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Calculation of the Efficiency of Contact Stages of Distillation Columns with Cocurrent Swirl Contact Devices

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Abstract—Using a diffusion model, a method for calculating the efficiency of contact stages of distillation columns with cocurrent swirl contact devices is developed.

A promising type of distillation columns is columns with cocurrent swirl contact devices [1, 2]. The flow-rate-average vapor velocity in such columns is 12–15 m/s, whereas this velocity in bubble columns does not exceed 1.5–2.0 m/s at a comparable efficiency of contact stages. This allows one to use distillation columns with cocurrent swirl contact devices in high-power plants and save significant capital costs [3]. Contact stages of such columns consist of cocurrent swirl contact elements 80–120 mm in diameter, which are arranged in staggered rows (Fig. 1). At contact stages, the liquid is recycled: after passing through contact devices of a stage, the liquid returns to the receiving tray and is mixed with the bulk of the liquid.

The efficiency E_{0y} of a contact stage and the efficiency E_y of a single contact device can be calculated by the expressions

$$E_{0y} = \frac{y_n - y_0}{y^*_{x_{\text{in}}} - y_n}, \quad E_y = \frac{y_{\text{out}} - y_0}{y^*_{x_{\text{out}}} - y_0}. \tag{1}$$

A widely used method for evaluating the efficiency of contact stages is to experimentally determine this efficiency on pilot and semi-industrial plants. This method is highly labor-, cost-, and time-consuming. Therefore, to calculate the efficiency of contact stages of distillation columns with cocurrent swirl contact devices, we assume that the efficiencies of all contact devices of a stage are equal. This assumption will allow us to derive expressions that relate the efficiency E_{0y} of a contact stage with the efficiency E_y of a single contact device.

Using this approach, we can avoid the experimental investigation of the efficiency of a contact stage and restrict our study to the efficiency of a single contact device for various flow rates and different physical properties of the phases. For the efficiencies of all contact devices of a stage to be equal under real conditions, it is necessary that the vapor before entering the contact stage should be well dispersed and that the vapor and liquid should be uniformly distributed over the contact devices.

In small-diameter columns, where the number of rows of contact devices is small, the liquid is well dispersed; therefore, the composition of the liquid entering all the contact devices is the same and coincides with the composition of the liquid leaving the contact stage.

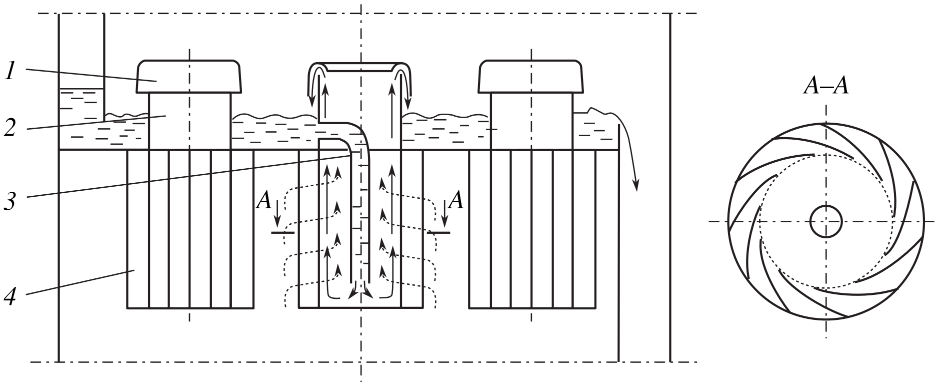

Fig. 1. Schematic of a contact stage with cocurrent swirl contact devices (solid and dotted arrowed lines show the liquid and vapor flows, respectively): (*1*) separator, (*2*) separation cylinder, (*3*) inlet tube for feeding the liquid to the element, and (*4*) tangential swirler.

