



A composite channel hopping algorithm for blind rendezvous in heterogeneous cognitive radio networks

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Abstract

In cognitive radio networks (CRNs), rendezvous is the vital step prior to the communication between two unlicensed secondary users (SUs), where the SUs hop on the same channel at the same time to establish a link. With the dramatic fall in the cost and size of wireless transceivers, it becomes more reasonable to apply multiple radios to achieve significant improvement in the rendezvous performance. However, most of the existing multiradio rendezvous algorithms are proposed for homogeneous CRNs where all the SUs are equipped with an equal number of radios and do not possess backward compatibility to SU with a single radio. In reality, the CRNs are heterogeneous in nature as SUs may have different numbers of radios. In this paper, a composite CH algorithm is proposed for an asynchronous and heterogeneous network to achieve blind rendezvous with full rendezvous diversity. An SU with m number radios are categorized into three groups those follow different channel hopping (CH) algorithms. The upper bound of the rendezvous latency is being evaluated with a brief theoretical and mathematical analysis. Extensive simulations have conducted for different performance metrics, and the results are compared with the state-of-art algorithms. Overall, the proposed algorithm shows better performance in heterogeneous CRNs.

Keywords Cognitive radio network · Blind rendezvous · Multiple radios · Channel hopping · Difference sets

1 Introduction

With the exponential development of smart wireless technology and the explosive spread of wireless devices, the unlicensed spectrum has become overcrowded and a scarce resource. At the same time, a significant portion of the licensed spectrum remains underutilized. Cognitive radio (CR) has gained attention in the past decade for efficiently utilizing this underutilized licensed spectrum. The unlicensed or secondary users (SUs) in the cognitive radio networks (CRNs) opportunistically share the temporarily vacant spectrum of the primary users (PUs), also called the licensed users [1]. Each SU in the CRN is equipped with a cognitive transceiver, or CR, that is used to sense and access the idle channels of PUs for data transmission. However, before the data transmission between any pair of SUs, they must share control information and establish a link on the available channel, called the rendezvous of SUs. The conventional rendezvous is achieved using a centralized unit

or dedicated common control channels (CCCs), which have security issues and also face scalability problems [2, 3]. Hence, blind rendezvous is widely adopted by the CRNs, where SUs hop on their respective available channels following certain channel hopping sequences (CHSs) until they find the same channel at the same time. This channel rendezvous is blind, as the SUs do not have any prior information about others' presence and available channel sets. The design of the CH sequence that can achieve blind rendezvous also has the following challenges [4–9]:

- *Asynchronous clock* Clock synchronization is unrealistic and difficult to maintain for spatially scattered SUs in distributed CRNs. CHSs must handle asynchronous scenarios while ensuring rendezvous.
- *Heterogeneity in CRN* It is implicitly assumed that CRNs are homogeneous, where each SU is equipped with an equal number of CRs, and those can sense and access all the available channels in the network. Nevertheless, different SUs may have a different number of CRs, and the available channels may also differ because of different sensing capabilities. Hence, heterogeneous CRNs are needed and should be focused on.

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- *Symmetric CHS generation* Many rendezvous algorithms use asymmetric CHSs for a pair of SUs with the preassigned roles of transmitter and receiver. This method can offer the lowest rendezvous latency, but the prior knowledge or assignment of roles is unrealistic. In practice, symmetric CHSs are generated for each SU with symmetric roles.
- *Anonymous SUs' information* To achieve better rendezvous results, ID-based CH algorithms are designed with unique identifications of SUs, where the CHS length is the function of the ID string. However, with increasing numbers of SUs in the network, the rendezvous time would also increase with the increasing length of ID strings. Moreover, for security concerns, an ID independent rendezvous algorithm is preferred.

With the challenges discussed above, any randomly designed blind rendezvous algorithm cannot guarantee rendezvous. To achieve guaranteed and fast rendezvous within a finite time, the construction of CH sequences must focus on the degree and latency of rendezvous. Three primary performance metrics are *Average Time to Rendezvous (ATTR)*, *Maximum Time to Rendezvous (MTTR)*, and *rendezvous diversity*. When a pair of SUs start to hop with their CHSs, which might be asynchronous, the time required for the first rendezvous is called *time to rendezvous (TTR)*. The expected and maximum time required for the rendezvous under all possible clock drifts between the SUs are considered as ATTR and MTTR respectively. The upper bound of MTTR is denoted as *maximum conditional time to rendezvous (MCTTR)* in most of the literature, and it is the MTTR value at the worst condition when only one channel is common between sensed available channel sets of SUs. Rendezvous diversity is the minimum number of distinct channels on which two SUs can rendezvous. If they can rendezvous on all the channels, they achieve optimum rendezvous diversity. Hence, with full rendezvous diversity, a pair of SUs guarantee rendezvous even in the worst conditions.

Most of the rendezvous schemes in the literature assume that SUs are equipped with a single transceiver or radio [5, 6, 10–13]. The cost of transceivers has dropped dramatically in recent years [7, 14, 15], and multiradio rendezvous has become feasible to implement. In multiradio rendezvous, SUs can hop in parallelly on multiple channels at the same time, and the TTR can be shortened significantly. Some previous works [16–24] attempted to design multiradio rendezvous algorithms. However, many disadvantages are still present in these works that need to be explored, such as (i) All the SUs in the CRN are assumed to have an equal number of radios, which is unrealistic for heterogeneous networks. (ii) Each SU is implicitly assumed to be equipped with more than one radio. In other words, the designed algorithms are not backward compatible with single radio rendezvous. If at least

one SU is using one radio, rendezvous for a pair of SUs cannot be guaranteed. (iii) Even though some multiradio rendezvous algorithms possess backward compatibility, they have a very large MTTR for single radio rendezvous. (iv) With multiradios, a SU may hop on the same radio at the same time, which is just a waste of radio resources. Hence, by accessing different distinct channels with the radios at the same time, the possibility of rendezvous will increase in each time slot. (v) For some of the algorithms [16, 24], MTTR value is infinite as they cannot guarantee rendezvous within finite time.

CHS period describes the repeating pattern of the CHS. Studies show that it is faster to rendezvous for an SU pair with multiple radios within a short CHS period, which results in a shorter TTR value. However, if an SU only has one radio, the CHS period of the SU increases significantly, increasing TTR value and computational complexity. In this paper, the novelty lies in the use of a DDS-based CHS for a single radio as the period length can be reduced by using a disjoint difference sets with minimum elements. A composite channel hopping algorithm is introduced for heterogeneous radios, focusing on the disadvantages discussed above for guaranteed blind rendezvous. For the realistic network scenario, the SUs are considered with no pre-assigned roles, and the SUs have asynchronous local clocks, different numbers of radios, and asymmetric available channel sets. Internet of Things devices presently are constrained by their low memory and processing power, which restricts the type of computations they can perform. The limitation of resources may serve as the primary constraint that imposes restrictions on IoT devices. The suggested CHRR method would be more beneficial for device-to-device communication and IoT applications that can save time and memory in small sensors and mobile devices due to the shortest CHS period and rendezvous latency.

The contribution of this paper is summarized as follows.

- (1) A composite CHS is proposed for the two-user rendezvous, where the SUs may be equipped with a heterogeneous number of radios. For this, a DDS-based CH algorithm and a stay-jump based CH algorithm are proposed. The first one is used for single radio rendezvous, while the combined form of the two CHSs is used for multiradio rendezvous.
- (2) The upper bounds of MTTR are derived for the CHRR algorithm under all possible combinations of radios in a heterogeneous CRN.
- (3) The performance of the CHRR in terms of ATTR and MTTR is evaluated through extensive simulations and compared with other state-of-the-art multiradio and heterogeneous radio rendezvous algorithms.

The rest of this paper is organized as follows: In Sect. 2, existing multiradio rendezvous algorithms are briefly reviewed.

Section 3 introduces the system model and problem formulation of our work. Section 4 presents the proposed composite multiradio rendezvous algorithm. In Sect. 5, the theoretical analysis has been done for rendezvous diversity and the upper bound of MTTR for heterogeneous networks. Section 6 demonstrates the simulation and the comparison with other algorithms. Finally, the paper is concluded in Sect. 7.

2 Related works

The existing channel-hopping rendezvous algorithms are implicitly classified based on the number of radios that the SUs have. Most of the previous works have considered single radio-based rendezvous. Several researchers are concentrating on multiple radios-based CH algorithms since multiple radios have become more realistic in recent years. The CH algorithms can be further characterized based on the CRN type: homogeneous or heterogeneous. All the SUs in the homogeneous networks are equipped with an equal number of radios, whereas the number of radios is not the same in the heterogeneous networks. The design of a CH algorithm for heterogeneous networks is more challenging since it must be a multiradio rendezvous method with single radio rendezvous backward compatibility. In this section, a detailed review of recent multiradio rendezvous algorithms is presented.

The role-based parallel sequence (RPS) algorithm is proposed in [16] for multiradio rendezvous in homogeneous CRNs. The primary idea behind this article is to think of one radio as a dedicated radio and the rest as jump radios for an SU with m radios. The dedicated radio stays on a specific channel for a certain number of timeslots, while the $(m - 1)$ jump radios hop continuously on the available channels in a round robin manner. The ATTR and MTTR performance of the RPS algorithm is further improved in adjustable multiradio rendezvous (AMRR) [21] by adjusting the number of dedicated radios of an SU. If N_1 and N_2 be the number of available channels of SU_1 and SU_2 respectively in a CRN with N number of total available channels, then MTTR value of RPS and AMRR depends on N and (N_1, N_2) , respectively. This shows that, unlike RPS, AMRR takes heterogeneity into account. The general construction for rendezvous (GCR) in [17] divides the total radios of an SU into pairs and the available channels are distributed among them. Two-prime modular clock concept is utilized here for rendezvous. The disadvantage of this method is that an even number of radios is necessary. Moreover, bounded MTTR is guaranteed for RPS, AMRR and GCR only when the number of radios is greater than one, and MTTR tends to infinity for a single radio environment.

