

Simultaneous Heat and Momentum Transfer in Two-Phase Systems at Low Pressure and High Temperature: Vapor Phase

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Received March 25, 2004

Abstract—The heat and momentum transfer in a liquid–vapor two-phase film system at high temperature and low pressure is studied by the example of the removal of highly volatile fractions from high-boiling liquids in film evaporators. The temperature and velocity distributions in the vapor phase are determined.

The theoretical investigation of an important chemical engineering process—the removal of highly volatile fractions from high-boiling liquids by means of evaporating a free-falling film at low pressure and high temperature in a cocurrent flow—was begun in a previous work [1], where, in the approximation of weak hydrodynamic and thermal interaction, the velocity and temperature distributions in a liquid film falling down the inner surface of a tube were determined.

At realistic temperature differences in an evaporator ($\Delta T' \sim 50\text{--}100^\circ\text{C}$), the temperature dependence of μ_g , λ_g , ρ_g , λ , ρ , c_p , c_{pg} can be ignored, unlike the temperature dependence of the equilibrium pressure $P^*(T')$ and the liquid viscosity $\mu(T')$, which can be represented in the form [1, 2]

$$P^*(T') = P_0^* \exp(at), \quad \mu(T) = \mu_0 \exp(-bt). \quad (1)$$

Here, $a = A(T_w - T_0)/T_0^2$ and $b = B(T_w - T_0)/T_0^2$ are dimensionless parameters, and A and B are empirical constants.

It was shown that, in a short initial region of size $x' \sim h_0 \text{Pe}_r \ll l$, the velocity and the film thickness vary mainly because of the temperature dependence of the viscosity, and outside of this region, they vary because of the evaporation of the liquid solution. At an evaporator length of $l' \sim (q_0/V'_{nw})$, the film completely evaporates. The mass transfer of the solvent and highly volatile components through the interface occurs in the main evaporation region, where $T' \equiv T_w$.

In the case of weak hydrodynamic and thermal interaction between the phases, the velocity components and the temperature of the liquid and vapor are related as [1]

$$u_s'^g = u_s' - \left(\frac{\rho v_n'}{\rho_g} \right) \frac{dh}{dx'},$$

$$v_s'^g = - \left(\frac{\rho v_n'}{\rho_g} \right) - u_s' \frac{dh}{dx'}, \quad (2)$$

$$T'_{gs} = T'_s.$$

Consequently, it can be supposed that, in solving the momentum- and energy-transfer equations in the vapor phase, the values of u'_z , v'_r , T'_g at the interface are known.

A schematic of an evaporator and the coordinate system in the liquid and gas phases are presented in Fig. 1. The energy- and momentum-transfer equations can be represented in the form

$$\rho_g c_{pg} \left[u'_z \frac{\partial T'_g}{\partial z'} + v'_r \frac{\partial T'_g}{\partial r'} \right] = \lambda_g \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial T'_g}{\partial r'} \right),$$

$$\rho_g \left(u'_z \frac{\partial u'_z}{\partial z'} + v'_r \frac{\partial u'_z}{\partial r'} \right) = - \frac{\partial P'_g}{\partial z'} + g \rho_g + \mu_g \frac{1}{r'} \frac{\partial}{\partial r'} \left(r' \frac{\partial u'_z}{\partial r'} \right), \quad (3)$$

$$\frac{1}{r'} \frac{\partial}{\partial r'} (r' v'_r) + \frac{\partial v'_z}{\partial z'} = 0.$$

The material-balance equation has the form

$$2\pi \int_0^{x'} v'_n dx' = 2\pi \int_0^{R_0} u'_z r' dr'. \quad (4)$$

At the center of the evaporator, the symmetry conditions for the velocities and temperatures are met:

$$\frac{\partial u'_z}{\partial r'} = v'_r = \frac{\partial T'_g}{\partial r'} = 0 \quad \text{at } r' = 0.$$

Since all of the vapor flowing down forms by liquid evaporation, in the upper part of the evaporator, there is

a stagnant zone in which there is almost no motion. Consequently, the initial conditions at the inlet of the evaporation region can be written in the form

$$T'_g = T_0, \quad v'_g = u'_g = 0 \quad \text{at } x' = 0.$$

INITIAL EVAPORATION REGION

Similar to the analysis of the liquid film [1], here the velocity distributions in the vapor in the initial evaporation region are studied. The temperature of the liquid and, hence, at the interface in this region, varies most strongly; more specifically, it increases from T_0 virtually up to the wall temperature T_w .

