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THE WELFARE CONSEQUENCES OF STRATEGIC VOTING IN TWO COMMONLY USED PARLIAMENTARY AGENDAS

ABSTRACT. This paper studies the welfare consequences of strategic voting in two commonly used parliamentary agendas by comparing the average utilities obtained in simulated voting under two behavioural assumptions: expected utility maximising behaviour and sincere behaviour. The average utility obtained in simulations is higher with expected utility maximising behaviour than with sincere voting behaviour under a broad range of assumptions. Strategic voting increases welfare particularly if the distribution of preference intensities correlates with voter types.

KEY WORDS: agendas, counterbalancing, simulation, strategic voting, welfare

JEL CLASSIFICATION NUMBERS: D71, D81

1. INTRODUCTION

This paper investigates whether strategic voting is beneficial or harmful in two commonly used parliamentary voting rules; amendment and elimination agendas. It is widely acknowledged that strategic voting may be beneficial because it may contain the power of an agenda-setter[1] but usually the possibility of strategic voting is considered an undesirable characteristic of a social decision mechanism.[2] Thus far, however, the welfare consequences of strategic voting have not been studied by explicitly comparing strategic voting behaviour with sincere voting behaviour (but see Chen and Yang (2002)).[3]

The welfare consequences of strategic voting are evaluated by comparing voters' *average utility* obtained with *Expected*

Utility maximising voting behaviour (EU behaviour) and with *Sincere Voting behaviour* (SV behaviour). In SV behaviour *all* voters always vote sincerely. In EU behaviour, voters may vote strategically or sincerely in any given stage of voting depending on the expected utility of the choice options. If the average utility obtained with EU behaviour is higher than with SV behaviour, strategic voting is said to be *welfare-increasing*. Otherwise it is *welfare-decreasing*.

The idea that strategic voting may result in better outcomes than sincere voting on the aggregate level may be surprising because strategic voting means voting for an alternative that is not highest in one's preference order. The mechanism of *counterbalancing of strategic votes* explains why, when, and how strategic voting may lead to desirable outcomes on the aggregate level. In a large group of voters, there are usually incentives to vote strategically both for and against a given alternative. Strategic votes for an alternative are counterbalanced by strategic votes against this same alternative. An *intensively* supported alternative gets *more* strategic votes than a less intensively supported alternative.[4] Strategic voting thus increases the chance that an intensively supported alternative beats an alternative which has less intense support but a broader base of supporters. If an intensively supported alternative would lose against an alternative with a larger number of supporters in a sincere pair-wise first-round vote between the two, strategic voting may increase welfare by increasing the chance that an intensively supported alternative is selected in an early stage of voting.

Some scholars have lamented that the widespread use of majority rule has not been properly explained, particularly in view of the negative impossibility and instability (McKelvey (1976), Schofield (1978)) results in social choice theory. It has been widely acknowledged that preference intensities are relevant for social welfare judgements,[5] but there are very few models that explicitly try to study how these intensities affect voting outcomes (but see Blais and Nadeau (1996)). The traditional criticism of majority rule is that it does not take into account preference intensities. The results presented here provide a more positive perspective on majority rule than many previous results in voting theory because it will be shown that strategic voting not only may, but is likely to be beneficial in the sense that the outcomes reflect preference intensities *if* and only if *voters vote strategically*.

Preference intensities in agenda voting can be explicitly modelled only in a model with incomplete information. The model of incomplete information is based on statistical *signal extraction* since voters obtain *noisy signals* of the true structure of the game, and formulate beliefs on the basis of these signals.[6] The signal extraction model is explained in more detail in Lehtinen (2006a). The model has been applied to Borda rule (Lehtinen forthcoming) and to plurality and run-off rules (A. Lehtinen (2006b), Unpublished data).

Instead of presenting an analytical model, computer simulations are used for modelling voters' belief formation and behaviour. Simulations are used for the following reasons. First, welfare-increasing strategic voting is what the literature on computer simulations calls an "emergent property", it emerges only when the individual votes are combined. The mechanism of counterbalancing strategic votes explains why an "invisible hand" result is obtained. Although it may be possible to derive such a result analytically, it is very difficult to analyse the interaction of hundreds of heterogeneous voters with an analytical model. Second, the purpose of the simulations is to examine *how much* voters' preference intensities must correlate with voter types, and *how reliable* must voters' signals be, in order for strategic voting to be welfare-increasing. This is why the degree of reliability and the degree of correlation are taken as exogenous parameters.

The existence of a *Condorcet winner* (CW) is usually considered sufficient for satisfactory performance in majority rule. If there is a CW among the alternatives, this alternative will be the outcome under amendment agendas if all voters vote sincerely Black (1958), or if they maximise utility with complete information.[7] However, various results have established that the existence of a CW is highly unlikely, especially if the number of alternatives and/or voters is large.[8]

Simulation approaches to voting have evaluated and compared voting rules by investigating how frequently a CW is chosen in a voting rule (assuming that it exists), or by investigating how frequently a *utilitarian winner* (the alternative with the largest sum of utility) is chosen (see, e.g., Merrill (1988)).

All well-known incomplete information models of strategic voting in majority rule (Enelow (1981); Jung (1987); Ordeshook and Palfrey (1988)) assume that voters condition their choices on the possibility that they are pivotal in the sense that they make their choices by comparing the expected utility of voting for each of the alternatives. Enelow's model differs from the other models, however, in that it does not assume that the voters *formulate beliefs* by conditioning on the assumption of being pivotal in the first round of voting. If a voter who conditions her choices on being pivotal has poor knowledge of the type distribution of voters, she may well obtain a worse outcome for herself by voting strategically than she would have obtained by voting sincerely. Therefore, while conditioning one's *choice* on being pivotal is rational, conditioning one's *beliefs* on being pivotal is irrational.

The paper is organised as follows. EU behaviour under amendment agendas is based on Enelow (1981) expected utility model, which is introduced in Section 2.1. Section 2.2 presents a similar expected utility model for elimination agendas. These basic building blocks are sufficient for understanding the logic of the welfare consequences of strategic voting. Simple examples in Section 3 show that the utilitarian winner rather than the CW may be selected if voters engage in strategic voting under incomplete information.

In the rest of the paper, the circumstances in which strategic voting increases or diminishes welfare are investigated using computer simulation. A model of incomplete information is introduced in Section 4 by describing the assumptions related to voters' signals and beliefs.

Section 5 explains in detail how the counterbalancing of strategic votes affects the welfare consequences of strategic voting. Section 6 describes the structure of the simulation framework. The behavioural assumptions of EU behaviour and SV behaviour are analysed with *setups*. A setup is a collection of assumptions on voters' preferences, beliefs, behaviour and the institutional structure. This section also establishes the criteria for evaluating voting outcomes.

Section 7 presents simulation results. Since the results depend on a utilitarian welfare function, it will be necessary to discuss interpersonal comparisons of utilities. Section 7.3 presents simulation results with various different interpersonal comparisons. The results from these various setups indicate that strategic voting increases welfare irrespective of what kinds of interpersonal comparisons are made. The purpose of these simulations is thus to show that the results are robust with respect to interpersonal comparisons. Section 8 presents the conclusions.

2. EXPECTED UTILITY MODELS FOR AGENDAS

2.1. *Amendment agendas: Enelow's model*

Let $X = \{x, y, z\}$ denote a set of available alternatives,[9] $I = \{1, 2, \ldots, i, \ldots, N\}$ a set of voters, and U_i voters i's utility. Let U_1, U_2 and U_3 denote the utilities for the best, second-best and the worst alternatives, respectively. (The subscript i denoting the individual is dropped here in order to avoid clutter.) The possible voter types are displayed in Table I.

Let us say that U_2 denotes a voter's *intensity* of preference. There are six different *types of voters*, $t^1, t^2, \ldots, t^6$. A voter's type refers only to his or her order of preferences here, it does not include a specification of his or her beliefs. All preferences are assumed to be strict.

Alternatives are put to a sequence of pair-wise majority comparisons in an *amendment agenda* or in an *elimination agenda*.[10] An amendment agenda is constructed as follows: two alternatives (say x and y) are put to a majority vote against each other in the first round of voting. The winner of

TABLE I
Voter types and utilities with three alternatives

Type of voter						
t^1	t^2	t^3	t^4	t^5	t^6	utility
x	y	z	x	y	z	U_1
y	z	x	z	x	y	U_2
z	x	y	y	z	x	U_3

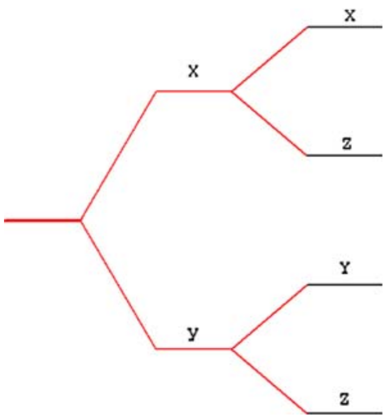

Figure 1. An amendment agenda with three alternatives.

this first contest is then put to vote against the third alternative (z) in a second round of voting. Fig. 1 presents this amendment agenda. Since path-dependence is not studied in this paper, other possible voting orders in amendment agendas are not shown here.