In [18], multiradio sunflower-set based rendezvous (MSS) algorithm is suggested, which is inspired by the single-radio sunflower-set based rendezvous (SSS) algorithm. For a sin-

gle radio, a periodic CH sequence is generated using the SSS algorithm. For multiradio, the SSS-generated CH sequence is assigned to the first radio, and for the remaining radios, CH sequences are generated sequentially by cyclically rotating the sequence of the previous radio by $2P$ timeslots. P is the smallest prime integer that is not less than three or the number of available SU channels. Though MSS can be applied to heterogeneous CRNs, the MTTR is high and increases significantly with the increase in the number of available channels. Further, two SUs cannot guarantee rendezvous when they start the rendezvous process simultaneously or when any SU has only one available channel. A Chinese remainder theorem-based multiradio rendezvous (CMR) is proposed in [25]. This algorithm guarantees rendezvous when at least two different prime numbers are used by an SU for the generation of multiradio CH sequences. If both SUs are equipped with a single radio, CMR cannot guarantee a rendezvous when both employ the same prime integer to generate their CH sequences. The adaptive rendezvous (AR) algorithm in [19] guarantees rendezvous with a high probability, but the number of radios in each SU is assumed to be equal, which makes it difficult to implement in heterogeneous CRNs.

In the algorithms [16–19], many radios of an SU may hop on the same channel at the same time, wasting radio resources. Hybrid radio rendezvous (HRR) algorithm, enhanced HRR(EHRR) algorithm, and modified enhanced heterogeneous radio rendezvous (MEHRR) algorithm in [22, 23, 26] respectively, ensure that different radios of an SU access different channels at a given time-slot. The three algorithms fully address the heterogeneity of the CRNs by designing two separate algorithms for SUs with a single radio and SUs with multiple radios. Here, rendezvous is also guaranteed when one SU is with one radio and the other is with multiple radios. However, when the two SUs are equipped with single radios, in the worst case, the upper bound of MTTR would be $5P^3$, $4P^3$, and $3P^3$ in HRR, EHRR, and MEHRR respectively. Despite the fact that these algorithms work extremely well for multiradio rendezvous, the MTTR values for single radio rendezvous scenario are notably very high. Here, P is same as refereed in other algorithms.

The computational complexities of these rendezvous algorithms are directly related to the period of the CH sequences and the upper bound of MTTR. In [16, 17, 21], faster rendezvous is guaranteed with a short CHS period, where the SUs are with multiple radios and hence, complexities are relatively lower as compared to others. In the case of heterogeneous radios, if one of the SUs is equipped with a single radio, the period length of the CH sequences for HRR, MEHRR, and CHRR would be $5P^2$, $3P^2$, and $2P^2 + \lfloor \frac{P}{2} \rfloor \times P$, respectively. However, despite the fact that the period lengths of HRR and MEHRR are $O(P^2)$, their corresponding MTTR with full rendezvous diversity is $O(P^3)$, as seen in Table 1. In contrast, CHRR achieves full ren-

Table 1 Performance comparison of multi-radio rendezvous schemes

Algorithms	Upper bound of MTTR	Heterogeneous multiple radios	Compatible to single radio	Complexity for heterogeneous radios
RPS [16]	$O(\frac{N^2}{\min(m_A, m_B)})$	Yes	No	$O(\frac{N^2}{\min(m_A, m_B)})$
AMRR [21]	$O(\frac{N_A N_B}{m_A m_B})$	Yes	No	$O(\frac{N_A N_B}{m_A m_B})$
GCR [17]	$O(\frac{N_A N_B}{m_A m_B})$	Yes	No	$O(\frac{N_A N_B}{m_A m_B})$
MSS [18] [†]	$O(\frac{\max(P_A^2 P_B, P_B^2 P_A)}{m_A m_B})$	Yes	Yes	$O(\frac{\max(P_A^2 P_B, P_B^2 P_A)}{m_A m_B})$
AR [19]	Unknown	No	No	–
DS-ACH [20]	1	No	No	–
HRR [22]	(i) $5P^{3*}$ (ii) $5P^2 + w_B^{**}$ (iii) $2\lfloor \frac{N_A - G}{m_{AS}} \rfloor w_A + 2w_B^\bullet$	Yes	Yes	$O(P^3)$
MEHRR [23]	(i) $3P^{3*}$ (ii) $3P^2 + w_B^{**}$ (iii) $\lfloor \frac{N_A}{m_{AS}} \rfloor w_A + w_B^\bullet$	Yes	Yes	$O(P^3)$
CHRR	(i) $2P^2 + \lfloor \frac{P}{2} \rfloor \times P^\ddagger$ (ii) $\lfloor \frac{N_A}{m_{AS}} \rfloor w_A + w_B^\diamond$	Yes	Yes	$O(P^2)$

N : Total number of channels in the CRN; P : Smallest prime greater than N ; N_A, N_B : Number of available channels of SU_A and SU_B , respectively; G : Number of commonly available channels between SU_A and SU_B ; m_A, m_B : Number of radios of SU_A and SU_B , respectively; m_{AS}, m_{BS} : Number of stay radios of SU_A and SU_B , respectively; P_A, P_B : Smallest primes not smaller than N_A and N_B , respectively

[†]Rendezvous is guaranteed only when both SUs do not start their CH process at the same time

* $m_A = 1, m_B = 1$

** $m_A = 1, m_B > 1$

• $m_A > 1, m_B > 1$

‡ $m_A = 1$ or $2, m_B \geq 1$

◊ $m_A > 2, m_B > 2$

dezdvous diversity within the period for which its complexity is $O(P^2)$ as compared to others.

Table 1 compares state-of-the-art multiradio rendezvous algorithms in terms of the upper bound of MTTR, complexity, heterogeneity, and compatibility with single radio rendezvous.

3 System model and problem formulation

3.1 System model

We consider a heterogeneous CRN with N non-overlapping licensed channels, where the channel set $C = \{0, 1, 2, 3, 4, \dots, N - 1\}$. A subset of C is said to be accessible to an SU if the user can communicate over these channels without interfering with the PUs. The SU_u is considered to be equipped with m_u radios, where $m_u \geq 1$. The set of available channels sensed idle by the SU_u is $C_u \subseteq C$ and $|C_u| = N_u$. The network divides time into timeslots of length $2t_s$, where t_s is the time necessary to create a link between two users by completing the beaconing and handshaking operations. According to IEEE 802.22 standard, $t_s = 10$ ms [27].

In the heterogeneous network, if SU is equipped with a single radio, i.e., $m = 1$, the radio is named as *main radio*. When $m = 2$ for an SU, first radio is termed as *main radio* with CHS same as that of $m = 1$, and the second radio termed as *stay radio*. For the SU with $m \geq 3$, the radios are divided into three groups, one *main radio*, k *stay radios*, and

$(m - 1) - k$ *jump radios*. The CHS generation algorithms for different groups of radios are discussed in Sect. 4.

The rendezvous is considered between a pair of SUs, SU_u and SU_v . Here, m_u and C_u may not be equal to m_v and C_v , respectively. The available common channels between SU_u and SU_v is denoted by the set $G_{uv} = C_u \cap C_v$. Rendezvous is possible between the users if $G_{uv} \neq \emptyset$. The CHS of SU_u is denoted as $S_u = \{S_u^1, S_u^2, S_u^3, \dots, S_u^{i_u}, \dots, S_u^{m_u}\}$, where $S_u^{i_u} = \{S_u^{i_u}(1), S_u^{i_u}(2), S_u^{i_u}(3), \dots, S_u^{i_u}(t), \dots, S_u^{i_u}(T)\}$ is the CHS of i th radio with a length of T timeslots. Hence, $S_u^{i_u}(t)$ represents the channel accessed by the i th radio of SU_u at t th timeslot. A clock drift of δ is considered between the CHSs of two SUs.

3.2 Problem formulation

The blind rendezvous problem for the heterogeneous CRN can be formulated as follows:

For a pair of SUs, SU_u and SU_v , design the channel access strategy for their radios at different timeslots $S_r^{i_r}(t) \in C_r$, where $r \in \{u, v\}$ and $\forall C_u, C_v, \delta$, and $\forall R \in G_{uv}, \exists t', i_u, i_v$ such that $S_u^{i_u}(t' + \delta) = S_v^{i_v}(t') = R$

Here, the CH process of SU_v starts δ timeslots later than that of SU_u , and the rendezvous is said to be achieved on channel R at t' th timeslot. The ATTR is the average times required for the first occurrence of rendezvous and can be represented as $ATTR = E(\min_{\forall R} t')$.

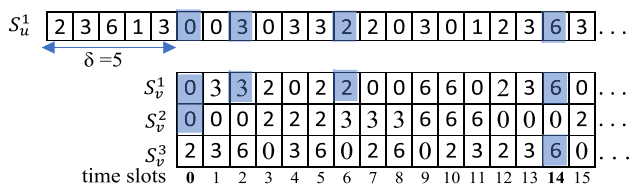


Fig. 1 Example of rendezvous process

Let $\Gamma(G_{uv}, \delta)$ is the minimum TTR on all the commonly available channels. Then MTTR with full rendezvous diversity can be represented as:

$$MTTR = \max_{\forall \delta} (\Gamma(G_{uv}, \delta))$$

In the worst-case scenario, where $G_{uv} = 1$, the MTTR would have its upper bound. Short MTTR represents two SUs can rendezvous quickly even at the worst condition, and short ATTR means two SUs can rendezvous quickly on average. Hence, both are significant metrics for the design of a CH rendezvous algorithm.

In Fig. 1, a rendezvous process is illustrated between SU_u and SU_v with radios $m_u = 1$ and $m_v = 3$ respectively. Here, $C = \{0, 1, 2, 3, 4, 5, 6\}$, $C_u = \{0, 1, 2, 3, 6\}$, $C_v = \{0, 2, 3, 6\}$, and $G_{uv} = \{0, 2, 3, 6\}$. The CH process of SU_v starts after $\delta = 5$ timeslots. As TTR is calculated from the instant when both the users start their CH, local clock of SU_v is considered for the TTR calculation. The rendezvous at channels $\{0, 2, 3, 6\}$ occurred for the first time at 1^{st} , 3^{rd} , 7^{th} , and 15^{th} timeslots. Hence, $\min_{\forall R} t'$ or the minimum $TTR = 1$, and $\Gamma(G_{uv}, 5) = 15$.

4 Composite CH-based heterogeneous radio rendezvous algorithm

To achieve fast rendezvous in heterogeneous CRN, a DDS-based CH algorithm is proposed for SU with a single radio. For SU with multiple radios, a composite CH algorithm is introduced, where the DDS-based CH algorithm is used for the generation of the CHS of its first radio and a Stay-Jump CH algorithm is used for the CHSs of the remaining radios.