Let us represent the velocity components in the gas at the interface with regard to Eq. (4) in the form

$$\begin{aligned} u'_{gs} &= v_{n0}^g \left[\left(\frac{u_0}{v_{n0}^g} \right) u_s(x, y) - \left(\frac{1}{Pe_T} \right) e^{at_s} \frac{dy_s}{dx} \right], \\ v'_{gs} &= -v_{n0}^g \left[e^{at_s} + \left(\frac{1}{Pe_T} \right) \left(\frac{u_0}{v_{n0}^g} \right) u_s(x, y) \frac{dy_s}{dx} \right], \end{aligned} \quad (5)$$

where u_s , t_s , and y_s are dimensionless functions and coordinates for the liquid phase, which were derived previously [1] and are on the order of unity. Expressions (5) show that both velocity components in the vapor, u'_g and v'_g , are proportional in order of magnitude to the normal surface velocity v_{n0}^g . Assuming that the vapor is an ideal gas and the film flow is laminar ($Re_0 \leq 400$), one can take

$$\begin{aligned} \rho_g &= P_g/RT', \quad h_0 = (9v_{n0}^2/g)^{1/3} Re_0^{1/3}, \\ \rho &= 10^3 \text{ kg/m}^3. \end{aligned} \quad (6)$$

Since the accommodation coefficient α^* for most liquids is 10^{-4} – 10^{-5} , it is easy to show that, at sufficiently high temperatures ($T' \geq 400$ K) and low pressures ($P_g \leq 10$ mm Hg), the ratio u_0/v_{n0}^g is small.

Previously [1], it was demonstrated that, in the initial evaporation region, the dimensionless temperature t in the liquid and the dimensionless film thickness y_s mainly depend on two dimensionless parameters, a and b , which take into account the change in the surface velocity (P^*) and viscosity with temperature (see expressions (1)). Along with the two effects, the effect of the heat ΔQ of evaporation at the interface on the processes in the film is also studied. However, this effect is usually so weak that it can be taken into account by expanding the corresponding dimensionless functions (u , v , and t) as series in the small parameter $\alpha = (\Delta T'_s/\Delta T'_w) \ll 1$ [1]:

$$\begin{aligned} t &= t^{(0)}(b, x, y) + \alpha t^{(1)}(a, b, x, y) + \dots, \\ y_s &= y_s^{(0)}(b, x) + \alpha y_s^{(1)}(a, b, x) + \dots, \end{aligned}$$

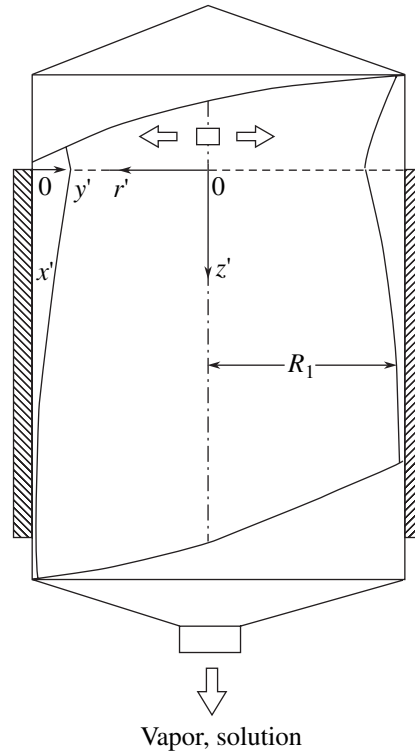


Fig. 1. Schematic of an evaporator.

$$\begin{aligned} u &= u^{(0)}(b, x, y) + \alpha u^{(1)}(a, b, x, y) + \dots, \\ v &= v^{(0)}(b, x, y) + \alpha v^{(1)}(a, b, x, y) + \dots, \end{aligned}$$

where the zeroth-order approximations depend only on a single dimensionless parameter, b [1], and the higher-order approximations depend on both a and b .

Let us formulate the heat- and momentum-transfer problem in the vapor phase in dimensionless form. For this purpose, the following dimensionless functions t_g , u_g , and v_g and coordinates x_g and r_g should be introduced:

$$\begin{aligned} T' &= T_0 + (T_w - T_0)t_g, \\ u'_g &= v_{n0}^r u_g(x_g, r_g), \quad v'_g = v_{n0}^r v_g(x_g, r_g), \\ x' &= R_0 x_g, \quad r' = R_0 r_g. \end{aligned}$$

Let us transform heat-transfer, momentum-transfer, and continuity equations (3) to dimensionless form:

$$u_g \frac{\partial t_g}{\partial x_g} + v_g \frac{\partial t_g}{\partial r_g} = \left(\frac{1}{Pe_{g0}} \right) \frac{1}{r_g} \frac{\partial}{\partial r_g} \left(r_g \frac{\partial t_g}{\partial r_g} \right), \quad (7)$$

$$u_g \frac{\partial u_g}{\partial x_g} + v_g \frac{\partial u_g}{\partial r_g} = -F_0 + \left(\frac{1}{Pe_{g0}} \right) \frac{1}{r_g} \frac{\partial}{\partial r_g} \left(r_g \frac{\partial u_g}{\partial r_g} \right), \quad (8)$$

$$\frac{\partial u_g}{\partial x_g} + \frac{1}{r_g} \frac{\partial}{\partial r_g} (r_g v_g) = 0. \quad (9)$$

Material-balance equation (4) takes the form

$$\int_0^1 u_g r_g dr_g = \int_0^{x_g} \exp(at_s(x)) dx_g.$$

The function F_0 is defined as

$$-F_0 \equiv (-\partial P/\partial x' + \rho_g g) R_0 / \rho_g (v_{0n}^g)^2$$

and is assumed to depend only on the longitudinal coordinate x_g . This function has the meaning of a dimensionless force and should be calculated in the cause of solving the problem.

