoffs*: Acts f_2 and g_2 ($\hat{f}_2$ and $\hat{g}_2$) offer €0 on the same events R (G) and €50 on equally ambiguous events Y and B. The difference between f_2 and g_2 ($\hat{f}_2$ and $\hat{g}_2$) lies in the fact that f_2 ($\hat{g}_2$) offers the intermediary outcome €25 with probability one-half while g_2 ($\hat{f}_2$) offers the same outcome with a probability that can range from 0 to 1. When considering individual payoffs, the main difference between f_2 ($\hat{g}_2$) and g_2 ($\hat{f}_2$) is based on the nature of the intermediary outcome. For acts f_2 ($\hat{g}_2$), intermediate outcome is not ambiguous while this is the case for acts g_2 ($\hat{f}_2$). Thus, a decision maker who is ambiguity-averse in terms of individual payoffs would rather choose $f_2\hat{g}_2$. If ambiguity is defined as *uncertainty about probability, created by missing information that is relevant and could be known* (Frisch and Baron 1988), then the probability of winning €25 is the missing information that is relevant and could be known. In that sense, ambiguity in terms of individual payoffs coincides with Camerer and Weber's (1992) "ambiguity about probability". The difference is that ambiguity about probability is defined between urns in the Ellsberg's (1961) two-color problem whereas ambiguity in terms of individual payoffs is defined within the reflection example.
- *Decumulative payoff events*: As before, the best outcome €50 is equally ambiguous under f_2 and g_2 ($\hat{f}_2$ and $\hat{g}_2$) and the worst outcome €0 is placed on the same event R (G). A closer look at decumulative payoff events shows that f_2 and g_2 yield €25 or more on the same event $Y \cup B \cup G$. In terms of decumulative payoff events, this event is equally ambiguous across f_2 and g_2 (and $R \cup Y \cup B$ is also equally ambiguous between $\hat{f}_2$ and $\hat{g}_2$), the missing information being the same between acts. As a consequence, if ambiguity is defined in terms of decumulative payoff events a decision maker would be indifferent between $f_2(\hat{f}_2)$ and $g_2(\hat{g}_2)$. We note that CEU maximizers who follow informational symmetry fall in this category. Decision makers who are indifferent between acts exhibit consistent beliefs, and hence reveal no preference reversal. If one considers that following informational symmetry is a necessary condition for a rational choice then indifference is the only behavior consistent with CEU. If individuals are not sensitive to informational symmetry but CEU maximizers, they would exhibit strict preferences ($f_2\hat{f}_2$ or $g_2\hat{g}_2$).
- *Exposure to ambiguity*: f_2 ($\hat{g}_2$) concentrates ambiguity on the 10 yellow or red balls, whereas g_2 ($\hat{f}_2$) concentrates this amount over the 20 balls. The missing information that is relevant to the decision is concentrated within the set of 10 yellow or red balls in f_2 ($\hat{g}_2$) whereas it is distributed over the whole urn in g_2 ($\hat{f}_2$). Thus, an individual who is averse to exposure to ambiguity minimize the concentration of missing information and prefers to span ambiguity over the 20 balls rather than over only 10 balls. Then, she will choose $g_2\hat{f}_2$.

It is worth noticing that, while the second criterion allows for behavior consistent with CEU, the first and third criteria violate weak tail-separability. If one considers that informational symmetry is inherent to the urn and this, independently of any specification of the acts, then a CEU maximizer should be indifferent between both pairs of acts and should satisfy the second criteria only. It appears, therefore, that CEU (and other models aiming to model ambiguity) may not be appropriate for dealing with all patterns of preferences.

The following subsection describes an experimental study mainly based on Machina's (2008) proposal which aims at testing the validity of the first and third criteria. The validity of the second criteria is discussed in Sect. 5.1 through a specific replication of the main experiment.

3 Experiment

Four groups of subjects (94 students: 39 females, and 55 males) enrolled in economics courses at IUFM and Ecole Centrale Paris participated in this experiment. Most of the students were acquainted with probability theory but they had no explicit training in decision theory. The experiment consisted of a pencil and paper questionnaire. Subjects were presented with choice-situations described in the above three tables; each choice-situation was described as the corresponding urn with balls of different colors, and a picture of the urn was also displayed. Subjects could read the composition of the urn and were asked to choose between two options labeled A and B (See Fig. 1 for typical display).