4.1 DDS-based channel hopping algorithm

4.1.1 Disjoint difference set

Definition 1 A set $D = \{a_0, a_1, a_2, \dots, a_{k-1}\} \subset Z_n$ is referred as a relaxed cyclic difference set or simply (n, k) -DS, if an order pair (a_i, a_j) exists $\forall d \in (0, n)$ such that $(a_i - a_j) \equiv d \pmod n$. Here, $a_i, a_j \in D$ and Z_n is the set of positive integers denoted as $\{0, 1, 2, \dots, n - 1\}$.

Corollary 1 If D is a (n, k) -DS, under Z_n then $rotate(D, i)$ is likewise a (n, k) -DS, where $i \in (0, n)$ and $rotate(D, i) = \{a_0 + i, a_1 + i, \dots, a_{k-1} + i\} \pmod n$,

Proof The corollary has been proved in our previous work [28]. As an illustration, if $D = \{1, 4, 5\} \subset Z_6$ is a DS, then $rotate(D, 3) = \{4, 1, 2\}$ is also a DS.

Definition 2 A set $Q = \{A_0, A_1 \dots A_{n-1}\}$ is referred as a cyclic quorum system (CQS), if each set $A_i \subseteq Z_n$ and Q has the following two properties for all $i, j \in [0, n - 1]$:

- i) $A_i = rotate(A_0, i)$,
- ii) $A_i \cap A_j \neq \emptyset$

Corollary 2 A CQS under Z_n can be formed with a group of n sets $\{A_0, A_1, \dots, A_{n-1}\}$, if $A_i = \{(a_0 + i) \pmod n, (a_2 + i) \pmod n, \dots, (a_{k-1} + i) \pmod n\} \forall i \in [0, n)$ and set $D = \{a_0, a_1, \dots, a_{k-1}\}$ is a (n, k) -DS.

Proof The corollary has been proved in our previous work [28].

Definition 3 A set $S = \{D_0, D_1, \dots, D_{m-1}\}$ denotes a group of disjoint difference sets (DDS) under Z_n , where D_i is a DS and $D_i \cap D_j = \emptyset, \forall i, j \in [0, m - 1]$.

Corollary 3 If $S = \{D_0, D_1, \dots, D_{m-1}\}$ is a DDS with $D_i \subset Z_n$, then the set $rotate(S, i)$ is likewise a DDS, where $i \in (0, n)$ and $rotate(S, i) = \{rotate(D_0, i), rotate(D_1, i), \dots, rotate(D_{m-1}, i)\}$.

Proof The corollary has been proved in our previous work [28].

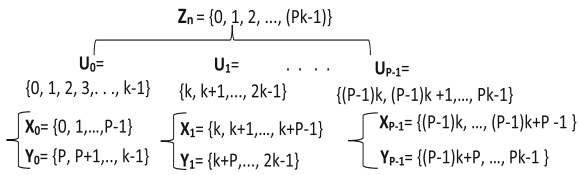
Corollary 4 If $S = \{D_0, D_1, \dots, D_{m-1}\}$ is a DDS with $D_i \subset Z_n$, then S can be used to create a set of m number of CQS.

Proof The corollary has been proved in our previous work [28].

4.1.2 Construction of DDS

An algorithm is established for the construction of a DDS under Z_n . In this paper, $n = Pk$, where P is the smallest prime number greater than equal to the total number of licensed channels in the CRN. k is the average number of elements per DS. The set $S = \{D_0, D_1, D_2, \dots, D_{P-1}\}$ is constructed with P disjoint DSs, where each DS is of size k and $k = 2P + \lfloor \frac{P}{2} \rfloor$.

To start building a DDS under Z_n , the Z_n is partitioned into P distinct sets of size k and is designated as U_0, U_1, \dots, U_{P-1} . This procedure is depicted in Fig. 2. Here, $U_l = \{lk, lk+1, \dots, (lk+k-1)\}, l \in [0, P-1]$. Each U_l is further split into two subsets, X_l and Y_l . The subsets X_l



$$D_i = D_{xi} \cup D_{yi} = \{i, k+i, 2k+i, \dots, (P-1)k+i, k+(T_0+i)+P, k+(T_1+i)+P+1, \dots, k+(T_{k-N-1+i})+P+(k-P-1)\}$$

Fig. 2 Construction of the i th difference set D_i under $Z_n, \forall i \in [0, P - 1]$

and Y_l each have P and $(k - P)$ number of elements, respectively. Now, the i th DS of size k is constructed with the first P elements selected from $\{X_0, X_1, \dots, X_{P-1}\}$ represented with a set, D_{X_i} and the rest $(k - P)$ elements are selected from $\{Y_0, Y_1, \dots, Y_{P-1}\}$ by using the concept of triangular number. The selected $(k - P)$ elements are presented with the set, D_{Y_i} . The term T_j , used in Fig. 2 is a triangular number and is defined as $\frac{j(j+1)}{2}$ for any non negative integer j . Therefore, D_i is given by

$$D_i = D_{X_i} \cup D_{Y_i},$$

where

$$D_{X_i} = \{x_{ip}\}, x_{ip} = kp + i,$$

$$D_{Y_i} = \{y_{iq}\}, y_{iq} = k(T_q + i) + P + q,$$

$$T_q = \frac{q(q+1)}{2} \bmod P, 0 \leq p \leq P - 1 \text{ and } 0 \leq q \leq k - P - 1$$

Algorithm 1 Construction of Disjoint Difference Sets

```

Require: P
Ensure: DDS  $S = \{D_0, D_1, D_2, \dots, D_{P-1}\}$ 
1:  $S = \emptyset$ 
2:  $k = 2P + \lfloor \frac{P}{2} \rfloor$ 
3: for  $i=0$  to  $(P - 1)$  do
4:   for  $p=0$  to  $(P - 1)$  do
5:      $x_{ip} = k \times p + i$ 
6:      $D_{X_i} = \{x_{ip}\}$ 
7:   end for
8:   for  $q = 0$  to  $(k - P) - 1$  do
9:      $v = \frac{q \times (q+1)}{2} \bmod P + i$ 
10:     $y_{iq} = k \times v + P + q$ 
11:     $D_{Y_i} = \{y_{iq}\}$ 
12:   end for
13:    $D_i = D_{X_i} \cup D_{Y_i}$ 
14:    $S = S \cup \{D_i\}$ 
15: end for
    
```

In Algorithm 1, the construction of D_{X_i} and D_{Y_i} are mentioned respectively in lines 4 ~ 6 and lines 8 ~ 11. The i th difference set, D_i is constructed in line 13 by combining D_{X_i} and D_{Y_i} and the final set S , obtained in line 14 is a *DDS*. The proof for the constructed D_i 's being the disjoint difference sets is given in the ‘‘Appendix’’.

4.1.3 CH sequence generation based on DDS for the rendezvous

As stated in Algorithm 2, a DDS-based CH sequence is generated for SU_u . If $C = \{0, 1, 2, \dots, P - 1\}$ is the set of

licensed channels in the network, and C_u is the available channel set of SU_u then the DDS S has P number of disjoint sets $D_i, i \in C$. These DSs are used to construct CH sequence for the first radio with time period of n time slots as shown in lines 6 ~ 13.

Algorithm 2 Generation of DDS-based CH sequence

```

Require:  $C, C_v \setminus \setminus$  for  $SU_v, C =$  licensed channels set
Ensure:  $S_v^1$ 
1:  $P =$  smallest prime number  $\geq |C|$ 
2: Invoke Alg. 1 to construct a DDS  $S$  with  $P$ 
3:  $n =$  Total elements in DDS,  $S$ 
4:  $x =$  a random number in  $[0, n - 1]$ 
5:  $S =$  rotate  $(S, x)$ 
6: for  $t = 0$  to  $n - 1$  do
7:   find  $D_i \in S$  such that  $t \in D_i$ 
8:   if  $i \in C_v$  then
9:      $CH_v[t] = i \setminus \setminus i$  is accessed in slot  $t$ 
10:  else
11:     $CH_v[t] =$  Random channel  $\in C_v$ 
12:  end if
13:   $S_v^1 = CH_v$ 
14: end for
    
```

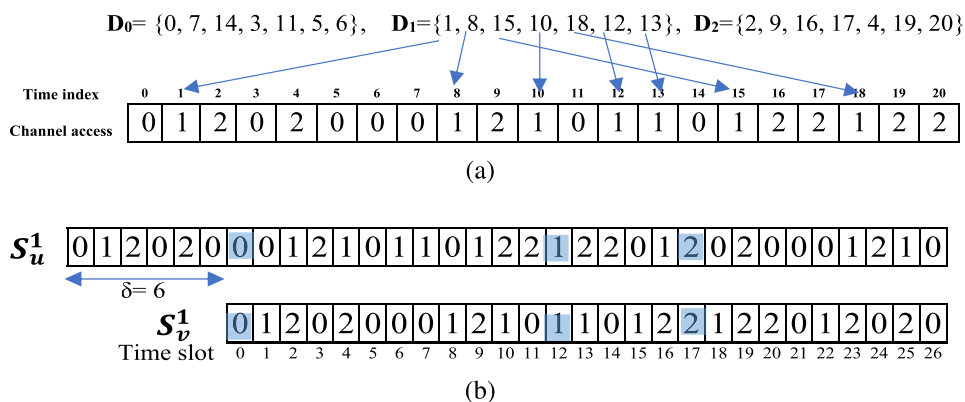
Example A CRN network is considered with the licensed channel set $C = \{0, 1, 2\}$. $P = 3$ for the given C . The SUs SU_u and SU_v with $C = C_u = C_v$ generate their CH sequences using Algorithm 1 and 2. Figure 3 illustrates the relevant CH sequences and the rendezvous between them in an asynchronous environment.

