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UNANIMITY AND RESOURCE MONOTONICITY

ABSTRACT. In the context of indivisible public objects problems (e.g., candidate selection or qualification) with "separable" preferences, unanimity rule accepts each object if and only if the object is in everyone's top set. We establish two axiomatizations of unanimity rule. The main axiom is *resource monotonicity,* saying that resource increase should affect all agents in the same direction. This axiom is considered in combination with *simple Pareto* (there is no Pareto improvement by addition or subtraction of a single object), *independence of irrelevant alternatives,* and either *path independence* or *strategy-proofness.*

KEY WORDS: unanimity rule, resource monotonicity, simple Pareto, path independence, strategy-proofness

1. INTRODUCTION

Unanimity and majority decisions are two best-known schemes of group decision making in democratic society. While a number of studies offer justifications for majority decision,[1] few justifications exist for unanimity decision (Gordon, 2001; Berga et al., 2004). In the context of indivisible objects problem, studied by Barberà et al. (1991), we establish two axiomatizations for unanimity decision. Our key axiom is *resource monotonicity* saying that when there is an increase or decrease in available resources, everyone should be affected in the same direction. This axiom is studied by Chun and Thomson (1988), Moulin and Thomson (1988), and Thomson (1994) in other social choice models.[2]

The social choice model we consider in this paper takes the following abstract form. There is a set of indivisible public objects such as candidates, laws, public projects, public facilities, etc., referred to as an *agenda*. A group of agents need to decide which objects in the agenda to accept. There is no constraint for the choice and any subset of the agenda is an (social) *alternative.* All agents have strict preferences (no indifference between alternatives) and a (social choice) *problem* is characterized by a profile of individual preferences and an agenda. A social choice rule, or simply, a *rule* associates with each problem an alternative. *Unanimity rule* gives the alternative containing all objects which everyone wants; that is, this rule selects only those objects that are in everyone's choice set, or *top set.*

We first study *resource monotonicity* in conjunction with the Pareto principle, or briefly, *Pareto* (the choice for each problem should not allow any Pareto improvement making all agents better off). A weaker principle, called *simple Pareto,* is that the choice for each problem should not allow any Pareto improvement by either addition or subtraction of a single object. When there is no restriction on the domain of admissible preferences, we show that no rule satisfies both *resource monotonicity* and *simple Pareto* (so, the same impossibility with *Pareto*). This motivates us to focus on a restricted, yet, interesting domain of preferences, known as "separable" preferences (Barberà et al., 1991). When an agent has separable preferences, each object affects his welfare separately from other objects.

On the separable preferences domain, we first investigate rules satisfying *resource monotonicity* and *Pareto*. When there are at most two objects, we show that there exists only one rule satisfying the two axioms as well as the standard axiom of independence of irrelevant alternatives, or briefly, *IIA*. When there are more than two objects, however, we show that *resource monotonicity* and *Pareto* are incompatible. Thus, we relax *Pareto* and consider a weaker axiom, *simple Pareto.* In our first characterization of unanimity rule, we impose *resource monotonicity*, *simple Pareto*, *IIA*, and *path independence* (the choice should be independent of the successive decision path, generated by partitioning the agenda; Plott, 1973). Our second characterization of unanimity rule is obtained by replacing both *IIA* and *path independence* in the first characterization with *strategy-proofness* (no agent can benefit by misrepresenting his preferences).

The rest of this paper is organized as follows. In Section 2, we define the model and axioms. We state our main results in Section 3 and offer the proofs of the main results in Section 4.

2. THE MODEL AND AXIOMS

Let $N \equiv \{1, \ldots, n\}$ be a society of n agents with $n \geqslant 2$. The society faces a problem of choosing a subset from a set of indivisible public objects. These objects are elements of a finite set A with at least two elements. Each nonempty subset of A is a potential *agenda* and the set of all agendas is denoted by $\mathcal{A} \equiv \{X \subseteq A : X \neq \emptyset\}$. Given an agenda $X \in \mathcal{A}$, society can choose any subset of X, including the empty set. Thus, we call each of these subsets an *alternative*. Each agent has complete, transitive, and strict preferences over the set of all potential alternatives, namely the set of all subsets of A, denoted by 2^A. Generic notation for preferences is P_0 and notation for agent i's preferences P_i. We write $X\, P_i\, Y$ when X is preferred to Y according to P_i. Let $\mathcal{P}$ be the family of all such preferences and $\mathcal{P}^N$ the family of all profiles of preferences in $\mathcal{P}$. A social choice problem, or simply, a *problem* is a pair of a preference profile $P \in \mathcal{P}^N$ and an agenda $X \in \mathcal{A}$.[3] Let $\mathcal{D}_{\mathcal{P}} \equiv \mathcal{P}^N \times \mathcal{A}$ be the family of all these problems, called, the *unrestricted domain*.

Given a subset $\mathcal{D} \subseteq \mathcal{D}_{\mathcal{P}}$, a social choice rule, or simply a *rule*, on $\mathcal{D}$ is a function $\varphi\colon \mathcal{D} \to 2^A$ associating with each problem $(P, X) \in \mathcal{D}$ a subset of the agenda $\varphi(P, X) \subseteq X$.

