

The effect of Weibull fading channel on cooperative spectrum sensing network using an improved energy detector

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Abstract This paper analyses the performance of proposed cooperative spectrum sensing (CSS) network in Weibull fading environment. First, we have derived the novel analytic expressions for probabilities of missed detection and false alarm in Weibull fading channel, assuming an improved energy detector (IED), selection combining diversity scheme and multiple antennas at each cognitive radio (CRs). Next, performance is analyzed using complementary receiver operating characteristics curves, total error rate, average channel throughput, and network utility function curves for the proposed CSS network. The optimal performance of CSS network is achieved by optimizing the CSS network parameters. The closed form of expressions for the optimum value of number of CRs, arbitrary power of received signal, and detection threshold at each CR are derived using *OR-Rule* and *AND-Rule* at fusion center (FC). The average channel throughput and network utility function analysis are evaluated using $k = 1 + n$ and $k = N - n$ fusion rules at FC. Finally, the impact of several network parameters such as, multiple antennas at each CR (M), number of CRs (N) in CSS network, Weibull fading parameter (V), arbitrary power of received signal (p), and sensing channel SNR ($\bar{\gamma}$) on the performance of proposed CSS network are investigated using the simulation results. The performance comparison between conventional energy detector and an IED has been highlighted with the simulations.

Keywords Improved energy detector · Weibull fading · Total error rate · Multiple antennas · Throughput · Network utility function

1 Introduction

In present days, the uses of radio spectrum have become crowded due to increase in the number of communication networks and services. Various reports on spectrum utilization have shown an inefficient usage of spectrum statistics [1]. Hence, new method of allocation policy has to be introduced for the proper usage of radio spectrum. The cognitive radio (CR) concept is a better solution for an efficient utilization of available radio spectrum. The CR users called secondary users (SUs) [2] can utilize the available radio spectrum without interfering primary user's (PU) operation. Spectrum sensing (SS) [3] is an important task to monitor the radio frequencies continuously. Notably, vacant bands present in the spectrum can be identified with the help of detection techniques. Conventional energy detection (CED) technique [4] is frequently used detection technique to identify the existence of PU by calculating the energy of the received signal. CED is the simplest one, non-coherent in nature, and also adds less complexity to CR network [5]. When a single CR based CED technique employed in the system, it might be facing a hidden terminal problem and thereby limiting its performance due to shadowing and fading effects. These drawbacks can be overcome by the cooperative spectrum sensing (CSS) [6] technique which introduces multiple CRs in the network to sense the spectrum and identify the existence of PU. The CSS network gives better detection probability values though shadowing and fading effects are present in the nature [7]. The performance of spectrum-sensing-based energy detection (ED) in cognitive

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radio networks (CRNs) over generalized fading channels and the fading channel is modelled by the extended generalized-K (EGK) distribution [8]. The average probability of detection of the energy detector (ED) over $\alpha - \mu$ generalized fading channels with selection combining (SC) diversity reception using CSS network is discussed in [9]. An improved energy detection (IED) scheme [10] is introduced to improve the detection probability and to overcome the limitations present in the CED scheme. More precisely, the detection performance can be further improved significantly by replacing CED with an IED at each CR in CSS network [11]. The IED measures the received signal amplitude (i.e. PU's transmitted signal) with an arbitrary positive power (p). In [12], an experimental approach of IED based spectrum sensing for CR network is proposed.

Sometimes, it is required to optimize the CSS network parameters to achieve the better performance and to minimize the complexity of the network. Optimization of CSS network using CED technique in CR network is discussed in [13]. In [14], optimized performance of CSS network is achieved by using a single antenna at each CR with an IED scheme over Rayleigh fading channel. In [15], performance is analyzed in CSS network using multiple antennas at each CR with an IED scheme over Rayleigh fading channel. The CSS network with an IED scheme is considered in [16] which uses optimization techniques to minimize the total error rate (It is the sum of missed detection and false alarm probabilities). The performance of CSS network with a single antennas at each CR with CED scheme over Rician fading channel is analyzed in [17]. Optimization of CSS network parameters with an IED scheme using the multiple antennas at each CR is investigated over AWGN and Rayleigh fading channels [18]. Similar analysis is carried out in Nakagami- m and Weibull fading channel with an IED scheme in [19–21].

It is also important to maximize the average channel throughput and network utility function to improve the detection performance of PUs. Due to fading effect in the environment sometimes PUs are not detected correctly, this may cause sever interference problem. This issue can be overcome, and PUs are exactly identified if the sensing time of each SU user is increases, this reduces the throughput value of the network. Hence, there is a trade-off between sensing time and throughput value of the network [22]. Throughput value can be increased if the sensing time decreases but it degrades the accurate detection of PUs. Hence, average channel throughput value will be increased by performing sensing and transmission simultaneously [23]. In [24], throughput maximization is considered over erroneous control channel using CED scheme. Similarly, network utility function should be maximized to improve the detection performance of PUs and to improve the spectral efficiency. The network utility function is maximized using an optimal number of

SUs is addressed in [25]. Maximization of average channel throughput and network utility function performance analysis is evaluated using multiple antennas at each CR with an IED scheme over AWGN channel in [26].

It is an important to study optimal detection performance of IED based CSS network in Weibull fading environment because this channel has been developed in modelling multi-path waves propagating in non-homogeneous communication environments [27]. Weibull distribution is very flexible in both indoor and outdoor communication environments. Rayleigh and exponential distributions are considered as special cases of Weibull fading distribution for a certain fading parameter values. In urban communication environments, the distributions have the capability of accounting for propagation if the Rayleigh distribution fails e.g., digital enhanced cordless telecommunications (DECT) system. However, in spite of the usefulness of this distribution, the work related to CSS network with an IED scheme using multiple antennas at each CR over Weibull fading is not reported in the literature.

These above-stated literature papers are driven us to evaluate the performance of CROC curves, total error rate, average channel throughput and network utility function using the multiple antennas at each CR with an IED scheme over Weibull faded channel in this paper. Finally, with this paper, our contributions to an existing literature are as follows:

- We have derived the novel closed-form of expression for missed detection probability (P_m), probability of false alarm (P_f) using the multiple antennas at each CR in Weibull fading channel.
- The novel analysis of complementary receiver operating characteristics (CROC) is discussed with the firm support of analytical expressions and support of MATLAB simulations.
- Selection combining (SC) diversity scheme is used at each CR to select the maximum value of decision statistics obtained from an IED at all antennas. The closed form of expressions for a threshold value using a single and multiple antennas at each CR for the CSS network are derived.
- The novel closed-form of expressions are derived for optimized network parameters such as an optimum number of CR users (N_{opt}), the optimum value of threshold (λ_{opt}), and the optimum value of arbitrary power of received signal (p_{opt}) for a single and multiple antennas case.
- The optimized expressions of network parameters are derived using hard decision fusion rules (*OR-Rule* and *AND-Rule*) at FC. The total error rate ($Q_m + Q_f$) analysis also discussed using these fusion rules.
- Average channel throughput and maximization of network utility function analysis are evaluated using $k =$

$1 + n$ and $k = N - n$ fusion rules at FC over Weibull fading channel.

- The impact of several network parameters such as fading parameter (V), multiple antennas at each CR (M), number of CR users (N), threshold value (λ), and average S-channel SNR ($\bar{\gamma}$) on the proposed CSS network are investigated.
- Comparison between CED and IED with ($M > 1$) and without ($M = 1$) diversity antenna case is also evaluated.

2 System model

The system model for an IED scheme is shown in Fig. 1. The functioning of IED is similar to CED. In [5], functioning of CED is described, it calculates the energy of the received signal. In case of IED [19], it measures the received signal strength with an arbitrary power (p) rather than squaring device. This extra feature in IED block diagram improves the detection performance about the PU.

