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POLICY STABLE STATES IN THE GRAPH MODEL FOR CONFLICT RESOLUTION

ABSTRACT. A new approach to policy analysis is formulated within the framework of the graph model for conflict resolution. A policy is defined as a plan of action for a decision maker (DM) that specifies the DM's intended action starting at every possible state in a graph model of a conflict. Given a profile of policies, a Policy Stable State (PSS) is a state that no DM moves away from (according to its policy), and such that no DM would prefer to change its policy given the policies of the other DMs. The profile of policies associated to a PSS is called a Policy Equilibrium. Properties of PSSs are developed, and a refinement is suggested that restricts DMs to policies that are credible in that they are in the DM's immediate interest. Relationships with existing stability definitions in the graph model for conflict resolution are then explored.

KEY WORDS. Game equilibrium, graph model, policy equilibrium, policy stable state, strategic conflict

1. INTRODUCTION

In a strategic conflict, a decision maker (DM) may declare in advance what it intends to do at each state that could arise. For example, in a potential military confrontation, one country may announce in advance that it will go to war only if it is invaded, while another may proclaim that it will launch an attack if hostile troops are massed close to its border. Recently, to eliminate nuclear development programs in the Korean peninsula, the Secretary of State of the United States, Colin Powell, stated on September 23, 2004 that "the sooner they return to the six-party format and begin discussions again at the fourth round of the six-party meeting, the sooner we will able to help North Korea deal with its very serious economic problems"(Bloomberg News Series, 2004). Such declarations, or *policies*, are clearly intended to influence the outcome of the conflict.

The objective of this paper is to design a unique paradigm for defining stable states that are a direct consequence of DMs' announced policies. To accomplish this in a realistic and convenient fashion, the new approach to policy analysis uses the graph model for conflict resolution (Fang et al., 1993) as a foundation on which to construct the paradigm. Following an overview of the graph model in Section 2, a Policy Equilibrium and an associated Policy Stable State (PSS) are defined in Section 3. In Section 4, some theorems are presented that relate PSSs to existing stability concepts within the graph model framework. A refinement is suggested in Section 5 that reduces the number of equilibria. Section 6 considers the existence of policy equilibria for conflict models in graph form.

2. THE GRAPH MODEL FOR CONFLICT RESOLUTION

The graph model for conflict resolution, a unique methodology for modeling and analyzing real-world conflicts (Fang et al., 1993; Kilgour et al., 1990), has some connections to game theory (Hamilton and Slutsky, 1993; von Neuman and Morgenstern, 1953). A graph model for a conflict consists of a directed graph and a preference structure on the set of all states for each DM who can affect the dispute. Although the graph model is defined for general conflict having n DMs, for simplicity this paper considers conflicts with two DMs only. Let $N = \{1, 2\}$ denote the set of DMs. Sometimes a representative DM of N is denoted by i and the other DM is denoted by j. Furthermore, let S be the set of states or possible scenarios in the model where $|S| = u$. A finite directed graph $D_i = (S, A_i)$, $i \in N$, keeps track of the movements among states that DM i can make in one step. The vertices of each graph are the possible states of the conflict and therefore the vertex set, S, is common to both directed graphs. If DM i can unilaterally move (in one step) from state s_1 to state s_2, there is an arc with orientation from s_1 to s_2 in A_i and state s_2 is therefore reachable from state s_1 by DM i. For $i \in N$, DM i's reachable list for state $s \in S$ is the set $R_i(s)$ of all states to which DM i can move (in one step) from state s.

The preference structure in the graph model is expressed in terms of a pair of binary relations $\{\succ_i, \sim_i\}$ on S, where $s_1 \succ_i s_2$, for $s_1, s_2 \in S$, indicates that DM i prefers s_1 to s_2, and $s_1 \sim_i s_2$ that DM i is indifferent between s_1 and s_2, or equally prefers s_1 and s_2. The following properties are assumed:

(1) $\succ_i$ is *asymmetric*, i.e., $s_1 \succ_i s_2$ and $s_2 \succ_i s_1$ cannot hold true at the same time, where $s_1, s_2 \in S$.
(2) $\sim_i$ is *reflexive*, i.e., $s \sim_i s$ for any $s \in S$, and *symmetric*, i.e., if $s_1 \sim_i s_2$ then $s_2 \sim_i s_1$, where $s_1, s_2 \in S$.
(3) $\{\succ_i, \sim_i\}$ is *strongly complete*, i.e., if $s_1, s_2 \in S$, then exactly one of $s_1 \succ_i s_2$, $s_2 \succ_i s_1$ and $s_1 \sim_i s_2$ is true.

Sometimes, the notation $s_1 \succeq_i s_2$ is used to indicate either $s_1 \succ_i s_2$ or $s_1 \sim_i s_2$. Note that transitivity of preferences is not assumed, so that the results in this paper are valid for both intransitive and transitive preferences.