Examples of rules are in order. Given an agenda X, since each preference $P_0 \in \mathcal{P}$ is *strict*, there exists a unique best alternative in 2^X, or the *top set*, denoted by $T(P_0, X)$.

Unanimity rule, φ^U. For all $(P, X) \in \mathcal{D}$ and all $x \in X, x \in \varphi^U(P, X)$ if and only if for all $i \in N, x \in T(P_i, X)$.

Majority rule, φ^M. For all $(P, X) \in \mathcal{D}$ and all $x \in X, x \in \varphi^M(P, X)$ if and only if $|\{i \in N : x \in T(P_i, X)\}| \geqslant (n+1)/2$.

A rule φ is *dictatorial* if there exists $i \in N$ such that for all $(P, X) \in \mathcal{D}, \varphi(P, X) = T(P_i, X)$.

The above rules are examples of the following family of rules. For each agenda $X \subseteq A$ and each object $x \in X$, a *committee structure associated with x and X* is a non-empty class of groups, $\mathcal{C}_{x,X}$, such that (i) $\emptyset \notin \mathcal{C}_{x,X}$ and (ii) if $C \in \mathcal{C}_{x,X}$ and $C' \supseteq C$, then $C' \in \mathcal{C}_{x,X}$. A rule is a *scheme of extended voting by committees* if there is a list of committee structures $((\mathcal{C}_{x,X})_{x \in X})_{X \subseteq A}$ such that for all $(P, X) \in \mathcal{D}$ and all $x \in X, x \in \varphi(P, X)$ if and only if $\{i \in N : x \in T(P_i, X)\} \in \mathcal{C}_{x,X}$. This definition is a natural extension of schemes of "voting by committees" studied by Barberà et al. (1991) in the fixed agenda model. Note that in our definition, committee structures depend not only on the object but on the agenda. Unanimity rule has the constant committee structure consisting of only the entire group N. Majority rule also has the constant committee structure consisting of groups whose cardinalities are greater than or equal to $(n+1)/2$.

Axioms

The most important axiom for our results pertains to the situation where agenda expands or contracts and no one is responsible for this change. Then, it is appealing to require that agents should be affected in the same direction. It is formulated by Chun and Thomson (1988) in the context of bargaining problem.

Resource monotonicity. For all $P \in \mathcal{P}^N$ and all $X, X' \in \mathcal{A}$ with $X \subseteq X'$, if $\varphi(P, X) \neq \varphi(P, X')$, then either (i) for all $i \in N, \varphi(P, X') P_i \varphi(P, X)$ or (ii) for all $i \in N, \varphi(P, X) P_i \varphi(P, X')$.

We also consider the following weaker axiom pertaining to one-object variations.[4]

Simple resource monotonicity. For all $(P, X) \in \mathcal{D}$ and all $y \in A \backslash X$, if $\varphi(P, X) \neq \varphi(P, X \cup \{y\})$, then either (i) for all $i \in N, \varphi(P, X \cup \{y\}) P_i \varphi(P, X)$ or (ii) for all $i \in N, \varphi(P, X) P_i \varphi(P, X, \cup\{y\})$.

The next axiom requires that there should be no Pareto improvement making all agents better off than at the chosen alternative.

Pareto. For all $(P, X) \in \mathcal{D}$, there exists no $X' \subseteq X$ such that for all $i \in N$, $X' P_i \varphi(P)$.

We show in Proposition 2 that no *resource monotonic* rule satisfies *Pareto*. Therefore, we consider the following weaker axiom. It says that no Pareto improvement should be possible either by addition or by subtraction of a *single* object.

Simple Pareto. For all $(P, X) \in \mathcal{D}$, there exists no $x \in X$ such that either (i) for all $i \in N, \varphi(P, X) \backslash \{x\} P_i \varphi(P, X)$ or (ii) for all $i \in N, \varphi(P, X) \cup \{x\} P_i \varphi(P, X)$.

A common practice for making a choice out of a large agenda X is through sequential decisions over subsets of X. The original agenda X is partitioned into subsets and each subset is considered independently. Then the union of the choices for these subsets is considered for the final decision. Sequential decisions can be useful, but they can cause conflicts of interests if the final decision depends on the path that has been taken. The next axiom prevents such potential conflicts.

Path independence. For all $(P, X) \in \mathcal{D}$ and all $X' \subseteq X$ with $\varphi(P, X') \cup \varphi(P, X \backslash X', \neq \emptyset$,

$$\varphi(P, \varphi(P, X') \cup \varphi(P, X \setminus X')) = \varphi(P, X).$$ [5]

A stronger version of this axiom was introduced by Plott (1973) in the Arrovian social choice model. Note that *path independence* is implied by *division invariance* saying that for all $(P, X) \in \mathcal{D}$ and all $X' \subseteq X, \varphi(P, X) = \varphi(P, X') \cup \varphi(P, X \setminus X')$.[6]

The next axiom requires that truthful representation of one's preference should result in an outcome preferred to any other outcomes from possible misrepresentations, independently of others' representations.