The proposed model of CSS network is shown in Fig. 2, it consist of multiple number of CR users (N), each CR is equipped with multiple antennas (M), a fusion center (FC), and a primary user (PU). In sensing channel (S-channel),

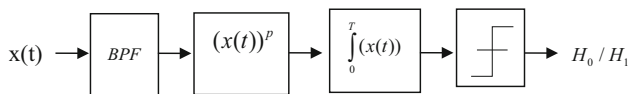


Fig. 1 System model of an improved energy detector

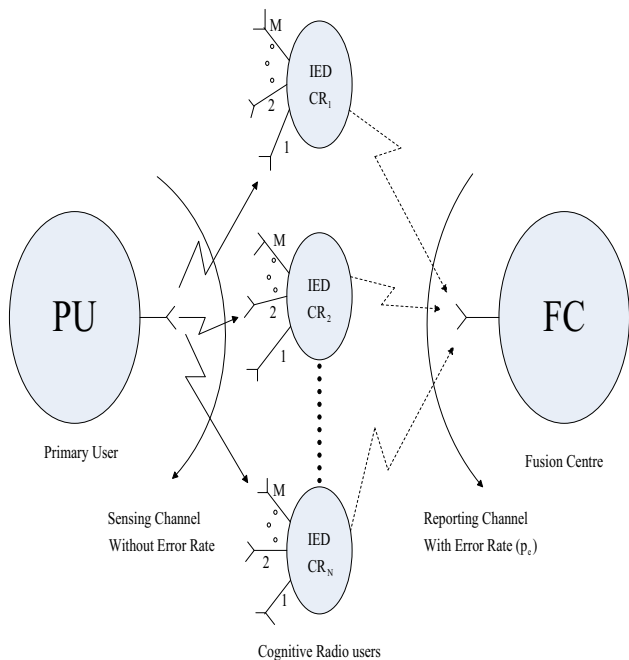


Fig. 2 Proposed model of cooperative spectrum sensing network

each CR senses and stores the information about the PU. The sensing information available with each CR is reported to the FC through reporting channel (R-channel). The complete information is collected at FC, and final decision about the PU is taken at FC using fusion rules such as hard decision rules (*OR-Rule* and *AND-Rule*), k -out of N rules ($k = 1 + n$ and $k = N - n$), where k represents the selected number of CRs, N is total number of available CRs, and n is a positive integer ($n = 2$).

Based on received signal at j -th CR, the existence of PU (absence of PU: H_0 and presence of PU: H_1) can be decided using [5]:

$$y(t) = \begin{cases} n_j(t) : & H_0 \\ h_j * s(t) + n_j(t) : & H_1 \end{cases} \quad (1)$$

In above expression $s(t)$ represent the received signal at the input of IED, $n_j(t)$ is the noise value at j -th CR and h_i is fading coefficient. In the proposed CSS network each CR equipped with multiple antennas (M), an IED scheme is used to detect the spectrum hole. Finally, expression at i -th antenna to make a local decision about the PU is given by [11]:

$$W_i = |y_i|^p \quad p > 0 \quad (2)$$

For IED, p value should be more than 2, i.e., ($p > 2$) to achieve better detection probability than CED.

Each CR uses an IED technique to get the decision statistics from all ($i = 1 \dots M$) antennas. With the help of selection combining, the largest value of w_i can be selected from all available w_i values and it is denoted as Z . Finally, to make decision about the PU, the value of Z is compared with detection threshold (λ) as given in [15],

$$Z > \lambda : H_1 \ \& \ Z < \lambda : H_0 \quad (3)$$

where λ can be obtained from the expression:

$$\lambda = \lambda_n \sigma_n^p. \quad (4)$$

where λ_n represents the fixed normalized detection threshold value and σ_n^2 represents the noise power for $p = 2$.

The missed detection probability (P_m) expression for Weibull fading channel is calculated with the help of pdf given in [26]:

$$f_{w_i|H_1}(y) = \frac{2y^{(2/p)-1} C}{p} \left[\frac{\Gamma(P)}{(E_s \sigma_h^2 + \sigma_n^2)} \right] \left(y^{2/p} \right)^{C-1} \exp \left(- \left\{ \frac{y^{2/p} \Gamma(P)}{(E_s \sigma_h^2 + \sigma_n^2)} \right\}^C \right) \quad (5)$$

where $C = V/2$, $P = 1 + 1/C$, and V —is the Weibull fading parameter.

The missed detection probability expression for Weibull fading channel can be calculated as follows [14]:

$$P_m = pr(y < \lambda|H_1) = \int_0^\lambda f_{w_i|H_1}(y)dy \tag{6}$$

$$= \int_0^\lambda \frac{2y^{(2/p)-1} C}{p} \left[\frac{\Gamma(P)}{(E_s \sigma_h^2 + \sigma_n^2)} \right] \left(y^{2/p} \right)^{C-1} \exp \left(- \left\{ \frac{y^{2/p} \Gamma(P)}{(E_s \sigma_h^2 + \sigma_n^2)} \right\}^C \right) dy \tag{7}$$

$$= \int_0 \left(\frac{\Gamma(P) \lambda^{2/p}}{(E_s \sigma_h^2 + \sigma_n^2)} \right) \exp(-t) dt \tag{8}$$

$$P_m = \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \bar{\gamma})} \right\}^C \right) \right] \tag{9}$$

where $\bar{\gamma} = E_s \sigma_h^2 / p \sigma_n^2$ is S-channel SNR, E_s is the signal energy, σ_h^2 is the fading channel coefficient variance.

We have considered that each CR is having multiple antennas and selection combining (SC) scheme is used to select the maximum value of antenna among all antennas (M).

Using the SC diversity scheme maximum value from all branches can be calculated as described in [15]

$$P_m = Pr[\max(W_1, W_2, W_3...W_M)] \tag{10}$$

Using the SC diversity under the hypothesis H_1 , the missed detection probability can be calculated as:

$$P_m = Pr[\max(W_1, W_2, W_3...W_M) \leq \lambda|H_1] \tag{11}$$

The closed form of P_m expression for Weibull fading channel using an IED scheme with multiple antennas (M) at each CR is given in [20]:

$$P_m = \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \bar{\gamma})} \right\}^C \right) \right]^M \tag{12}$$

The P_m expression is different for different fading channels because it depends upon S-channel SNR. Similarly, expression for probability false alarm (P_f) with a single antenna at each CR can be calculated using the pdf given in [26]:

$$f_{w_i|H_0}(y) = \frac{2y^{(2/p)-1} C}{p} \left[\frac{\Gamma(P)}{(\sigma_n^2)} \right] \left(y^{2/p} \right)^{C-1} \exp \left(- \left\{ \frac{y^{2/p} \Gamma(P)}{(\sigma_n^2)} \right\}^C \right) \tag{13}$$

where $C = V/2$, $P = 1 + 1/C$, and V – is the Weibull fading parameter.

The probability of false alarm expression for Weibull fading channel can be calculated as [14]:

$$P_f = pr(y > \lambda|H_0) = 1 - \int_0^\lambda f_{w_i|H_0}(y)dy \tag{14}$$

$$= 1 - \int_0^\lambda \frac{2y^{(2/p)-1} C}{p} \left[\frac{\Gamma(P)}{(\sigma_n^2)} \right] \left(y^{2/p} \right)^{C-1} \exp \left(- \left\{ \frac{y^{2/p} \Gamma(P)}{(\sigma_n^2)} \right\}^C \right) dy \tag{15}$$

$$= 1 - \int_0 \left(\frac{\Gamma(P) \lambda^{2/p}}{(\sigma_n^2)} \right) \exp(-t) dt \tag{16}$$

$$P_f = \left[\exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right] \tag{17}$$

If we consider multiple antennas (M) at each CR using selection combining (SC) diversity scheme, then expression for (P_f) will be:

$$P_f = 1 - \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right]^M \tag{18}$$

2.1 Threshold calculation

Threshold value is more important to decide the presence or absence of primary user. It is also important to draw the complementary receiver operating characteristics (CROC) in the presence of Weibull fading channel. The closed form of expression for threshold value with a single antenna at each CR can be calculated using the P_f expression given by Eq. (17). Applying *logarithm* on both sides to Eq. (17), then it reduces to:

$$\ln(P_f) = - \left\{ \lambda^{2/p} \Gamma(P) / \sigma_n^2 \right\}^C \tag{19}$$

$$\lambda = \left(\frac{\sigma_n^2}{\Gamma(P)} (\ln(1/P_f))^{1/C} \right)^{p/2} \tag{20}$$

The above expression is useful to set the threshold value for the identification of PU in case of single antenna at each CR.

Similarly, the closed form of expression for threshold value using the multiple antennas at each CR can be calculated with the help of Eq. (18). Applying *logarithm* on both sides to Eq. (18) then it reduces to:

$$\implies M \ln \left(1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) = \ln (1 - P_f) \tag{21}$$

after simplification and step by step analysis is provided in ‘‘Appendix’’ section. Finally, the expression for threshold value is:

$$\lambda = \left(\frac{\sigma_n^2}{\Gamma(P)} (\ln (1/(1 - \exp (\ln(1 - P_f)/M))))^{1/C} \right)^{p/2} \tag{22}$$

With the help of above expression, the threshold can be set for the identification of PU when the multiple antennas are used at each CR.