The set of Eqs. (7)–(9) should be solved under the following additional conditions:

on the film surface ($r_g = 1$),

$$\begin{aligned} t_{gs} &= t_s = t_s^{(0)}(b, x) + \alpha t_s^{(1)}(a, b, x) + \dots, \\ u_{gs} &= (u_0/v_{0n}^g) u_s^{(0)}(b, x), \\ v_{gs} &= e^{at_s} = -\exp(at_s^{(0)}) \\ &\quad - \alpha \exp(at_s^{(0)}) at_s^{(1)}(a, b, x) + \dots, \end{aligned} \quad (10)$$

on the channel axis ($r_g = 0$),

$$\partial t_g / \partial r_g = \partial u_g / \partial r_g = v_g' = 0,$$

at the inlet section of the evaporator ($x_g = 0$),

$$t_g = u_g = 0.$$

All this means that, in the general case, the expansions of the solution of set (7)–(9) up to the first order in the small parameters α and u_0/v_{0n}^g have the form

$$\begin{aligned} t_g &= t_g^{(0)}(x_g, r_g) + \alpha t_g^{(1)}(x_g, r_g) \\ &\quad + (u_0/v_{0n}^g) t_g^{(11)}(x_g, r_g) + \dots, \\ u_g &= u_g^{(0)}(x_g, r_g) + \alpha u_g^{(1)}(x_g, r_g) \\ &\quad + (u_0/v_{0n}^g) u_g^{(11)}(x_g, r_g) + \dots, \\ v_g &= v_g^{(0)}(x_g, r_g) + \alpha v_g^{(1)}(x_g, r_g) \\ &\quad + (u_0/v_{0n}^g) v_g^{(11)}(x_g, r_g) + \dots, \\ F_0(x_g) &= F^{(0)}(x_g) + \alpha F^{(1)}(x_g) \\ &\quad + (u_0/v_{0n}^g) F^{(11)}(x_g) + \dots \end{aligned} \quad (11)$$

Expansions (11) could be written not only in α and u_0/v_{0n}^g but also in another small parameter, $1/Pe_T$. However, at realistic values $Re_0 > 1$, effects related to Pe_T are usually ignored [3, 4]. If it is taken into account that, for the vapor phase, $Pr_g \cong 1$ [2], then it is easy to see that, in the general case, the zeroth-order approximations t_g , u_g , v_g , and F_0 and also higher-order terms in the expan-

sions of these functions in the small parameters α and u_0/v_{0n}^g depend on three dimensionless parameters, a , b , and Re_{g0} ; the ranges of a and b are usually finite ($a \leq 1$, $b \leq 10$) and Re_{g0} can formally vary from zero to infinity.

Let us present the solution of hydrodynamic equations (8), (9) at low Reynolds numbers. As $Re_{g0} \rightarrow 0$, set (8), (9) can be simplified to the form

$$-F_0 Re_{g0} + \frac{1}{r_g} \frac{\partial}{\partial r_g} \left(\frac{\partial u_g}{\partial r_g} \right) = 0,$$

$$\frac{\partial u_g}{\partial x_g} + \frac{1}{r_g} \frac{\partial}{\partial r_g} (r_g v_g) = 0.$$

The zeroth-order approximation of the solution of this set is

$$\begin{aligned} u^{(0)}(x_g, r_g) &= 4(1 - r_g^2) \frac{x_g}{x} \int_0^x \exp[at_s^{(0)}(x)] dx, \\ v^{(0)}(x_g, r_g) &= -\exp[at_s^{(0)}(x)] (2r_g - r_g^3), \end{aligned} \quad (12)$$

$$F_0^{(0)} = -\frac{16}{Re_{g0}} \frac{x_g}{x} \int_0^x \exp[at_s^{(0)}(x)] dx.$$

At an arbitrary Re_{g0} value, set (8), (9) can be numerically solved by a method that was first used for sets of this kind by Simuni [5, 6]. Let us consider the simplest case $a = 0$. In this case, the temperature $t_s(x)$ at the interface does not affect the solution and conditions (10) at the interface are considerably simplified:

$$r_g = 1: u_{gs}^{(0)} = 0, \quad v_{gs}^{(0)} = -1.$$

Under such boundary conditions, the solution of the problem can be sought in the form

$$\begin{aligned} u_g &= x_g u(Re_{g0}, r_g), \quad F_g = x_g f(Re_{g0}), \\ v_g &= v(Re_{g0}, r_g). \end{aligned}$$

Substitution of these functions into Eq. (9) yields the relation between v and u :

$$v = -\frac{1}{r_g} \int_0^{r_g} u r_g dr_g.$$

For the condition for v_s at the interface to be met, the following equality should be satisfied:

$$\int_0^1 u r_g dr_g = 1. \quad (13)$$

The velocity u is found from the nonlinear equation

$$u^2 + \left(\frac{1}{r_g} \int_0^{r_g} u r_g dr_g \right) \frac{du}{dr_g} = -f + \left(\frac{1}{Re_{g0}} \right) \frac{1}{r_g} \frac{\partial}{\partial r_g} \left(r_g \frac{\partial u}{\partial r_g} \right) \quad (14)$$