Strategy-proofness. For all $(P, X) \in \mathcal{D}$, all $i \in N$, and all $P_i' \in \mathcal{P}$, if $\varphi((P_i, P_{-i}), X) \neq \varphi((P_i', P_{-i}), X)$,

$$\varphi((P_i, P_{-i}), X) P_i \varphi((P_i', P_{-i}), X).$$

Our final axiom says that the choice should be made independently of preferences information on irrelevant alternatives. Two preferences $P_0, P_0' \in \mathcal{P}$ are *identical on agenda* $X \in \mathcal{A}$, if for all $Y, Y' \in 2^X$, $Y P_0 Y'$ if and only if $Y P_0' Y'$. In this case, we write $P_0|_X \equiv P_0'|_X$. Two profiles P, P' are *identical on agenda* X, denoted by $P|_X \equiv P'|_X$, if for all $i \in N$, $P_i|_X \equiv P_i'|_X$.

Independence of irrelevant alternatives, briefly **IIA**. For all $(P, X), (P', X') \in \mathcal{D}$, if $X = X'$ and $P|_X \equiv P'|_X$, then $\varphi(P, X) = \varphi(P', X)$.

The unrestricted domain: An impossibility result

We first show that on the unrestricted domain $\mathcal{D}_{\mathcal{P}}$, *simple resource monotonicity* and *simple Pareto* are incompatible.

PROPOSITION 1. *There exists no rule on the unrestricted domain $\mathcal{D}_{\mathcal{P}}$ satisfying (simple) resource monotonicity and (simple) Pareto.*

Proof. For simplicity, let $N \equiv \{1, 2\}$ and $A \equiv \{a, b\}$. Let φ be a rule on $\mathcal{D}_{\mathcal{P}}$ satisfying *simple resource monotonicity* and *simple Pareto*. Let P_1 and P_2 be such that $\{a\} P_1 \{b\} P_1 \emptyset P_1 \{a, b\}$ and $\{b\} P_2 \{a\} P_2 \emptyset P_2 \{a, b\}$. Then, by *simple Pareto*, $\varphi(P, \{a\}) = \{a\}$ and $\varphi(P, \{b\}) = \{b\}$. If $\varphi(P, \{a, b\}) = \{a\}$, then when the agenda changes from $\{b\}$ to $\{a, b\}$, agent 1 is better off and agent 2 is worse off, contradicting *simple resource monotonicity*. Hence $\varphi(P, \{a, b\}) \neq \{a\}$. Similarly, $\varphi(P, \{a, b\}) \neq \{b\}$. On the other hand, by *simple Pareto*, $\varphi(P, \{a, b\}) \neq \emptyset$ and $\varphi(P, \{a, b\}) \neq \{a, b\}$. Therefore, there is no choice φ can make for $(P, \{a, b\})$ without violating either one of the two axioms. □

In what follows, we focus on a restricted family of preferences, known as separable preferences (Barberà et al., 1991).

The separable preferences domain

A preference $P_0 \in \mathcal{P}$ is *separable* if for all $X \subseteq A$ and all $x \notin X, [X \cup \{x\}]P_0 X$ if and only if $\{x\}\, P_0\, \emptyset$. Let $\mathcal{S}$ be the family of all separable preferences. Let $\mathcal{D}_{\mathcal{S}} \equiv \mathcal{S}^N \times \mathcal{A}$ be the family of problems with separable preferences, referred to as the *separable domain*.

The following notation is useful, For each separable preference $P_0 \in \mathcal{S}$, A is partitioned into two subsets. An object $x \in A$ is a *good for* P_0 if $\{x\} P_0\, \emptyset$. It is a *bad for* P_0 if $\emptyset P_0 \{x\}$. Similarly, for all $P \in \mathcal{S}^N$ and all $x \in A, N$ is partitioned into the group of agents for whom x is a good, $N_x^G(P)$, and the group of agents for whom x is a bad, $N_x^B\ (P)$. Using this notation, unanimity rule and majority rule can be defined as follows: for all $(P, X) \in \mathcal{D}_{\mathcal{S}}$ and all $x \in X$,

$$x \in \varphi^U(P, X) \Leftrightarrow N_x^G(P) = N;$$
$$x \in \varphi^M(P, X) \Leftrightarrow |N_x^G(P)| \geqslant \frac{n+1}{2}.$$

3. RESULTS

We first show that when there are only two objects, there is a unique rule satisfying *resource monotonicity, Pareto*, and *IIA*. This rule coincides with unanimity rule for all problems with a *singleton* agenda. For other problems, the choice is based also on the idea of unanimity, however, in a somewhat different manner from unanimity rule. Formally, *agenda unanimity rule* φ^{AU} is defined as follows: for all $(P, X) \in \mathcal{D}_{\mathcal{S}}$, (i) if there is $x \in X$ with $N_x^G(P) \in \{\emptyset, N\}$, then $\varphi(P, X) \equiv \cup_{x \in X} \varphi^U(P, \{x\})$ and (ii) otherwise, if $N_X^G(P) = N, \varphi(P, X) = X$ and if $N_X^G(P) \neq N, \varphi(P, X) = \emptyset$, where $N_X^G(P) \equiv \{i \in N : X\, P_i \emptyset\}$.

THEOREM 1. *Assume that there are only two objects, that is,* $|A| = 2$. *Then the agenda unanimity rule is the only rule on the separable domain satisfying* (*simple*) *resource monotonicity, Pareto, and IIA.*[7]

The proof is given in Section 4.

Although Theorem 1 has only limited applicability, it exhibits an interesting connection between *resource monotonicity* and the unanimity principle.