Voter i's subjective probability that a given alternative j beats k $(j, k \in X)$ in a pair-wise second-round contest is denoted $p_i\,(jBk)$. In the first round of voting, voters' choice options are lotteries on the second-round outcomes.

In the first round of voting, voters choose by evaluating lotteries $(x, z;\, p_i\,(xBz)\,, 1 - p_i\,(xBz))$ and $(y, z;\, p_i\,(yBz)\,, 1 - p_i\,(yBz))$. Maximizing expected utility implies giving one's vote for the branch of the voting tree with the greatest

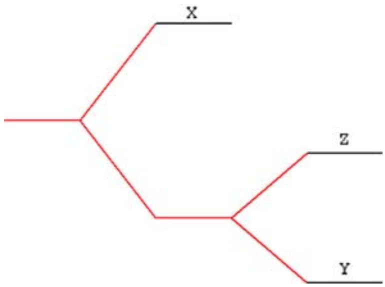

Figure 2. An elimination agenda with three alternatives.

expected utility. A voter will vote for the upper branch (i.e. for x) if

$$p_i(xBz)\,U_i(x) + (1 - p_i(xBz))U_i(z) \geq p_i(yBz)\,U_i(y) + (1 - p_i(yBz))U_i(z). \qquad (1)$$

If the expected utility is the same for the two branches, the voter is assumed to vote sincerely. Voters of types 2 and 4 have a dominant strategy to vote sincerely (Farquharson 1969). Enelow uses a zero-one normalisation for utilities for formulating the model. Although this normalisation is not used in the simulations, the examples presented in later sections are formulated using this normalisation in order to simplify the presentation.

2.2. *Elimination agendas*

Although most of this paper is concerned with amendment agendas, elimination agendas are also briefly considered. Under an elimination agenda, alternative x first put to vote against the other alternatives. If x wins it is elected, if not, the winner is decided by a pairwise vote between y and z. This agenda is denoted $([x]\,yz)$, and is shown in Fig. 2.

The expected utility of voting for the upper branch is $U_i(x)$, and the expected utility of voting for the lower branch is $p_i(yBz)\,U_i(y) + [1 - p_i(yBz)]\,U_i(z)$. A voter thus votes for the upper branch if

$$U_i(x) \geq p_i(yBz)\,U_i(y) + [1 - p_i(yBz)]\,U_i(z)\,. \qquad (2)$$

Voter types 1, 2, 4 and 6 have dominant strategies to vote sincerely (see, e.g., Miller, 1995, pp. 48–52).

3. THE LOGIC OF THE WELFARE CONSEQUENCES OF STRATEGIC VOTING: THREE EXAMPLES

In the examples that follow, we will say that a voter's beliefs are *reasonable* if they could have been derived from relatively reliable signals. The purpose of using the term "reasonable" here is that it is merely a shorthand for "could have been derived from relatively reliable signals". The examples below are meant to provide an intuitive understanding of how the quality of the beliefs affects the voting outcomes. The term "reasonable beliefs" does not have any role in the theory and nothing depends on it. It is thus introduced merely for the heuristic purpose of making the logic of the model more salient. The following examples involve only three voters, but the model of incomplete information is based on applying the Central Limit Theorem, and it is thus not directly applicable for a society of three voters.[11] This is why the signal extraction model is not used in discussing the examples here. It is hoped, however, that these examples provide the reader with an easier access to the intuition of the model than one with a large amount of voters. The signal extraction model is introduced in Section 4. Section 5.1 will then present another example with 29 voters in which this signal extraction model is used for determining voters' beliefs.

If the CW is not the same alternative as the utilitarian winner (UW), the latter ought to be selected according to the utilitarian welfare criterion. Strategic voting may lead to the choice of the UW even if some other alternative is a CW, but this usually requires that most voters' beliefs are reasonable.

Example 1 illustrates such a situation. Assume that the preferences of three voters A, B and C, can be described with Table II.

x is the UW here. The numbers in parentheses denote voters' utilities for the alternatives. If they vote sincerely, y will

TABLE II
Example 1

A	B	C
y (1)	y (1)	x (1)
x (0.9)	x (0.9)	z (0.9)
z (0)	z (0)	y (0)

beat x in the first round and z in the second round, and the CW, y, is chosen.

Assume now that all three voters have identical beliefs such that $p(xBz) = 0.9$, and $p(yBz) = 0.7$. Voters thus consider it likely that y beats z, but even more likely that x beats z in the second round of voting.

Let $U^t(j)$ denote a type t voter's utility for alternative j. Voters A and B are of type 5. They will vote strategically for x in the first round if $p(xBz)U^5(x) + 0 > p(yBz) \cdot 1 + 0 \leftrightarrow U^5(x) > \frac{p(yBz)}{p(xBz)}$, i.e. if $0.9 > \frac{0.7}{0.9} = 0.7778$. Since this is true, A and B will vote strategically for x in the first round of voting. Voter C has a weakly dominant strategy to vote for x in the first round of voting. x is the outcome if all voters maximise expected utility because it beats y in the first round and z in the second round. The UW x is thus chosen if voters maximise expected utility, but the CW y is chosen if all voters vote sincerely. Example 1 also shows that a CW is not necessarily chosen in majority rule, and that this may happen under fairly reasonable assumptions on voters' beliefs. This result has already been proven by Ordeshook and Palfrey (1988), but their model is based on implausible assumptions. In particular, given that they assume incomplete information, it is implausible to assume that the players condition their beliefs on the assumption that exactly three of the six possible types of players may be playing the voting game.

Consider now an example in which voters' beliefs are not reasonable. Let the preferences of three voters D, E and F be as follows (Table III):

TABLE III
Example 2

D	*E*	*F*
x (1)	y (1)	x (1)
y (0.9)	z (0.9)	z (0.9)
z (0)	x (0)	y (0)

Here $x = \text{UW} = \text{CW}$. Let us now assume that voters have identical beliefs such that $p(xBz) = 0.3$ and $p(yBz) = 0.7$. They now believe that z will beat x in the second round even though $x = \text{CW} = \text{UW}$, and z is the worst alternative in utilitarian terms. $p(yBz)$ is reasonable, because y beats z in the second round if it survives the first. Voter D will vote strategically for y in the first round, because $U^1(y) = 0.9$ is larger than $\frac{p(xBz)}{p(yBz)} = 0.428$. Voter E has a weakly dominant strategy to vote for y in the first round. Voter F has a weakly dominant strategy to vote sincerely for x in the first round. Thus, if all voters maximise expected utility, y beats x in the first and z in the second round, and emerges as the outcome. Here strategic voting leads to an outcome (y), which is worse in utilitarian terms than the outcome if all voters vote sincerely (x).

Examples 1 and 2 illustrate that the welfare consequences of strategic voting depend on how accurate voters' beliefs are. If they are clearly inaccurate, as in example 2, strategic voting can diminish welfare, but if they are relatively accurate, as in example 1, strategic voting may increase welfare. If voters have complete information, the CW wins in both cases. Hence, strategic voting with incomplete information may increase welfare when compared to strategic voting with complete information (example 1). However, strategic voting with complete information never has the catastrophic consequences that strategic voting with incomplete and poor information may have (example 2).

These examples also show that if a voter thinks that her information is highly unreliable, she *should not* take the risk of voting strategically because she might well obtain a worse outcome for herself. In example 2, voter D obtained a worse outcome (y) by voting strategically than she would have obtained if she had voted sincerely (x). Furthermore, the *benefit* from this "foolish" strategic voting *accrued to voter E* (*who voted sincerely*), who obtained a better outcome than she would have obtained if D had voted sincerely.

In example 1, the strategic voting of A and B resulted in an outcome that has a lower utility for them than the alternative that would have been chosen if they had voted sincerely. Nevertheless, their actions increased the average utility of *all* voters because voter C's utility increases more than their own utility decreases. Hence, EU behaviour may be welfare-increasing on the aggregate level even though those who vote strategically may diminish their own utility.

Strategic voting may also be welfare-increasing and increase the utility of those who engage in it. The famous Condorcet paradox in example 3 illustrates such a case (Table IV).