4.2 Stay-jump channel hopping algorithm

When an SU is equipped with multiple radios, and $|C_v| \leq m_v$, each radio by default will act as a stay radio and access the same channel at every timeslots as shown in Fig. 4b. However, for $|C_v| > m_v$, the CHS for the first radio is generated based on DDS-based CH algorithm, and CHSs for the remaining radios are generated using *Jump-Stay CH* algorithm. One period of CHSs generated by the latter algorithm is consist of η number of inner periods, each of length τ_v . Based on the number of stay radios (m_{vs}) and the jump radios (m_{vj}), length of the periodic CHSs is $T = \eta \times \tau_v$, where $\eta = \frac{LCM(C_v, m_{vs})}{m_{vs}}$ and $\tau_v = \lceil \frac{C_v - m_{vs}}{m_{vj}} \rceil$ as shown in Fig. 4a, and the channel accessed by the radios in each time slot is derived in Algorithm 3.

For each inner period, the set C_v is divided into distinct disjoint sets, C_v^S and C_v^J , where $|C_v^S| = m_{vs}$. Throughout an inner period, each stay channel stays on a particular channel, $c \in C_v^S$, and each jump radio hops to a different channel, $c \in C_v^J$ at different timeslots. In line 8, C_v^S and C_v^J are initialized to null sets for each inner period. In lines 11 ~ 12, $|C_v^S| = m_{vs}$ distinct channels from C_v are assigned to the stay radios and the selected channels are taken into C_v^S

Fig. 3 Rendezvous process using DDS-based CH sequences. **a** Illustration for channel access at different time slots using DDS S , Disjoint difference sets when $P = 3$, **b** Rendezvous between two SUs while using DDS-based CHSs with 6 time slots drift



Algorithm 3 Generation of Stay-Jump CH sequence

Require: $C_v, m_v^*, m_{vs}, m_{vj} \setminus \setminus$ for SU_v , and $m_v^* = m_v - 1$
Ensure: $S_v^* = \{S_v^2, S_v^3, \dots, S_v^{m_v^*}\}$

- 1: $\eta = \frac{LCM(C_v, m_{vs})}{m_{vs}}$
- 2: **if** $m_{vj} = 0$ **then**
- 3: $\tau = C_v$
- 4: **else**
- 5: $\tau = \lceil \frac{C_v - m_{vs}}{m_{vj}} \rceil$
- 6: **end if**
- 7: **for** $t = 1$ to η **do**
- 8: $C_v^S = \emptyset, C_v^J = \emptyset$
- 9: **for** $q = 1$ to τ **do**
- 10: **for** $r = 1$ to m_{vs} **do**
- 11: $i_1 = ((t - 1) * m_{vs} + (r - 1)) \bmod |C_v| + 1$
- 12: $S^r = C_v(i_1)$
- 13: $C_v^S = C_v^S \cup C_v(i_1)$
- 14: **end for**
- 15: $C_v^J = C_v \setminus C_v^S$
- 16: $z = (i_1 - m_{vs}) \bmod |C_v| + 1$
- 17: **if** $z > |C_v|$ **then**
- 18: **for** $r = m_{vs} + 1$ to m_v^* **do**
- 19: $i_2 = (r - (m_{vs} + 1) + (q - 1) * m_{vj}) \bmod |C_v^J| + 1$
- 20: $S^r = C_v^J(i_2)$
- 21: **end for**
- 22: **else**
- 23: **for** $r = m_{vs} + 1$ to m_v^* **do**
- 24: $i_2 = ((i_1 - 1) + r - (m_{vs} + 1) + (q - 1) * m_{vj}) \bmod |C_v^J| + 1$
- 25: $S^r = C_v^J(i_2)$
- 26: **end for**
- 27: **end if**
- 28: **end for**
- 29: $S_v^* = S^r$
- 30: **end for**

in line 13. C_v^J is derived in line 15, and in lines 17 ~ 25, the jump radios hop on different channels of C_v^J during the inner period in such a way that their accessed channels are different at one time slot. Finally, the required one period of the CHSs of all the $(m_v - 1)$ radios is generated in line 29.

Algorithm 4 CHRR Rendezvous Algorithm

Require: $C, C_v, m_v, m_{vs}, m_{vj} \setminus \setminus$ for SU_v
Ensure: $S_v = \{S_v^1, S_v^2, S_v^3, \dots, S_v^{m_v}\}$

- 1: $m_v^* = m_v - 1$
- 2: **if** $m_v = 1$ **then**
- 3: Invoke Algorithm 2 to generate CHS,
 $S_v = \text{Algorithm 2}(C, C_v)$
- 4: **end if**
- 5: **if** $m_v \geq |C_v|$ **then**
- 6: **for** $r = 1$ to m_v **do**
- 7: $i = r \bmod m_v$
- 8: $S^r = C_v(i)$
- 9: **end for**
- 10: $S_v = S^r$
- 11: **end if**
- 12: **if** $m_v < |C_v|$ **then**
- 13: Invoke Algorithm 2 to generate CHS of first radio,
 $S_{v1} = \text{Algorithm 2}(C, C_v)$
- 14: Invoke Algorithm 3 to generate CHS for the rest $m_v - 1$ radios,
 $S_{v2} = \text{Algorithm 3}(C_v, m_v^*, m_{vs}, m_{vj})$
- 15: $S_v = \{S_{v1}, S_{v2}\}$
- 16: **end if**
- 17: Attempt rendezvous using S_v

4.3 CHRR algorithm

Lastly, as previously described, the composite-CH based heterogeneous radio rendezvous (CHRR) method is constructed in Algorithm 4 to achieve full rendezvous diversity. The SUs continue to access channels using their respective created CHSs using Algorithm 4 and accomplish rendezvous regardless of their asynchronous clocks.

An example is discussed in Fig. 4 to make the algorithms easier to follow. Here, the licensed channel set $C = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ and two secondary users, say SU_u and SU_v are considered which are equipped with multiple radios of m_u and m_v respectively. The available channel sets of SU_u and SU_v are $C_u = \{0, 1, 3, 4, 5, 7, 8\}$ and $C_v = \{3, 4, 6\}$ respectively. Since $m_v > |C_v|$, each radio of SU_v acts as a stay radio, and the channels in C_v are assigned to each radio using the lines 6 ~ 10 of Algorithm 4. The generated CHS is shown in Fig. 4b. As $m_u < |C_u|$ for SU_u ,

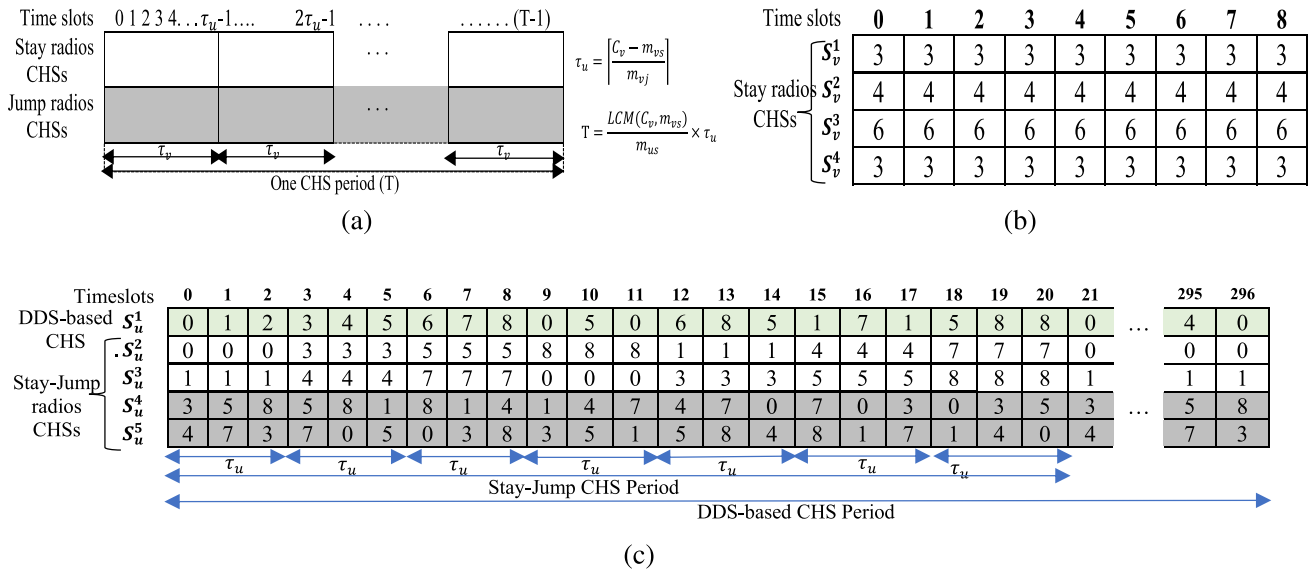


Fig. 4 Generation of the CHS for an SU having multiple radios. **a** The structure of one period of the CHSs generated by Stay-Jump CH Algorithm. **b** An example of of CHS generated by CHRR Algorithm when

$m_v > |C_v|$ for SU_v . **(c)** An example of of CHS generated by CHRR Algorithm when $m_u < |C_u|$ for SU_u

the CHS of each radio is generated using the lines 12 ~ 15 of Algorithm 4. In this case, a DDS-based CHS with the period of $2P^2 + \lfloor \frac{P}{2} \rfloor \times P = 297$ timeslots is generated using Algorithm 2 for the first radio, where $P = 11$ as $|C| = 9$. For the rest 4 radios Stay-Jump CHS is generated using Algorithm 3 with $m_{us} = 2$ and $m_{uj} = 2$. Here, the length of each inner period is $\tau_u = \lceil \frac{|C_u| - m_{us}}{m_{uj}} \rceil = \lceil \frac{7-2}{2} \rceil = \lceil \frac{5}{2} \rceil = 3$. The length of one outer period of the Stay-Jump CHS = $\frac{LCM(|C_u|, m_{us})}{m_{us}} \times \tau_u = \frac{LCM(7, 2)}{2} \times 3 = 21$ timeslots. For the first inner period, $C_u^S = \{0, 1\}$ and $C_u^J = \{3, 4, 5, 7, 8\}$. Each stay radio stays on a distinct channel of C_u^S during the inner period using lines 10 ~ 13 of Algorithm 3. Then channels of $\{3, 4, 5, 7, 8\}$ are assigned to the two jump radios sequentially using lines 16 ~ 27 of Algorithm 3. Similarly, for the next inner period the channels are assigned to the radios of SU_u with $C_u^S = \{3, 4\}$ and $C_u^J = \{0, 1, 5, 7, 8\}$.