In the two-objects case, there are only two possible resource expansions. More variety of resource expansions exist when there are more than two objects, and so *resource monotonicity* has more bite. In this case, we show that *resource monotonicity* and *Pareto* are incompatible. The result holds even after weakening *resource monotonicity* to its simple version.

THEOREM 2. *Assume that there are at least three objects, that is,* $|A| \geqslant 3$. *Then there exists no rule on the separable domain* $\mathcal{D}_{\mathcal{S}}$ *satisfying* (*simple*) *resource monotonicity and Pareto.*

The proof is given in Section 4.

The incompatibility no longer holds when we replace *Pareto* with *simple Pareto*. For example, unanimity rule satisfies *resource monotonicity* and *simple Pareto*. We show that it is the only rule satisfying, in addition, *path independence* and *IIA*.

THEOREM 3. *A rule on the separable domain* $\mathcal{D}_{\mathcal{S}}$ *satisfies* (*simple*) *resource monotonicity, simple Pareto, path independence, and IIA if and only if it is unanimity rule.*

The proof is given in Section 4.

Next, we consider *strategy-proofness*. We use the main result by Barberà et al. (1991) in the fixed agenda model. They show that schemes of voting by committees are the only rules satisfying *strategy-proofness* and the axiom of "voter sovereignty". Let $X \subseteq A$ and $\mathcal{D}_{\mathcal{S}}(X) \equiv \{(P, X) : P \in \mathcal{S}^N\}$. A rule φ over $\mathcal{D}_{\mathcal{S}}(X)$ satisfies *voter sovereignty* if for all $Y \in 2^X$, there exists $P \in \mathcal{S}^N$ such that $\varphi(P, X) = Y$. Note that *simple Pareto* implies *voter sovereignty* over $\mathcal{D}_{\mathcal{S}}(X)$. To apply the result in Barberà et al. (1991) in our variable agenda model, we show that *strategy-proofness* implies *IIA*, and we obtain the following result.

PROPOSITION 2. *A rule on the separable domain $\mathcal{D}_{\mathcal{S}}$ satisfies strategy-proofness and simple Pareto if and only if it is a scheme of extended voting by committees.*

The proof is given in Section 4.

We show that when a scheme of voting by committees satisfies *resource monotonicity*, its committees are composed of only the entire group N, that is, it is unanimity rule. Thus we obtain the following characterization.

THEOREM 4. *A rule on the separable domain $\mathcal{D}_{\mathcal{S}}$ satisfies (simple) resource monotonicity, simple Pareto, and strategy-proofness if and only if it is unanimity rule.*

The proof is given in Section 4.

Examples 1–4 show independence of the axioms in each of Theorems 3 and 4.

EXAMPLE 1. We define a rule φ satisfying *resource monotonicity, simple Pareto*, and *IIA*, but violating both *path independence* and *strategy-proofness*. For simplicity, let $A \equiv \{a, b, c\}$. For all $P \in \mathcal{S}^N$ and all $x \in A, x \in \varphi(P, \{x\}) \Leftrightarrow N_x^G(P) = N$. Let $\varphi(P, \{a, b\}) \equiv \varphi(P, \{a\}) \cup \varphi(P, \{b\})$ and $\varphi(P, \{a, c\}) \equiv \varphi(P, \{a\}) \cup \varphi(P, \{c\})$. If $N_b^G(P) \in \{\emptyset, N\}$ or $N_c^G(P) \in \{\emptyset, N\}$, then let $\varphi(P, \{b, c\}) \equiv \varphi(P, \{b\}) \cup \varphi(P, \{c\})$. If $N_b^G(P) \notin \{\emptyset, N\}$ and $N_c^G(P) \notin \{\emptyset, N\}$, then (i) when $N_{\{b,c\}}^G(P) = N$, let $\varphi(P, \{b, c\}) \equiv \{b, c\}$; (ii) when $N_{\{b,c\}}^G(P) \neq N$ let $\varphi(P, \{b, c\}) \equiv \emptyset$. Finally, let $\varphi(P, \{a, b, c\}) \equiv \varphi(P, \{a\}) \cup \varphi(P, \{b, c\})$.

EXAMPLE 2. We define a rule φ satisfying *resource monotonicity, path independence, strategy-proofness*, and *IIA*, but violating *simple Pareto*. For all $(P, X) \in \mathcal{D}_{\mathcal{S}}$, let $\varphi(P, X) \equiv \emptyset$.

EXAMPLE 3. Majority rule satisfies *simple Pareto, path independence, strategy-proofness*, and *IIA*, but violates *resource monotonicity*.

EXAMPLE 4. We define a rule φ satisfying *resource monotonicity, simple Pareto*, and *path independence*, but violating

IIA. For each $P \in \mathcal{S}^N$, (i) if all agents rank objects in the same order and no object is either a good for everyone or a bad for everyone, then for all non-empty $X \subseteq A$, let $\varphi(P, X) \equiv \{x : x$ is the best object in X for all agents$\}$; (ii) otherwise, let $\varphi(P, X)$ equal the choice made by unanimity rule. It is easy to show that φ satisfies *resource monotonicity* and *simple Pareto*. To show *path independence*, let $X \subseteq A$ and $X' \subseteq X$ be non-empty. Consider P in part (i) in the definition of φ. Then the best object out of X is also the best object between the best object out of X' and the best object out of $X \backslash X'$.