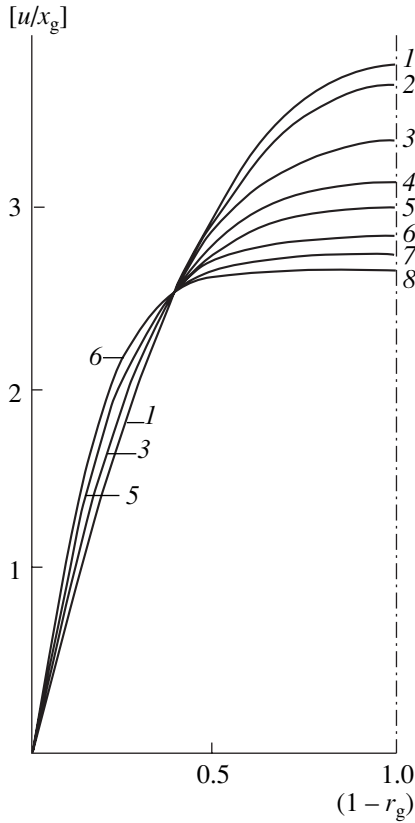


Fig. 2. Zeroth-approximation longitudinal velocity profile $u^{(0)}$ at $Re_g = (1) 0, (2) 1, (3) 3, (4) 6, (5) 10, (6) 20, (7) 40,$ and $(8) \infty$.

under the boundary conditions

$$r_g = 0: du/dr_g; \quad r_g = 1: u = 0. \quad (15)$$

For any fixed $f(Re_{g0})$, problem (14), (15) has a single solution. The f value should be such that material-balance condition (13) is met. Integro-differential equation (14) can hardly be solved analytically. Therefore, it should be solved numerically [7]. The results of numerical calculations of $u, v,$ and F_0 are presented in Figs. 2–4. It is observed that the entire diversity of the functions $u^{(0)}$ and $v^{(0)}$ lie between the limiting functions that correspond to $Re_{g0} = 0$ (Figs. 2, 3, curves 1) and $Re_{g0} = \infty$ (Figs. 2, 3, curves 8, 5). In practice, the range of Re_{g0} is finite: $0.3 \leq Re_{g0} \leq 40$. With an increase in Re_{g0} , the velocity profile $u^{(0)}(r_g)$ flattens in the central region of the channel; i.e., a significant part of the vapor phase is occupied by a zone (flow core) with constant longitudinal velocity inherent in high Reynolds numbers. Interestingly, the presence of a transverse velocity component at the interface at high Re_{g0} probably does not disturb the laminar vapor flow, since the gradients of the velocity u_g in the transverse direction near the interface are finite even at $Re_{g0} = \infty$.

In the general case, at any Re_{g0} , the effect of the temperature $t_s(x)$ at the interface on the velocity and pres-

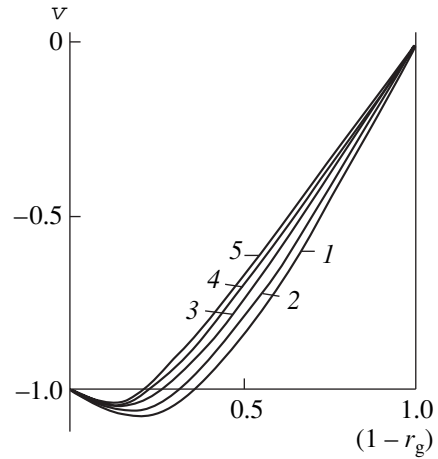


Fig. 3. Zeroth-approximation transverse velocity profile $v^{(0)}$ at $(1) Re_g \leq 1; Re_g = (2) 3, (3) 10,$ and $(4) 20;$ and $(5) Re_g > 40$.

sure distributions in the vapor is noticeable only within a short region of length $x' \cong h_0 Re_T (x \cong 1)$ (see, e.g., solution (12) at $Re_{g0} = 0$). Outside of this region, the transverse velocity v_s is constant again:

$$r_g = 1: v_s \Rightarrow e^a, \quad u_s = 0,$$

and the corresponding asymptotic values of the zeroth-order approximations of the velocity in the vapor are

$$u_g^{(0)} \Rightarrow u(r_g, Re_{gw}) x_g \exp a,$$

$$v_g^{(0)} \Rightarrow v(r_g, Re_{gw}) \exp a,$$

$$F^{(0)} \Rightarrow f(Re_{gw}) x_g \exp(2a),$$

where the dimensionless functions $u, v,$ and f coincide with the corresponding functions (Figs. 2–4) after the obvious substitution $Re_{g0} \Rightarrow Re_{gw} \equiv Re_{g0} \exp a$. It will be proven further that all these asymptotic functions continuously join the corresponding functions for the main evaporation region.