A unilateral improvement from a particular state for a specific DM is any preferred state to which the DM can unilaterally move. The unilateral improvement list for DM i from state s is denoted as $R_i^+(s) = \{s_1 \in R_i(s) | s_1 \succ_i s\}$.

In a graph model, a strategic conflict begins at a *status quo* state and progresses from state to state via state transitions controlled by various DMs, who may act whenever they want to. As described above, a graph model represents the state transitions controlled by each DM as a directed graph with the set of states as the vertex set. The graph model incorporates a number of distinct submodels of how DMs decide whether to move the conflict from its current state. These submodels, called *stability definitions*, allow for variation in several aspects of decision style, such as level of foresight, risk aversion, and so on. Among these submodels, metarational stability (general and symmetric), sequential stability and limited-move stability have computational advantages, and are widely used to analyze real-world conflicts.

EXAMPLE 1. Consider a simple conflict of two interest groups over the planned construction of a bridge to span a river. DM 1, representing drivers of vehicles who will use the bridge, would prefer that the bridge be built. However, DM 2, representing local residents, would prefer that the bridge not be built to avoid increased noise and air pollution. As depicted in Figure 1, there are two states in this illustrative conflict—state s_1 in which a bridge is built and state s_2 in which there is no bridge. The pair of numbers given in brackets for each state represents the preferences of each DM, where a higher number means more preferred, and the first and second entries stand for the preferences of DMs 1 and 2, respectively. Hence, state s_1 is preferred by DM 1 to s_2, while state s_2 is preferred to s_1 by DM 2. In the left-hand graph of Figure 1 (DM 1's graph), there are two vertices representing the two states; the direction of the arc between them indicates that DM 1 has the ability to build the bridge. The right-hand graph in Figure 1 is DM 2's graph; the direction of the arc means that DM 2 can destroy the bridge or block its construction. In other words, DM 1 can stay at any state or move from s_2 to s_1, while DM 2 can stay at any state or move from s_1 to s_2.

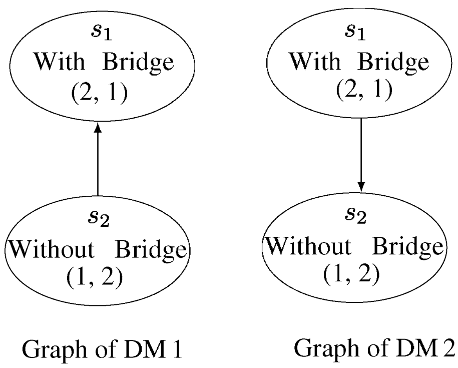

Figure 1. The bridge conflict in graph form.

3. POLICY IN THE GRAPH MODEL PARADIGM

3.1. *Definition of a Policy*

A *policy* for DM $i \in N$ is a function $\mathcal{P}_i : S \rightarrow S$ such that $\mathcal{P}_i(s) \in R_i(s) \cup \{s\}$ for all $s \in S$. The policy can therefore be written $\mathcal{P}_i = \{\mathcal{P}(i)(s) : s \in S\}$. Note that a policy for DM i specifies a plan of action for DM i from each state s, where i can stay or move to any state reachable by i from s. We interpret a policy as specifying what a DM will do at a state if that state arises.

In the example in Figure 1, $\mathcal{P}_1(s_1) \in \{s_1\}$ and $\mathcal{P}_1(s_2) \in \{s_1, s_2\}$. Therefore, there are two policies for DM i, $\{s_1, s_1\}$ and $\{s_1, s_2\}$. For instance, at the first policy, $\{s_1, s_1\}$, DM i intends to stay at state s_1 if he is at state s_1, and to move to state s_1 if he is at state s_2.

3.2. *Policy Equilibrium*

It is assumed that no DM can move consecutively. In other words, only alternating sequences of DMs are considered. Given an initial state s^*, an originating DM i, i's policy $\mathcal{P}_i$, and j's policy $\mathcal{P}_j$, a sequence of moves and counter moves is completely specified as follows:

$$\mathcal{P}_0(s^*) \equiv s^*, \mathcal{P}_i(s^*), \mathcal{P}_j(\mathcal{P}_i(s^*)), \ldots,$$
$$\mathcal{P}_i(\mathcal{P}_j(\cdots \mathcal{P}_i(s^*) \cdots)), \mathcal{P}_j(\mathcal{P}_i(\cdots \mathcal{P}_i(s^*) \cdots)), \ldots$$

The above sequence can be rewritten as a series of elements. Each element (s, i) is composed of a state (s) and a DM (i) who moves at that state. If DM i stays at state s according to its policy, the sequence terminates at element (s, i) and is called a *terminated sequence*. The *result* of a terminated sequence is defined to be the state in the last element.