Remark 1. Our results are not affected after strengthening *resource monotonicity* into *resource monotonicity*$_+$: for all $P \in \mathcal{P}^N$ and all $X, X' \in \mathcal{A}$ with $X \subseteq X'$, if $\varphi(P, X) \neq \varphi(P, X')$, then for all $i \in N, \varphi(P, X') P_i \varphi(P, X)$. This is because unanimity rule satisfies this stronger axiom. Independence of axioms in Theorems 3 and 4 is not affected either because *resource monotonic* rules in Examples 1–4 also satisfy *resource monotonicity*$_+$.

4. PROOFS

We first establish several useful lemmas. Let $x, y \in A$. Two preferences P_0 and $P_0' \in \mathcal{S}$ exhibit *conflicting interests associated with x and y* if $\{x\} P_i \emptyset P_i \{y\}$ and $\{y\} P_j \emptyset P_j \{x\}$. A profile $P \in \mathcal{S}^N$ exhibits *conflicting interests associated with x and y*, if there are at least two agents whose preferences exhibit conflicting interests associated with x and y.

LEMMA 1. *Let φ be a rule satisfying simple resource monotonicity and simple Pareto. For all $P \in \mathcal{S}^N$ and all $x \in A$, if there exists $y \in A \backslash \{x\}$ such that P exhibits conflicting interests associated with x and y, then $x \notin \varphi(P, \{x\})$. If φ satisfies, in addition, IIA, then for all $P \in \mathcal{S}^N$ and all $x \in A$,*

$$x \in \varphi(P, \{x\}) \Leftrightarrow N_x^G(P) = N. \tag{1}$$

Proof. Let φ be a rule satisfying *simple resource monotonicity* and *simple Pareto*. Let $P \in \mathcal{S}^N$ and $x, y \in A$ be given as above. Let $i, j \in N$ be such that $\{x\}P_i\emptyset P_i\{y\}$ and $\{y\}P_j\emptyset P_j\{x\}$. We first show

$$x \in \varphi(P, \{x\}) \Leftrightarrow x \in \varphi(P, \{x, y\}). \tag{2}$$

Suppose by contradiction that $x \in \varphi(P, \{x\})$ and $x \notin \varphi(P, \{x, y\})$. Then $\varphi(P, \{x\}) = \{x\}$ and either $\varphi(P, \{x, y\}) = \emptyset$ or $\varphi(P, \{x, y\}) = \{y\}$. In either case, when the agenda changes from $\{x\}$ to $\{x, y\}$, agent i is worse off and agent j is better off, contradicting *simple resource monotonicity*. This shows (2).

Suppose $x \in \varphi(P, \{x\})$. Then by (2), $x \in \varphi(P, \{x, y\})$. Thus, $\varphi(P, \{x, y\}) = \{x\}$ or $\{x, y\}$. If $\varphi(P, \{x, y\}) = \{x\}$, a contradiction to *simple resource monotonicity* occurs when the agenda changes from $\{y\}$ to $\{x, y\}$ (whether $\varphi(P, \{y\}) = \{y\}$ or $\emptyset$). If $\varphi(P, \{x, y\}) = \{x, y\}$, the same contradiction occurs when the agenda changes from $\{x\}$ to $\{x, y\}$.

To prove (1), assume that φ also satisfies *IIA*. If $N_x^G(P) = N$, then by *simple Pareto*, $x \in \varphi(P, \{x\})$. In order to prove the converse, suppose $N_x^G(P) \neq N$. Then $N_x^G(P) = \emptyset$ or $N_x^G(P) \neq \emptyset$. In the former case, by *simple Pareto*, $x \notin \varphi(P, \{x\})$. In the latter case, there are $P' \in \mathcal{S}^N$ and $y \in A \backslash \{x\}$ such that $N_x^G(P') = N_x^G(P)$ and P' exhibits conflicting interests associated with x and y. Therefore by the previous result, $x \notin \varphi(P', \{x\})$. By *IIA*, $\varphi(P, \{x\}) = \varphi(P', \{x\})$ and so $x \notin \varphi(P, \{x\})$. □

LEMMA 2. *Let φ be a rule satisfying simple resource monotonicity and simple* Pareto. *Let $\{a, b\} \subseteq A$ and $P \in \mathcal{S}^N$ be such that for all $x \in \{a, b\}$, (1) holds. If $N_a^G(P) \in \{\emptyset, N\}$ or $N_b^G(P) \in \{\emptyset, N\}$, then $\varphi(P, \{a, b\}) = \varphi(P, \{a\}) \cup \varphi(P, \{b\})$.*