The temperature distribution $t_g(x_g, r_g)$ in the vapor in the initial evaporation region at any Re_{g0} can readily be imagined by studying the heat-transfer equation for low ($Re_{g0} \rightarrow 0$) and high ($Re_{g0} \rightarrow \infty$) Reynolds numbers. In the first case ($Re_{g0} \rightarrow 0$), the solution of heat-transfer equation (7) under all of the boundary conditions has the form

$$t_g^{(0)} = t^{(0)}(x, b),$$

i.e., as $Re_{g0} \rightarrow 0$, there is virtually no change in the temperature of the vapor in the transverse direction. As $Re_{g0} \rightarrow \infty$, the right-hand side of Eq. (7) can be neglected and, hence, $t_g^{(0)}$ satisfies the homogeneous first-order partial differential equation

$$x_g u(r_g) (\partial t_g / \partial x_g) + v(r_g) (\partial t_g / \partial r_g) = 0$$

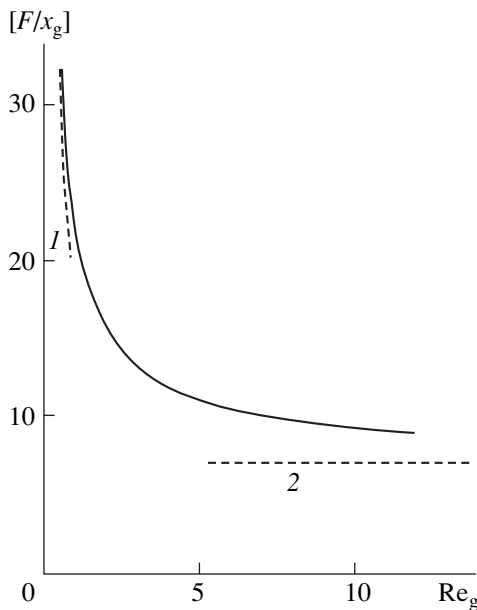


Fig. 4. Pressure $F^{(0)}/x_g$ versus Re_{gw} as (1) $Re_g \rightarrow 0$ and (2) $Re_g \rightarrow \infty$.

under the additional conditions

$$x_g = 0: t_g^{(0)} = 0; \quad r_g = 1: t_g^{(0)} = t_s^{(0)}(x).$$

The solution of this problem that was obtained by the method of characteristics [8] has the form

$$t_g(x_g, r_g) = t_s \left[x \exp \left(\int_{r_g}^1 \frac{u(r_g)}{v(r_g)} dr_g \right) \right],$$

where the ratio u/v virtually coincides with $u^{(0)}/v^{(0)}$, since both functions, u and v , are proportional to $\exp[at_s(x)]$ (see, e.g., expression (12)). Taking into account that $v(r_g) < 0$, one can readily see that the temperature in the vapor phase at any section obeys the inequality $t_g(x_g, y_g) \leq t_s(x)$. This temperature monotonically decreases with distance from the interface and tends to zero at the center of the channel, since $v \cong r_g$ as $r_g \rightarrow 0$. One can assume that, at $x \gg 1$, regardless of the Re_g value, the temperature profile in the vapor attains the limiting curve $t_s(x)$ and the asymptotic value up to the first order in the small parameter α has the form [1]

$$t_g \Rightarrow t_s(x) = 1 - \alpha \exp(a - b/3). \quad (16)$$

VELOCITY AND TEMPERATURE IN THE MAIN EVAPORATION REGION IN THE VAPOR PHASE

The length $x' \sim h_0 Pe_T$ of the initial evaporation region usually constitutes a small part of the total evaporator length, whose maximal size is $H_{\max} = q^0/v'_{nw}$ [1]. The velocity distribution in the film and the film thickness in the region $x = x'/h_0 Pe_T \gg 1$, i.e., in the main evapora-

tion region, were found previously [9]. The zeroth-order approximations of these functions in the expansions in the small parameter α have the form

$$u = 3 \exp(b/3)(z - z^2/2)(1 - x_1)^{2/3},$$

$$v_1 = z^2/2, \quad y_s = \exp(-b/3)(1 - x_1)^{1/3},$$

where $z \equiv y/y_s$, $x_1 = x'/H_{\max}$, $v'_y = v'_{nw} v_1$, and $u'_x = u_0 u$.

The zeroth-order approximation of the temperature in the film at $x \gg 1$ is trivial ($t_s^{(0)} = 1$); therefore, the temperature profile was determined up to the first order in the expansion in the small parameter α :

$$t = 1 - (\Delta t_s/y_s)y,$$

where $\Delta t_s \equiv 1 - t_s = \alpha \exp(a - b/3)(1 - x_1)^{1/3}$.

The velocity components on the film surface on the vapor side in the main evaporation region can be represented in the form (see Eqs. (2))

$$u'_s{}^g = u'_s - \frac{\rho}{\rho_g} v_n \frac{dh}{dx'} \cong v_{gw} \left(\frac{u_0}{v_{gw}} \right) u_s(x_1),$$

$$v'_s{}^g = -\frac{\rho}{\rho_g} v_n - \frac{dh}{dx'} u'_s \cong -v_{gw} \exp(-a \Delta t_s),$$

where, as above (expressions (6)), the parameter $(u_0/v_{gw}) \ll 1$ and the terms on the order of $1/Pe_g$ are neglected.