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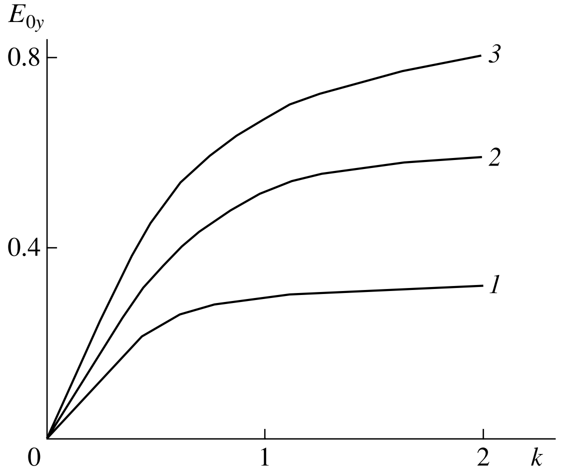

Fig. 2. Dependence of the efficiency E_{0y} of a contact stage on the recycle ratio k of the liquid at the stage at $mG/L = 1$ and E_y = (*1*) 0.4, (*2*) 0.6, and (*3*) 0.8.

In this case, the efficiency of a contact stage is expressed as [1]

$$E_{0y} = E_y/\left(1 + \frac{mG}{kL}E_y\right). \tag{2}$$

Here, k is the recycle ratio, which is the ratio of the amount of the liquid passing through the contact devices per unit time to the amount of the liquid passing through the stage per unit time.

With an increase in the number of rows of contact devices in an apparatus, the liquid concentration varies in the direction from overflow to overflow. The efficiency of such contact stages can be calculated using the sectioning principle, which is widely used in evaluating the efficiency of bubble columns [1]. According to this principle, each stage is divided into several sections, in each of which the liquid is well dispersed, and the liquid concentration varies stepwise from section to section. Each contact stage is divided into sections along rows of contact devices that are normal to the liquid flow direction at the stage. Within each section, the liquid coming from the previous section is mixed with the liquid leaving the contact devices of this section. In this case, the efficiency of a contact stage is represented as [1]

$$E_{0y} = \frac{L}{mG}\left[\left(1 + \frac{E_y mG/sL}{1 + E_y mG/kL}\right)^s - 1\right]. \tag{3}$$

The dependence of E_{0y} on the recycle ratio k of the liquid through contact devices of a stage is illustrated in Fig. 2.

The main drawback of the sectioning model is the formal introduction of the number of sections in which the liquid is well dispersed. This number is difficult to find because, with a change in the liquid recycle ratio, the degree of liquid dispersion at a stage varies and, hence, so does the number of sections in which the liquid is well dispersed. Therefore, we propose to describe the degree of liquid dispersion in continuous flows by a diffusion model whose main advantage is the possibility of experimentally determining the Peclet number Pe.

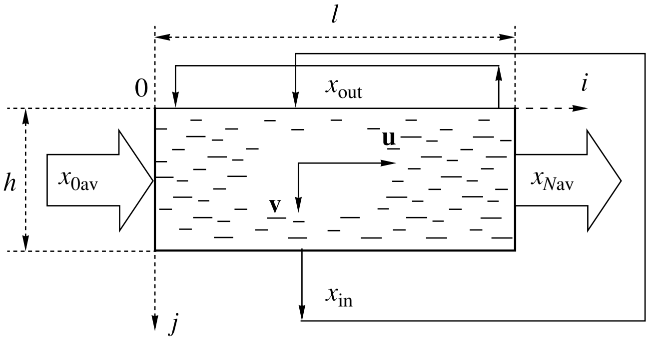

Fig. 3. Flow chart of a contact stage.

The fresh liquid with concentration $x_{0\rm av}$ enters a contact stage and flows over the tray at flow-rate-average velocity u (Fig. 3). The longitudinal direction of the liquid flow is taken to be the direction of the main flow from the overflow through which the liquid enters the stage to the overflow through which the liquid leaves the stage. As the liquid flows over the tray, it is recycled through mass-transfer elements. Passing through them, the liquid contacts the vapor phase and returns to the liquid layer on the tray with concentration $x_{\rm out}$.