Proof. If $N_a^G(P) \in \{\emptyset, N\}$ and $N_b^G(P) \in \{\emptyset, N\}$, then by *simple Pareto*, $\varphi(P, \{a, b\}) = \varphi(P, \{a\}) \cup \varphi(P, \{b\})$. Now suppose, without loss of generality, that $N_a^G(P) \in \{\emptyset, N\}$ and $N_b^G(P) \notin \{\emptyset, N\}$. Then $N_a^G(P) = N$ or $\emptyset$. We consider the case $N_a^G(P) = N$ and skip the same argument for the other case. By *simple Pareto*, $a \in \varphi(P, \{a, b\})$ and $\varphi(P, \{a\}) = \{a\}$. Since $N_b^G(P) \neq N$ then by (1), $\varphi(P, \{b\}) = \emptyset$. Hence we only have to show $b \notin \varphi(P, \{a, b\})$. Suppose, by contradiction, $b \in \varphi(P, \{a, b\})$. Then $\varphi(P, \{a, b\}) = \{a, b\}$. Since $N_b^G(P) \notin \{\emptyset, N\}$, there exist $i, j \in N$

such that $i \in N_b^G(P)$ and $j \in N_b^B(P)$. Therefore, since preferences are separable, then when the agenda changes from $\{a\}$ to $\{a, b\}$, agent i is better off and agent j is worse off, contradicting *simple resource monotonicity.* □

LEMMA 3. *Let φ be a rule satisfying simple resource monotonicity and simple Pareto. Let $\{a, b\} \subseteq A$ and $P \in \mathcal{S}^N$ be such that for all $x \in \{a, b\}$,* (1) *holds. If $N_a^G(P) \notin \{\emptyset, N\}$ and $N_b^G(P) \notin \{\emptyset, N\}$, then $\varphi(P, \{a, b\}) = \{a, b\}$ or $\emptyset$.*

Proof. Suppose, by contradiction, $\varphi(P, \{a, b\}) \notin \{\{a, b\}, \emptyset\}$. Then $\varphi(P, \{a, b\}) = \{a\}$ or $\{b\}$. We consider the former case and skip the same argument for the latter case. Since $N_a^G(P) \notin \{\emptyset, N\}$, there exists i, j such that $\{a\} P_i \emptyset$ and $\emptyset P_j \{a\}$. By (1), $\varphi(P, \{a\}) = \emptyset$. Hence when agenda changes from $\{a\}$ to $\{a, b\}$, agent i is better off and agent j is made worse off, contradicting *simple resource monotonicity.* □

LEMMA 4. *Let φ be a rule satisfying simple resource monotonicity and Pareto. Let $\{a, b\} \subseteq A$ and $P \in \mathcal{S}^N$ be such that for all $x \in \{a, b\}$,* (1) *holds. If $N_a^G(P) \notin \{\emptyset, N\}$ and $N_b^G(P) \notin \{\emptyset, N\}$, then*

$$N_{\{a,b\}}^G(P) = N \Rightarrow \varphi(P, \{a, b\}) = \{a, b\};$$
$$N_{\{a,b\}}^G(P) \neq N \Rightarrow \varphi(P, \{a, b\}) = \emptyset.$$

Proof. By Lemma 3, $\varphi(P, \{a, b\}) = \{a, b\}$ or $\emptyset$. Thus, by *Pareto*, if $N_{\{a,b\}}^G(P) = N$, then $\varphi(P, \{a, b\}) = \{a, b\}$ and if $N_{\{a,b\}}^G(P) = \emptyset$ then $\varphi(P, \{a, b\}) = \emptyset$.

Now consider $N_{\{a,b\}}^G(P) \notin \{\emptyset, N\}$. Then there exist i, j such that $\{a, b\} P_i \emptyset$ and $\emptyset P_j \{a, b\}$. Suppose by contradiction that $\varphi(P, \{a, b\}) \neq \emptyset$. Thus $\varphi(P, \{a, b\}) = \{a, b\}$. By (1), $\varphi(P, \{a\}) = \emptyset$. Therefore when the agenda changes from $\{a\}$ to $\{a, b\}$, agent i is better off and agent j is worse off, contradicting *simple resource monotonicity.* □

Proof of Theorem 1. Theorem 1 follows directly from Lemmas 1–4. □

Proof of Theorem 2. Let $|A| \geqslant 3$. Suppose by contradiction that there exists a rule φ satisfying *simple resource monotonicity* and *Pareto*. Let $a, b, c \in A$ be three distinct objects. Let $\bar{P}_1$ and $\bar{P}_2$ be such that (i) $\{a\}\bar{P}_1\{a,b\}\bar{P}_1\{a,c\}\bar{P}_1\emptyset\bar{P}_1\{a,b,c\}\bar{P}_1\{b\}\bar{P}_1\{c\}\bar{P}_1\{b,c\}$; (ii) $\{b,c\}\bar{P}_2\{a,b,c\}\bar{P}_2\{c\}\bar{P}_2\{a,c\}\bar{P}_2\{b\}\bar{P}_2\{a,b\}\bar{P}_2\emptyset\bar{P}_2\{a\}$.[8] Thus $\bar{P}_1$ and $\bar{P}_2$ exhibit conflicting interests associated both with a and b and with a and c. Let $P \in \mathcal{S}^N$ be such that $P_1 = \bar{P}_1, P_2 = \bar{P}_2, N^G_{\{a,b\}}(P) = N$, and $N^G_{\{a,c\}}(P) = N$. Note that $N^G_a(P) \notin \{\emptyset, N\}, N^G_b(P) \notin \{\emptyset, N\}, N^G_c(P) \notin \{\emptyset, N\}, N^G_{\{b,c\}}(P) \notin \{\emptyset, N\}$, and $N^G_{\{a,b,c\}}(P) \notin \{\emptyset, N\}$. Then by Lemma 1,

$$\varphi(P,\{a\}) = \varphi(P,\{b\}) = \varphi(P,\{c\}) = \emptyset.$$

And by Lemma 4,

$$\varphi(P,\{a,b\}) = \{a,b\}; \varphi(P,\{a,c\}) = \{a,c\}; \varphi(P,\{b,c\}) = \emptyset.$$

Since $N^G_{\{a,b\}}(P) = N$, then by *Pareto*, $\varphi(P,\{a,b,c\}) \neq \emptyset$.