Let us introduce the following dimensionless velocity components in the vapor, u_g and v_g , and pressure F_w :

$$u'_g = v_w{}^g u_g(r_g, x_g, Re_{gw}),$$

$$v'_g = v_w{}^g v_g(r_g, x_g, Re_{gw}),$$

$$F_w \equiv (\partial p / \partial x' - \rho_g g) / \rho_g (v_{wn}{}^g)^2.$$

Obviously, u_g and v_g satisfy the same Eqs. (8), (9), in which the following change of variables should be made:

$$v_{g0} \Rightarrow v_{gw}, \quad Re_{g0} \Rightarrow Re_{gw} \equiv R_0 v_{wn}{}^g / v_g,$$

$$F_0 \Rightarrow F_w.$$

The functions $u_g(r_g, x_g, Re_{gw})$ and $v_g(r_g, x_g, Re_{gw})$ in the main evaporation region, as well as the initial evaporation region, can be sought in the form of expansions (11) in the small parameters α and $u_0/v_{wn}{}^g$. Obviously, the zeroth-order approximation of the velocities and pressures are

$$u_g^{(0)} = x_g u(r_g, Re_{gw}),$$

$$v_g^{(0)} = v(r_g, Re_{gw}), \quad F_w^{(0)} = x_g f(Re_{gw}),$$

where u , v , and f are found from the plots in Figs. 2–4 and $Re_{gw} \Rightarrow Re_{g0}$.

Let us find the temperature in the main evaporation region. Taking into account that the zeroth-order approximation of $t_g^{(0)}$ is trivial ($t_g^{(0)} = 1$), one can obtain the temperature distribution in the vapor in the main evaporation region up to the first order in the expansions in the small parameters α and u_0/v_w^g . Substituting expansions (11) into heat-transfer equation (7), where $Pe_0^g \Rightarrow Pe_w^g = Re_w^g$, one can readily demonstrate that the first-order approximation dimensionless functions, $t^{(1)}$, obey the equation

$$\begin{aligned} u_g^{(0)} \frac{\partial t_g^{(1)}}{\partial x_g} + v_g^{(0)} \frac{\partial t_g^{(1)}}{\partial r_g} + \left(u_g^{(1)} \frac{\partial t_g^{(0)}}{\partial x_g} + v_g^{(1)} \frac{\partial t_g^{(0)}}{\partial r_g} \right) \\ = \frac{1}{Pe_w^g} \frac{\partial}{\partial r_g} \left(r_g \frac{\partial t_g^{(1)}}{\partial r_g} \right). \end{aligned}$$

Here, $(u_g^{(1)} \partial t_g^{(0)} / \partial x_g + v_g^{(1)} \partial t_g^{(0)} / \partial r_g) = 0$, since the zeroth-order approximation $t_g^{(0)} \equiv 1$. A similar equation is also valid for $t^{(1)}$. Thus, to calculate the temperature in the evaporation region up to the first order, it is unnecessary to calculate the corresponding first-order approximations of the velocities: $u_g^{(1)}$, $v_g^{(1)}$ and $u_g^{(11)}$, $v_g^{(11)}$. Under the temperature boundary conditions for $t_g^{(1)}$, the following differential problem is obtained:

$$\begin{aligned} x_1 u(r_g) \frac{\partial t_g^{(1)}}{\partial x_1} + v(r_g) \frac{\partial t_g^{(1)}}{\partial r_g} &= \frac{1}{Pe_w^g} \frac{1}{r_g} \frac{\partial}{\partial r_g} \left(r_g \frac{\partial t_g^{(1)}}{\partial r_g} \right), \\ r_g = 0: \partial t_g^{(1)} / \partial r_g &= 0; \\ x_1 = 0: t_g^{(1)} &= -\exp(a - b/3), \\ r_g = 1: t_{gs}^{(1)} &= -\exp(a - b/3)(1 - x_1)^{1/3}. \end{aligned} \quad (17)$$

It is easy to show that $t_g^{(11)} = 0$: the corresponding boundary conditions are zero conditions, because the temperature in the liquid is independent of the parameter u_0/v_w^g (see conditions (10)).