If voters engage in SV behaviour, x beats y in the first round, and z beats x in the second. If they maximise expected utility with $p(xBz) = 0.1$ and $p(yBz) = 0.9$, G votes strategically for y in the first round ($0.8 > \frac{0.1}{0.9}$), and the others continue to vote sincerely. y beats x in the first round and z in the second. Voter G obtains a better result for herself than she would have obtained by voting sincerely. y is also better than z in terms of the sum of utility. Notice, however, that y

TABLE IV

Example 3

G	*H*	*I*
x (1)	y (1)	z (1)
y (0.8)	z (0.1)	x (0.9)
z (0)	x (0)	y (0)

is not a UW. Strategic voting resulted in a clearly better outcome than sincere voting, but the UW was not selected.

4. A MODEL OF INCOMPLETE INFORMATION IN SIMULATED VOTING GAMES

The previous section showed that in some situations strategic voting is welfare-increasing and in some others it is not. These examples may provide some insight into the logic of strategic voting, but it will be important to know whether strategic voting is *typically* beneficial or not. The examples were also silent on how voters are assumed to formulate their beliefs. Let us now give an account of the voters' beliefs in a framework of simulated voting games.

A standard Bayesian model of incomplete information would assume that the players start with common priors and update them with Bayes' rule. Voters may be able to update their beliefs after the first round of voting, but they are not able to *benefit* from these updated beliefs when there are only three alternatives because all voters vote sincerely in the second round of voting. The model can be extended to four or more alternatives, but introducing an updating model is beyond the scope of this paper because four or more alternatives also bring other complications that should be dealt with.[12]

A model that starts with common priors does not provide interesting results under amendment agendas with three alternatives because all priors before the first round of voting would be equal to $\frac{1}{2}$ if voters knew that all voter types are equally likely. It can be checked that all voters will vote sincerely if this value $\frac{1}{2}$ is inserted into the condition that determines strategic voting presented in Equation (1). For these reasons, voters need to have some information on the preferences of the other voters *before* the *first* round of voting.

Voters are thus assumed to obtain *perturbed signals* of the other voters' preferences before the first round of voting. They formulate beliefs on the basis of these noisy signals. This information model is embedded in simulated games for which the voter types are generated with the *impartial anonymous culture* (IAC) assumption. This assumption means that each voter type is equally likely. If the preferences for a pair of alternatives j and k is considered, it means that each voter is equally likely to prefer j to k as the reverse. The IAC assumption over-emphasises the prevalence of strategic voting when compared to real-world situations. The use of this assumption is legitimate in this model, however, because the purpose is not to evaluate how common strategic voting is, but rather what its consequences are when it occurs and is significant. The IAC assumption is the best possible assumption for this purpose because it generates the maximum amount of very tight elections and thereby a maximum amount of cases in which strategic voting matters.[13]

A *simulated game* g consists of a set of utilities created by a random number generator, beliefs based on these utilities, voters' perturbed signals and voting outcomes under the different behavioural assumptions. Let $\succ_i^g$ denote voter i's preference relation in a simulated game g. Let $n^g(j \succ k)$ denote the number of voters who prefer alternative j to alternative k in simulated game g, and $n^g(k \succ j)$ the amount of voters with opposite preferences. If alternatives j and k are put to vote against each other in the last round, j beats k if $n^g(j \succ k) > n^g(k \succ j)$.

Since all the symbols to be defined in what follows concern a single simulated game g, the superscript will be omitted in the sequel. Let $n_i(j \succ k) = 1$, if voter i prefers j to k, and $n_i(j \succ k) = 0$, if voter i prefers k to j. Then $n(j \succ k)$ can be viewed as a sum of N Bernoulli trials. The total number of supporters for j against k is thus given by $n(j \succ k) = \sum_{i=1}^{N} n_i(j \succ k)$. Let p denote the probability that such a Bernoulli trial results in the outcome that $n_i(j \succ k) = 1$. The impartial culture implies that $p = \frac{1}{2}$. $n(j \succ k)$ can thus be viewed as a random variable with a binary distribution $n(j \succ k) \sim B(N, \frac{1}{2})$.

4.1. *Signals*

The voters are assumed to obtain a perturbed signal of the number of voters who prefer j to k. It will be more convenient to use a standardised sum of Bernoulli trials $Q(j \succ k)$ instead of the variable $n(j \succ k)$ itself:

$$Q(j \succ k) = \frac{n(j \succ k) - Np}{\sqrt{Np^2}}. \tag{3}$$

Since $p = \frac{1}{2}$, this is $Q(j \succ k) = \frac{2n(j \succ k) - N}{\sqrt{N}}$. A *signal* of voter i concerning the preferences of all voters for alternatives j and k , $S_i(j,k)$, is given by

$$S_i(j,k) = \frac{2n(j \succ k) - N}{\sqrt{N}} + \varepsilon \cdot r_i\,(j,k)\,, \tag{4}$$

where $r_i\,(j,k)$ is a realisation of an i.i.d. standard normal random variable, and ε is a scaling factor that reflects the *reliability* of the signals. Let $R_i\,(j,k) = \varepsilon \cdot r_i\,(j,k)$. The signal can then be written as follows:

$$S_i(j,k) = Q(j \succ k) + R_i\,(j,k)\,. \tag{5}$$

The brief term "signal" is used here, even though the longer expression "a voter's conception of an aspect of the game to be played" would be more accurate. A voter's conception of the game may be the result of several observations.

A signal is the only constraint imposed on a voter's beliefs. In particular, beliefs that constitute a cycle are allowed; $p_i\,(xBz) > \frac{1}{2}$, $p_i\,(zBy) > \frac{1}{2}$, and $p_i\,(yBx) > \frac{1}{2}$. The reason for this is that if the underlying preferences are cyclical, the beliefs for them may well be cyclical as well.

Deriving beliefs from these signals involves applying the Central Limit Theorem and standard statistical inference. Voters are thus modelled as amateur econometricians involved in a *signal extraction* problem. Lehtinen (2006a) shows that voters' beliefs are given by Equations (6) and (7).

$$p_i(xBz) = 1 - \Phi\left(\frac{-s_i(x,z)}{\varepsilon\sqrt{1+\varepsilon^2}}\right) \tag{6}$$

and

$$p_i(yBz) = 1 - \Phi\left(\frac{-s_i(y,z)}{\varepsilon\sqrt{1+\varepsilon^2}}\right). \tag{7}$$

Voters are assumed to know that the voter types are drawn from a uniform distibution. Hence, they cannot use their own type for deriving a belief about others because their own type does not provide them with new information.

Let us say that ε is the *reliability of the signals*. The smaller ε is, the more reliable a voter's signals are. In this paper, voters are assumed to know the reliability of their signals. This assumption can be relaxed as explained in Lehtinen (2006a).

5. COUNTERBALANCING OF STRATEGIC VOTES

The mechanism of *counterbalancing* strategic votes explains when and why strategic voting is welfare-increasing. Four different types of voters may vote strategically under amendment agendas. Voters of types 5 and 6 may vote strategically for x, while voters of types 1 and 3 may vote strategically for y. Let us now reformulate Equation (1) as follows:

$$\begin{aligned} L_i = {} & p_i\,(xBz)\,U_i(x) + (1 - p_i\,(xBz))U_i(z) \\ & - p_i\,(yBz)\,U_i(y) - (1 - p_i\,(yBz))U_i(z). \end{aligned} \tag{8}$$

This equation says that if $L_i \geq 0$, the voter votes for the upper branch (x). It is easy to see that $\frac{\partial L_i}{\partial U_i(y)} < 0$, and that $\frac{\partial L_i}{\partial U_i(x)} > 0$. The signs of these derivatives mean that the higher is the utility of y for voters of type 1 and 3, the more likely they are to vote strategically for y. Similarly, the higher the utility of x for voters of type 5 and 6, the more likely they are to vote strategically for x.

Hence, if the utility for x is *almost as high* as the utility of y for many voters of types 5 and 6, and if the utility of y is *significantly lower* than the utility of x for many voters of types 1 and 3, a larger number of voters of types 5 and 6 than of types 1 and 3 vote strategically. This means that x

gets *more* strategic votes than y. Furthermore, strategic votes for x are at the same time strategic votes *against* y.

Ceteris paribus, if many $U^5(x)$ and $U^6(x)$ are almost as high as $U^5(y)$ and $U^6(y)$, respectively, and if many $U^1(x)$ and $U^3(x)$ are significantly higher than $U^1(y)$ and $U^3(y)$, respectively, the sum of utility for alternative x is relatively large, and the sum of utility for alternative y is relatively small. Hence, under these assumptions on individual utilities, x is likely to have a larger sum of utility than y. Counterbalancing means that both x and y will obtain strategic votes, but x is likely to obtain more strategic votes than y if it has a larger sum of utilities than y.