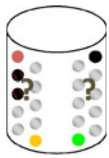
As an introduction, subjects were told there were no right or wrong answers, and they had to choose the alternative they preferred. In order to increase motivation, we introduced a random incentive mechanism similar to the one used by Camerer and Ho (1994), Harrison et al. (2007a,b). The mechanism worked as follows. In each of the four groups, one of the subjects was randomly selected from that group. Only for

Choice Task n°3

The following urn contains 20 balls:

-10 red or yellow balls in unkown proportion

-10 black or green balls in unkown proportion



A ball will be drawn at random within this urn

Which situation do you choose?

Situation A		Situation B	
If the ball is **yellow**,	you receive 50 €	If the ball is **black**,	you receive 50 €
If the ball is **black or green**,	you receive 25 €	If the ball is **green or yellow**,	you receive 25 €
If the ball is **red**,	you receive 0 €	If the ball is **red**,	you receive 0 €

☐ ☐

Fig. 1 A typical display used in the experiment (indifference not allowed) these subjects one of their task was selected and their choice was played for real, and each selected subject could win up to €75 depending on her responses. Subjects were informed about the mechanism prior to the experiment . There was no time constraint. We controlled for order effects, permuting situations A and B (two groups: 49 students for one and 45 for the other). Moreover, for each subject, we also controlled for color effect in order to guarantee that we effectively captured a preference toward an alternative rather than a preference for a particular color. Thus, we replicated choice situations of Tables 2 and 3 by reversing payoffs between colors. This prevented subjects from thinking that the ratios of colors were chosen to bias the bets in favor of the experimenters.

In this specific experiment, indifference between options A and B was not allowed. Two reasons justify such a protocol that forces subjects to express outright choice for one of the two options. First, using strict preference patterns, the protocol generates a sharp distinction between behavior consistent with CEU and behavior consistent with informational symmetry. By making these two behaviors mutually exclusive, we obtained a direct and clear test on the possibility—or the difficulty—to observe a paradox for CEU. Second, the absence of indifference avoids certain choice behaviors such as randomization or indecisiveness. If individuals are subject to randomization or indecisiveness, indifference may be viewed as a way to escape from the choice problem and not as an equivalence judgment between the two options. Not allowing for indifference is therefore a first step to test whether CEU could be prone to a paradox. Section 5.1 discusses the importance of the indifference hypothesis and presents results from a replication of the experiment on another set of 42 subjects. We found that, even if indifference is allowed, more than 90% of subjects still express strict preferences.

4 Results

4.1 Confirming Ellsberg paradox

A first result is that 65% of subjects exhibit a preference reversal against the SEU prediction. This result confirms the classic Ellsberg paradox and replicates the most commonly observed choice pattern in the three-color example. For example, Slovic and Tversky (1974) also find that 65% of subjects ($n = 29$) violate SEU (percentage raises to 72% after subjects have received arguments pro and con SEU). McCrimmon and Larsson (1979) found 79% of answers ($n = 19$) inconsistent with SEU.

4.2 Informational symmetry

Table 4 summarizes subjects' choices between acts in Tables 2 and 3 described above. For each pair of bets, the table gives the number of subjects that chose each of the four possible patterns of choice. The table also provides the proportion of preference reversals observed under weak tail-separability and the significance of this proportion as compared to one-half through the p-value of a binomial test.

Informational symmetry, which is due both to the symmetric structure of the urn and the symmetry between acts, is not violated by a significant proportion of subjects. Indeed, information symmetric behavior corresponding to following patterns $f_t\hat{g}_t$ or $g_t\hat{f}_t$ is exhibited by 74% of subjects in the lower tail case and 86.5% in the upper tail case. A necessary condition for CEU to accommodate informational symmetry is $f_t \sim g_t$ and $\hat{f}_t \sim \hat{g}_t$. The results provide evidence against weak tail-separability and highlight the relevance of informational symmetry in Machina's reflection example.

4.3 A paradox for Choquet expected utility

Table 4 shows that violations of weak tail-separability in Machina's reflection example are greater than those observed in Ellsberg urns under SEU. All percentages of preference reversals are above 70% and all are significantly different from 0.5. In the same vein, Wu (1994) empirically finds that more than 50% of subjects violate the ordinal independence axiom under risk, and consequently, that the rank-dependent expected utility model is not sufficient to explain the observed behavior. At odds with our results, Fennema and Wakker (1996) find that only 25% of subjects violate upper tail-separability under uncertainty. We used the Conlisk's D statistic (Conlisk 1989) to test and compare preference reversals between upper and lower-tail separability. Interestingly, we observe a greater amount of preference reversal under CEU with upper tail-separability than with lower tail-separability (Conlisk's D statistic is $D = 2.12$, $p = 0.02$ for treatment 1, $D = 2.28$, $p = 0.01$ for treatment 2). An ANOVA with repeated measures rejects the equality of preference reversals proportions across the four situations (p-value$\,=\,$0.0027).