Let us seek solution (17) in the form

$$t_g^{(1)} = -\exp(a - b/3)t(r_g, x_1). \quad (18)$$

The dimensionless temperature $t(r_g, x_1)$ obviously satisfies Eq. (17). The additional conditions for $t(x_1, r_g)$ are the following:

$$\begin{aligned} r_g = 0: \partial t / \partial r_g &= 0; \\ x_1 = 0: t &= 1; \quad r_g = 1: t = (1 - x_1)^{1/3}. \end{aligned}$$

Following the approximate Galerkin method [7], the solution $t(x_1, r_g)$ meeting the conditions at the boundaries is sought in the form

$$t(x_1, r_g) = (1 - x_1)^{1/3} + b(x_1)(1 - r_g^2), \quad (19)$$

where $b(x_1)$ is an arbitrary function of x_1 . Let us rewrite the transfer equations in the following equivalent form, using continuity equation (9):

$$\frac{\partial}{\partial x_1} (x_1 u(r_g) r_g t) + \frac{\partial}{\partial r_g} (v(r_g) r_g t) = \frac{1}{Pe_w^g} \frac{\partial}{\partial r_g} \left(r_g \frac{\partial t}{\partial r_g} \right).$$

Integration with respect to r_g from 0 to 1 gives

$$\frac{\partial}{\partial x_1} (\bar{t}) + (v_s t_s) = \frac{1}{Pe_w^g} \left(\frac{\partial t}{\partial r_g} \right)_s. \quad (20)$$

Here, the average temperature \bar{t} is

$$\begin{aligned} \bar{t} &= \int_0^1 x_1 u(r_g) r_g t dr_g \\ &= x_1 \int_0^1 u r_g [(1 - x_1)^{1/3} + b(1 - r_g^2)] dr_g \\ &= x_1 (1 - x_1)^{1/3} + (x_1 b) k(Re_{gw}), \end{aligned}$$

where

$$k(Re_{gw}) \equiv \int_0^1 u r_g (1 - r_g^2) dr_g.$$

Calculations showed that this function depends slightly on Re_{gw} and varies within the range $0.60 \leq k \leq 0.66$. Using the equalities $v_s = 1$, $t_s = (1 - x_1)^{1/3}$, and $(dt/dr_g)_s = -2b$ and substituting expression (19) into expression (20) for determining the unknown function $b(x_1)$, one can obtain the inhomogeneous first-order linear ordinary differential equation

$$[(1 - x_1)^{1/3} x_1]' + k(x_1 b)' - (1 - x_1)^{1/3} = -2b/Pe_w^g,$$

where the primes mean differentiation with respect to x_1 . Simple transformations yield

$$k(x_1 b)' + \frac{2}{Pe_w^g} \frac{(x_1 b)}{x_1} = \frac{1}{3} \frac{x_1}{(1 - x_1)^{2/3}}.$$

This equation, which meets the obvious condition $b(0) = 0$ (see solutions (18), (19)), can be solved by quadratures [8]:

$$b = \frac{x_1}{3k} \int_0^1 \frac{z^{1+2/(kPe_{gw})}}{(1 - x_1 z)^{2/3}} dz > 0. \quad (21)$$

As follows from expression (19), in its physical meaning, $b \equiv (t_c - t_s)$ is the difference of the dimensionless temperatures on the channel axis and on the film. In the general case, as follows from solution (21), this difference depends on two decisive parameters, x_1 and Pe_{gw} . The calculated function $b(x_1)$ is presented in Fig. 5. The function $b(x_1)$ at any Pe_{gw} everywhere in the

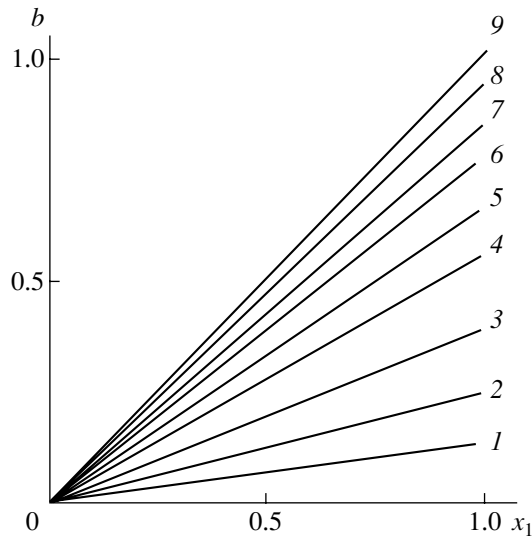


Fig. 5. Function $b(x_1, Pe_{gw})$ at $Re_{gw} = (1) 0.2, (2) 0.5, (3) 1, (4) 2, (5) 3, (6) 5, (7) 10, (8) 20, (9) \infty$.

range $0 \leq x_1 \leq 1$ is observed to be virtually linear. This means that the effect of the factor $(1 - x_1 z)^{2/3}$ in integral (21) is not very substantial; consequently, one can write in approximation

$$b(x_1, Pe_{gw}) = \frac{x_1}{3k} \int_0^1 z^{1+2/(kPe_{gw})} dz = \frac{x_1}{6k} \left(\frac{0.6Pe_{gw}}{1 + 0.6Pe_{gw}} \right).$$

The effect of the parameter Pe_{gw} on $b(x_1)$ is generally monotonic: $b \rightarrow 0$ as $Pe_{gw} \rightarrow 0$ and $b \rightarrow x_1/3.6$ as $Pe_{gw} \rightarrow \infty$ (see Fig. 5, curves 1, 7, respectively).