5.1. *An example of counterbalancing*

Consider now an example that purports to show how counterbalancing affects the voting results. There are 29 voters whose utilities are the result of a simulation. Their signals were formulated with $\varepsilon = 1$. Table V on page 12 displays voters' types (t), decisions (D), preference intensities $U_{2,i}$, beliefs ($p_i(xBz)$ and $p_i(yBz)$), perturbation terms ($R_i(x,z)$ and and $R_i(y,z)$) and expected utilities for the two branches of a voting tree [$\mathrm{EU}_i(U)$ for Upper (a vote for x) and $\mathrm{EU}_i(L)$ for Lower (a vote for y)]. When a voter votes sincerely $D = S$, and when a voter votes strategically $D = T$.

The sums of utilities are $U(x) = 15.43$, $U(y) = 13.88$ and $U(z) = 12.85$. y is the CW because

$$n_1 + n_3 + n_4 = n(x \succ y) = 6 + 2 + 5 = 13,$$
$$n_2 + n_5 + n_6 = n(y \succ x) = 7 + 4 + 5 = 16,$$
$$n_1 + n_4 + n_5 = n(x \succ z) = 6 + 5 + 4 = 15,$$
$$n_2 + n_3 + n_6 = n(z \succ x) = 7 + 2 + 5 = 14,$$
$$n_1 + n_2 + n_5 = n(y \succ z) = 6 + 7 + 4 = 17 \text{ and}$$
$$n_3 + n_4 + n_6 = n(z \succ y) = 2 + 5 + 5 = 12.$$

The standardised numbers of voters are $Q(x \succ z) = \frac{2n^g(x \succ z) - N}{\sqrt{N}} = \frac{2 \cdot 15 - 29}{\sqrt{29}} = 0.185\,70 \approx 0.19$, and $Q(y \succ z) = \frac{2n^g(y \succ z) - N}{\sqrt{N}} = 0.928\,48 \approx$

TABLE V
Example 4

Number	t	D	$U_{2,i}$	$p_i(xBz)$	p_i(yBz)	R_i(x,z)	R_i(y,z)	EU_i(U)	EU_i(L)
1	1	S	0.22	0.41	0.86	−0.53	0.59	0.41	0.19
2	1	T	0.6	0.37	0.62	−0.66	−0.49	0.37	0.37
3	1	S	0.42	0.68	0.52	0.47	−0.87	0.68	0.22
4	1	S	0.23	0.96	0.79	2.3	0.22	0.96	0.18
5	1	S	0.25	0.7	0.7	0.57	−0.18	0.7	0.17
6	1	S	0.58	0.76	0.83	0.83	0.44	0.76	0.49
7	2	S	0.67	0.49	0.96	−0.22	1.55	0.34	0.99
8	2	S	0.56	0.86	0.4	1.35	−1.3	0.08	0.73
9	2	S	0.66	0.66	0.96	0.41	1.63	0.22	0.99
10	2	S	0.81	0.53	0.86	−0.08	0.6	0.38	0.97
11	2	S	0.57	0.44	0.88	−0.41	0.71	0.32	0.95
12	2	S	0.39	0.41	0.83	−0.52	0.41	0.23	0.90
13	2	S	0.14	0.68	0.82	0.49	0.36	0.04	0.84
14	3	S	0.74	0.46	0.32	−0.33	−1.58	0.88	0.68
15	3	S	0.61	0.95	0.96	2.14	1.54	0.63	0.04
16	4	S	0.36	0.9	0.55	1.6	−0.76	0.93	0.16
17	4	S	0.05	0.44	0.73	−0.41	−0.04	0.47	0.01
18	4	S	0.5	0.58	0.87	0.09	0.66	0.79	0.07
19	4	S	0.8	0.86	0.45	1.37	−1.09	0.97	0.44
20	4	S	0.34	0.78	0.76	0.9	0.07	0.85	0.08
21	5	T	0.72	0.83	0.5	1.18	−0.93	0.6	0.50
22	5	T	0.88	0.66	0.52	0.4	−0.85	0.58	0.52
23	5	S	0.85	0.18	0.86	−1.48	0.6	0.15	0.86
24	5	S	0.63	0.31	0.46	−0.88	−1.09	0.2	0.46
25	6	S	0.53	0.68	0.74	0.47	−0.01	0.32	0.65
26	6	T	0.42	0.38	0.86	−0.63	0.63	0.62	0.50
27	6	T	0.03	0.13	0.4	−1.77	−1.28	0.87	0.61
28	6	S	0.08	0.36	0.35	−0.71	−1.47	0.64	0.68
29	6	S	0.52	0.74	0.33	0.73	−1.54	0.26	0.84

0.93. If a voter would have obtained a *perfectly reliable signal* ($R_i = 0$), he or she would have formulated the following probabilities $p(xBz) = 1 - \Phi\left(\frac{-0.19}{1\sqrt{1+1^2}}\right) = 0.55$, and $p(yBz) = 1 - \Phi\left(\frac{-0.93}{1\sqrt{1+1^2}}\right) = 0.74$. Probabilities that are *close* to these values could be considered "reasonable". It should now be easier to understand why the inexact notion of reasonable beliefs was used and what it could mean. One might argue that reasonable beliefs are those that correspond to reality, and that this would mean that reasonable beliefs must be degenerate zeros or ones. But if you know that the signals on which your probabilities are based are not fully reliable, it is not rational to assign probabilities one and zero to anything of concern to you. Furthermore, we have seen that if a voter engages in strategic voting with poor information, she may lose rather than gain in utility by doing so. It is natural to take perfectly reliable signals as a measuring rod for what counts as a reasonable belief. A voter's belief is the more reasonable, the closer her signals are to being perfectly reliable. It seems plausible to say that there is a continuum of beliefs from reasonable to (highly) unreasonable between the extremes of, say, ($\varepsilon = 0.01, R = 0$) and ($\varepsilon = 100,\ R = 100$) or ($\varepsilon = 100,\ R = -100$), even though there are no non-arbitrary values of R_i and ε that make a belief based on these parameters reasonable.

To see how the actual beliefs are derived in this example, consider voter 2 as an example. Applying Equation (4), it is seen that $s_i(x,z) = \frac{2n(x \succ z) - N}{\sqrt{N}} + \varepsilon \cdot r_i(x,z) = 0.19 + 1 \cdot (-0.66) = -0.47$. Applying Equation (6) it is seen that $p_2(xBz) = 1 - \Phi\left(\frac{-s_i(x,z)}{\varepsilon\sqrt{1+\varepsilon^2}}\right) = 1 - \Phi\left(\frac{0.47}{\sqrt{2}}\right) = 0.3698 \approx 0.37$. A similar calculation applies to $p_2(yBz)$.

This example is analogous to example 1 in that the CW y is chosen with SV behaviour, but the UW x is chosen with EU behaviour. Voter 2 gives a strategic vote for y, but this is counterbalanced by four strategic votes for x by voters 21, 22, 26 and 27. The fact that x receives more strategic votes is not a coincidence. The average preference intensity for x

(0.7383) is clearly higher than that for y (0.3527). In contrast, the perturbations are distributed relatively equally for all voter types. What matters for the voter's choice is not only the size of the perturbations, but also whether the perturbation for $Q(x \succ z)$ mutually reinforces the perturbation for $Q(y \succ z)$, i.e. whether the sign of the two perturbations is the same or not. If $|R(x,z) - R(x,z)| > 1$ is taken as a criterion, voters 1, 3, 7, 8, 9, 11, 14, 16, 19, 21–23, 26 and 29 have mutually reinforcing perturbations.

Of these, voters 1, 14, 21, 22 and 26 have perturbations that increase the probability of voting strategically as compared with zero perturbations. Considering only the beliefs, voters 1 and 14 *could have voted strategically* for y, but they voted sincerely. Notice that voter 1's intensity for y (0.22) is relatively low, and voter 14's intensity for x (0.74) is relatively high. In contrast, voters 21, 22 and 26 do vote strategically because the intensities for x ($U_{21}(x) = 0.72$, and $U_{22}(x) = 0.83$) are relatively high, and the intensity for y ($U_{26}(y) = 0.42$) relatively low. Counterbalancing thus implies that alternatives with high average utility will get more and lose less strategic votes than other alternatives.

6. SIMULATION AND SETUPS

A *simulated EU-game* g consists of a profile of utilities, $\Pi^g(\Psi) = \{U_1^g, U_2^g, \ldots, U_N^g\}$, as determined by a rule Ψ, and a profile of beliefs computed on the basis of ε and $\Pi^g(\Psi)$. All simulations had $N = 201$ voters.