5 Performance analysis

In this section, the theoretical analysis of the MTTR with optimal rendezvous diversity is obtained in heterogeneous CRNs, and the upper bound of the TTR is derived for the worst-case scenario. For a better understanding, Lemma 2 is presented first, and the performance is analyzed for the rendezvous between SU_u and SU_v under two scenarios:

- (i) $m_u \leq 2, m_v \geq 1$, ii) $m_u > 2, m_v > 2$.

For Lemma 2, some notations are used in the structure of two adjacent inner periods of the Stay-Jump CHS and is

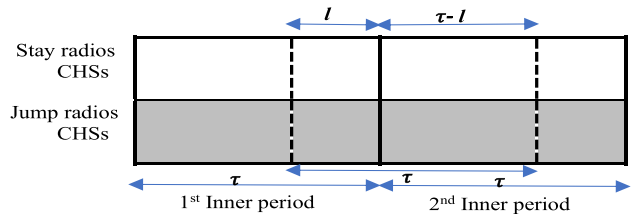


Fig. 5 Rendezvous between SU_u and SU_v when $m_u > 2, m_v > 2$

shown in Fig. 5. To analyze any contiguous τ time slots in the CHS, two adjacent inner periods are considered in Fig. 5, where l number of time slots fall in the front inner period and $(\tau - l)$ timeslots fall in the subsequent inner period. The channel sets covered by the Stay radios and Jump radios in the front l time slots are denoted as C_{fS} and C_{fJ} . Similarly C_{bS} and C_{bJ} are the notations used for the back $(\tau - l)$ timeslots. Here, τ is the length of one inner period and $l \in [1, \tau - 1]$.

Lemma 2 In the Stay-Jump CHS of an SU, all the available channels of the SU are visited at least once during any contiguous τ timeslots, where τ is the length of one inner period.

Proof According to the Algorithm 3, the total distinct channels visited by the radios of the SU in one inner period are $|C|$, where C is the available channel set of SU. It is cleared from the Algorithm that $|C|$ is equivalent to $(S + J(\tau - 1) + (|C| - S - 1) \bmod J + 1)$. Hence, all the channels of set C with indexes $\{0, 1, 2, 3, \dots, (|C| - 1)\}$ are assigned to stay and jump radios of SU in one inner period and the channel assignment to different radios varies in different inner periods. In the Fig. 5, let i is the index of the channel

slot index	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
CHS 0	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2
CHS 1	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2
CHS 2	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1
CHS 3	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2
CHS 4	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2
CHS 5	2	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1
CHS 6	1	2	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1	0
CHS 7	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1	1
CHS 8	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0	1
CHS 9	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1	0
CHS 10	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0	1	2	1
CHS 11	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0	0	1
CHS 12	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0	1
CHS 13	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0	0
CHS 14	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0	0
CHS 15	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2	0
CHS 16	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0	2
CHS 17	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2	0
CHS 18	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1	2
CHS 19	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0	1
CHS 20	1	2	0	2	0	0	0	1	2	1	0	1	1	0	1	2	2	1	2	2	0

Fig. 6 All possible CHSs constructed with Algorithm 2 with the DDS S as mentioned in Fig. 4a for available channel set, {0, 1, 2}

assigned to the first stay radio of the front inner period. Then according to Algorithm 3, the indexes of the channels in the following sets would be,

- (1) for set $C_{fS} \Rightarrow i$ to $(i + (S - 1)) \bmod |C|$
- (2) for set $C_{fJ} \Rightarrow (i + S + J(\tau - l)) \bmod |C|$ to $(i + S + J(\tau - 1)) \bmod |C|$
- (3) for set $C_{bS} \Rightarrow (i + S) \bmod |C|$ to $(i + (2S - 1)) \bmod |C|$
- (4) for set $C_{bJ} \Rightarrow (i + 2S) \bmod |C|$ to $(i + S + J(\tau - l) - 1) \bmod |C|$

For any value of l and i , the channel indexes covered by the set $C_{fS} \cup C_{fJ} \cup C_{bS} \cup C_{bJ}$ will be $\{i, (i + 1) \bmod |C|, \dots, (i + S + J(\tau - 1) + (|C| - S - 1)) \bmod |C|\}$ which is equivalent to $\{i, (i + 1) \bmod |C|, \dots, (i + (|C| - 1)) \bmod |C|\}$ or $\{0, 1, \dots, (|C| - 1)\}$. Hence, it is proved that during any τ timeslots in the CHS, all the channels in C are visited at least once.

Theorem 2 The MTTR of the proposed composite algorithm is upper bounded by $2P^2 + \lfloor \frac{P}{2} \rfloor \times P$ time slots when $m_u \leq 2$, $m_v \geq 1$ with full rendezvous diversity.

Proof At least one radio of every SU follows DDS-based CHS. The DDS S is constructed in Algorithm 1, and by using $rotate(S, x)$, $\forall x \in [0, n - 1]$ in Algorithm 2, n distinct CH sequence can be generated that the SUs can employ. Let two SUs generate their CH sequences using $rotate(S, x)$ and $rotate(S, y)$ respectively, where $x, y \in [0, n - 1]$. Let the second SU's CH sequence be d time slots behind the first SU's CH sequence in an asynchronous environment. The two non-synchronized CH sequences are equivalent to the two synchronized CH sequences generated by $rotate(S, x)$ and $rotate(S, y + d \bmod n)$. These two synchronized CH

sequences can be simply examined using the example in Fig. 6. According to Algorithm 2, the n number of (n, k) -DSs $rotate(D_i, 0), rotate(D_i, 1), \dots, rotate(D_i, n - 1)$ are assigned to channel i , and in accordance with corollary 4, the n DSs would form a CSQ under $Z_n, \forall i \in [0, P - 1]$. Now the two CH sequences generated using $rotate(S, x)$ and $rotate(S, y)$ will rendezvous on the channel i at the timeslot $t \in rotate(D_i, x) \cap rotate(D_i, y + d \bmod n), \forall i \in [0, P - 1]$ within one time period of CH sequence.

The proposed rendezvous algorithm ensures rendezvous during the CH period of n timeslots on all N licensed channels. In this paper, n is the total number of elements of DDS S , which is $2P^2 + \lfloor \frac{P}{2} \rfloor \times P$. As the algorithm has N degree of rendezvous, in the worst-case scenario, when there is just one channel common between the available channel sets of two SUs, rendezvous takes place on that channel during a CHS period. Hence, $MCTTR = n = 2P^2 + \lfloor \frac{P}{2} \rfloor \times P$.

Theorem 3 For $m_u > 2, m_v > 2$, the MTTR of the proposed composite algorithm with full rendezvous diversity is upper bounded by $\tau_u \times \lceil \frac{C_u}{m_{us}} \rceil + \tau_v$ time slots when $\tau_v < \tau_u$ or $\tau_v = \tau_u$ and $\lceil \frac{C_u}{m_{us}} \rceil \leq \lceil \frac{C_v}{m_{vs}} \rceil$.

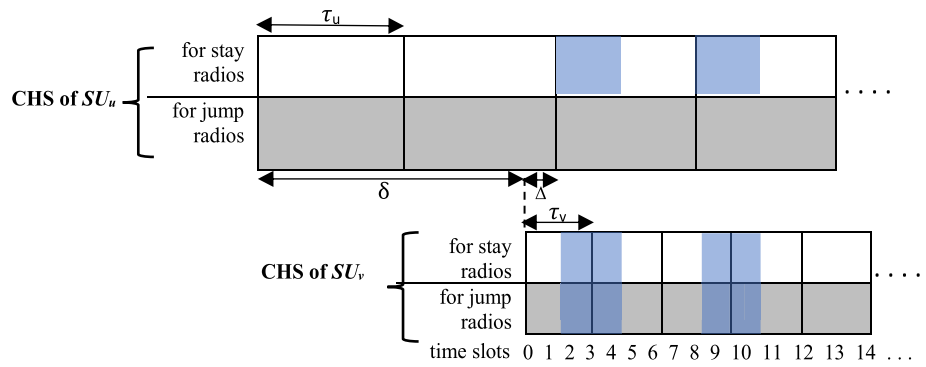
Proof The inner period length of SU_v is smaller than SU_u as shown in the Fig. 7. All the available channels of SU_v are visited during any continuous τ_v time slots as proved in Lemma 2, and a constant channel is accessed by the stay radio of SU_u during one inner period. Hence, rendezvous is guaranteed on the stay channel during the τ_v timeslots if it is a common channel between the SUs. Similar manner rendezvous occurs on all the commonly available channels during the stay periods of SU_u as the stay channels are changed in different stay periods. The possibilities of rendezvous are shown by the blue-shaded regions in the figure. With $\lceil \frac{C_v}{m_{vs}} \rceil$ numbers of inner periods containing all the distinct channels, TTR for full rendezvous diversity will be $\lceil \frac{C_v}{m_{vs}} \rceil \times \tau_u$. For some of the cases, the starting overlapping timeslots between the SUs are not enough to achieve rendezvous as shown in Fig. 7 with duration Δ . Hence the MTTR is upper bounded by $\tau_u \times \lceil \frac{C_v}{m_{vs}} \rceil + \tau_v$.

For $\tau_v = \tau_u$ if $\lceil \frac{C_u}{m_{us}} \rceil \leq \lceil \frac{C_v}{m_{vs}} \rceil$ all the available channels of SU_v will be accessed by the stay radios within $\tau_v \times \lceil \frac{C_v}{m_{vs}} \rceil$ time slots. For SU_u , all the available channels are visited during any τ_u time slots. Hence, the upper bound of MTTR to achieve rendezvous on all commonly available channels will be $\tau_u \times \lceil \frac{C_v}{m_{vs}} \rceil + \tau_v$.

6 Performance evaluation and simulation results

In this section, the performance of the proposed CHRR algorithm is evaluated. Extensive simulations are conducted using

Fig. 7 Rendezvous between SU_u and SU_v when $m_u > 2$, $m_v > 2$



Matlab, and the correctness of the theoretical analysis is verified with the results. A CRN with N licensed channels denoted with channel set C is taken into account. In the network, the SU pair trying to rendezvous are denoted as SU_u and SU_v and they are equipped with m_u and m_v radios, respectively. For the heterogeneous CRN, different possibilities of m_u and m_v pair are considered for the simulations and the SUs may have different available channel sets denoted as C_u and C_v respectively, where $C_u, C_v \subseteq C$. The SUs achieve rendezvous on channels $c \in C_u \cap C_v$ by using their respective CHSs. Again, all the simulations are conducted in an asynchronous environment by considering random time-slots drift between the CHSs of the SUs. Thus, for a heterogeneous and asynchronous CRN, the rendezvous performance of the proposed CHRR scheme is observed under a different number of radios. In the multi radio rendezvous case, the effect of allocated jump radios and stay radios on the TTR value is evaluated. Moreover, the performance of CHRR is compared with other state-of-the-art multi-radio CH algorithms in terms of ATTR and MTTR with full rendezvous diversity.