3 Optimization of CSS network parameters

3.1 Optimization of threshold value using P_f and P_m expressions

An optimum value of threshold (λ_{opt}) is required to decide the existence of PU with minimum value of threshold. The closed form of expression for λ_{opt} for a single antenna case can be calculated by differentiate Eqs. (9 and 17) w.r.t to λ :

$$\begin{aligned} \frac{\partial P_f}{\partial \lambda} + \frac{\partial P_m}{\partial \lambda} &= 0 \tag{23} \\ \implies - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \frac{2\lambda^{(2/p)-1} \Gamma(P)}{p\sigma_n^2} \\ &+ \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1+\gamma)} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{(1+\gamma)\sigma_n^2} \right\}^{C-1} \frac{2\lambda^{(2/p)-1} \Gamma(P)}{p\sigma_n^2 (1+\gamma)} = 0 \end{aligned} \tag{24}$$

after simplification and step by step analysis is provided in Appendix. Finally, expression for λ_{opt} is:

$$\lambda_{opt} = \left(\frac{\sigma_n^2}{\Gamma(P)} \left(C \ln (1 + \gamma) / \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{p/2} \tag{25}$$

Similarly, the closed form of expression for λ_{opt} with the multiple antennas ($M = 3$) at each CR case can be calculated using the above procedure and final expression for λ_{opt} is:

$$\lambda_{opt} = \left(\frac{\sigma_n^2}{\Gamma(P)} \left(C \ln (1 + \gamma) / \left(2 \left\{ \frac{\gamma}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{p/2} \tag{26}$$

3.2 Optimization of arbitrary power of received signal (p) using P_f and P_m expressions

It is also necessary to optimize the arbitrary power of the received signal. The optimum value of p (p_{opt}) can be calculated by differentiating Eqs. (9) and (17) w.r.t p for a single antenna case:

$$\frac{\partial P_f}{\partial p} + \frac{\partial P_m}{\partial p} = 0 \tag{27}$$

$$\begin{aligned} \frac{\partial P_m}{\partial p} &= -M \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \right]^{M-1} \\ &\exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^{C-1} \\ &\frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2 (1 + \gamma)} \end{aligned} \tag{28}$$

$$\begin{aligned} \frac{\partial P_f}{\partial p} &= M \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right]^{M-1} \\ &\exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \\ &\frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2} \end{aligned} \tag{29}$$

after simplification and step by step analysis is provided in ‘‘Appendix’’. Finally, expression for p_{opt} is:

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left\{ \frac{C \ln(1+\gamma)}{\left[1 - \left(\frac{1}{1+\gamma} \right)^C \right]} \right\}^{1/C} \left(\frac{\sigma_n^2}{\Gamma(P)} \right) \right)} \tag{30}$$

Similarly, the closed form of expression for p_{opt} with the multiple antennas ($M = 3$) at each CR case can be calculated using the above procedure and final expression is:

$$P_{opt} = \frac{2 \ln \lambda}{\ln \left(\left\{ \frac{C \ln(1+\gamma)}{(0.5) \left[1 - \left(\frac{1}{1+\gamma} \right)^C \right]} \right\}^{1/C} \left(\frac{\sigma_n^2}{\Gamma(P)} \right) \right)} \tag{31}$$

4 Total error calculations using hard decision rules

We have assumed that R-channel is perfect channel (i.e. error free R-channel) to calculate total error probability ($Q_m + Q_f$) in different fading channels. Firstly, we require probability of false alarm (P_f) and probability of missed detection (P_m) expressions for each CR. Secondly, we can calculate Q_m and Q_f values using the below expressions, which are missed detection and false alarm probabilities of all CRs. The Q_m and Q_f expressions with an error rate (p_e) in R-channel are given as [14]:

$$\begin{aligned} Q_f &= 1 - [(1 - P_f)(1 - p_e) + p_e P_f]^N \\ Q_m &= [P_m(1 - p_e) + p_e(1 - P_m)]^N \end{aligned} \tag{32}$$

Finally, total error rate $Z(N)$ is calculated by taking the sum of Q_f and Q_m as [14]:

$$Z(p, \lambda, N) \cong Q_f + Q_m \tag{33}$$

Expressions for Q_m and Q_f with perfect R-channel ($p_e = 0$) when *OR-logic* is used at FC:

$$\begin{aligned} Q_f &= 1 - [(1 - P_f)]^N \\ Q_m &= [P_m]^N \end{aligned} \tag{34}$$

Similarly, expressions for Q_m and Q_f when *AND-logic* is used at FC:

$$\begin{aligned} Q_m &= 1 - [(1 - P_m)]^N \\ Q_f &= [P_f]^N \end{aligned} \tag{35}$$

4.1 Optimization of network parameters using ‘OR-Rule’ at FC

4.1.1 Optimization of number of CRs (N_{opt}):

Optimization of number of CRs is required to achieve better performance with minimum number of CR numbers. Here we are calculating N_{opt} value for perfect channel. The closed form of expression for N_{opt} value for *OR-Rule* can be calculated using Eq. (33) as follows:

$$\begin{aligned} \text{Total error rate : } Z(N) &= Q_m + Q_f \\ \Delta Z(N) &= Z(N + 1) - Z(N) \end{aligned}$$

$$\begin{aligned} \implies & 1 - [(1 - P_f)]^{N+1} - 1 + [(1 - P_f)]^N \\ & + [P_m]^{N+1} - [P_m]^N = 0, \end{aligned} \tag{36}$$

step by step analysis is provided in ‘‘Appendix’’ section. Finally, the expression for N_{opt} using *OR-Rule* at FC is

$$N_{opt} = \left\lceil \frac{\ln \left(\frac{1 - P_m}{P_f} \right)}{\ln \left(\frac{1 - P_f}{P_m} \right)} \right\rceil \tag{37}$$

4.1.2 Optimization of threshold value (λ_{opt})

The closed form of expression for λ_{opt} value can be calculated by differentiating Eq. (33) w.r.t to λ , and equating to zero.

$$\frac{\partial Q_m}{\partial \lambda} + \frac{\partial Q_f}{\partial \lambda} = 0 \tag{38}$$

$$\implies N(1 - P_f)^{N-1} \frac{\partial P_f}{\partial \lambda} + N(P_m)^{N-1} \frac{\partial P_m}{\partial \lambda} = 0 \tag{39}$$

For a single antenna case ($M = 1$), $\partial P_f / \partial \lambda$ and $\partial P_m / \partial \lambda$ expressions are given in Eq. (24), substituting Eqs. (24) in (39), applying logarithm on both sides and solving algebraic expressions, the above expression reduces to

$$\begin{aligned} \lambda_{opt} &= \left(\frac{\sigma_n^2}{\Gamma(P)} \left((N - 1) \left(\ln \left(\frac{1 - P_f}{P_m} \right) \right) \right. \right. \\ &\quad \left. \left. + C \ln(1 + \gamma) / \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{P/2} \end{aligned} \tag{40}$$

The above expression represents the optimum value of λ with a single antenna at each SU, multiple number of SUs (N), and using *OR-Rule* at FC in the proposed CSS network.

Similarly, using the multiple antennas ($M = 3$) at each CR and multiple number of SUs (N) in the proposed CSS network, the closed form of expression for λ_{opt} is:

$$\begin{aligned} \lambda_{opt} &= \left(\frac{\sigma_n^2}{\Gamma(P)} \left((N - 1) \left(\ln \left(\frac{1 - P_f}{P_m} \right) \right) \right. \right. \\ &\quad \left. \left. + C \ln(1 + \gamma) / \left(2 \left\{ \frac{\gamma}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{P/2} \end{aligned} \tag{41}$$

4.1.3 Optimization of arbitrary power of the received signal (p_{opt})

The closed form of expression for p_{opt} can be calculated by differentiating Eq. (33) w.r.t to p ,

$$\frac{\partial Q_m}{\partial p} + \frac{\partial Q_f}{\partial p} = 0 \tag{42}$$

$$\implies N(1 - P_f)^{N-1} \frac{\partial P_f}{\partial p} + (P_m)^{N-1} \frac{\partial P_m}{\partial p} = 0 \tag{43}$$

For a single antenna case ($M = 1$), $\partial P_f / \partial p$ and $\partial P_m / \partial p$ expressions are given in Eqs. (28) and (29), substituting these equations in Eq. (42), apply *logarithm* on both sides, after simplification, the closed form of expression for p_{opt} is:

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left(\frac{(N-1) \left(\ln \left(\frac{1-P_f}{P_m} \right) \right) + C \ln(1+\gamma)}{\left(1 - \left(\frac{1}{1+\gamma} \right)^C \right)} \right)^{1/C} \frac{\sigma_n^2}{\Gamma(P)} \right)} \tag{44}$$

The above expression represents an optimum value of p with a single antenna at each SU, multiple number of SUs (N), and using *OR-Rule* at FC in the proposed CSS network.