An expected utility *setup* (EU-setup) is a collection of assumptions $S = \{\mathbb{I}, X, \Pi(\Psi), A, \varepsilon, C, IPC\}$. There are $G = 10000$ simulated games in a setup. $\mathbb{I} = \{I^1, I^2, \ldots, I^g, \ldots, I^G\}$ is a collection of G sets of voters, and $\Pi(\Psi) = \{\Pi^1, \Pi^2, \ldots, \Pi^G\}$ is a collection of utility profiles, one set for each simulated game. A is an agenda. C and IPC denote parameters that will be explained shortly.

In what will be called *uniform setups*, the rule Ψ that determines voters' types and preference intensities is a combination of the impartial culture assumption and the assumption that the utilities are derived from a uniform distribution on [0,1]. Since the logic of counterbalancing suggests that strategic voting should be more welfare-increasing if there are systematic differences between voters' relative utilities that are not reflected in the preference orderings, setups in which the preference intensities for the second-best alternatives are systematically different for different voter types will be studied. In order to generate such *setups with correlation between preference intensities and voter types* without affecting the interpersonal comparisons or the preference orderings, the individual utilities were derived in the following way.

U_1, U_2 and U_3 were first generated from the uniform distribution on [0,1] for each voter, but U_2 was not used for any purpose. Instead, a standardised utility $\widetilde{U}_2$ for the second-best alternative was generated from the uniform distribution on [0,1]. This standardised utility expresses what a voter's utility for the second-best alternative would be if his or her scale of utility was [0,1]. These standardised second-best utilities will be referred to as *intra*personal *intensities*. In setups with intensity correlation, these intensities were multiplied with a parameter C, $0.5 < C \leq 1$ for those who put y second (voter types 1 and 6) so that the new correlated intensities $\widetilde{U}_2^{C,1}$ and $\widetilde{U}_2^{C,6}$ were given by

$$\widetilde{U}_2^C = C\widetilde{U}_2.$$

To compensate the decreases in utility for voter types 1 and 6, the intensities for voters of types 3 and 5 (i.e. for x) were given by

$$\widetilde{U}_2^C = 1 - C\widetilde{U}_2.$$

These adjustments make the average utilities for x higher and the average utilities for y lower than in the uniform setups while keeping the overall average utility fixed. In uniform setups, $C = 1$. C thus denotes the *degree of correlation* between preference intensities and voter types.

These standardised intensities were then scaled back into the original $[U_3, U_1]$ utility scale. Let U_2^* denote a voter's intensity expressed in terms of the original $[U_3, U_1]$ scale. Since the relationship between the standardised intrapersonal utility for the second-best alternative and the original scale of utility is given by

$$\widetilde{U}_2^C = 1 - \frac{U_1 - U_2^*}{U_1 - U_3} \tag{9}$$

U_2^* is given by:

$$U_2^* = U_3 + \widetilde{U}_2^C (U_1 - U_3). \tag{10}$$

6.1. *Criteria for evaluating the welfare consequences of strategic voting*

The shorter expression $\mathcal{S}(\varepsilon, C)$ will be used to refer to an EU-setup, because an EU-setup is essentially a set of simulated games in which the reliability of signals ε and the degree of correlation C are the same for all voters. The winner of voting is denoted W_{SV}^g in a simulated SV-game, and $W_{\text{EU}}^g(\varepsilon, C)$ in a simulated EU-game. Let $U^i(W_{\text{EU}}^g, \varepsilon, C)$ and $U^i(W_{\text{SV}}^g)$ denote voter i's utility in simulated game g in an EU-setup and SV-setup, respectively. The *Average Utility in an EU-setup* $\mathcal{S}(\varepsilon, C)$, $\text{AU}_{\text{EU}}(\varepsilon)$, is:

$$\text{AU}_{\text{EU}}(\varepsilon, C) = \frac{\sum_{g=1}^{G} \sum_{i=1}^{N} U^i(W_{\text{EU}}^g, \varepsilon, C)}{G * N}. \tag{11}$$

The *Average Utility in the SV-setup*, AU_{SV}, is:

$$AU_{SV} = \frac{\sum_{g=1}^{G} \sum_{i=1}^{N} U^i(W_{\text{SV}}^g)}{G * N}. \tag{12}$$

EU behaviour is **welfare-increasing** *in setup $S(\varepsilon, C)$ if the average utility of all voters is larger in this EU-setup than in the SV-setup:*

$$\text{AU}_{\text{EU}}(\varepsilon, C) > \text{AU}_{\text{SV}}. \tag{13}$$

If the converse holds, EU behaviour is **welfare-decreasing**. Let us also say that *strategic voting is welfare-increasing* in a setup if EU behaviour is welfare-increasing in that setup.

7. SIMULATION RESULTS

7.1. *Amendment agendas*

Fig. 3 displays average utilities from setups with $\varepsilon = [0, 0.4, \ldots, 1.6]$ and $C = [1, .., 0.5]$.[14]

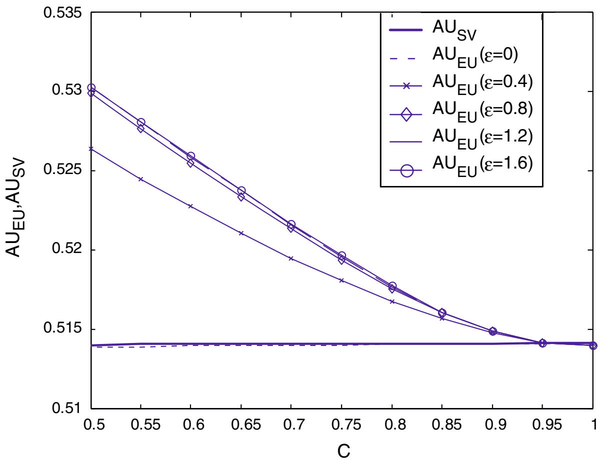

Figure 3. Average utilities in amendment agendas.

Since the variance of $Q(j \succ k)$ is 1, the reliability of the voters' signals with $\varepsilon = 1.6$ is very low; chance is more important in determining the signal than the real preference profile in setups with $\varepsilon > 1$. Such a large range of parameter values were studied in order to ensure that the relevant parameter range, and more, is covered.

The following observations can be made from these simulation results. EU behaviour increases welfare in almost all setups. In uniform setups the average utilities are virtually the same under the two behavioural assumptions. As expected, welfare-increasing strategic voting becomes more and more important as the correlation between voter types and preference intensities increases. EU behaviour with complete information ($\varepsilon = 0$) yields lower average utilities than EU behaviour with incomplete information. As long as information is not complete, the quality of voters' information does not seem to

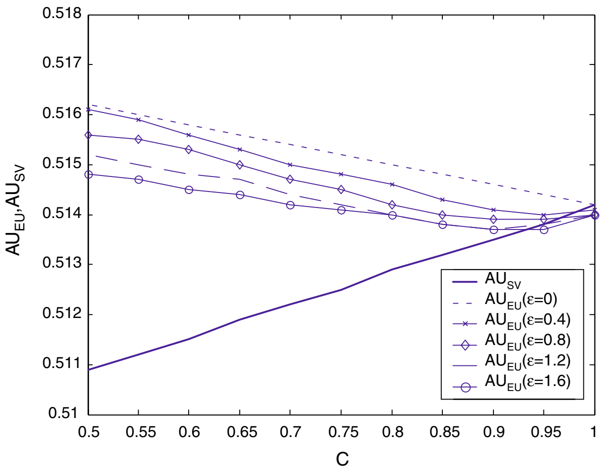

Figure 4. Average utilities in setups in which the intensities of x are high and the intensities of z low.

be particularly important for the results. In fact, the average utilities are highest when the perturbations are large and when the correlation is strong.

What happens if the intensity for z rather than y is decreased (or increased) in setups with intensity correlation? The results from such a setup are presented in Fig. 4. The difference in average utilities between SV behaviour and EU behaviour is now considerably lower. Furthermore, the average utilities are lower under both behavioural assumptions. These results can be explained as follows. Since the utility of z is low, it is natural that the average utilities are lower; z always participates in the second-round contest, and wins one-half of them. The difference in average utility between the two behavioural assumptions is now lower because in the setups with low utilities for y, strategic voting is effective in eliminating y in the first voting round, but in the latter setups this matters less because the low-utility z is always waiting in the second round of voting. If the roles of z and x are reversed by decreasing the intensities for x and increasing the intensities for z, the results are again similar to the ones presented in Figure 3. They are presented in Fig. 5.