An element (s, i) in a sequence is said to be *repeating* if the same element (s, i) appeared earlier. Evidently, there is no repeating element in a terminated sequence. If there is a repeating element in a sequence, then there exists a unique cycle of even length containing the repeating element. Once a sequence encounters the first repeating element, the sequence cycles among all the repeating elements.

A sequence having h elements is called a sequence of length h. This paper concerns sequences of infinite length. Because the number of all states in a conflict is finite, there must exist a repeating element in an infinite sequence. The *result* of an infinite sequence is defined to be the state in the first repeating element. This definition can be justified by considering a move to have an infinitesimal cost as reflected in the inertia assumption (see Brams, 1994; Kilgour and Zagare, 1987; Zagare, 1984). Therefore, for a sequence of infinite length, the subsequence between the initial element and the second appearance of the first repeating is substantial, and we call this part a *complete sequence*. The result of a sequence of infinite length is also called the *result* of its complete sequence.

DEFINITION 1. Policies $\mathcal{P}_1, \mathcal{P}_2$ form a policy equilibrium with respect to status quo state s^* if

(i) $\mathcal{P}_i(s^*) = s^*$ holds for both DM $i = 1, 2$,
(ii) $\forall i = 1, 2$, $\forall \mathcal{P}'_i$ such that $\mathcal{P}'_i(s^*) \neq s^*$, the result of any terminated sequence or any complete sequence is not preferred to s^* by DM i.

A state s^* satisfying the above two conditions is called a PSS.

The following procedure can be employed to determine whether a state $s^* \in S$ is a PSS under policies $\mathcal{P}_i$ and $\mathcal{P}_j$. First, if $\mathcal{P}_i(s^*) \neq s^*$ or $\mathcal{P}_j(s^*) \neq s^*$, then s^* is not a PSS under policies $\mathcal{P}_i$ and $\mathcal{P}_j$. If $\mathcal{P}_i(s^*) = s^*$ and $\mathcal{P}_j(s^*) = s^*$, then given DM j's policy $\mathcal{P}_j$, we construct a tree of all terminated or complete sequences to see whether there is a more preferred result for DM i. At the same time, given DM i's policy $\mathcal{P}_i$, we also construct a tree of all terminated or complete sequences to ascertain if there is a more preferred result for DM j. If, in both trees, there does not exist a more preferred state, then s^* is a PSS under $\mathcal{P}_i$ and $\mathcal{P}_j$. Otherwise, s^* is not a PSS.

Figure 2 illustrates a tree for the case of DM i, given j's policy $\mathcal{P}_j$. First, DM i tries every possible deviation from s^* to

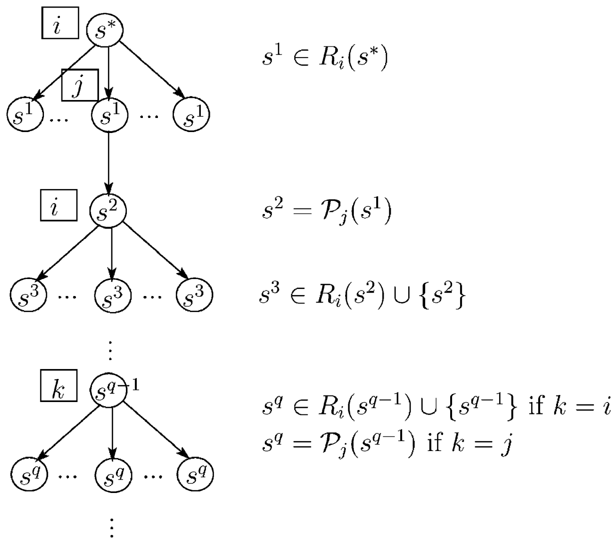

Figure 2. Constructing all complete sequences.

s^1. Then, a move by DM j is determined by policy $\mathcal{P}_j$. If $\mathcal{P}_j(s^1) = s^1$, then we obtain a terminated sequence. If $\mathcal{P}_j(s^1) = s^2 \neq s^1$, then DM i can either stay at s^2 or choose to move to $s^3 \in R_i(s^2)$. If $s^3 = s^*, s^1$, or s^2, we obtain a complete sequence. Otherwise, $\mathcal{P}_j$ determines the next state $s^4 = \mathcal{P}_j(s^3)$. After proceeding in this manner for at most $u - 1$ steps, we obtain all terminated or complete sequences.

The total number of policies for DM i is

$$\pi_i = \underset{i \in S}{\pi}(|R_i(s)| + 1).$$

Therefore, the total number of pairs of policies for a conflict is $\pi_1 \times \pi_2$.