Similarly, using the multiple antennas ($M = 3$) at each CR and multiple number of SUs (N) in the proposed CSS network, the closed form of expression for p_{opt} is:

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left(\frac{(N-1) \left(\ln \left(\frac{1-P_f}{P_m} \right) \right) + C \ln(1+\gamma)}{\left(0.5 \left(\frac{\gamma}{1+\gamma} \right)^C \right)} \right)^{1/C} \frac{\sigma_n^2}{\Gamma(P)} \right)} \tag{45}$$

4.2 Optimization of network parameters using ‘AND-Rule’ at FC

4.2.1 Optimization of number of CRs (N_{opt})

The closed form of expression for N_{opt} value using *AND-logic* at FC can be calculated using Eqs. (35) and (36) as follows:

$$\implies 1 - [(1 - P_m)]^{N+1} - 1 + [(1 - P_m)]^N + [P_f]^{N+1} - [P_f]^N = 0 \tag{46}$$

after simplification, above expression reduces to

$$\implies \left((1 - P_f) / P_m \right) = \left((1 - P_m) / P_f \right)^N,$$

applying *logarithm* on both sides, the final expression for N_{opt} is:

$$N_{opt} = \left\lceil \frac{\ln \left(\frac{1-P_f}{P_m} \right)}{\ln \left(\frac{1-P_m}{P_f} \right)} \right\rceil \tag{47}$$

4.2.2 Optimization of threshold value (λ_{opt})

The closed form of expression for λ_{opt} value can be calculated using Eqs. (24), (35), (38), and (39) and, applying *logarithm* on both sides and solving algebraic expressions, the expression for λ_{opt} is:

$$\lambda_{opt} = \left(\frac{\sigma_n^2}{\Gamma(P)} \left((N - 1) \left(\ln \left(\frac{P_f}{1 - P_m} \right) \right) + C \ln(1 + \gamma) / \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{P/2} \tag{48}$$

The above expression represents an optimum value of λ with single antenna at each SU, multiple number of SUs (N), and using *AND-Rule* at FC in the proposed CSS network.

Similarly, using the multiple antennas ($M = 3$) at each CR and multiple number of SUs (N) in the proposed CSS network, the expression for λ_{opt} is:

$$\lambda_{opt} = \left(\frac{\sigma_n^2}{\Gamma(P)} \left((N - 1) \left(\ln \left(\frac{P_f}{1 - P_m} \right) \right) + C \ln(1 + \gamma) / \left(2 \left\{ \frac{\gamma}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{P/2} \tag{49}$$

4.2.3 Optimization of arbitrary power of the received signal (p_{opt})

The closed form of expression for p_{opt} value can be calculated using Eqs. (28), (29), (35), (42), and (43), applying *logarithm* on both sides and solving algebraic expressions, the expression for λ_{opt} is:

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left(\frac{(N-1) \left(\ln \left(\frac{P_f}{1-P_m} \right) \right) + C \ln(1+\gamma)}{\left(1 - \left(\frac{1}{1+\gamma} \right)^C \right)} \right)^{1/C} \frac{\sigma_n^2}{\Gamma(P)} \right)} \quad (50)$$

The above expression represents an optimum value of p with a single antenna at each SU, multiple number of SUs (N), and using *AND-Rule* at FC in the proposed CSS network.

Similarly, using the multiple antennas ($M = 3$) at each CR and multiple number of SUs (N) in the proposed CSS network, the expression for p_{opt} is:

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left(\frac{(N-1) \left(\ln \left(\frac{P_f}{1-P_m} \right) \right) + C \ln(1+\gamma)}{\left(0.5 \left(\frac{\gamma}{1+\gamma} \right)^C \right)} \right)^{1/C} \frac{\sigma_n^2}{\Gamma(P)} \right)} \quad (51)$$

5 Throughput and network utility function analysis

The effective probability expressions for the proposed CSS network are calculated by considering an error rate (p_e) in R-channel. The expressions for effective false alarm and detection probabilities are given in [25] as:

$$p_{fe} = P_f (1 - p_e) + p_e (1 - P_f) \quad (52)$$

$$p_{de} = P_d (1 - p_e) + p_e (1 - P_d) \quad (53)$$

The information associated with CRs are transferred to FC through R-channel in the form of binary decisions (either 0 or 1). The final decision about the PU is taken at FC using the fusion rules ($k = 1 + n$ and $k = N - n$) with the help of following expressions given in [25]:

$$P_{F,k} = \sum_{l=k}^N \binom{N}{l} p_{fe}^l (1 - p_{fe})^{N-l} \quad (54)$$

$$P_{D,k} = \sum_{l=k}^N \binom{N}{l} p_{de}^l (1 - p_{de})^{N-l} \quad (55)$$

where k represent the number of selected SUs, N is the total number of SUs, and n is the positive arbitrary integer (we have assumed $n = 2$ in our simulations).

5.1 Average channel throughput analysis

The average channel throughput (C_{avg}) for a given CSS network is calculated by an expression given in [28]:

$$C_{avg,k}(N) = \varphi_0 + \varphi_1 P_{D,k}(N) - \varphi_2 P_{F,k}(N) \quad (56)$$

where

$$\varphi_0 = P_1 \left(\tilde{C}_p + \tilde{C}_s \right) + P_0 C_s \quad (57)$$

$$\varphi_1 = P_1 \left(C_p - \tilde{C}_p + \tilde{C}_s \right) \quad (58)$$

$$\varphi_2 = P_0 C_s \quad (59)$$

\tilde{C}_s, C_s are the throughput of the secondary system in the presence and absence of PU respectively. Similarly, \tilde{C}_p, C_p represents the throughput of the primary system in the presence and absence of SU.

With the help of above expressions, the average channel throughput expressions for fusion rules ($k = 1 + n$ and $k = N - n$) can be written as [25]:

$$C_{avg,1+n}(N) = \varphi_0 + \varphi_1 P_{D,1+n}(N) - \varphi_2 P_{F,1+n}(N) \quad (60)$$

$$C_{avg,N-n}(N) = \varphi_0 + \varphi_1 P_{D,M-n}(N) - \varphi_2 P_{F,M-n}(N) \quad (61)$$

5.2 Network utility function analysis

The closed form of expression for maximizing the network utility function (NUF) of CSS network is given in [26]:

$$J(N) = u_1 \left((1 - P_{F,k}(N)) p(H_0) + P_{m,k}(N) p(H_1) \right) - u_2 P_{m,k}(N) p(H_1) - u_3 N \quad (62)$$

The above mentioned closed form of NUF expression is a combination of three parts, in which first part gives the information regarding the amount of usage of spectrum. Sometimes, PUs are not detected accurately this may cause interference problem (PUs with SUs), this interference information is given by the second part of above expression. Finally, third part represents the utilization of resources in the network. μ_1, μ_2 and μ_3 are the cost functions for the three sections. The network utility function increases with the cooperation SUs.

6 Results and discussions

The simulation results and their discussions are presented in this section. The effect on performance for different values of network parameters such as $\lambda_n, p, \bar{\gamma}, M, N$, and V are discussed when the proposed system is effected by Weibull fading environment. The performance is evaluated using total error rate ($Q_m + Q_f$), complementary receiver operating characteristics (CROC), probability of detection (Q_d), average channel throughput (C_{avg}), and maximization of network utility function.

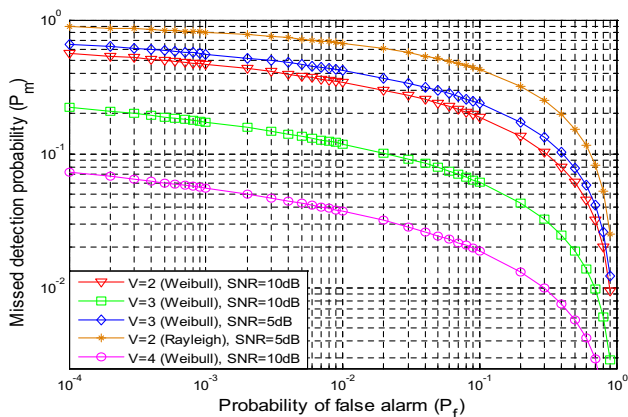


Fig. 3 CROC curves for Weibull fading channel with a single antenna at each CR

Figure 3 is drawn between probability of false alarm (P_f) and missed detection probability (P_m) values for Weibull fading channel. This MATLAB simulation is evaluated for various values of $\bar{\gamma}$ namely (10 and 5 dB), V namely ($V = 2, 3$, and 4), $M = 1$, and $p = 3$. From the simulation it can be observed that P_m value decreases, as P_f value increases. Similar nature also observed other simulation parameters $\bar{\gamma}$ and V . As the fading parameter increases, fading effect present between transmitter and receiver is decreases, and there by P_m value decreases.