6.1 Performance under heterogeneous numbers of radios

The performance of CHRR is evaluated for different combinations of radios (m_u, m_v) . The simulations are conducted by varying the total licensed channel N from 10 to 50. The number of available channels for each SU is set as $0.7N$ and while considering the asymmetric model, the number of common channels between the SUs is set at $0.5N$. The simulated ATTR and MTTR values of the CHRR are shown in Fig. 8a,b, respectively for the radio combinations of (1, 1), (1, 3), (1, 5), (2, 2), (2, 3), (3, 3). For the multiple radios case, the number of stay radios of the SU_u and SU_v is set as $m_{us} = \lfloor \frac{m_u}{2} \rfloor$ and $m_{vs} = \lfloor \frac{m_v}{2} \rfloor$, respectively. The correctness of the theoretical analysis is verified with the simulated results, as the values shown in the graphs are less than the derived upper bound of the MTTR in the previous section. The ATTR and MTTR values increase as the value of N increases. The reason is that ATTR and MTTR are directly

proportional to the length of one period of the CHS, and the length of the CHS of any SU is directly proportional to the number of licensed channels and the number of available channels to that SU.

Likewise, for a given N , the ATTR and MTTR for the SU pair are relatively shorter when at least one is equipped with a single radio than when both are equipped with a single radio. If we consider three distinct cases, the *first* case, where both SUs are equipped with single radios, the rendezvous is performed with the DDS-based CHSs of both users, and the TTR values are large since the CHS length is $O(N^2)$. In the *second* case, when one SU has a single radio and the other has multiple radios, guaranteed rendezvous happens owing to the DDS-based CHSs of both SUs. However, there is a possibility of a rendezvous between the first SU's DDS-based CHS and the other SU's stay radios. In the *third* case, if both SUs are equipped with multiple radios, the stay radios of one SU have a high chance of colliding with the radios of the other SU. Apart from these, both SUs also achieved rendezvous with their first radio using DDS-based CHSs. Furthermore, as the value of m_u and m_v increases in the third case, the period length of stay-jump CHSs decreases significantly, shortening the ATTR and MTTR. The correctness of the rendezvous performance for these three cases is reflected in Fig. 8a,b.

6.2 Performance with different allocation of radios

The influence of varied radio allocation on the performance of the CHRR is investigated in this subsection. The notation (m_{us}, m_{vs}) denotes the number of stay radios for SU_u and SU_v , and the number of jump radios may be determined using the provided notation. As a result, the notation (m_{us}, m_{vs}) might be interpreted as different radio allocations. In the simulation, the total number of radios in SU_u and SU_v is set to 5 and 5, respectively. We have six distinct allocation choices here, marked as (1, 1), (1, 2), (1, 3), (2, 2), (2, 3), and (3, 3).

The performance of all the six types of allocations is compared in Fig. 9a, b for the different numbers of licensed channels varying from 10 to 50. It is observed from the graphs

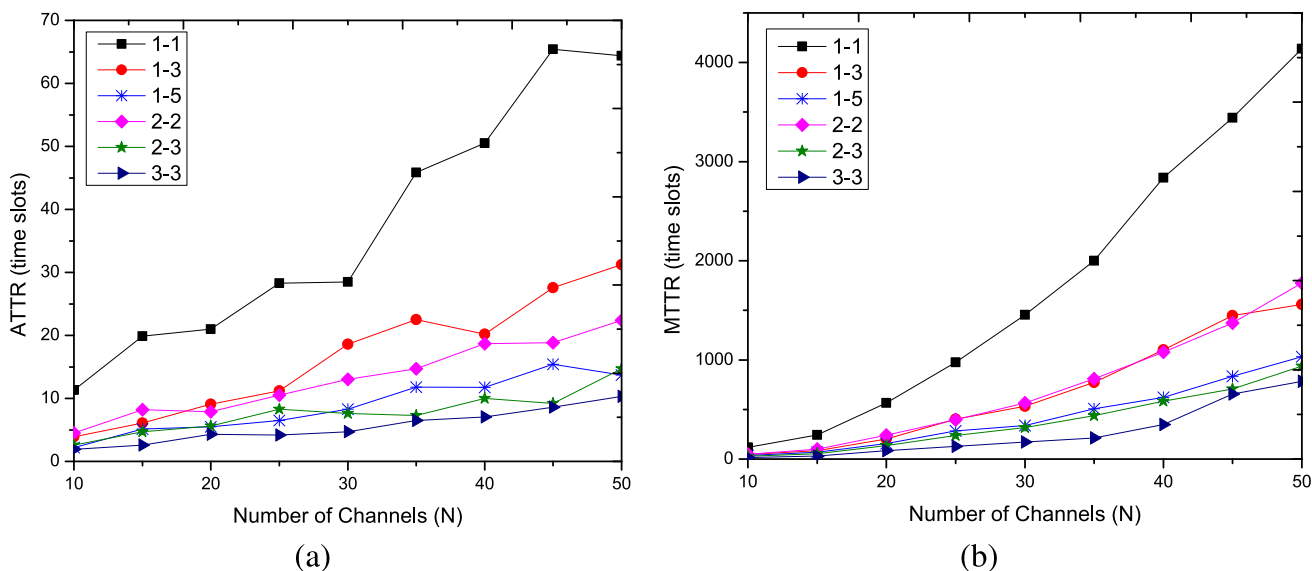


Fig. 8 Performance comparison for different numbers of radios

that the ATTR and MTTR are comparatively higher when one of the SUs is equipped with the maximum possible stay radii or the minimum possible jump radii. Here, the radio allocations (3, 3), (2, 3) and (1, 3) are showing higher values, while the allocations (1, 1), (1, 2), and (2, 2) are showing shorter TTR values. The reason is that the increase in jump radii leads to a decrease in the length of the CHS period, and MTTR is directly related to the period length. For a symmetrical model, the available channels set for all SUs are the same. Hence, the (1, 1) radio allocation can be used to achieve minimum TTR. However, the SUs have a different set of available channels in an asymmetric model. Though, the (3, 3) and (1, 1) radio allocations in Fig. 9a show higher and shorter ATTR, respectively, the same allocations may not have the maximum and minimum MTTR respectively, as shown in Fig. 9b.

6.3 Comparison with other algorithms

In this section, the proposed CHRR algorithm’s performance is compared to that of other current blind rendezvous methods. When one of the SU pairs is equipped with a single radio, the heterogeneous radio rendezvous techniques, the MEHRR algorithm [23], and the HRR algorithm [22] are evaluated for comparison. However, when both the SUs are equipped with multiple radios, in addition to [22, 23], the RPS algorithm [16] and the AMRR algorithm [21] are compared with the proposed algorithm. An asymmetric model is considered, where the number of available channels of each SU and the number of commonly available channels between any two SUs are set at $0.7N$ and $0.5N$, respectively. The simulation

results are obtained for various N values ranging from 10 to 50.

Figure 10a–f show the comparisons of MTTR with full rendezvous diversity for the (m_u, m_v) combinations (1, 1), (1, 3), (1, 5). When both SUs have a single radio, the MTTR of HRR and MEHRR are much higher than CHRR, as illustrated in Fig. 10a). The reason for this is that the CHSs of HRR and MEHRR for a single radio rendezvous have a longer period duration than CHRR, and the length of the CHS period grows significantly for HRR and MEHRR as N increases. Under the worst-case scenario of an asymmetric model, the upper bound of the MTTR is related to the CHS period, as shown in Table 1. By adopting DDS-based CHS for single radio rendezvous, the MTTR of CHRR is considerably lowered. In Fig. 10b, c, it is seen that when one of the SUs has multiple radios, the MTTR values of the algorithms are decreased, and the values decrease further as the number of multiple radios increases. Despite the fact that the MTTR values of HRR and MEHRR for the radio combinations (1, 3) and (1, 5) have reduced dramatically when compared to the combination (1, 1), they are still greater than CHRR, as seen in the graphs.

The comparisons of MTTR with complete rendezvous diversity of several multiradio rendezvous algorithms for the radio combinations (2, 2), (2, 3), (3, 3) are shown in Fig. 10d, f. In all graphs, the RPS scheme has a greater MTTR than the other schemes. The reason for this is that in PRS, CHSs are generated based on all licensed channels, but in the other four schemes, CHSs are generated based on the SUs’ available channel sets. As a result, the period length of CHS is substantially longer than that of PRS, resulting in a high MTTR. The AMRR scheme performs better with a