EXAMPLE 2. Consider the bridge example shown in Figure 1. When state s_1 is taken as the status quo state, select the following policies for DMs 1 and 2:

$$\mathcal{P}_1 = \{s_1, s_1\} \quad \text{and} \quad \mathcal{P}_2 = \{s_1, s_2\}.$$

The first condition in Definition 1 is satisfied because

$$\mathcal{P}_1(s_1) = s_1 \quad \text{and} \quad \mathcal{P}_2(s_1) = s_1.$$

For the second condition, one must examine other possible policies from the status quo state s_1 such that $\mathcal{P}_1(s_1) \neq s_1$, $\mathcal{P}_2(s_1) \neq s_1$. Notice for DM i that every policy for state s_1 is $\mathcal{P}_1(s_1) = s_1$. However, for DM 2, there is another policy from state s_1 given by $\mathcal{P}'_2 = \{s_2, s_2\}$. Hence, $\mathcal{P}'_2(s_1) = s_2 \neq s_1$. There is no terminated sequence and there is only one complete sequence, which is given by $(s_1, 2)$, $(s_2, 1)$ and $(s_1, 2)$. The result of this complete sequence is $\mathcal{P}_1(\mathcal{P}'_2(s_1)) = s_1$. Accordingly, no result of a terminated or complete sequences is better than state s_1 for DM 2 (DM 2 equally prefers state s_1 to state s_1). Thus, state s_1 constitutes a PSS and the policies $\mathcal{P}_1$, $\mathcal{P}_2$ form a policy equilibrium.

4. RELATIONSHIPS WITH STABILITY CONCEPTS IN THE GRAPH MODEL

A range of stability concepts has been defined for use within the graph model for conflict resolution in order to calculate the stability of a state for a particular DM. A state that is stable according to a specific stability concept for all DMs is compared to a PSS in this section. The concept of Nash Stability is based on the definition originally given by Nash (1950).

DEFINITION 2. A state $s^N \in S$ is a Nash stable state for DM i iff $R_i^+(s^N) = \phi$. A state is called Nash stable for the conflict iff it is Nash stable for all DMs.

THEOREM 1. *A Nash stable state for the conflict is a PSS.*

Proof. Let s^N be a Nash stable state for the conflict and $\mathcal{P}_i$ be the policy of DM $i = 1, 2$ of staying at each state. Since $\mathcal{P}_i(s^N) = s^N$, condition (i) of Definition 1 is satisfied. Furthermore, if DM i moves from s to another state s_1, and DM j uses policy $\mathcal{P}_j$, then the resulting state will become s_1, which is not preferred to s for DM i according to the definition of a Nash stable state. Therefore, the second condition of Definition 1 is also satisfied. Hence, $\mathcal{P}_1, \mathcal{P}_2$ form a policy equilibrium with respect to status quo state s^N and s^N is a PSS. □

The original definitions for general and symmetric metarationality were put forward by Howard (1971).

DEFINITION 3. A state s^{GMR} is general metarational for DM i iff for every $s_1 \in R_i^+(s^{GMR})$ there exists at least one $s_2 \in R_j(s_1)$ with $s^{GMR} \succeq_i s_2$. A state is called general metarational for the conflict iff it is general metarational for all DMs.

Hence, for general metarationality, each DM i expects the opponent will respond by hurting i if i moves to a better state.

The next result shows that policy stability is more restrictive than general metarational stability.

THEOREM 2. *A PSS is general metarational for the conflict.*

Proof. The theorem can be proven by contradiction. Assume that a PSS s^* is not general metarational for the conflict. Without loss of generality, we suppose that there exists $s_1 \in R_i^+(s^*)$ such that DM i cannot be sanctioned even if it moves from s^* to s_1. In other words, for every $s_2 \in R_j(s_1)$, $s_2 \succ_i s^*$. Then, DM i can change its policy to a new policy in which $\mathcal{P}_i(s^*) = s_1$ and DM i stays at all other states. This change creates a more preferred state for DM i and, hence, s^* is not a PSS. □

As demonstrated in the following example, the converse of Theorem 2 is not true.

EXAMPLE 3. For each state in Figure 3, the DMs' preferences are shown in brackets. By referring to these graphs, it is

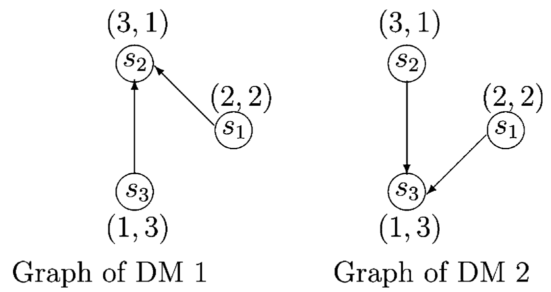

Figure 3. A general metarational stable state for the conflict which is not PSS.

easy to explain why s_1 is general metarational for DM 1. Specifically, as indicated in the left graph, DM 1 has a unilateral improvement from s_1 to s_2. However, from the right graph DM 2 can move from s_2 to s_3. Because state s_3 is less preferred than state s_1 by DM 1, DM 1's original unilateral improvement is sanctioned and hence s_1 is general metarational for this DM. Similarly, s_1 is general metarational for DM 2. However, one can now show that state s_1 is not a PSS according to Definition 1. In fact, if DM 1 stays at s_1, then DM 2 can change to the policy $\mathcal{P}'_2 = \{s_3, s_3, s_3\}$. For state s_3, DM 1 can either stay at s_3 or move to s_2. If DM 1 stays at s_3, then s_3 is the final result, which is preferable for the deviating DM 2. If DM 1 moves to s_2, then DM 2 can move to s_3 again. So according to the inertia assumption, s_3 is the result. In conclusion, given any policy of DM 1 for which it stays at s_1, it will be better for DM 2 to deviate from s_1 to s_3. This shows that s_1 is not a PSS by Definition 1.