When V value increases in Fig. 3 from $V = 3$ to $V = 4$, P_m value decreases by 68.5%, at $P_f = 0.01$, $\bar{\gamma} = 10$ dB, $M = 1$, and $p = 3$. Similarly, as the S-channel SNR value increases, noise value decreases in S-channel and it improves the detection probability of PU. As $\bar{\gamma}$ value increases from 5 to 10 dB, P_m value decreases by 71.6% at $P_f = 0.01$, $M = 1$, $V = 3$, and $p = 3$. Finally, our simulation results are exactly matches with Rayleigh fading channel when the Weibull fading channel parameter value chosen as $V = 2$ (Fig. 4).

Similar analysis (CROC analysis) is carried out in Fig. 5 also, considering multiple antennas at each CR in proposed CSS network over Weibull fading channel. Various values of M namely ($M = 1, 2$ and 3), V namely ($V = 2$ and 3), $\bar{\gamma} = 10$ dB, and $p = 3$ are used in the simulation. The effect of multiple antennas on missed detection probability is shown using CROC curves in Fig. 5. As the multiple antennas at each CR increases, it improves the diversity order, hence, detection probability of PU is increases. The P_m value decreases by 87.7%, as M value increases from $M = 1$ to $M = 3$ at $P_f = 0.01$, $\bar{\gamma} = 7$ dB, $p = 3$, and $V = 3$. When $\bar{\gamma}$ value increases from 4 to 7 dB, P_m value decreases by 83.1% at $P_f = 0.01$, $p = 3$, $M = 3$, and $V = 3$. Finally, when V value increases from $V = 2$ (Rayleigh) to $V = 3$ (Weibull), P_m value decreases 62.4% at $P_f = 0.01$, $p = 3$, $M = 3$, and $V = 3$. All the simulation results are drawn with strong

support of theoretical expressions and the simulation results are in perfect accordance with theoretical results.

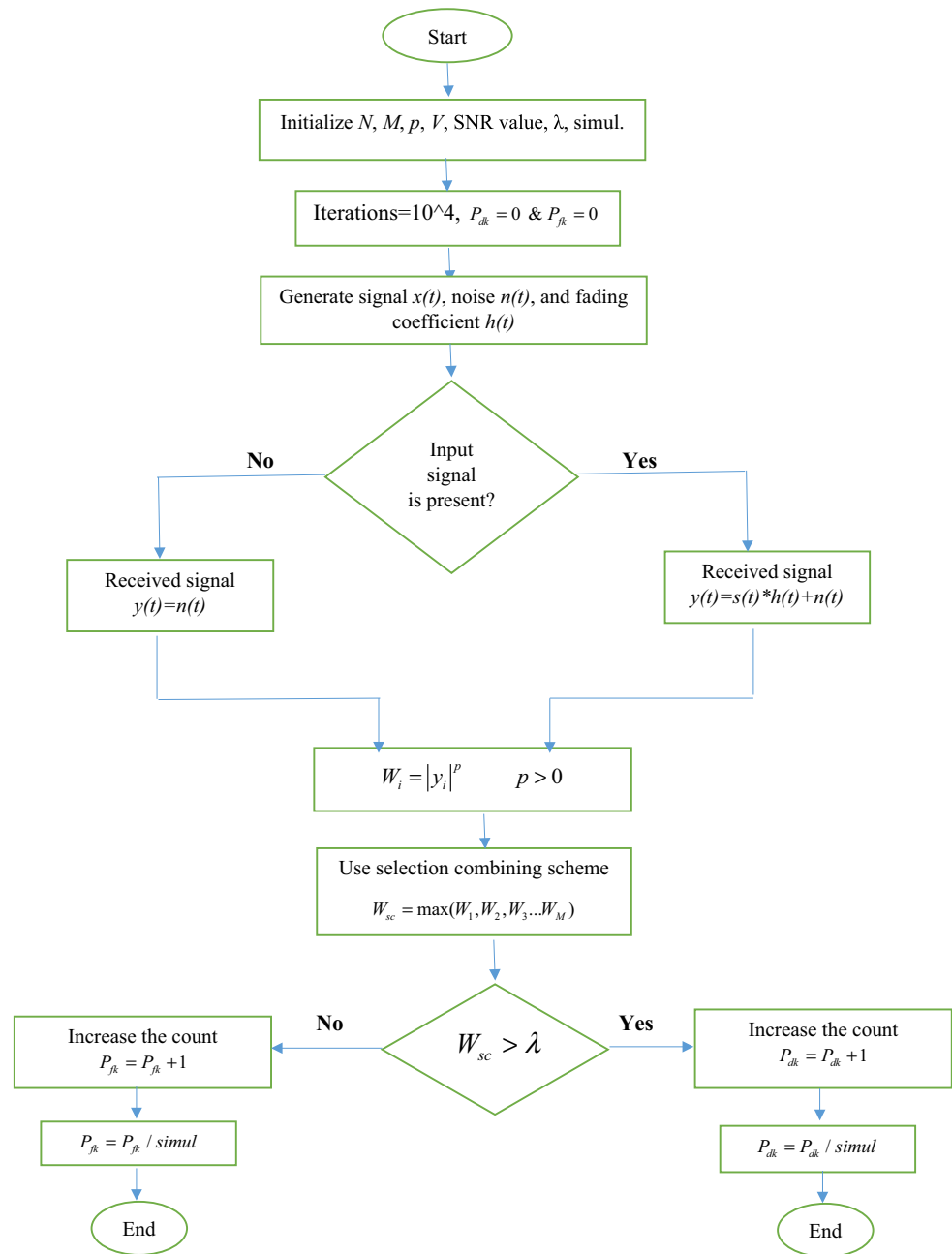
Figure 6 shows the performance evaluation of a single CR user-based spectrum sensing as function of p for different values of M (namely $M = 1$ and 3). The performance is evaluated using P_m versus p curves when S-channel is effected by Weibull fading effect. In Fig. 6, P_m value decreases and P_f value increases, as p value increases.

As we know that the CED operations would be obtained for $p = 2$, the P_m value is almost equal to 1 for $p = 2$ and it reduces significantly for $p > 2$ (i.e. CR with IED). In other words, when CED is replaced with IED, there is a significant improvement in the missed detection performance. It can be observed from the simulation that for a particular value of p ($p = 3$) when M value increases from 1 to 3, P_m value reduces from 0.5060 to 0.1296 (decreases by 74.3%) at $\bar{\gamma} = 10$ dB, and $\lambda = 30$. Similarly, the curves for P_f versus p are also shown in Weibull fading channel. When M value increases from $M = 1$ to $M = 3$, P_f value increases from 0.02963 to 0.08628 (increases by 65.6%) at $p = 5$, $\bar{\gamma} = 10$ dB, and $\lambda = 30$. It can be observed that there is a trade-off between P_f and P_m with respect to parameter p . From the simulation, it can be concluded that higher and lower values of p cannot be considered due to the rise in P_f and fall in P_m values, respectively. As p value increases, an optimum value of p should be considered in order to maintain both P_m and P_f at a particular level and this optimum value can be obtained when both P_m and P_f are minimum.

Figure 7 is drawn between optimum value of threshold (λ_{opt}) versus S-channel SNR ($\bar{\gamma}$) values. This graph is simulated for various values of V namely ($V = 2$ and 3) and p namely ($p = 2, 3$ and 4). It can be observed from simulation that λ_{opt} value increases with the increment of $\bar{\gamma}$ value. As S-channel SNR value increases, noise variance value decreases, the reduced value of noise variance, improves the signal strength. When p value decreases from $p = 4$ (IED) to $p = 2$ (CED), λ_{opt} value decreases by 52.8% at $\bar{\gamma} = 6$ dB and $V = 3$, it decreases by 49.2% at $\bar{\gamma} = 6$ dB and $V = 2$. Similarly, λ_{opt} value also depends upon fading environment present in S-channel. When V value decreases from $V = 3$ to $V = 2$, λ_{opt} value decreases by 10.1% at $\bar{\gamma} = 6$ dB, and $p = 4$. This simulation is evaluated with strong support of theoretical expression analysis and is in perfect accordance with the theoretical results.

Figure 8 is drawn between optimum value of arbitrary power of received signal (p_{opt}) versus $\bar{\gamma}$ values. The graph is simulated for various values of V namely ($V = 2$ and 3) and λ namely ($\lambda = 5, 10$ and 15). It can be observed from the simulation that p_{opt} value decreases with the increment of S-channel SNR. As $\bar{\gamma}$ value increases, noise variance value decreases which improves the signal strength. When λ value decreases from $\lambda = 15$ to $\lambda = 5$, p_{opt} value decreases by 40.5% at $\bar{\gamma} = 5$ dB and $V = 2$, it decreases by 43.2%

Fig. 4 Methodology of simulation process



at $\bar{\gamma} = 6$ dB and $V = 3$. Similarly, p_{opt} value also depends upon fading environment present in S-channel i.e., p_{opt} value decreases as fading effect is increases in S-channel. When V value increases from $V = 2$ to $V = 3$, p_{opt} value decreases by 11.2% at $\bar{\gamma} = 4$ dB and $\lambda = 15$. This simulation is evaluated with strong support of theoretical expression analysis and it is exactly matches with theoretical results. In the proposed CSS network, if the number of CR users increases, there is a chance to occur larger delays while making a decision about the PU. Hence, it is necessary to find out the number of CR users that are exactly contributing in the pro-

cess of making a decision about the PU, these CRs are known as the optimal number of CR users.