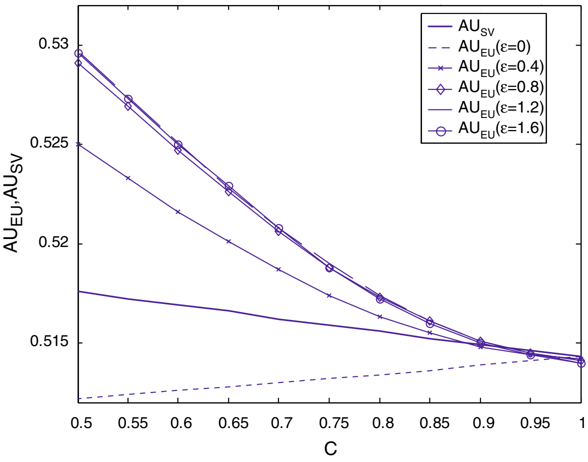

Figure 5. Average utilities in amendment agendas when the intensities of z are high and the intensities of x are low.

As expected, the average utilities under EU behaviour remain similar to what they were in previous setups, but now the average utility under SV behaviour increases slightly with an increase in the degree of correlation.

7.2. *Counterbalancing once again*

Uniformly distributed preference intensities generate very small differences in intensities between the different voter types. This is why uniform setups provide the *least favourable* comparison between EU behaviour and SV behaviour.

It can be seen from Figure 3 that even a very weak correlation between intensities and voter types makes strategic voting welfare-increasing in all setups. It is reasonable to assume that typically some amendments are widely endorsed as second-best alternatives. The setups with somewhat high correlation may well represent the reality better than the setups with weak correlation.

Let us now look at the logic of counterbalancing by considering some comparisons between uniform and correlated setups. Let $V(\sim x)^m$ denote the number of voters who prefer x to y, but who vote for y in simulated game m. The

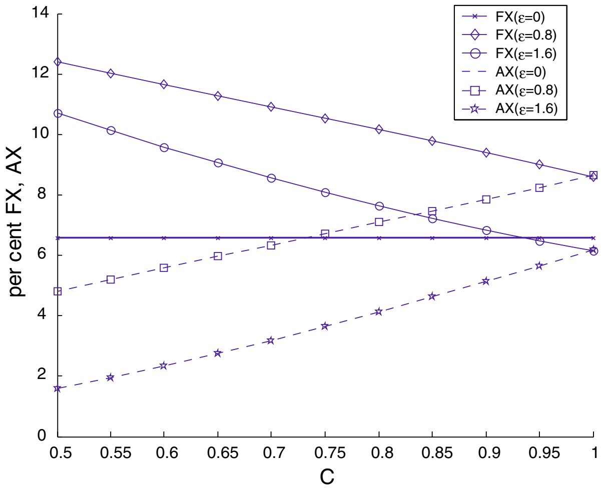

Figure 6. Strategic votes for and against x and y in various setups.

average percentage of votes against candidate x is the relative frequency of voters who prefer x to y but who vote strategically for y. The average percentage of votes *against* alternative x, AX, is thus given by

$$AX = \sum_{m=1}^{G} \frac{V(\sim x)^m}{G * N} * 100. \tag{14}$$

Let $V(\sim y)^m$ denote the number of voters who prefer y to x, but who vote for x in simulated game m. The average percentage of votes *for* alternative x, FX, is given by

$$FX = \sum_{m=1}^{G} \frac{V(\sim y)^m}{G * N} * 100. \tag{15}$$

Since a strategic vote for x is simultaneously a strategic vote against y, AX and FX also provide the percentages of strategic votes for and against y. Fig. 6 displays the average percentages of strategic votes for and against x in various setups. It is to be expected that in uniform setups where all preference intensities are taken from the uniform distribution, all candidates should obtain and lose about the same amount of strategic votes. Figure 6 shows that this is indeed the case. Furthermore, the more there is correlation between voter types and preference intensities, the more y loses and the more x gains strategic votes.

7.3. *Robustness with respect to interpersonal comparisons*

Since the results are based on average utilities, it is necessary to make interpersonal comparisons of utilities. Furthermore, *interpersonal comparisons of preference intensities* are also needed because it is necessary to assume that one person's utility may be added to another person's utility. In the simulations conducted thus far, random interpersonal comparisons of preference intensities have been used because the utilities have been derived from the uniform distribution on the [0,1] interval. This particular assumption creates some variation in the minimum and maximum values of utilities for different voters. If it is considered likely and important that different individuals in fact attach different importance to the different issues, this way of modelling is justifiable. Another possibility is to derive the utilities in such a way that the maximum and minimum utilities are given the values 1 and 0, respectively, and the utility for the second-best alternative is something in between these extremes. This way of modelling may be justified on the normative grounds that each voter should have the same weight in determining the best outcome. It could be seen as an expression of the one-man one-vote principle that takes preference intensities into account.

Irrespective of the way of modelling chosen, it may be argued that our choice of interpersonal comparisons is arbitrary. This arbitrariness ultimately derives from the fact that it is impossible to obtain exact information on individual differences in utilities. Epistemological considerations thus indicate that we will never know which interpersonal comparison is correct. Unfortunately, the results depend crucially on interpersonal comparisons of preference intensities. These are generally considered as the most suspect kinds of comparisons.

Strategic voting is beneficial *only* because it allows voters to express intensities indirectly even in voting rules in which such information is not explicitly collected. Therefore, if only ordinal welfare measures are used, it is to be expected that strategic voting is welfare-decreasing. Consider, however, what using only ordinal welfare measures implies if the expected utility model of voter behaviour is accepted. It implies that using intensity-based welfare measures are not accepted even though one acknowledges the relevance of intensities for individual voters' behaviour. But if intensities are important for the individuals, they should be normatively important for the whole electorate.

Fortunately, it is possible to accommodate the criticism that our choice of interpersonal comparisons is arbitrary. If the result that strategic voting increases average utility obtains with *all* different and at least mildly reasonable interpersonal comparisons, then this result does not depend on any particular interpersonal comparison. If the result is *robust* to interpersonal comparisons in such a way, we can be assured that we know something more about the consequences of strategic voting even though we do not know which interpersonal comparison is correct.

Several different variations on interpersonal comparisons were thus tried in order to see whether the results are robust or not. In order to retain comparability to previous results, all these variations need to change interpersonal comparisons without changing the preference orderings, the *intra* individual preference intensities, or the average utility of all alternatives. It is thus necessary to hold the parameters that determine individual *behaviour* fixed in evaluating robustness to interpersonal comparisons.

One interpersonal comparability variation is to preserve the original preference orderings and relative intraindividual intensities but redraw the minimum and maximum values for the utility scales (i.e. U_3 and U_1) randomly from the same uniform distribution as before. The results from this variation are almost identical to those presented before, and will therefore not be presented. This variation is admittedly quite slight because it merely changes the realisations of the random variables in one particular random assignment of utilities.

In order to make more dramatic changes, the utility scales must be changed in such a way that they are systematically different between different voter types. The utilities of voters of types 1, 3, 5 and 6 were again changed. The average utility for each voter type was retained, but the utility scale, i.e. the difference between the maximum and minimum utilities was made smaller (larger) for voters of types 1 and 6, and the utility scale for voters of types 3 and 5 was made larger (smaller). The utility scales of those who put alternative *y* second were thus shrunk and the utility scales of those who put alternative *x* second were stretched. Bearing in mind that in setups with correlated intensities the intensities for *x* are higher than for *y* on average, this variation effectively diminishes the importance of those who put *y* second and increases the importance of those who put *x* second. This variation on interpersonal comparisons will be referred to as the "mutually reinforcing correlation setup" because the *intra* personal intensities are high on average for the *same* voter types whose *inter* personal intensities weigh most in the sum of utilities. A second variation reverses the interpersonal correlation but retains the intrapersonal correlation by stretching the scales for voters of types 1 and 6, and shrinking the scales for voters of types 3 and 5. The second variation will be referred to as the "negative correlation setup".