DEFINITION 4. A state $s^{SMR} \in S$ is symmetric metarational for DM i iff for every $s_1 \in R_i^+(s^{SMR})$, there exists $s_2 \in R_j(s_1)$ such that $s^{SMR} \succeq_i s_2$ and $s^{SMR} \succeq_i s_3$ for all $s_3 \in R_i(s_2)$. A state is called symmetric metarational for the conflict iff it is symmetric metarational for all DMs.

Thus, symmetric metarationality is like general metarationality except that each DM i expects to have a chance to counter-respond to j's response to i's original move.

The following result shows that a PSS lies somewhere between general metarational stability and symmetric stability for the conflict.

THEOREM 3. *A symmetrical metarational for the conflict is a PSS.*

Proof. Let s^{SMR} be symmetric metarational for the conflict. Then for any $s_1 \in R_i^+(s^{SMR})$, there exists $s_2 \in R_j(s_1)$ such that

$$s^{SMR} \succeq_i s_2 \quad \text{and} \quad s^{SMR} \succeq_i s_3 \quad \text{for all } s_3 \in R_i(s_2). \tag{1}$$

Then we specify a policy $\mathcal{P}_j$ of DM j by moving from all such s_1 to the corresponding s_2, and staying at all other states. Similarly, we can define a policy $\mathcal{P}_i$ of DM i to stay at all states except for those actions to sanction possible improvements of DM j from s^{SMR}. Given this kind of policy $\mathcal{P}_j$, DM i has no policy that can create a loop in a complete sequence. In fact, given two movements specified by $\mathcal{P}_j$: $s_2 = \mathcal{P}_j(s_1)$, $s_2' = \mathcal{P}_j(s_1')$, we have $s_1 \succ_i s^{SMR}$, $s^{SMR} \succ_i s_1$, $s'_1 \succ_i s^{SMR}$ and $s^{SMR} \succ_i s'_2$. Hence, $s_1' \notin R_i(s_2)$ and $s'_2 \notin R_i(s_1)$ from (1). In conclusion, given the other DM's specified policy, if a DM deviates from s^{SMR} to a more preferred state, each complete sequence does not contain any loop, and the final result is not more preferred to the deviating DM. If a DM deviates from s^{SMR} to a less or equally preferred state, then the other DM's specified policy is to stay at the state and the result is not better than s^{SMR} for the deviating DM. Therefore, these two policies form an equilibrium and s is a PSS. □

A PSS is not necessarily symmetrical metarational for the conflict as shown by the following example.

EXAMPLE 4. As displayed in the conflict in Figure 4, from status quo state s_1, DM 1 can move to s_2 and DM 2 may sanction DM 1 by moving from s_2 to s_3. However, DM 1 can further move to s_4, which is more preferred by DM 1 to s_1. Hence, s_1 is not symmetrical metarational for the conflict. However, since DM 2 can ultimately move from s_4 to s_5, which is worse for DM 1 than s_1, the first move of

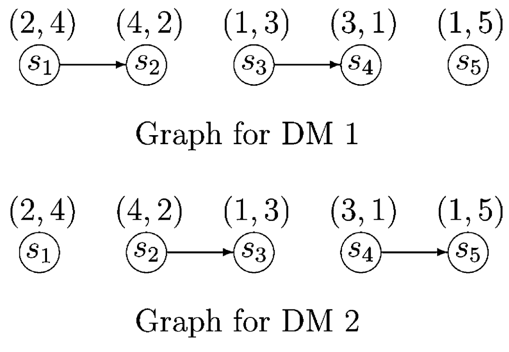

Figure 4. A graph model in which a PSS is not symmetrical metarational for the conflict.

DM 1 is not a good choice if DM 2 uses a policy of moving from s_2 to s_3 and moving from s_4 to s_5. When combined with a policy of DM 1 to stay at every state, these two policies form an equilibrium and s_1 is a PSS.

The above relationships established in Theorems 1–3 are illustrated in Figure 6.