4.4 An empirically consistent approach for ambiguity

The experiment provides an empirical complement to the three criteria proposed by Machina to analyze ambiguity. Table 4 shows that the prevailing pattern of observed choice is $f_t\hat{g}_t$ (for 50% of the sample on average), agreeing with the idea of ambiguity aversion based on individual payoffs. Moreover, about 30% of the subjects exhibit the

Table 4 Subjects' choices and preference reversals

Modified Ellsberg acts	Color treatment	$f_t\hat{f}_t$	$f_t\hat{g}_t$	$g_t\hat{g}_t$	$g_t\hat{f}_t$	Reversal (%)[a]	p-value
Table 2 (Lower tail shifts)	1	11	44	15	24	72	0.000
	2[b]	10	43	13	28	76	0.000
Table 3 (Upper tail shifts)	1	8	47	6	33	85	0.000
	2[b]	4	54	7	29	88	0.000

[a] Percentage of reversal is given by the percentage of subjects exhibiting $f_t\hat{g}_t$ or $g_t\hat{f}_t$. The p-value corresponds to a binomial test of the difference between the preference reversal proportion and 0.5

[b] Treatment 2 proposes similar acts to those described in Table 2 but reverses payoffs between colors

Table 5 Panel random-effect probit regression of preference reversals

Variable	Age	Gender	Order	Students type	Color-treatment
Coefficient	0.0004	0.525	0.035	0.456	0.168
t-statistic	0.00	1.66	0.13	2.97	0.97

Notes: The Log-likelihood value is −170.27; The Wald test for the null hypothesis that all coefficients are equal to zero has a chi-square value of 54.43 with four degrees of freedom (p-value = 0.00). The fraction of the total variance due to random individual effects is estimated to be 0.446, with a standard error of 0.097

pattern $g_t \hat{f}_t$ which is compatible with an approach of ambiguity in terms of exposure to ambiguity. In order to evaluate to which extent violations of tail-separability are systematic rather than random, we used the Conlisk's Z statistic. This statistic tests whether the percentage of $f_t \hat{g}_t$ is significantly different from the percentage of $g_t \hat{f}_t$. In all the Machina's choice problems we found large values of Z ($Z = 3.55$, $p = 0.002$, and $Z = 2.41$, $p = 0.007$ for lower tail shifts, and $Z = 1.86$, $p = 0.03$, and $Z = 3.26$, $p < 0.001$ for upper tail shifts). As a consequence, one can conclude that violations of tail-separability are not only frequent but also systematic. Observed choices cannot be justified by errors made by subjects close to indifference.

4.5 Other effects

In order to identify possible effects from order, age, gender, color, and treatment group, Table 5 displays estimates from a panel random-effect probit regression of preference reversals for the four modified Ellsberg choice situations. We find no effect of age and gender on preference reversals and no significant order effect. Moreover, the colors used in the experiment have no effect on preference reversals. The only significant variable is group treatment suggesting that, in our sample, engineers from Ecole Centrale ($N = 56$) are more prone to preference reversals.

5 Discussion

5.1 Choquet expected utility versus informational symmetry

Our results provide experimental evidence for the generalized Ellsberg paradox following Machina (2008). Informational symmetry is an important feature of preferences that calls for a reassessment of rank-dependence and specifically of weak tail-separability implications. The most common observed pattern of choice (47% on average) suggests that individuals are ambiguity averse in terms of individual payoffs. We also observed a significant proportion of choices (29% on average) compatible with the hypothesis of ambiguity aversion in terms of exposure to ambiguity.

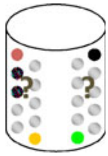
At this stage, our experiment did not account for indifference between acts. This has two main consequences. First, the experimental framework did not permit to observe the case where a CEU subject may decide to treat the various reflected events as informationally symmetric. In such a case, according to cumulative dominance (Sarin and Wakker 1992) she will be indifferent between each pair of acts. Second, since indiffer-

Choice Task n°3

The following urn contains 20 balls:

-10 red or yellow balls in unkown proportion

-10 black or green balls in unkown proportion



A ball will be drawn at random within this urn

Which situation do you choose?