Thus, the longitudinal dispersion of the liquid is caused by the turbulent dispersion directly in the liquid layer and by the distribution of the liquid flowing out from contact elements. The liquid dispersion over the layer thickness is induced by the liquid convection from the surface of the liquid layer on the tray to the inlet tubes of contact elements at effective velocity v.

Let us write the equation of the diffusion model for the case under consideration:

$$u\frac{\partial x}{\partial i} + v\frac{\partial x}{\partial j} = D_i\frac{\partial^2 x}{\partial i^2} + D_j\frac{\partial^2 x}{\partial j^2}. \tag{4}$$

We introduce the characteristic scales l and h of the length and thickness, respectively, of the liquid layer on the tray and, using these scales, turn to dimensionless coordinates

$$i = \xi l, \quad j = \eta h. \tag{5}$$

In terms of coordinates (5) and the reduced admixture concentration in the liquid

$$\varphi = x - x^*(y_{\rm out}), \tag{6}$$

Eq. (4) takes the form

$$\frac{\partial\varphi}{\partial\xi} + K\frac{\partial\varphi}{\partial\eta} = \frac{1}{{\rm Pe}_\xi}\frac{\partial^2\varphi}{\partial\xi^2} + \frac{K}{{\rm Pe}_\eta}\frac{\partial^2\varphi}{\partial\eta^2}. \tag{7}$$

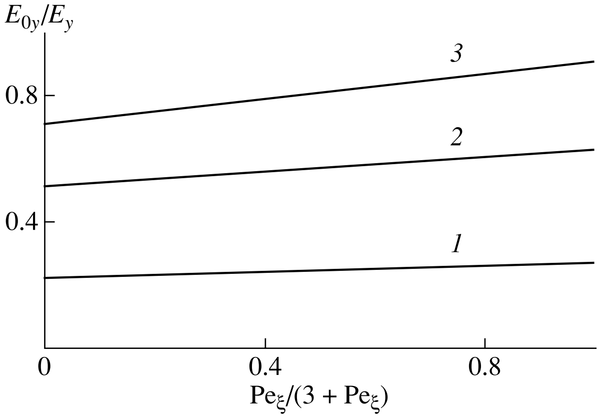

Fig. 4. Dependence of the efficiency of a contact stage on the liquid recycle at the stage at $\lambda E_y = 0.5$ and $K =$ (*1*) 0.5, (*2*) 1.0, and (*3*) 4.0.

The liquid with composition $\varphi_{0\mathrm{av}}$ enters the contact zone, and the liquid with average concentration $\varphi_{N\mathrm{av}}$ leaves the contact zone. From above, the liquid with composition φ_{out} from separators of mass-transfer devices enters the layer under consideration, and from below, the liquid with concentration φ_{in} is directed to recycle. Therefore, the boundary conditions with allowance for the inlet stepwise change are written as

$$\begin{gathered} \xi = 0, \quad \varphi(0, \eta) - \frac{1}{\mathrm{Pe}_\xi}\left(\frac{\partial\varphi}{\partial\xi}\right)_{\xi = 0} = \varphi_{0\mathrm{av}}; \\ \xi = 1, \quad (\partial\varphi/\partial\xi)_{\xi = 1} = 0; \end{gathered} \tag{8}$$

$$\begin{gathered} \eta = 0, \quad \varphi(\xi, 0) - \frac{1}{\mathrm{Pe}_\eta}\left(\frac{\partial\varphi}{\partial\eta}\right)_{\eta = 0} = \varphi_{\mathrm{out}}(\xi, 0); \\ \eta = 1, \quad (\partial\varphi/\partial\eta)_{\eta = 1} = 0 \quad (\varphi(\xi, 1) = \varphi_{\mathrm{in}}(\xi, 1)). \end{gathered} \tag{9}$$