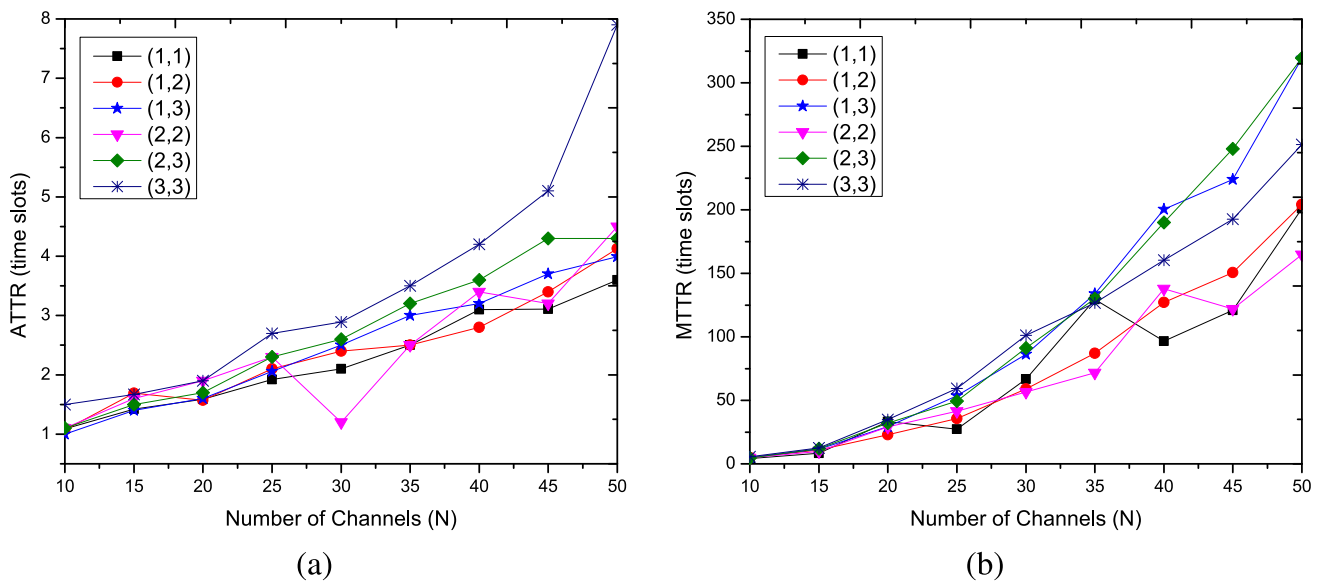


Fig. 9 Performance comparison with different allocations of radios

shorter MTTR. Nevertheless, both the RPS and AMRR algorithms are inapplicable to heterogeneous CRNs where each SU may have a single radio. The figures show that MTTR grows as N increases, since the inner period length of the CHS increases as well. The inner period length of the SUs is longer in the case of the HRR algorithm than in the MEHRR and CHRR for a particular radio combination, allocation, and available channel sets. As a result, the MTTR for HRR is greater. MEHRR has a shorter MTTR for multiradio rendezvous than the proposed CHRR scheme. The reason for this is that the greater the number of radios used to generate CHS for an SU, the shorter the inner period of the CHS. In the MEHRR scheme, all SU radios participate in the creation of stay-jump based CHS, whereas in the CHRR scheme, one SU radio follows DDS-based CHS and the remaining radios follow stay-jump based CHS. As a result, the inner period of the stay-jump based CHS is somewhat shorter in the case of MEHRR than in the case of CHRR. But, if the number of radios increases, the difference will become insignificant. Due to the small MTTR values as seen in the graphs of Fig. 10, the overall rendezvous latency for CHRR would be significantly reduced in the situation of heterogeneous radio. This improves the scheme's suitability for device-to-device communication and Internet of Things applications by maintaining the time and memory of the tiny devices, which are constrained by their limited processing power and memory.

The ATTR of several rendezvous methods is evaluated in Fig. 11 for heterogeneous radio combinations (1, 1), (1, 3), (2, 5), (4, 4). Because the CHS period duration is substantially longer for the (1, 1) radio combination, the ATTR

values of the schemes are larger for SUs with a single radio than SUs with multiple radios. In Fig. 11a, CHRR performs better because to its shorter CHS time. The ATTR values of the schemes are more or less near to each other in Fig. 11b–d. Despite the ATTR of CHRR is greater for situations owing to the asymmetric model, as shown in the graphs, the difference is minimal since the ATTR values for multiradio rendezvous scenarios are comparatively small.

7 Conclusion

In this paper, we have proposed a blind rendezvous scheme for heterogeneous CRNs, where the SUs may own either a single or multiple radios. The proposed CHRR algorithm, in contrast to other multiradio rendezvous schemes, exhibits backward compatibility with the single radio rendezvous method. In this scheme, the radios of an SU were assigned a combination of DDS-based CHS and Stay-jump-based CHS for this objective. When compared to the HRR and MEHRR algorithms, the CHRR algorithm resulted in a considerable reduction in MTTR with complete rendezvous diversity for the case, where at least one SU of the SU pair was equipped with a single radio. Although the MTTR for the CHRR was greater than that of the HRR and MEHRR in the context of multiradio rendezvous, the difference was not significant when compared to the difference observed in rendezvous involving SU with only one radio. The CHRR algorithm has been examined theoretically for various possible combinations of radios within a pair of SU, and

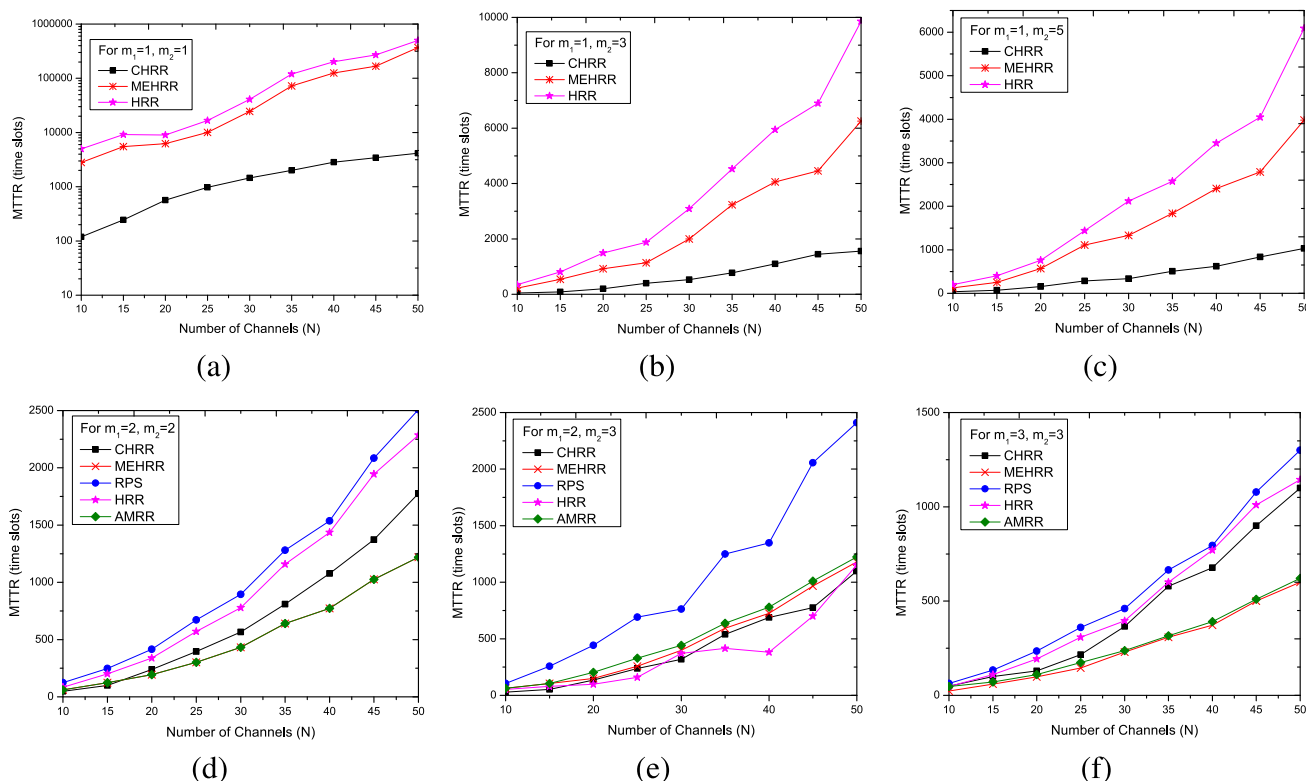


Fig. 10 MTTR comparison of different rendezvous algorithms

its performance was established by comprehensive simulations.

Author Contributions SS has formulated the problem. AM has supervised the work.

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Code availability Not applicable.

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

Appendix: Supporting Lemma and Theorem for DDS

Lemma 1 For a pair of numbers (a_1, a_2) under Z_n , with a difference value d such that $(a_1 - a_2) \bmod n = d$ and if $d \bmod k = d'$, there also exists a difference value $(n - d)$ for the same (a_1, a_2) pair, and $(n - d) \bmod k = (k - d')$.

Proof For any number pair, (a_1, a_2) under Z_n if $(a_1 - a_2) \bmod n = d$, then $(a_2 - a_1) \bmod n$ will be $(-d) \bmod n$ which is equal to $(n - d)$. If $d \bmod k = d'$ then for the other difference value $(n - d)$, we can prove that

$$\begin{aligned} (n - d) \bmod k &= (Pk \bmod k - d \bmod k) \bmod k \\ &= (0 - d') \bmod k \\ &= (k - d'). \end{aligned}$$

Theorem 1 In Algorithm 1, the constructed set $S = \{ D_0, D_1, D_2, \dots, D_{P-1} \}$ is a DDS.

Proof The set S must satisfy the two properties stated in Definition 3. For this, we need to demonstrate that each set in S is a difference set and all are disjoint from one another.

Triangular numbers and the modular idea are used to create a set that satisfies the DS properties using the numbers picked from Z_n . The required range of difference values, $d \in [1, n - 1]$ for each set, as stated in Definition 1 can be reduced to a narrower range of $d' \in [1, k - 1]$ using the modular approach $d' = d \bmod k$. Now, it has to be verified that $\forall d' \in [0, k - 1]$, there exists a pair of numbers (a_i, a_j) in each set of S such that $(a_i - a_j) \bmod n = d' + rk, \forall r \in [0, P - 1]$.