5. REFINEMENT OF POLICY

In Section 3.1, a policy for DM i from state s allows i to stay at state s or move to a state which is an element of $R_i(s)$, which may contain more preferred, equally preferred or less preferred states. A DM's policy is deemed to be credible, if it always moves to a more preferred state. Hence, a *credible policy* of DM i $\mathcal{P}_i^c$ is defined as $\mathcal{P}_i^c(s) \in R_i^+(s) \cup \{s\}$. Definition 1 can be modified using this concept. Therefore, the altered definition for Definition 1 defines a state to be a *credible PSS* if there is a credible policy equilibrium consisting of credible policies for staying at this state. Following a similar procedure to that described in Section 3.2, a procedure can be formulated to determine whether a state $s \in S$ is a credible PSS. Figure 5

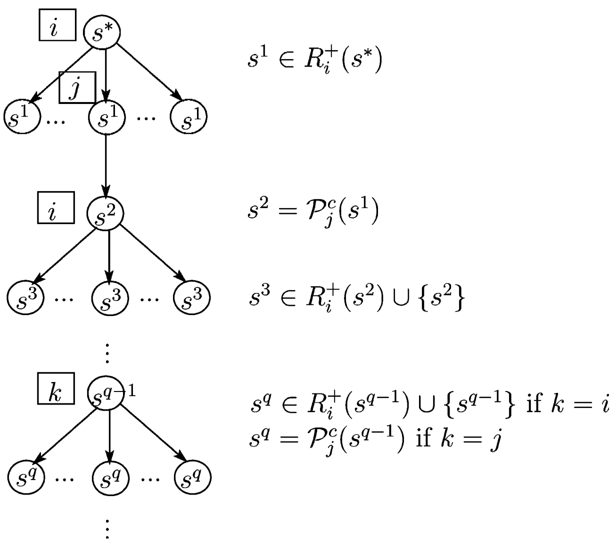

Figure 5. Constructing all complete sequences for credible policies.

depicts how to construct all terminated or complete sequences for credible policies. The proof of Theorem 1 actually shows that a Nash stable state for a conflict is a credible PSS.

EXAMPLE 5. The chicken game. In the game of chicken, two drivers, called DMs 1 and 2, are racing towards each other at high speed. Each DM has the choice of swerving or ‘‘chickening out’’ (denoted by C), thereby avoiding a collision, or continuing to drive straight ahead and thus selecting the strategy of ‘‘don’t swerve’’ (D). The normal form of chicken is shown below in which DMs 1 and 2 control the row and column strategies, respectively. The four possible states in the game are represented by the four cells in the matrix. The pair of numbers in each cell gives the relative preferences of DMs 1 and 2, respectively. Hence, the worst state for both DMs occurs at (D, D) and (D, C) is the most preferred state for DM 1.

		DM 2	
		D	C
DM 1	D	(1, 1)	(4, 2)
	C	(2, 4)	(3, 3)

States (D, C) and (C, D) are credible PSSs because they are Nash stable for the conflict. State (C, C) is a PSS with the following policies:

DM 1:	(D, D) ↓ stay	(C, D) ↓ (D, D)	(D, C) ↓ stay	(C, C) ↓ stay

DM 2:	(D, D) ↓ stay	(C, D) ↓ stay	(D, C) ↓ (D, D)	(C, C) ↓ stay

However, in the above policies, both DMs contain a move from a better state to a worse state (DM 1 from (C, D) to (D, D) and DM 2 from (D, C) to (D, D)). Hence, the policies are not credible. Fang et al. (1993) show that (C, C) is actually nonmyopically stable for both DMs, so we know that a nonmyopically stable state for the conflict may not be a credible PSS. In fact, in the definition of nonmyopic stability (Kilgour, 1984; Fang et al., 1993), each DM i is supposed to behave assuming that its opponent j will only maximize j's own utility. Therefore, anticipating j's action, i may temporarily move to a worse state on purpose.

The sequential stability concept (Fraser and Hipel, 1984) given below permits only credible moves (or unilateral improvements) by the DMs. More specifically,

DEFINITION 5. For a two-DM conflict having the set of DMs $N = \{i, j\}$, a state $S^{SEQ} \in S$ is sequentially stable for DM i iff for every $s_1 \in R_i^+(S^{SEQ})$ there exists $s_2 \in R_j^+(s_1)$ with $s^{SEQ} \succeq_i s_2$. A state is sequentially stable for the conflict iff it is sequentially stable for both DMs.

In the definition of a policy equilibrium, we require a sequence to be complete. The concept of a credible PSS is more restrictive than sequential stability in the sense that the latter concerns a sequence in which each DM appears at most once. However, the concept of PSS in Section 3 is weaker than the sequential stability because the other DM's policies are fixed when we check DM i's policy. The above example in Figure 3 also shows that a sequentially stable state for the conflict need not be a PSS. The following example shows that a PSS need not be sequentially stable for the conflict.