The solution of boundary-value problem (7)–(9) is sought in the form

$$\varphi = \bar{\varphi}(\xi) f(\eta). \tag{10}$$

In this case, the boundary-value problem is divided into two problems. For the function $\bar{\varphi}(\xi)$, we have

$$\bar{\varphi}'' - \mathrm{Pe}_\xi \bar{\varphi}' + \mathrm{Pe}_\xi c \bar{\varphi} = 0 \tag{11}$$

under the boundary conditions

$$\begin{gathered} \xi = 0, \quad \bar{\varphi}(0, \eta) - \frac{1}{\mathrm{Pe}_\xi}\bar{\varphi}'(0, \eta) = \varphi_{0\mathrm{av}}, \\ \xi = 1, \quad \bar{\varphi}'(1, \eta) = 0. \end{gathered} \tag{12}$$

The solution of Eq. (11) is known [4]:

$$\begin{gathered} \bar{\varphi} = A_1 e^{P_1\xi} + A_2 e^{P_2\xi}, \\ \bar{\varphi}(1) = \frac{\varphi_{0\mathrm{av}}(p_1 e^{P_1 + P_2} - p_2 e^{P_1 + P_2})}{(p_2 + c)e^{P_2} - (p_1 - c)e^{P_1}}. \end{gathered} \tag{13}$$

For the function $f(\eta)$, we have

$$f'' - \mathrm{Pe}_\eta f' - \mathrm{Pe}_\eta c_1 f = 0 \tag{14}$$

under the boundary conditions

$$\begin{gathered} \eta = 0, \quad f(\xi, 0) - \frac{1}{\mathrm{Pe}_\eta} f'(\xi, 0) = \frac{\varphi_{\mathrm{out}}(\xi, 0)}{\varphi_{\mathrm{in}}(\xi, 0)} = 1 - E_x, \\ \eta = 1, \quad f'(\xi, 1) = 0. \end{gathered} \tag{15}$$

The solution of Eq. (13) is also known [4]:

$$\begin{gathered} f = B_1 e^{Z_1\eta} + B_2 e^{Z_2\eta}, \\ f(1) = \frac{(1 - E_x)(z_1 e^{Z_1 + Z_2} - z_2 e^{Z_1 + Z_2})}{(z_2 - c)e^{Z_2} - (z_1 - c)e^{Z_1}}. \end{gathered} \tag{16}$$

To find the constant c, we integrate Eq. (7) under boundary conditions (9) with respect to the liquid layer thickness. Taking into account boundary conditions (15), we obtain

$$\frac{1}{\mathrm{Pe}_\xi}\frac{\partial^2\bar{\varphi}}{\partial\xi^2} - \frac{\partial\bar{\varphi}}{\partial\xi} - \frac{KE_x}{\aleph}\bar{\varphi} = 0. \tag{17}$$

Thus, solution (10) with expressions for $\bar{\varphi}$ and f determines the concentration field on the tray. The following expressions for the efficiency of a contact stage were derived:

$$E_{0x} = \frac{x_{0\mathrm{av}} - x_{N\mathrm{av}}}{x_{0\mathrm{av}} - x^*(y_{\mathrm{out}})} = 1 - \Phi, \tag{18}$$

$$E_{0y} = \frac{y_{\mathrm{out}} - y_0}{y^*(x_{\mathrm{out}}) - y_0} = \frac{1}{\lambda}\left(\frac{1}{\Phi} - 1\right), \tag{19}$$

where

$$\Phi = \frac{\varphi_{N\mathrm{av}}}{\varphi_{0\mathrm{av}}} = \frac{\varphi(1)}{\varphi_{0\mathrm{av}}} = \frac{4q}{(1 + q)^2 t_1 - (1 - q)^2 t_2}.$$