Suppose that there exists $x \in \{a,b,c\}$ such that $\varphi(P,\{a,b,c\}) = \{x\}$. Then since $\varphi(P,\{b,c\}) = \emptyset$ and $N^G_x(P) \notin \{\emptyset, N\}$, then when the agenda changes from $\{b,c\}$ to $\{a,b,c\}$, there exist an agent who is better off and an agent who is worse off. This contradicts *simple resource monotonicity*.

Suppose $\varphi(P,\{a,b,c\}) \in \{\{a,b\},\{a,b\},\{a,c\}\}$. Consider the case $\varphi(P,\{a,b,c\}) = \{a,b\}$. Then when the agenda changes from $\{a,c\}$ to $\{a,b,c\}$, agent 1 is better off and agent 2 is worse off, contradicting *simple resource monotonicity*. We derive the same contradiction when $\varphi(P,\{a,b,c\}) = \{b,c\}$ and $\varphi(P,\{a,b,c\}) = \{a,c\}$, considering agenda changes from $\{b,c\}$ to $\{a,b,c\}$ and from $\{a,b\}$ to $\{a,b,c\}$, respectively.

Finally suppose $\varphi(P,\{a,b,c\}) = \{a,b,c\}$. Consider the agenda change from $\{a,b\}$ to $\{a,b,c\}$. Then agent 1 is worse off and agent 2 is better off, contradicting *simple resource monotonicity*. □

Proof of Theorem 3. It is easy to show that unanimity rule φ^U satisfies the four axioms. In order to show the converse, let φ be a rule satisfying the four axioms. We show the following statement S(k) with $k \in \{1, \ldots, |A|\}$ by induction with respect to k.

S(k): For all $(P, X) \in \mathcal{D}_{\mathcal{S}}, |X| \leqslant k \Rightarrow \varphi(P, X) = \varphi^U(P, X)$.

S(1) holds by Lemma 1. Suppose that for all $m \leqslant k-1$, S(m) holds. Let $X \in \mathcal{A}$ be such that $|X| = k$. Let $P \in \mathcal{S}^N$. If for all $x \in X, N_x^G(P) = N$, then by *simple Pareto*, $\varphi(P, X) = \varphi^U(P, X)$. Now suppose that there exists $x \in X$ such that $N_x^G(P) \neq N$. Then by Lemma 1, $\varphi(P, \{x\}) = \varphi^U(P, \{x\}) = \emptyset$. Thus $|\varphi(P, X \setminus \{x\}) \cup \varphi(P, \{x\})| \leqslant |X \setminus \{x\}| = k-1$ and by the induction hypothesis,

$$\varphi(P, \varphi(P, X \backslash \{x\}) \cup \varphi(P, \{x\})) = \varphi^U(P, \varphi(P, X \backslash \{x\}) \cup \varphi(P, \{x\})).$$

Again, by the induction hypothesis,

$$\varphi(P, X \backslash \{x\}) \cup \varphi(P, \{x\}) = \varphi^U(P, X \backslash \{x\}) \cup \varphi^U(P, \{x\}).$$

Hence

$$\varphi(P, \varphi(P, X \backslash \{x\}) \cup \varphi(P, \{x\})) \\ = \varphi^U(P, \varphi^U(P, X \backslash \{x\}) \cup \varphi^U(P, \{x\})).$$

Since both φ and φ^U satisfy *path independence*, then $\varphi(P, X) = \varphi^U(P, X)$. □

Proof of Proposition 2. Let φ be a rule satisfying the two axioms. If φ satisfies in addition *IIA*, then for every subdomain with a fixed agenda, the characterization of voting by committees by Barberà et al. (1991) applies. Thus, we only have to show that φ satisfies *IIA*. Let $X \in \mathcal{A}$ and $P, P' \in \mathcal{S}^N$ be such that for each $i \in N$, $P_i|_X \equiv P'_i|_X$. Let $Y \equiv \varphi(P, X)$ and $Y^1 \equiv \varphi((P'_i, P_{-1}), X)$. If $Y \neq Y^1$, then as preferences are strict, $Y\, P_1\; Y^1$ or $Y^1 P_1 Y$. In the former case, since $P_1|_X \equiv P'_1|_X$, $Y\;\; P'_1 Y^1$. Thus agent 1 with true preference P'_1 is better off by reporting P_1. In the latter case, agent 1 with true preference P_1 is better off by reporting P'_1. This contradicts *strategy-proofness*. Thus, $Y = Y^1$. Similarly, changing preferences of all other agents $i \in N \backslash \{1\}$ from P_i to P'_i successively, we can show that $Y = Y'$. □

Proof of Theorem 4. It is easy to show that φ^U satisfies the three axioms. In order to show the converse, let φ be a rule satisfying the three axioms. Then, by Proposition 2, φ is a scheme of extended voting by committees. Let $((\mathcal{C}_{x,X})_{x\in X})_{X\subseteq A}$ be the profile of committee structures. We show the following statement $\mathbf{S}(k)$ with $k\in\{1,\ldots,|A|\}$ by induction with respect to k.