EXAMPLE 6. Consider a normal form game as follows:

		DM 2	
		β_1	β_2
DM 1	α_1	$(1, 1)$	$(0, 0)$
	α_2	$(2, 0)$	$(-1, -1)$
	α_3	$(0, 0)$	$(0, 2)$

State (α_1, β_1) is a PSS with the policies $\mathcal{P}_1$ and $\mathcal{P}_2$ for DMs 1 and 2, respectively, as follows:

$\mathcal{P}_1(\alpha_2, \beta_2) = (\alpha_3, \beta_2)$ and $\mathcal{P}_1(s) = s$ for all other states.
$\mathcal{P}_2(\alpha_1, \beta_2) = (\alpha_1, \beta_1)$,
$\mathcal{P}_2(\alpha_2, \beta_1 = (\alpha_2, \beta_2)$, and $\mathcal{P}_2(s) = s$ for all other states.

However, (α_1, β_1) is not sequentially stable for the conflict, because α_2 is a unilateral improvement for DM 1 and DM 2 cannot sanction DM 1 in one step without moving to a less preferred state.

The following result shows that a credible PSS is a strictly stronger stability concept than sequential stability.

THEOREM 4. *If $s \in S$ is a credible PSS, then s is sequentially stable for the conflict.*

Proof. Suppose that a credible PSS s is not sequentially stable for DM i. Then there exists $s_1 \in R_i^+(s)$, such that for all $s_2 \in R_j^+(s_1)$, it holds that $s_2 \succ_i s$. Since $\mathcal{P}_j$ is credible, $\mathcal{P}_j(s_1) \in R_j^+(s_1) \cup \{s_1\}$. Therefore, if DM i changes its policy from $\mathcal{P}_i$ to another policy $\mathcal{P}_i'$ which moves from s to s_1 and stays at all such s_2, then the result is more preferred. This contradicts the fact that $\mathcal{P}_i$ and $\mathcal{P}_j$ form a policy equilibrium. □

The relationships discussed in this section among various stability concepts are shown in Figure 6.

In certain conflict situations, DMs in the process of considering moves and counter-moves may prefer never to harm

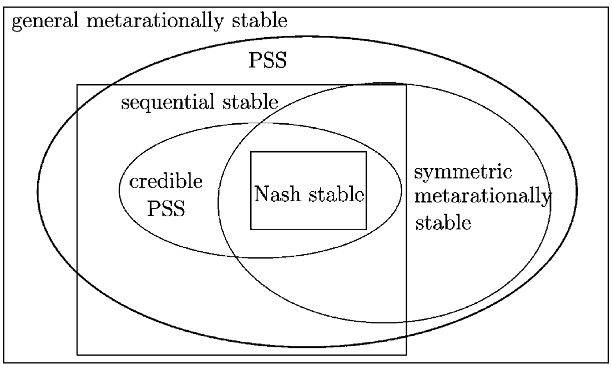

Figure 6. Relations among different stability concepts.

themselves. Why, for example, should a specific military policy be adhered to if it will result in a large number of casualties when a safer policy involving reduced losses could essentially produce the same result? Hence, the concept of credibility of actions can be important in many situations. Nevertheless, in certain circumstances, DMs may be willing to move to less preferred states in the hope of ending up at a better overall outcome. To allow for both of these types of situations, credible (Section 5) and regular (Section 3) policies are proposed in this paper. By comparing the results obtained using both procedures, one can ascertain any differences that may arise, which in turn may furnish insightful strategic advice. As explained above, in the graph model for conflict resolution, sequential stability captures the notion of credibility in DMs' behavior under conflict, while other stability concepts such as general and symmetric metarationality, limited-move stability and nonmyopic stability sometimes permit movement to less preferred situations.

6. EXISTENCE OF A PSS AND A SEQUENTIALLY STABLE EQUILIBRIUM

Fraser and Hipel (1984) prove that a conflict that can be modeled in option form in which every DM has transitive preferences contains at least one sequentially state for the conflict. The general graph model for conflict resolution does not assume transitivity of preferences or transitivity of movement. The counterexample in Figure 7 demonstrates that an existence theorem does not hold when transitivity of movement is not assumed. Specifically, no state is sequentially stable for both DMs. For example, as shown in the left graph in Figure 7, state s_1 is stable for DM 1, because this DM has no movement from s_1. However, as depicted in the right graph, DM 2 has a unilateral improvement from s_1 to s_2 and this unilateral improvement is not sanctioned by DM 1's unilateral improvement from s_2 to s_3, shown in the left graph in Figure 7. Thus, state s_1 is not sequentially stable for the conflict. Moreover, a counterexample can be easily constructed to demonstrate that a

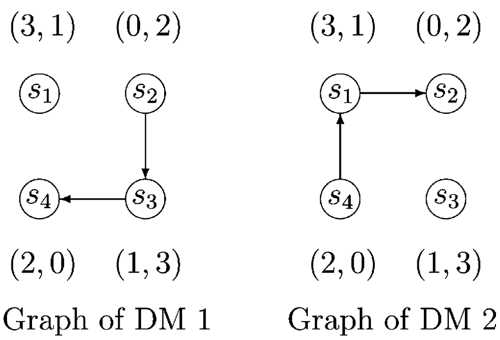

Figure 7. An example without any sequentially stable state for the conflict.

sequentially stable state for the conflict does not exist for a two-DM conflict having intransitive preferences but transitivity of movement.