Let IPC denote a parameter that reflects how much voters' scales are shrunk or stretched. The original utilities are U_1, U_2^* and U_3. Let $\underline{U}_1$ and $\overline{U}_3$ denote the maximum and minimum utilities for voters of types one and six after their scales have been shrunk ($\underline{U}_1 < U_1$ and $\overline{U}_3 > U_3$). Since the idea is to subtract as much from U_1 as is added to U_3, $\overline{U}_3 - U_3 = U_1 - \underline{U}_1$. $\underline{U}_1$ (and $\overline{U}_3$) is obtained by adding to (subtracting from) the midpoint of the utility scale $\frac{U_1+U_3}{2}$ a part of the individual's scale $\frac{\text{IPC}\cdot(U_1-U_3)}{2}$ so that

$$\underline{U}_1 = \frac{[U_1 + U_3 + \text{IPC}\cdot(U_1 - U_3)]}{2} \tag{16}$$

and

$$\overline{U}_3 = \frac{[U_1 + U_3 - \text{IPC} \cdot (U_1 - U_3)]}{2} = U_1 + U_3 - \underline{U}_1. \tag{17}$$

Similarly, let $\overline{U}_1$ and $\underline{U}_3$ denote the maximum and minimum utilities for voters of types three and five after their utility scales have been stretched. The idea now is to add as much, on average, to U_1 as was subtracted from voters of types one and six. Thus, the difference between $\frac{U_1 - U_3}{2}$ and $\frac{\text{IPC} \cdot (U_1 - U_3)}{2}$ is added to the original U_1 so that

$$\overline{U}_1 = U_1 + \frac{(1 - \text{IPC})(U_1 - U_3)}{2} = 2 \cdot U_1 - \underline{U}_1. \tag{18}$$

Again it is required that $U_3 - \underline{U}_3 = \overline{U}_1 - U_1$ so that

$$\underline{U}_3 = U_3 - \overline{U}_1 + U_1 = U_3 - U_1 + \underline{U}_1. \tag{19}$$

What remains is to rescale the interpersonal intensities in such a way that their *intrapersonal* relative values remain unchanged. Let U_2^{shrink} and U_2^{stretch} denote these two intensities. Then

$$U_2^{\text{shrink}} = \overline{U}_3 + U_2^*(\underline{U}_1 - \overline{U}_3) \tag{20}$$

and

$$U_2^{\text{stretch}} = \underline{U}_3 + U_2^*(\overline{U}_1 - \underline{U}_3). \tag{21}$$

In the mutually reinforcing correlation setup voter types 1 and 6 have utilities $(\underline{U}_1, U_2^{\text{shrink}}, \overline{U}_3)$, voter types 3 and 5 have utilities $(\overline{U}_1, U_2^{\text{stretch}}, \underline{U}_3)$ and voter types 2 and 4 have utilities (U_1, U_2, U_3).

The results from these two setups with $\varepsilon = 0.4$ are displayed in Figs. 7 and 8. The results are similar with other values of ε.

As expected, the more the interpersonal intensities correlate with the intrapersonal intensities (i.e. the smaller IPC and C are), the higher are the average utilities in the mutually reinforcing correlation setup. This result can be explained as follows. The lower IPC is, the more the utilities of voter types 3 and 5 weigh in the sum of utility. Since these voters put x second, and since the sum of utility of *all* these voters is higher for x than for y or z, the average utility is higher for x than

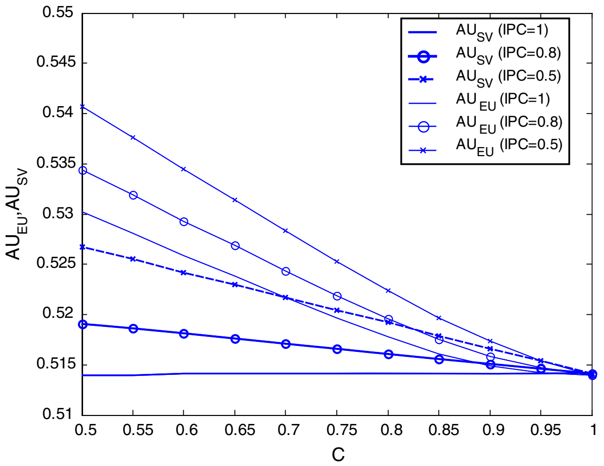

Figure 7. Average utilities in mutually reinforcing correlation setups.

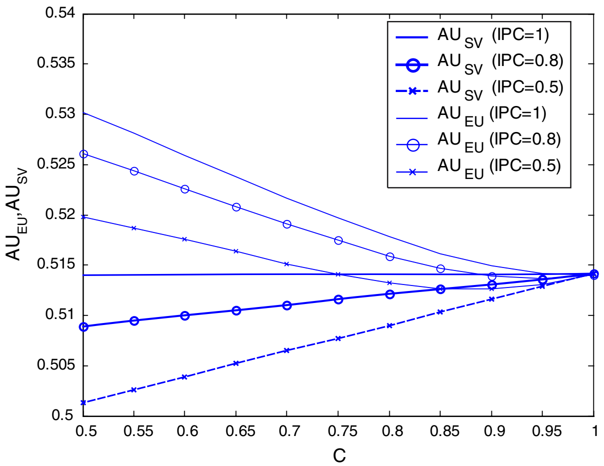

Figure 8. Average utilities in setups negative correlation setups.

it was without the reinforcing correlation. Since these voters also vote strategically for x, their actions make the average utility relatively high. As Figure 8 shows, reversing the interpersonal correlation while keeping the intrapersonal correlation makes average utilities lower. Notice, however, that EU behaviour remains welfare-increasing even in negative correlation setups.

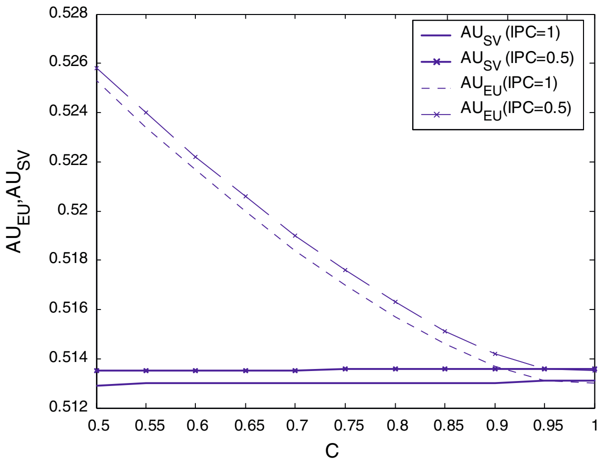

Figure 9. Average utilities for setups with an upward shift in utility for voter types 1 and 6.

Yet another interpersonal comparison consists in making all three utilities higher for some voter types than for some others. In “shift *x* upwards setups” the utility of voter types 3 and 5 was diminished by substracting the parameter IPC from their utilities and the utility of voter types 1 and 6 was increased by adding IPC to their utilities. In “shift *y* upwards setups” the roles of the voter types were again reversed. The results from these two setups are displayed in Figs. 9 and 10.

It is easy to see that the intrapersonal differences in these setups are much more important than the interpersonal ones. There is a slight difference however. Shifting the utilities of voter types 3 and 5 upwards, and those of types 1 and 6 downwards increases average utilities slightly. Reversing the voter types have the opposite effect. These results can be explained as follows. Voters of types 3 and 5 put alternative *x* second. Increasing their utilities increases their weight in the sum of utility. Such a shift slightly increases average utilities under EU behaviour because voter types 3 and 5 also vote strategically for *x*.

The simulation results from the setups studying different interpersonal comparisons can be summarised as follows. Making different interpersonal comparisons does change the

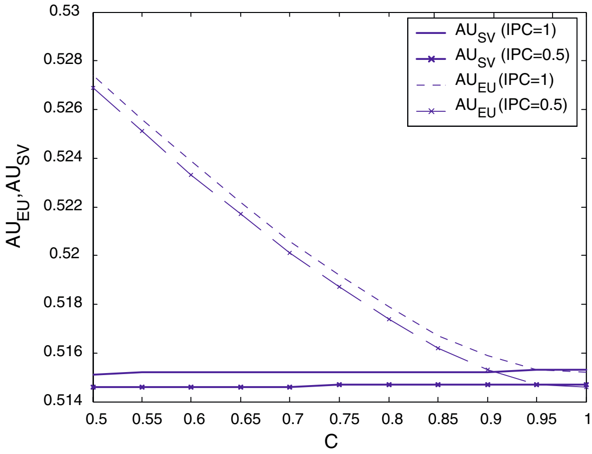

Figure 10. Average utilities in setups with an upward shift in the utilities of voter types 3 and 5.

results, but EU behaviour *remains welfare-increasing* with each different interpersonal comparison. The results are thus robust with respect to interpersonal comparisons.