An optimum number of CR users (N_{opt}) are calculated as a function of λ using hard decision logic called (*OR-Rule*) at FC in Fig. 9. The N_{opt} value calculated for error free R-channel, various values of M , p , $\bar{\gamma}$, and V . From Fig. 9, it is clear that N_{opt} value increases with λ and for a given $\bar{\gamma}$, p , and V values. Since we are using *OR-Rule* at FC, when λ value increases, FC needs binary decisions from more SUs to reduce the missed detection probability. For a particular case, when $\bar{\gamma}$ value increases from 5 to 7 dB, N_{opt} value decreases

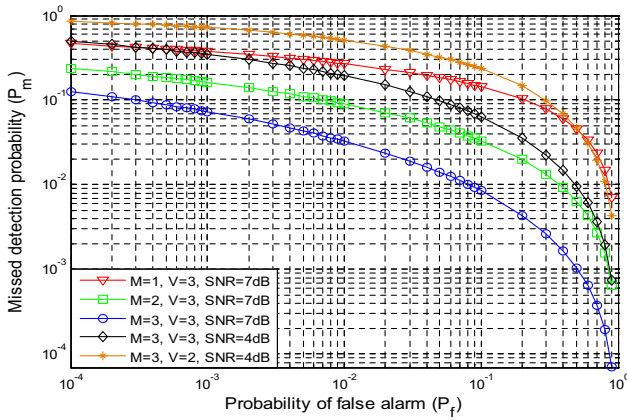


Fig. 5 CROC curves for Weibull fading channel with the multiple antenna at each CR

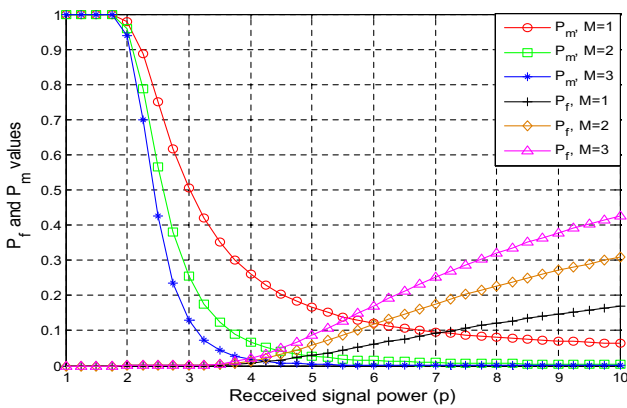


Fig. 6 P_f and P_m versus p curves for Weibull fading channel

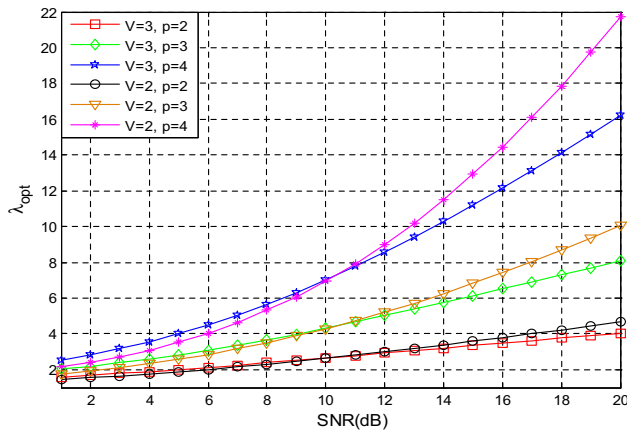


Fig. 7 λ_{opt} versus $\bar{\gamma}$ curves for Weibull fading channel with a single antenna at each CR

by 50.0% at $\lambda = 6$, $M = 2$, $p = 3$, and $V = 3$. The fading effect present in S-channel also influence in deciding the optimum number of CR users. As V value increases from $V = 2$ to $V = 3$, N_{opt} value decreases by 28.5% at $\lambda = 8$, $M = 1$, $p = 3$, and $\bar{\gamma} = 5$ dB.

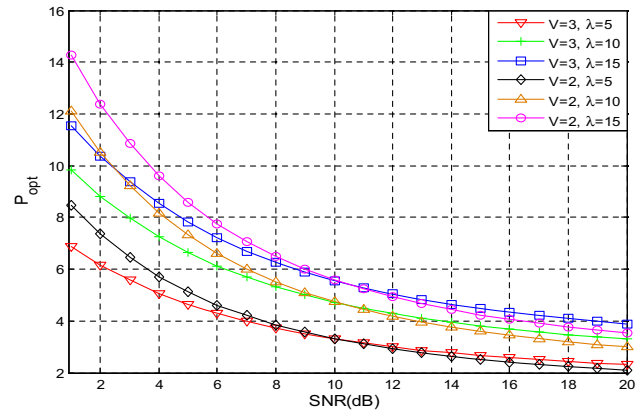


Fig. 8 p_{opt} versus $\bar{\gamma}$ curves for Weibull fading channel with a single antenna at each CR

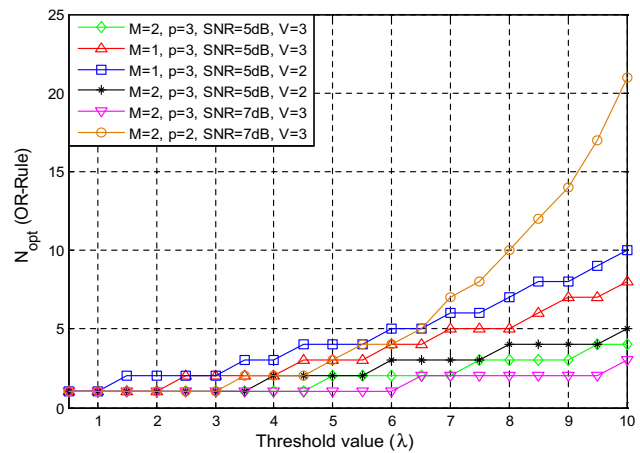


Fig. 9 N_{opt} versus λ using OR-Rule at FC

It can also observe from simulation that N_{opt} value is less for lower threshold values and it also depends upon p value. If the detection scheme changes from CED ($p = 2$) to IED ($p = 3$), N_{opt} value decreases by 80.0% at $\lambda = 8$, $M = 2$, $V = 3$, and $\bar{\gamma} = 5$ dB. Finally, number of antennas at each CR also effect the N_{opt} value, as M value increases from $M = 1$ to $M = 2$, N_{opt} value decreases by 50% at $\lambda = 7$, $p = 3$, $V = 3$, and $\bar{\gamma} = 5$ dB.

Figure 10 is drawn between Q_d and $\bar{\gamma}$ in Weibull fading channel for various values of p , N , and M namely ($p = 2$ and 3), ($N = 1, 2$, and 3) and ($M = 1$ and 2) using OR-Rule at FC. As $\bar{\gamma}$ value increases, Q_d value increases for any type of fading and for any values of p , N , and M . This is due to the fact that when S-channel SNR value increases, noise effect decreases in the S-channel connected to each CR, which leads to improvement in detection performance. For $M = 2$, $N = 3$, and $\bar{\gamma} = 6$ dB, as p value decreases from $p = 3$ (IED) to $p = 2$ (CED), Q_d value decreases by 72.5% in Weibull fading channel. This is due to increases in the received signal strength by an arbitrary power parameter (p).