Nonetheless, the following theorem establishes an existence theorem for sequential stability for the conflict when both the transitivity of movement and transitivity of preferences are assumed.

THEOREM 5. *If movements and preferences are transitive, then there is at least one sequentially stable state for the conflict in the graph model for a two-DM conflict.*

Proof. If the theorem is false, each state is not sequentially stable for at least one DM. Without loss of generality, let s_1 be a state most preferred by DM i which is not sequentially stable for it. Then there is at least one move of DM i to a more preferred state s_2. Since s_2 is preferred to s_1 by DM i, s_2 should be sequentially stable for DM i. If it is also sequentially stable for DM j, then the proof is complete. Otherwise, DM j has a move to a better state s_3. By use of the transitivity of preference of DM j, suppose that s_3 is the most preferred state by DM j among those deviations from s_2. It must hold that $s_3 \succ_i s_1$, since otherwise, a move from s_2 to s_3 is a credible sanction against DM i. Therefore, s_3 is again sequentially stable for DM i. If it is sequentially stable for DM j, then the proof is complete. If it is not sequentially stable for DM j, then DM j can move to a better state s_4. Because of the transitivity of movement, DM j has a move from s_2 to s_4 directly, which contradicts the assumption that s_3 is most preferred by DM j. □

However, even if a graph model is fully transitive, it may happen that there is no PSS. Consider the graph in Figure 8. Specifically, in this example, if the status quo state is s_1, then no policy of DM 1 can sanction deviations by DM 2 if DM 2 uses policy $\{s_2, s_2, s_3, s_2\}$. If the status quo state is s_2, then no policy of DM 2 can sanction deviations by DM 1 if DM 1 uses policy $\{s_1, s_3, s_3, s_4\}$. A similar result holds for s_3 and s_4.

In game theory, the concept of Nash equilibrium is extremely important because its existence is ensured in finite games (using mixed strategies). Since we do not assume utilities to represent DMs' preferences, we do not consider mixed strategies in our framework. Therefore, the existence of a sequentially stable equilibrium is very important, because it guarantees a solution of every graph model provided preferences and movements are transitive. Comparatively, the nonexistence of a PSS is due to the fact that a policy looks very far into the future. Within the stability concepts defined for the graph model, the same thing happens with nonmyopic stability—graph models exist with no nonmyopic equilibria. Intuitively, this means that contemplating moves and counter-moves far into the future can restrict the ways in which sanctioning and potential equilibria can arise, and there may be no existence theorem. To allow for the fact that moves further into the future should perhaps be discounted in

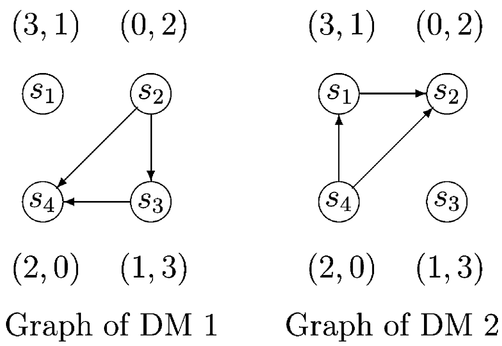

Figure 8. A fully transitive graph with no PSS.

terms of their effects on immediate preferences, one can examine stability concepts that do not have long foresight. In this way, one can appreciate the strategic impacts of alternative conflict behavior involving short, medium and long foresight.

Within the limited move and nonmyopic definitions of stability, backward induction is employed for defining and calculating stability. But backward induction is not without paradoxical features (Brams and Kilgour, 1998). If an analyst feels that this approach is not realistic in some conflict situations, the stability definitions of general and symmetric metarationality in which sanctioning DMs can move to less preferred states, or sequential stability, which permits only credible moves by all DMs, can be employed. For each of these stability definitions, backward induction is not used and states are compared only in a pairwise fashion when stability is calculated.

7. CONCLUSIONS

A novel approach to policy analysis is defined using the graph model for conflict resolution as a launching pad. To put this new methodology into perspective, comparisons are made to existing stability definitions and the idea of credibility of moves is entertained. The authors believe that continued research on policy analysis is of great import because policies are often adopted by DMs in practical situations. For example, a union may proclaim before negotiations that it will go on strike if the company does not meet its wage demands while the company may state that it will lock out the workers if the union demands high wage increases. In international trade, a country may threaten not to abide by rulings of the World Trade Organization that are detrimental to it. The foregoing and other examples reinforce the need for more research in policy analysis.

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