Thus, along the film flow in the evaporation region, the change in dimensionless temperature in the transverse direction in the liquid phase generally decreases, while in the vapor, it increases:

$$1 - t_s \equiv \Delta t_s = \alpha \exp(a - b/3)(1 - x_1)^{1/3},$$

$$t_s^g - t_c^g \equiv \Delta t_g = \alpha \exp(a - b/3)b(x_1, Pe_w^g).$$

Both functions Δt_s and Δt_g are proportional to the small parameter α .

In conclusion, we note that asymptotic expression (16) is actually equivalent to expressions (18) and (19); i.e., the solution of the heat-transfer equations in the initial region continuously transforms into the corresponding solution for the main evaporation region at sufficiently large x . At virtually any a and b , this takes place at $x \geq 2$. It was previously shown [1] that, on such short intervals, the solution evaporation can be ignored. In studying mass-transfer processes, it will be shown that, at $x \leq 1$, the evaporation of any highly volatile fraction can also be ignored. This means that, for efficient purification, the evaporator length l should satisfy the inequality $l \gg x_T = h_0 Pe_T$ and, hence, it is sufficient to

study the desorption of highly volatile fractions in the main evaporation region alone. In this sense, there is no need to accurately describe the velocity and temperature distributions in the region $x' \sim x_T$.

The obtained velocity distributions in the main evaporation region both in the liquid and the vapor phase will be further used to study the two-phase desorption of highly volatile components into the vapor phase in evaporators of the considered type. In this case, in the first approximation, one can consider that the temperature in the evaporator is T_w .

NOTATION

a_1, a_g —thermal diffusivities of the liquid and the vapor, respectively, m^2/s ;

C_p, C_{pg} —specific heats at constant pressure of the liquid and the vapor, respectively, $J/(kg K)$;

g —acceleration due to gravity, m/s^2 ;

H —evaporation region height, m ;

h —liquid film thickness, m ;

M —molecular weight of the liquid, kg/mol ;

P —pressure, $kg/(m s^2)$;

$P_{(T)}^*$ —equilibrium vapor pressure of the liquid, $kg/(m s^2)$;

Q —specific enthalpy, J/kg ;

$\Delta Q = Q_g - Q_l$ —heat of evaporation of the liquid solution, J/kg ;

$q_0 = u_0 h_0$ —film flux density at the evaporator inlet, m^2/s ;

R_0 —evaporator tube radius, m ;

(r', z') —cylindrical coordinate system in the vapor, m ;

T', T_g' —temperatures of the liquid and the vapor, respectively, K ;

$\Delta T_s' = (\rho V_n' \Delta Q h / \lambda)_0$ —temperature of the cooling of the surface, K ;

$\Delta T_w' = T_w - T_0$ —maximal temperature difference, K ;

$t = (T' - T_0)/(T_w - T_0)$, $t_g = (T_g' - T_0)/(T_w - T_0)$ —dimensionless temperatures of the liquid and the vapor, respectively;

u, v —dimensionless film velocity components;

u_0 —average velocity in the film at the evaporator film, m/s ;

$v_n', v_{gn}' = \rho v_n' / \rho_g$ —normal velocity components on the liquid and vapor sides, respectively, m/s ;

v_x', v_y' —velocity vector components in the film, m/s ;

v_z', v_r' —velocity vector components in the vapor, m/s ;

(x', y') —rectangular coordinate system in the thin film, m ;

$x = x'/(h_0 \text{Pe}_T)$, $y = y'/h_0$ —dimensionless coordinates in the film;

$x_g = x'/R_0$, $r_g = r'/R_0$ —dimensionless coordinates in the vapor;

$\alpha = \Delta T'_s / \Delta T'_w$ —parameter that takes into account the cooling of the surface;

λ , λ_g —thermal conductivities in the liquid and the vapor, respectively, (kg m)/(s³ K);

μ , μ_g —dynamic viscosities of the liquid and the vapor, respectively, kg/(m s);

$\nu = \mu/\rho$, $\nu_g = \mu_g/\rho_g$ —kinematic viscosities of the liquid and the vapor, respectively, m²/s;

ρ , ρ_g —densities of the liquid and the vapor, respectively, kg/m³;

$\text{Pe}_g = q_0/a_1$, $\text{Pe}_n = h v'_n/a_1$ —Peclet numbers in terms of the average film velocity and the evaporation velocity, respectively;

$\text{Pr} = \lambda/(C_p \mu)$, $\text{Pr}_g = \lambda_g/(C_{pg} \mu_g)$ —Prandtl numbers in the liquid and the vapor, respectively;

$\text{Re}_0 = q_0/\nu$, $\text{Re}_g = V_n^{*g} R_0/\nu_g$ —Reynolds and Peclet numbers for the vapor, respectively;

$\text{Re}_g = R_0 v_n^{*g} / \nu_g$, $\text{Pe}_g = R_0 v_n^{*g} / a_g$ —Reynolds numbers for the liquid and the vapor, respectively.

SUBSCRIPTS AND SUPERSCRIPTS

0—value at the initial temperature;

1—value in the evaporation region;

g—gas phase;

l—liquid phase;

n —vector component that is normal to the film;

s—film surface (interface);

w—wall.

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