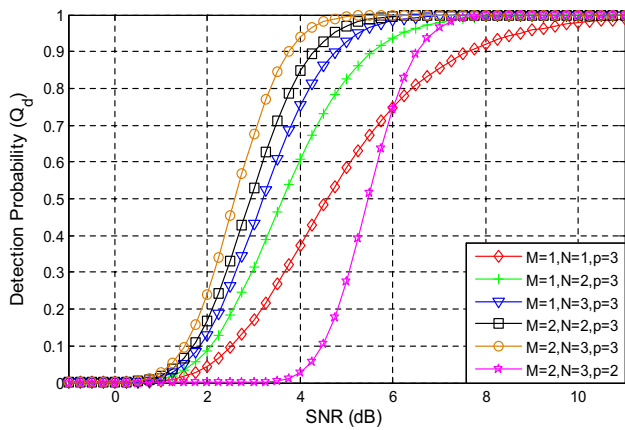


Fig. 10 Q_d versus $\bar{\gamma}$ using OR-rule at FC

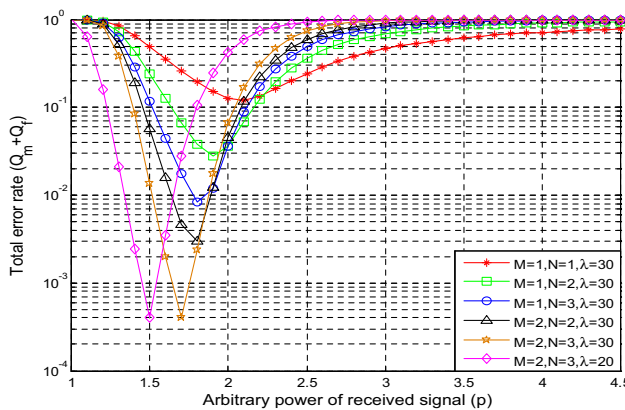


Fig. 11 $Q_m + Q_f$ versus p using OR-Rule at FC

Similarly, Q_d value also increases by increasing the M value at each CR, this is because of increase in the diversity order of the antenna at each CR. For a particular case, $p = 3$, $\bar{\gamma} = 4$ dB, and $N = 3$, when M value decreases from 2 to 1, Q_d value decreases by 19.6% using OR-Rule at FC. The detection probability also depends upon number of cooperative CR users in the network. When N value decreases from $N = 3$ to $N = 1$, Q_d value decreases by 50.4% for a single antenna case, $\bar{\gamma} = 4$ dB, and $p=3$. Similarly, Q_d value decreases by 9.9% for multiple antenna case, when N value decreases from $N = 3$ to $N = 2$, $\bar{\gamma} = 4$ dB, and $p = 3$.

In Fig. 11, total error probability ($Q_m + Q_f$) performance is analyzed as a function of p . Simulation results are in exact accordance with derived theoretical expressions. Three values of N (namely 1, 2, and 3), two values of M (namely 1 and 2), and two values of λ (namely 30 and 20) are considered as simulation parameters.

The performance is evaluated when S-channel effected by Weibull fading and OR-Rule is used at FC. It is evident from simulation that, as p value increases, the ($Q_m + Q_f$) value initially decreases, reaches to a minimum value, and then it increases. Similar variation is observed with all other

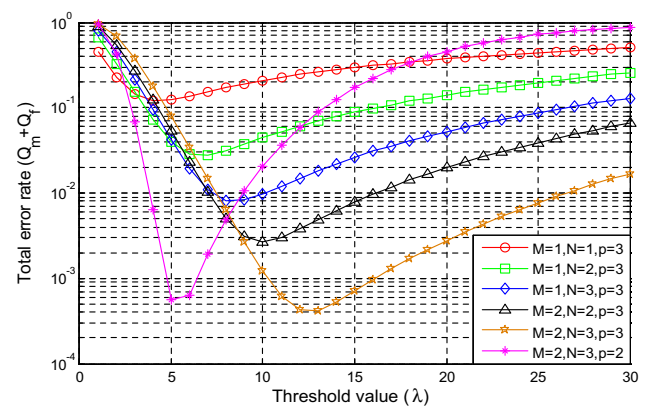


Fig. 12 $Q_m + Q_f$ versus λ using OR-Rule at FC

simulation parameters. This nature of the curve is due to the decrease in λ value for increasing p value according to ($\lambda = \lambda_n \sigma_n^p$). The increase in the P_f value at each CR and increase in Q_f value at FC are the reasons for improvement in the ($Q_m + Q_f$) value. Not only total error performance is calculated from Fig. 11, but also an optimum value of p (p_{opt}) can be calculated. The p value at which total error rate is minimum can be treated as p_{opt} . The optimum value of p also depends upon various network parameters. For a particular case, ($Q_m + Q_f$) value decreases by 26.8%, as N value increases from $N = 1$ to $N = 3$ at $p = 2.3$, $M = 1$, $\lambda = 30$, and $\bar{\gamma} = 10$ dB. The p_{opt} value decreases and also shifts towards left (moves towards origin) as N value increases. Similarly, the performance of CSS can be improved further by increasing the number of antennas at each CR user. As M value increases from $M = 1$ to $M = 2$, ($Q_m + Q_f$) value decreases by 55.9% at $p = 1.9$, $\lambda = 30$, $N = 2$, and $\bar{\gamma} = 10$ dB. It can be clearly observed from the graph that as the λ value increases, the total error rate curve shifts towards right i.e., moves away from the origin.

In Fig. 12, ($Q_m + Q_f$) performance is shown as a function of λ for various values of N , p , and M . The performance is evaluated for fixed values of $\bar{\gamma}$, V , and using OR-rule at FC. From the simulation we can justify the nature of the curve as follows; as λ value increases, initially the total error rate value decreases to a minimum value, and later it increases. The nature of the curve remains same with other simulation parameters also. Along with the total error rate, optimum value of λ (λ_{opt}) also calculated using this simulation. The λ value at which total error is minimum can be treated as optimum value of λ . The ($Q_m + Q_f$) value decreases by 95.3%, when N value increases from $N = 1$ to $N = 3$ at $\lambda = 10$, $M = 1$, $V = 3$, $\bar{\gamma} = 10$ dB, and $p = 3$. As N value increases, λ_{opt} value also increases and the curve moves away from the origin i.e. shift towards right. The proposed CSS network performance with SC diversity ($M > 1$) outperforms the CSS system performance without diversity ($M = 1$) under

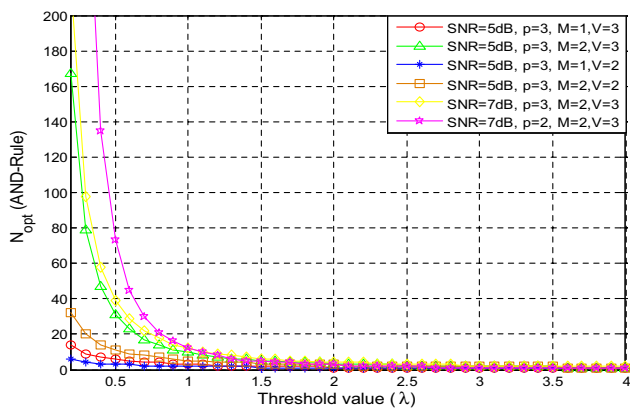


Fig. 13 N_{opt} versus λ using AND-Rule at FC

the similar values of network and channel parameters. When M value increases from $M = 1$ to $M = 2$, $(Q_m + Q_f)$ value decreases by 87.5% at $\lambda = 10$, $N = 3$, $p = 3$, $V = 3$ and $\bar{\gamma} = 10$ dB. It can be clearly observed from simulation that for fixed values of M and N , as p value decreases, total error rate curve shifts towards the origin.

In Fig. 13, N_{opt} value is calculated as a function of λ . The performance is evaluated using AND-Rule at FC for different values of p namely ($p = 2$ and $p = 3$), V namely ($V = 2$ and $V = 3$), M namely ($M = 1$ and $M = 2$), and $\bar{\gamma}$ namely ($\bar{\gamma} = 5$ dB and $\bar{\gamma} = 7$ dB). The N_{opt} value is calculated when S-channel is effected by Weibull fading and R-channel is considered as error free channel. From the simulation, it is clear that N_{opt} value decreases with the increment of λ value using AND-Rule at FC and similar nature is observed with other simulation parameters also. As $\bar{\gamma}$ value decreases from 7 to 5 dB, N_{opt} value decreases from 29 to 23 at $\lambda = 0.6$, $p = 3$, $M = 2$, and $V = 3$, this is due to the decrease in signal strength in S-channel reduces the N_{opt} value. Similarly, as V value increases, N_{opt} value increases due to the increase in fading value which causes a decrease in fading effect in S-channel. As the fading factor value increases from $V = 2$ to $V = 3$, N_{opt} value increases from 11 to 31 at $\lambda = 0.5$, $\bar{\gamma} = 5$ dB, $p = 3$, and $M = 2$. Using the diversity scheme (SC) in our proposed IED-based CSS network improves the N_{opt} value compared to non-diversity scheme. When the number of antennas at each CR is increased from $M = 1$ to $M = 2$, there is a significant improvement in N_{opt} value from 6 to 31 at $\lambda = 0.5$, $\bar{\gamma} = 5$ dB, $p = 3$, and $V = 3$ which implies that the system performance can be improved using the diversity technique in the network.

Two different fusion rules are operated individually at FC, namely OR-Rule and AND-Rule. The N_{opt} value is high at lower values of λ using AND-Rule at FC. Similarly, N_{opt} value is high at higher values of λ using OR-Rule at FC.