Let us consider particular cases of calculating the stage efficiency. Let $\mathrm{Pe}_\xi \longrightarrow 0$, which corresponds to the condition of perfect dispersion of the liquid at the stage; then, the stage efficiency can be calculated as

$$E_{0x} = 1 - \frac{1}{1 + KE_x/\aleph}, \quad E_{0y} = \frac{KE_x}{\lambda\aleph}; \tag{20}$$

as $\mathrm{Pe}_\xi \longrightarrow \infty$, there is a plug flow on the tray:

$$\begin{gathered} E_{0x} = 1 - \frac{1}{\exp(-KE_x/\aleph)}, \\ E_{0y} = \frac{1}{\lambda}\left[\exp\left(-\frac{KE_x}{\aleph}\right) - 1\right]. \end{gathered} \tag{21}$$

Calculations of the efficiency by Eqs. (18) and (19) showed that, with an increase in the degree of liquid dispersion over both the length and the thickness of the layer, the contact stage efficiency decreases. The contact stage efficiency can be increased by enhancing the liquid recycle at the stage (Fig. 4). For this purpose, it is necessary to increase the liquid capacity of contact devices and prevent, by design measures, the mixing of the initial and separated liquids.

Comparison of the efficiencies calculated by Eqs. (18) and (19) with the efficiencies in the limiting cases demonstrated that, in the ranges $\mathrm{Pe}_\xi < 0.5$ and $\mathrm{Pe}_\xi > 15$, the contact stage efficiency can be calculated with an accuracy of 0.4% by Eqs. (20) and (21), respectively.

NOTATION

D—dispersion coefficient, m^2/s;

$f(\eta)$—normalized admixture concentration distribution function in the liquid in the transverse direction $\left(\frac{1}{h}\int_0^h f(\eta)\,d\eta = 1\right)$;

G, L—vapor and liquid flow rates through a contact stage, respectively, m^3;

i, j—longitudinal and transverse coordinates, respectively;

$K = \frac{v}{u}\frac{l}{h}$—recycle coefficient;

k—recycle ratio of the liquid at a stage;

l, h—length and thickness of the contact zone, respectively, m;

m—equilibrium constant;

$p_{1,2} = \frac{\mathrm{Pe}_\xi}{2}\left[1 \pm \left(1 - \frac{4c}{\mathrm{Pe}_\xi}\right)^{1/2}\right]$, $z_{1,2} = \frac{\mathrm{Pe}_\eta}{2}\left[1 \pm \left(1 + \frac{4c_1}{\mathrm{Pe}_\eta}\right)^{1/2}\right]$;

$q = (1 + 4KE_x/\mathrm{Pe}_\xi N)^{1/2}$;

s—number of contact stage sections in which the liquid is well dispersed;

$t_1 = \exp\left[-\frac{\mathrm{Pe}_\xi}{2}(1-q)\right]$; $t_2 = \exp\left[-\frac{\mathrm{Pe}_\xi}{2}(1+q)\right]$;

u—flow-rate-average liquid velocity in the longitudinal direction, m/s;

v—effective liquid velocity in the transverse direction, m/s;

x_{in}, x_{out}—liquid concentrations at the inlet and outlet of a contact device, respectively, $\mathrm{kmol/m}^3$;

$x_{0\mathrm{av}}$, $x_{N\mathrm{av}}$—average liquid concentrations at the inlet and outlet of a tray, respectively, $\mathrm{kmol/m}^3$;

y_0, y_n, y_x^*—inlet, outlet, and equilibrium vapor concentrations, respectively, $\mathrm{kmol/m}^3$;

$\lambda = mG/L$—diffusion potential number;

ξ, η—longitudinal and transverse coordinates, respectively;

$\bar{\varphi}(\xi)$—admixture concentration distribution function in the liquid in the longitudinal direction;

$\aleph = \bar{\varphi}[\xi]/\varphi_{\mathrm{in}}(\xi)$;

$\mathrm{Pe}_\xi = ul/D_i$, $\mathrm{Pe}_\eta = vh/D_j$—Peclet numbers in the longitudinal and transverse directions, respectively.

SUBSCRIPTS AND SUPERSCRIPTS

av—average;

N—number of mass-transfer elements;

x, y—liquid and vapor phases, respectively;

0—initial;

*—equilibrium.

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