$\mathbf{S}(k)$: For all $X\in\mathcal{A}$ with $|X|\leqslant k$ and all $x\in X, \mathcal{C}_{x,X}=\{N\}$.

S(1) holds by Lemma 1. Suppose that for all $m\leqslant k-1$, S(m) holds. Let $X\in\mathcal{A}$ be such that $|X|=k$. Let $x\in X$. Suppose by contradiction that there exists $S\subsetneqq N$ such that $S\in\mathcal{C}_{x,X}$. Let $P\in\mathcal{S}^N$ be such that $N_x^G(P)=S$ and for all $y\in X\setminus\{x\}, N_y^G(P)=\emptyset$. Then by *simple Pareto*, for all $y\in X\backslash\{x\}, y\notin\varphi(R,X)$. Thus, $\varphi(P,X)=\{x\}$. By the induction hypothesis, $\varphi(P,X\setminus\{x\})=\emptyset$. Therefore, when the agenda changes from $X\setminus\{x\}$ to X, all agents in S are better off and all others are worse off, contradicting *simple resource monotonicity*. □

NOTES

1. See, for instance, May (1952) and Ching (1996) for axiomatizations in the Arrovian social choice model. Ju (2005) axiomatizes majority rule in the same model as we consider here.
2. Different versions of *resource monotonicity* have been studied by a number of authors, in particular by Moulin (1987), Chun and Thomson (1988), Moulin and Thomson (1988), Alkan (1994), Thomson (1994), etc. See Thomson (2001) for an extensive survey.
3. We use notation, $P, P', \bar{P}, \bar{P}'$, etc., for elements in $\mathcal{P}^N$. Following standard notational convention, we write i's component of P with P_i and i's component of P' with P_i'.
4. The same weakening of resource monotonicity has been investigated by Alkan (1994) for assignment problems.
5. We do not consider problems with the empty agenda. So we need the condition $\varphi(R,X')\cup\varphi(R,X\backslash X')\neq\emptyset$.
6. *Division invariant* rule φ can be described as follows: for all $(P,X)\in\mathcal{D}, \varphi(P,X)=\cup_{x\in X}\varphi(P,\{x\})$. Then φ is *path independent* because, applying *division invariance* iteratively, we get

$$\begin{aligned}\varphi(P,\varphi(P,X')\cup\varphi(P,X\backslash X'))&=\varphi(P,\varphi(P,X))=\varphi(P,\cup_{x\in X}\varphi(P,\{x\}))\\&=\cup_{y\in\cup_{x\in X}\varphi(P,\{x\})}\varphi(P,\{y\})\\&=\cup_{x\in X}\varphi(P,\{x\})=\varphi(P,X).\end{aligned}$$

7. Independence of the three axioms can be established easily.
8. Let $u_1 : 2^A \to \mathbb{R}$ be an additive numerical representation such that $u_1(a) = 4, u_1(b) = -2$, and $u_1(c) = -3$. Let $u_2 : 2^A \to \mathbb{R}$ be an additive numerical representation such that $u_2(a) = -1, u_2(b) = 2$, and $u_2(c) = 4$. Let R_1 be the additive preference represented by u_1 and R_2 the additive preference represented by u_2. Then R_1 and R_2 order alternatives in the same way as required.

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REFERENCES

Alkan, A. (1994), Monotonicity and envyfree assignments, *Economic Theory* 4, 605–616.

Barberà, S., Sonnenschein, H. and Zhou, L. (1991), Voting by committees, *Econometrica* 59(3), 595–609.

Berga, D., Bergantiños, G., Massò, J. and Neme, A. (2004), Voting by Committees with Exit, *Social Choice and Welfare*, 23(2): 229–247.

Ching, S. (1996), A simple characterization of plurality rule, *Journal of Economic Theory* 71, 298–302.

Chun, Y. and Thomson, W. (1988), Monotonicity Properties of Bargaining solutions when applied to economics, *Mathematical Social Sciences* 15, 11–27.

Gordon, S. (2001), Solidarity Conditions and Unanimity Rules in Attribute-Based Domains: A Characterization. Northwestern University, mimeo.

Ju, B.-G. (2005), An efficiency characterization of plurality social choice on simple preference domains, *Economic Theory* 26(1), 115–128.

May, K. O. (1952), A set of independent necessary and Sufficient conditions for simple majority decision, *Econometrica* 20(4), 680–684.

Moulin, H. (1987), Egalitarian Equivalent cost-sharing of a public good, *Econometrica* 55(4), 963–977.

Moulin, H. and Thomson, W. (1988), Can Everyone Benefit From growth?, *Journal of Mathematical Economics* 17, 339–345.

Plott, C.R. (1973), Path independence, rationality, and social choice, *Econometrica* 41, 1075–1091.

Thomson, W. (1994), Resource-monotonic solutions to the problem of fair division when preferences are single-peaked, *Social Choice and Welfare* 11, 205–223.

Thomson, W. (2001), Monotonic Allocation Rules: A Survey. University of Rochester, mimeo.

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