7.4. *Elimination agendas*

The important difference between elimination agendas and amendment agendas is that in the former fewer voter types may have an incentive to vote strategically. Fig. 11 displays the simulation results with an elimination agenda ($[x]\,yz$). It is easy to see from this figure that strategic voting is welfare-increasing in all setups under elimination agendas if the preference intensities for alternative x are systematically higher than for y and z. It is relatively easy to explain why strategic voting is welfare-increasing under elimination agendas when the intensities for x are high. The average utility under SV behaviour is relatively low because alternative x is seldom selected, but, at the same time, there is approximately an equal number of supporters for each of the three alternatives. Hence, under elimination agenda ($[x]\,yz$), strategic voting may cause x to be selected, and this is what increases average utility in the EU-setups when compared to the SV-setup. x is

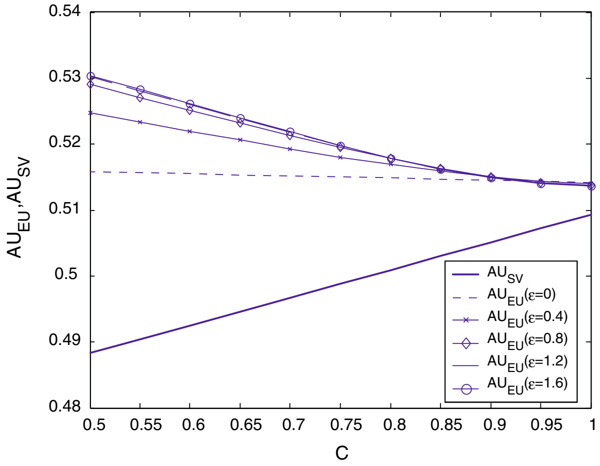

Figure 11. Average utilities in an elimination agenda when the intensities of x are high.

not the only alternative that obtains strategic votes, but under agenda $([x]\,yz)$, it is likely to be the only alternative for which the strategic votes matter; if y or z is intensively preferred, it will be selected also under sincere behaviour.

However, if the average utility for x is decreased, and that of y (or z) increased, the results are quite different. Fig. 12 shows average utilities under elimination agendas when the intensities correlate with the voter types such that the intensities of x are decreased and the intensities of y increased.

EU behaviour is now welfare-increasing only if the degree of correlation is not very high. Notice, however, that the average utilities under EU behaviour in two different cases are very similar. The main difference lies in the average utility under SV behaviour. The average utilities under SV behaviour are low under elimination agendas when the intensities of x are high because these high intensities are not reflected in voters' sincere behaviour in any way. Strategic voting thus at least gives a chance to an alternative that is introduced early under an elimination agenda. When the intensities of x are lower and the intensities of y higher, x is selected just as seldom as in all SV-setups. However, since the high-utility y always

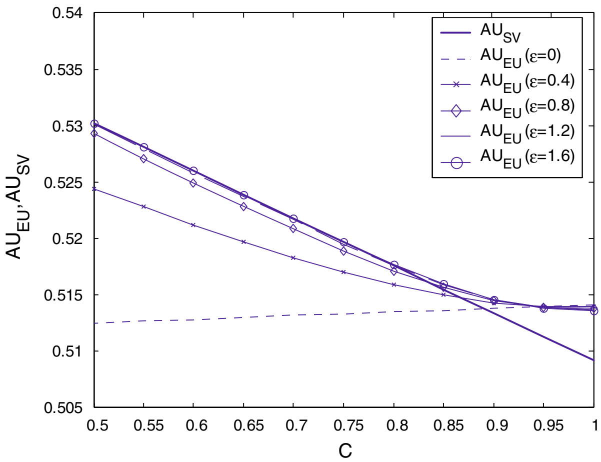

Figure 12. Average utilities in elimination agendas when the intensities for x are low.

participates in the second-round contest, the average utility becomes relatively high under SV behaviour. In setups with strong correlation, and in which the intensities of x are low, strategic voting decreases average utility because the strategic votes for the alternative that has a low sum of utility on the average are more likely to matter than the strategic votes for the other alternatives.

There is one important qualification to the results from both voting rules. If some voter types engage in EU behaviour and some in SV behaviour, the systematic absence of balancing strategic votes suggests that the welfare consequences of strategic voting are less beneficial or welfare-decreasing.[15] This is an important consideration, because it may well be reasonable to assume that some voter types are more prone to strategic voting than some others. The complexity of this matter, however, prevents us from presenting a discussion of it here.

8. CONCLUSIONS

The main conclusion that may be drawn from the simulation results is that welfare-increasing strategic voting is not a mere theoretical possibility in parliamentary voting. Indeed, it may well be the typical case.

The most important and widely discussed condition in Arrow's (1963) impossibility theorem is the Independence of Irrelevant Alternatives (IIA). IIA is closely connected to strategy-proofness.[16] The idea that strategic voting should be precluded in a voting rule is the only justification for strategy-proofness, and a crucial argument for IIA (e.g. Blin (1976)). The precise interpretation of these conditions may need to be re-evaluated in voting theory because the result presented here indicate that strategic voting may well be beneficial. Imagine that there was a strategy-proof voting rule. By definition, this would mean that voters would not have an incentive for changing their behaviour by voting strategically. Strategic voting could not be welfare-increasing or welfare-decreasing because the individuals would not have an incentive to engage in it. The point is that it is not possible to determine whether strategic voting and thereby strategy-proofness are desirable or not, a priori, without explicitly investigating the welfare consequences of strategic voting in each voting rule. The possibility that strategic voting is beneficial implies that the rationale for the so called *manipulability measures* (e.g. Saari (1990); Smith (1999)) is put into question. The important question to study is not which voting rules are best in selecting outcomes that are "close" to those that would have ensued from sincere voting, but rather which rules result in best outcomes when individuals vote strategically.

In some contexts (other than voting) strategy-proofness may be *intrinsically* important because it may be important to know the preferences of every agent. It should be borne in mind that the results here concern only two specific, although commonly used voting rules, whereas the scope of the impossibility theorems is considerably broader.

Strategic voting increases average utility compared to sincere voting because the former allows preference intensities to influence voting outcomes but the latter does not. Uniform setups yield the worst possible welfare consequences of strategic voting because the intensity differences between the alternatives are as small as they can possibly be. If the correlation between voter types and intensities is strong, strategic voting is very clearly welfare-increasing. This result also shows that if voters vote strategically, the criticism that majority rule does not take preference intensities into account is false. Furthermore, the larger the differences in the intensities are, the more welfare-increasing strategic voting is. If the correlation of intensities is strong, welfare-increasing strategic voting does not even require reliable signals.

A particular configuration of utilities under elimination agendas provides an exception. If the intensities for the alternative that may be eliminated on the first round are low on the average, strategic voting may be welfare-decreasing. We have seen, however, that even in this case the average utilities under EU behaviour are relatively high. It is just that the average utilities under SV behaviour are even higher because the unpopular alternative will often be eliminated with a sincere vote.

These findings suggest that strategic voting is a virtue rather than a vice in commonly used parliamentary agendas if all voters engage in expected utility maximising behaviour.

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NOTES

1. See Miller (1980), Shepsle and Weingast (1984) and Banks (1985).

2. However, Miller (1977) shows, by way of an example, that strategic voting may select the CW when sincere voting does not. The CW is an alternative that the majority of voters prefer to all other alternatives.
3. Vote-trading is also a way to vote strategically, and its welfare consequences have been investigated. See Shepsle and Weingast (1994) for a fairly recent review.
4. The literature on vote-trading has also acknowledged that strategic voting allows for expressing preference intensities (see, e.g., Stratmann (1997)).
5. See, e.g., Hildreth (1953), Coleman (1966) and Mackay (1980, p. 42).
6. This model of incomplete information is also similar to global games Carlsson and van Damme (1993). See Morris and Shin (2003) for a review. See also Frankel et al. (2003).
7. See McKelvey and Niemi (1978), Moulin (1979) and Sloth (1993).
8. See McKelvey (1990) and Austen-Smith and Banks (1999) for surveys on the analytical literature on the existence of a CW. Mueller (1989) and Gehrlein (2002) provide overviews of the simulation approaches.
9. Only the case with three alternatives is studied in this paper. Extending the model to any number of alternatives is possible but so complicated that it requires another paper Lehtinen (2002).
10. See Ordeshook (1986), Ordeshook and Schwartz (1987) and Miller (1995) for discussions on different agendas.
11. This approximation restricts the applicability of our model to situations with a fairly large number of voters. Thirty observations is sometimes given as a very rough guess on the validity of the Central Limit Theorem.
12. It is argued in Lehtinen (2002) that updating is difficult even if there are more than three alternatives. See also Enelow and Hinich (1983).
13. See Krehbiel and Rivers (1990), Eckel and Holt (1989), Calvert and Fenno (1994), Volden (1998), Wilkerson (1999) and Gilmour (2001) for discussions on the prevalence of strategic voting. See Tsetlin et al. (2003) and Gehrlein (2002) for recent discussions of impartial culture.
14. The results are presented only as graphs here. All numerical results in tabular form, as well as the FORTRAN codes to generate them are available from the author on request.
15. If some voters engage in EU-behaviour and some others in SV-behaviour, but engaging in EU-behaviour does not correlate with being of a certain voter type, the simulation results are similar (but weaker) to the results obtained here.
16. See Gibbard (1973), Satterthwaite (1975) and Blin and Satterthwaite (1978).

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