Situation A		Situation B	
If the ball is **yellow**,	you receive 50 €	If the ball is **black**,	you receive 50 €
If the ball is **black or green**,	you receive 25 €	If the ball is **green or yellow**,	you receive 25 €
If the ball is **red**,	you receive 0 €	If the ball is **red**,	you receive 0 €

Fig. 2 A typical display used in the replication with indifference allowed

ence captures ambiguity in terms of decumulative events, we obtained no information about this specific definition of ambiguity. In order to clarify this point, we ran a fourth experimental session with 42 students enrolled in economic courses at Ecole Normale Supérieure Cachan. Subjects faced the same questionnaire but have the possibility to express indifference between two acts (see Fig. 2). Overall only two subjects appear to be indifferent between all acts (4.7%) and thus satisfy both CEU and informational symmetry. Thus, ambiguity in terms of decumulative events was rarely found. Results on preference reversals and informational symmetry remain (79% and 69% for lower tail shifts, 79% and 83% for upper tail shifts, with a majority of subjects ambiguity averse in terms of individual payoffs). Consequently, this fourth session casts doubt on the possibility for informational symmetry to be an inherent part of CEU. Results from this fourth session confirm the preeminence of informational symmetry over CEU even when indifference is allowed. This rules out the possibility of "informational symmetric CEU preferences" being the most common observed pattern of choice. One may argue that our protocol involved no precise incentive to express indifference. This may appear as a drawback of our experiment. However, if indifference was the dominant pattern, we should have observed random choices rather than systematic choices at the aggregate level.

5.2 Informational symmetry and editing

Many violations in decision analysis can be explained by cognitive operations which forgo the evaluation of an act. For example, Wu (1994) explains observed violations of tail-separability under risk by a combination of editing and composition rules in a two-stage procedure. At the first stage, most people cancel common tails between prospects. Editing rule at hand is that people only cancel common tails that are directly apparent (unapparent common tails are said to be opaque). At the second stage, people use a composition rule that evaluate the lotteries, event by event. This composition rule used under risk is derived from original prospect theory (Kahneman and Tversky 1979), a theory which was not designed to deal with uncertainty. In our experiment, subjects may have used such an editing operation.

Table 2 shows that an editing rule could be applied to cancel out events R and G across acts f_2 and g_2 and between acts $\hat{f}_2$ and $\hat{g}_2$. With editing, f_2 is equivalent to $\hat{f}_2$ and g_2 is equivalent to $\hat{g}_2$. Such a cancellation of common tails must yield results consistent with weak tail-separability. Indeed the editing rule proposed by Wu (1994) is subtler: Subjects cancel apparent common tails but not opaque common tails. When we write acts of Table 2 $f_2 = (Y, 50; B \cup G, 25; R, 0)$ and $g_2 = (B, 50; Y \cup G, 25; R, 0)$, we observe that the only apparent common tail is on event R while the commonality of event G is opaque. The way we framed choices to the subjects (see Figs. 1, 2 for a typical display) promotes such canceling of the common tail (Erev et al. 1994, under risk, Fennema and Wakker 1996, under uncertainty). Once editing is performed, choice between f_2 and g_2 appears as a 50% chance to win exactly €25 plus an extra ambiguous chance to win €50 (f_2) or as a 50% chance to win at least €25 (possibly €50) plus an extra ambiguous chance to win €25 (g_2).

Similarly, after editing, choice between acts $\hat{f}_2$ and $\hat{g}_2$ appears also as a 50% chance to win exactly €25 plus an extra ambiguous chance to win €50 ($\hat{g}_2$) or a 50% chance to win at least €25 plus an extra ambiguous chance to win €25 ($\hat{f}_2$). Then, if people cancel only apparent common tails and not opaque tails they would identify f_2 and $\hat{g}_2$ (g_2 and $\hat{f}_2$) as similar. Such an editing operation *à la* Wu (1994) explains violations of weak tail-separability and is in agreement with informational symmetry. In Sect. 2.3 we showed that acts $\hat{f}_2$ and $\hat{g}_2$ are obtained from f_2 and g_2 through two lower tail replacements from $(R; 0)$ to $(R; 25)$, and from $(G; 25)$ to $(G; 0)$ under a weak tail-separability condition. This common lower tail replacements have been presented in a coalesced form to the subjects (see Figs. 1, 2) and then could not be considered as transparent. As Birnbaum et al. have shown for the special case of risk (see Birnbaum 2008 for a review), presenting choice problems in coalesced form leads to more frequent violations of tail-separability. This suggests that the observed violations of weak tail-separability are mainly a consequence of violation of coalescing.