- For $k = 2P + \lfloor \frac{P}{2} \rfloor$:
 - Case 1.1 $d' = 0$. All D_i has a number pair (a_{ij_1}, a_{ij_2}) such that $a_{ij_1}, a_{ij_2} \in D_{Xi}, 0 \leq j_1, j_2 \leq P - 1$ and the difference $d = (a_{ij_1} - a_{ij_2}) \bmod n$

$$\begin{aligned} d &= ((kj_1 + i) - (kj_2 + i)) \bmod Pk \\ &= (k(j_1 - j_2) + 0) \bmod Pk \\ &= (k(j_1 - j_2) + 0), \text{ for } 0 \leq j_1 - j_2 \leq P - 1 \\ &= kr + d' \end{aligned}$$

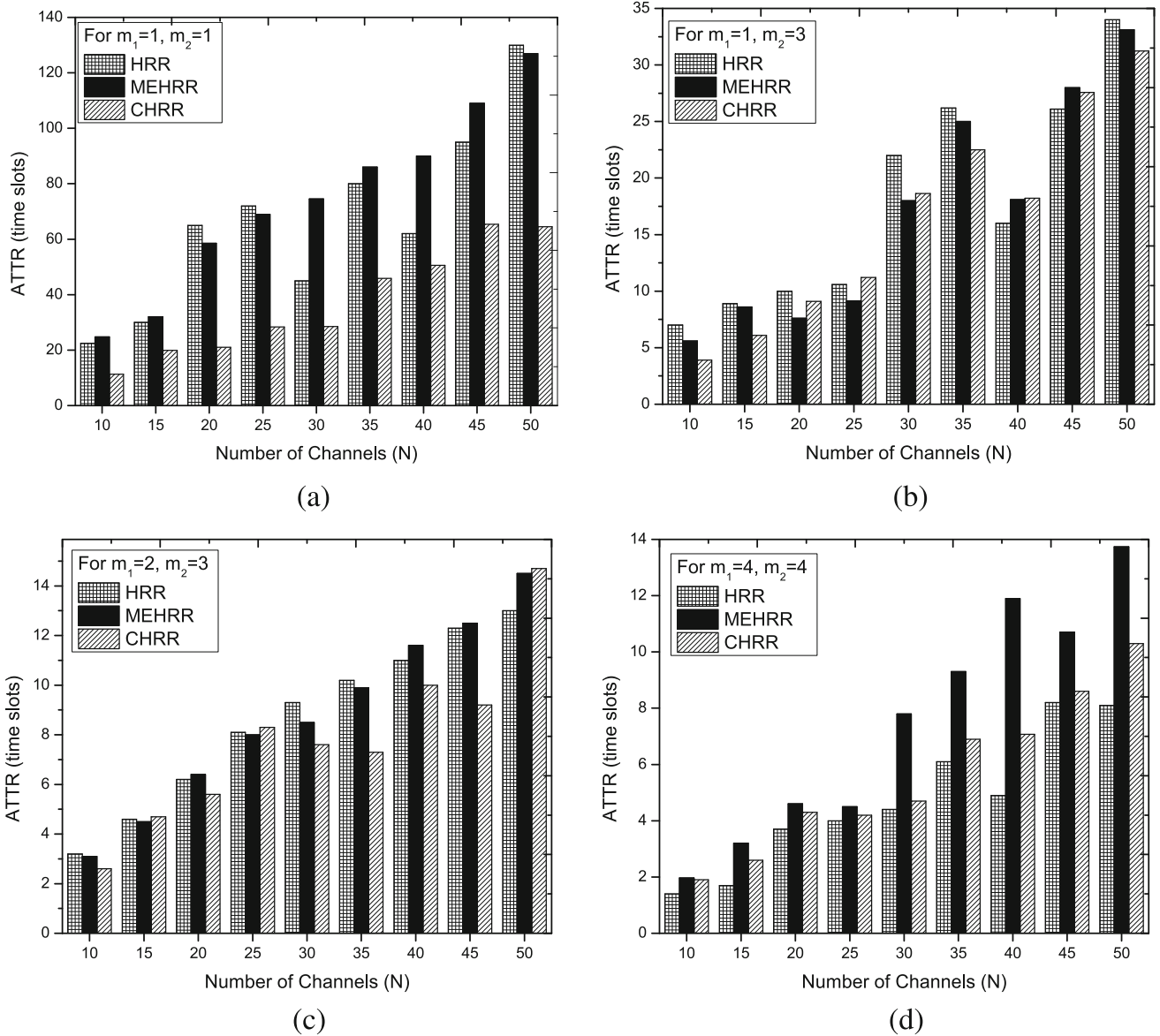


Fig. 11 ATTR comparison of different rendezvous algorithms

Case 1.2 $1 \leq d' \leq \lfloor \frac{P}{2} \rfloor$ and $2P \leq d' \leq 2P + \lfloor \frac{P}{2} \rfloor - 1$. All D_i has a number pair (b_{ij_1}, b_{ij_2}) such that $b_{ij_1}, b_{ij_2} \in D_{Yi}$ and $0 \leq j_1, j_2 \leq k - P - 1$ and the difference of the numbers b_{ij_1} and b_{ij_2} is $d = b_{ij_1} - b_{ij_2} \pmod n$

$$\begin{aligned}
 &= (k(T_{j_1} + i) + P + j_1) - (k(T_{j_2} + i) + P + j_2) \pmod n \\
 &= (k(T_{j_1} - T_{j_2}) + (j_1 - j_2)) \pmod{Pk} \\
 &= (k(T_{j_1} - T_{j_2}) \pmod{Pk} + (j_1 - j_2) \pmod{Pk}) \pmod{Pk} \\
 &= (k((T_{j_1} - T_{j_2}) \pmod{P}) + (j_1 - j_2)) \pmod{Pk} \\
 &= (kr + d') \pmod{Pk}
 \end{aligned}$$

Here $r = (T_{j_1} - T_{j_2}) \pmod{P}$

$$\begin{aligned}
 &= \left(\frac{j_1(j_1+1)}{2} - \frac{(j_1-d')(j_1-d'+1)}{2} \right) \pmod{P} \\
 &= (j_1 d' - \frac{d'^2-d'}{2}) \pmod{P}
 \end{aligned}$$

In this case, $\frac{d^2-d'}{2}$ is a whole number and P is a prime number making d' and P as co-prime. Hence, r will attain all the values from 0 to $P - 1$ for all P continuous values of j_1 . For a given d' , if $0 \leq j_2 \leq P - 1 \Rightarrow d' \leq j_1 \leq (P - 1) + d'$ and since the maximum value of j_1 is $P + \lfloor \frac{P}{2} \rfloor - 1$, therefore, the range of d' can be derived as $1 \leq d' \leq \lfloor \frac{P}{2} \rfloor$. Moreover, the range of d' can be expressed as $2P \leq d' \leq 2P + \lfloor \frac{P}{2} \rfloor - 1$ using Lemma 1.

Case 1.3 $\lfloor \frac{P}{2} \rfloor < d' \leq 2P - 1$. Every D_i has a pair of numbers, (a_{ij_1}, b_{ij_2}) such that $a_{ij_1} \in D_{Xi}, b_{ij_2} \in D_{Yi}$ and $0 \leq j_1 \leq P - 1, 0 \leq j_2 \leq k - P - 1$. The difference is given by $d = (b_{ij_2} - a_{ij_1}) \pmod n$

$$\begin{aligned}
 &= ((k(T_{j_2} + i) + P + j_2) - k(j_1 + i)) \pmod n \\
 &= (k(T_{j_2} + i - j_1) + (P + j_2 - i)) \pmod{Pk}
 \end{aligned}$$

$$= (k((T_{j_2} + i - j_1) \bmod P) + (P + j_2 - i)) \bmod Pk$$

$$= (kr + d') \bmod Pk$$

Here, $r = (T_{j_2} + i - j_1) \bmod P = (\frac{j_2(j_2+1)}{2} + i - j_1) \bmod P$ and $d' = (P + j_2 - i)$.

For a given i and j_2 , the expression $(\frac{j_2(j_2+1)}{2} + i)$ will be a constant and a whole number. Consequently, for P continuous values of j_1 , i.e., $0 \leq j_1 \leq P - 1$, r covers all values in $[0, P - 1]$. For a given range of j_2 in set D_{Y_i} , the range of d' is derived for different values of i . If $0 \leq i \leq \lfloor \frac{P}{2} \rfloor$ for a given d' , then $d' - P \leq j_2 \leq d' - P + \lfloor \frac{P}{2} \rfloor$, which denotes $P \leq d' \leq 2P - 1$. Further, the range of d' can be expressed as $\lfloor \frac{P}{2} \rfloor + 1 \leq d' \leq P + \lfloor \frac{P}{2} \rfloor$ using Lemma 1. Similarly, for a given d' if $\lfloor \frac{P}{2} \rfloor \leq i \leq P - 1$ then $d' - \lfloor \frac{P}{2} \rfloor - 1 \leq j_2 \leq d' - 1$, which implies $\lfloor \frac{P}{2} \rfloor + 1 \leq d' \leq P + \lfloor \frac{P}{2} \rfloor$. The range can also be expressed as $P \leq d' \leq 2P - 1$, using Lemma 1. The range of d' for all D_i is therefore, $\lfloor \frac{P}{2} \rfloor < d' \leq 2P - 1$.

Hence, all D_i are DS as each has a pair of numbers with $d = d' + rk$, $\forall d' \in [0, k - 1]$ and $\forall r \in [0, P - 1]$ with the three cases mentioned above.

Algorithm 1 states that $D_i = D_{X_i} \cup D_{Y_i}$ and $D_j = D_{X_j} \cup D_{Y_j}$ and with this the next three cases will be examined to demonstrate that $D_i \cap D_j = \emptyset$.

Case 1 Subsets X and Y are used to choose the elements of the subsets D_{X_i} and D_{Y_j} , respectively (Fig. 2) and $X \cap Y = \emptyset$. Hence, $D_{X_i} \cap D_{Y_j} = \emptyset$ and $D_{X_j} \cap D_{Y_i} = \emptyset$.

Case 2 $D_{X_i} = (kp_1 + i)$ and $D_{X_j} = (kp_2 + j)$, where $p_1, p_2 \in [0, P - 1]$. So any element in D_{X_i} and D_{X_j} are the i th and j th element of U_{p_1} and U_{p_2} respectively (Fig. 2). Since $i \neq j$, therefore, $D_{X_i} \cap D_{X_j} = \emptyset$.

Case 3 $D_{Y_i} = (k(T_{q_1} + i) + P + q_1)$ and in $D_{Y_j} = (k(T_{q_2} + j) + P + q_2)$, where $q_1, q_2 \in [0, k - P - 1]$. The elements in D_{Y_i} and D_{Y_j} are therefore $(q_1 + P)$ th and $(q_2 + P)$ th elements, respectively of the set $U_{T_{q_1}+i}$ and $U_{T_{q_2}+j}$. To prove via contraction, assuming that there is a common element in D_{Y_i} and D_{Y_j} , therefore, $(q_1 + P)$ th number of $U_{T_{q_1}+i}$ is equals to $(q_2 + P)$ th element of $U_{T_{q_2}+j}$ and since all U are disjoint, which implies $T_{q_1} + i = T_{q_2} + j$ and $q_1 = q_2$. Hence, the contraction $i = j$ is formed. It concludes, $D_{X_i} \cap D_{Y_j} = \emptyset$ for $i \neq j$.

As a result, the three scenarios above demonstrate that all sets in S are disjoint to one another.

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