5.3 Quality of the CEU model

Most of the empirical work on non-expected utility theories under uncertainty has been carried out with only two outcomes-lotteries (see Abdellaoui et al. 2005 for a review). Exceptions are McCrimmon and Larsson (1979), Tversky and Kahneman (1992), Fennema and Wakker (1996), Wu and Gonzalez (1999a,b), Hey et al. (2007) and Diecidue et al. (2007). Experiments involving two outcomes are well suited to the study of determinants and shapes of decision weights under uncertainty but miss middle-ranked positions. Such positions are important for a general study of rank-dependence. Although rank-dependence has been shown to be a major descriptive improvement when precise probabilities for events are given, things are less clear-cut under ambiguity. Fennema and Wakker (1996) test upper tail-separability with three-outcome acts in a more general setting than ours. According to their results, the CEU model does not provide any descriptive improvement over SEU. They concluded that "RDU can be of descriptive value in specific domains of decision making", i.e., if the certainty and possibility effect applies in the case of multi-outcomes gambles or within the Ellsberg paradox. Note that our experiment does not apply to risk (see L'Haridon and Placido 2008 on this topic). Under risk, RDU—and then rank-dependence—may still be descriptively superior when compared to Expected Utility. Using de Finetti's betting-odds system, Diecidue et al. (2007) elicited decision weights under uncertainty with three outcomes in a setting where the certainty effect applies and found evidence for rank-dependence. Using a British Bingo Blower to study individual attitudes toward ambiguity, Hey et al. (2007) also found support in favor of CEU. The results of the present experiment raise the question of the sensitivity of the weak tail separability to the number of outcomes and to the weight put on intermediate outcomes.

The problem investigated in this article reveals a deeper, more fundamental question: Is rank-dependence a general recipe for the study of ambiguity? In the Ellsberg paradox, subjects face a single source of ambiguity. In the reflection example tested in this article, there are two sources of ambiguity. The informational symmetric reflections of events within acts lead to violations of comotonicity because they induce specific ambiguity attitudes due to manipulation of ambiguity. By focusing on ambiguity on intermediate events, the reflection example shifts the decision maker attention away from extreme events. As a consequence, the CEU model with weak tail-separability, which aims precisely at focusing on these extreme events may not predict accurately the decision maker's choices. One may interpret Machina's reflection example in terms of sources of ambiguity (Siniscalchi 2008). In contrast to the Ellsberg paradox where only one source of ambiguity exists, the reflection example offers a choice between acts with two sources of ambiguity (R vs. Y and B vs. G for g_2 and $\hat{f}_2$, respectively) and acts with a unique source of ambiguity (R vs. Y for f_2 and B vs. G for $\hat{g}_2$). Our experimental results suggest that CEU keeps its descriptive value if a single source of ambiguity is considered but more refined approaches are needed to deal with multiple sources of ambiguity and the information attached to events. The model of Siniscalchi (2008) captures complementarities among ambiguous events and is able to explain the main pattern of preference observed in this experiment.

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Appendix 1

We consider the four following acts described in the main text:

$$h_1 = (G_1, z_1; \ldots; G_{j-1}, z_{j-1}; E_j, x_j; \ldots; E_n, x_n)$$

$$h_2 = (G_1, z_1; \ldots; G_{j-1}, z_{j-1}; F_j, y_j; \ldots; F_n, y_n)$$
$$h_3 = (G_1, x_1; \ldots; G_{j-1}, x_{j-1}; E_j, x_j; \ldots; E_n, x_n)$$
$$h_4 = (G_1, x_1; \ldots; G_{j-1}, x_{j-1}; F_j, y_j; \ldots; F_n, y_n)$$

Assuming CEU, preference for h_1 over h_2 gives

$$h_1 \succsim h_2 \Leftrightarrow CEU(G_1, z_1; \ldots; G_{j-1}, z_{j-1}; E_j, x_j; \ldots; E_n, x_n)$$
$$\geq CEU(G_1, z_1; \ldots; G_{j-1}, z_{j-1}; F_j, y_j; \ldots; F_n, y_n)$$

The value of the common term of h_1 and h_2 is

$$\sum_{k=1}^{j-2} v(\cup G_k)[u(z_k) - u(z_{k+1})] + v(\cup G_k)[u(z_{j-1})]$$

Replacing z_k by x_k for $k = 1 \ldots i - 1$ gives

$$\sum_{k=1}^{j-2} v(\cup G_k)[u(x_k) - u(x_{k+1})] + v(\cup G_k)[u(x_{j-1})]$$

It follows that

$$CEU(G_1, x_1; \ldots; G_{j-1}, x_{j-1}; E_j, x_j; \ldots; E_n, x_n)$$
$$\geq CEU(G_1, x_1; \ldots; G_{j-1}, x_{j-1}; F_j, y_j; \ldots; F_n, y_n) \Leftrightarrow h_3 \succsim h_4$$

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