Figure 14 shows the performance analysis of probability of detection (Q_d) at FC versus $\bar{\gamma}$ using AND-Rule at FC.

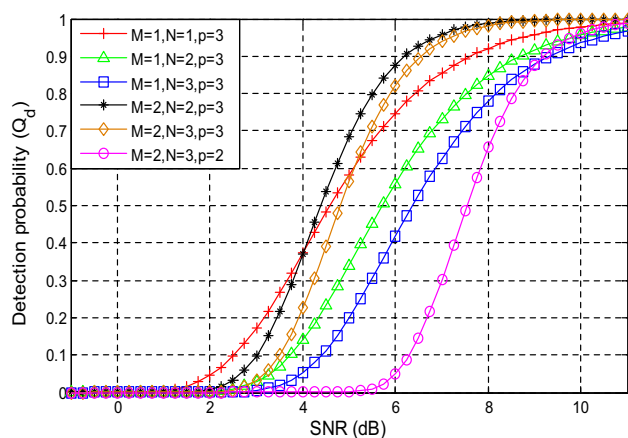


Fig. 14 Q_d versus $\bar{\gamma}$ using AND-Rule at FC

This performance is analysed when S-channel is affected by Weibull fading and R-channel is considered as the ideal (noiseless) channel. Various simulation parameters p , N , and M namely ($p = 2$ and $p = 3$), ($N = 1$, $N = 2$, and $N = 3$), and ($M = 1$ and $M = 2$) are used in the simulation. The Q_d value increases with increase in the $\bar{\gamma}$ value. Similar nature is observed with the other simulation parameters also, this is due to increase in $\bar{\gamma}$ value reduces the noise value which improves the P_d value, consequently Q_d value increases. For $M = 2$, $N = 3$, and $\bar{\gamma} = 7$ dB, when p value decreases from $p = 3$ (IED) to $p = 2$ (CED), Q_d value decreases by 67.8% in Weibull fading channel. The Q_d value increases using the diversity scheme ($M > 1$) in the proposed CSS network compared to non-diversity scheme ($M = 1$) and performance comparison between them is also shown in this simulation. For a particular case, $p = 3$, $\bar{\gamma} = 6$ dB, and $N = 3$, when M value decreases from $M = 2$ to $M = 1$, Q_d value decreases by 49.1% using AND-Rule at FC. It is also observed that Q_d value decreases with the increment of N value. This is due to the fact that the FC makes a final decision about the PU positively only when all CRs responds positively (i.e. PU is present), according to AND-Rule. If any one of the SU responds negatively (i.e. PU is absent), then FC declares that PU is absent. Hence, in case of AND-Rule, Q_d value decreases with the increment in number of CRs. For the fixed values of $\lambda = 30$ and $V = 3$, as N value increases from $N = 1$ to $N = 3$, Q_d value decreases by 44.1% at $M = 1$, $\bar{\gamma} = 6$ dB, and $p = 3$. Similarly, Q_d value decreases by 6.9% for multiple antenna case, when N value decreases from $N = 2$ to $N = 3$ at $\bar{\gamma} = 6$ dB, and $p = 3$.

Figure 15 is drawn between $(Q_m + Q_f)$ and p using AND-Rule at FC. This simulation is drawn with strong support of theoretical expressions and our simulation results are in perfect accordance with theoretical results. Three values of N (namely 1, 2, and 3), two values of M (namely 1 and 2), and two values of λ (namely 30 and 20) are used to simulate

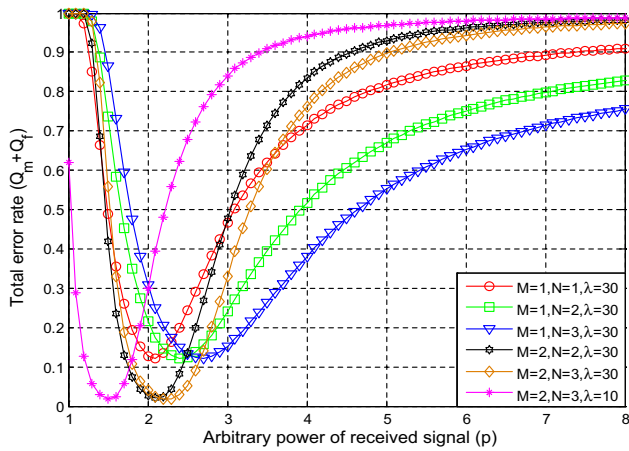


Fig. 15 $Q_m + Q_f$ versus p using AND-Rule at FC

Fig. 14 using MATLAB, with constant $V = 3$ and $\bar{\gamma} = 10$ dB values. It is clear from the simulation that, as p value increases, $(Q_m + Q_f)$ value initially decreases to minimum and it then increases. Similar variations also observed with all other simulation parameters. This nature of the curve is due to the decrease in λ value for increasing p value according to $(\lambda = \lambda_n \sigma_n^p)$. Hence, there is a scope for increment of P_f value at each CR and Q_f value at FC, which results in the improvement of $(Q_m + Q_f)$ value. For a particular case, when N value increases from $N = 1$ to $N = 3$, $(Q_m + Q_f)$ value decreases by 68.1% at $p = 4$, $M = 1$, $\lambda = 30$, and $\bar{\gamma} = 10$ dB. The optimum value of p also increases and shifts towards right as N value increases. Similarly, when M value at each CR increases from $M = 1$ to $M = 2$, the $(Q_m + Q_f)$ value decreases by 60.8% at $p = 2.5$, $\lambda = 30$, $N = 3$, and $\bar{\gamma} = 10$ dB. It can be observed from simulation that as the λ value increases, the total error rate curve shifts towards left i.e. moves towards the origin.

Figure 16 is drawn between average channel throughput (C_{avg}) versus detection threshold (λ) using $k = 1 + n$ fusion rule at FC. The performance is evaluated using an IED scheme in proposed CSS network over Weibull fading channel for different values of N , M and error rate (p_e) in R-channel. Comparison between perfect R-channel and imperfect R-channel (i.e. error rate present in R-channel) also provided to show the effect of an error rate on C_{avg} . It can be observed from Fig. 16 that the throughput value is maximum with error free R-channel and performance decreases as the error rate increases in R-channel.

As p_e value increases from $p_e = 0$ to $p_e = 0.2$, C_{avg} value decreases by 6.1% at $M = 2$ (multiple antennas case), $N = 3$, $\bar{\gamma} = 10$ dB, and $\lambda = 8$. It is also observed that initially throughput value increases for lower values of detection threshold and it reaches to the maximum point after that it decreases for higher values of detection thresholds. The C_{avg} value increases with the cooperation of multiple numbers of

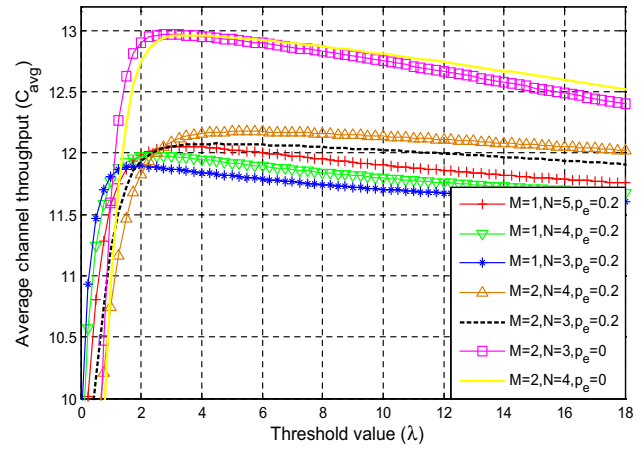


Fig. 16 C_{avg} versus λ using $k = 1 + n$ rule at FC

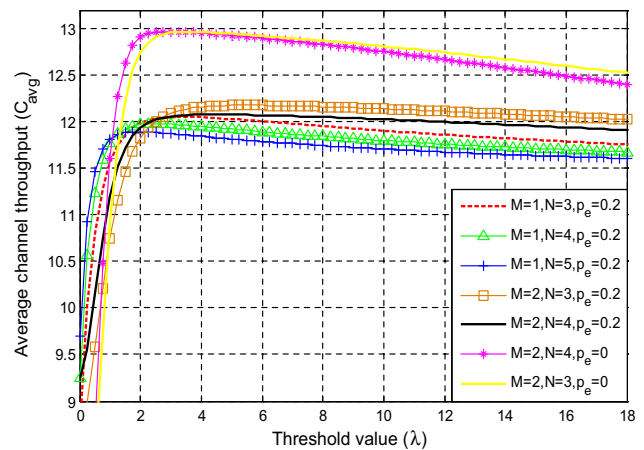


Fig. 17 C_{avg} versus λ using $k = N - n$ rule at FC

SUs using the $k = 1 + n$ fusion rule at FC. As N value increases from $N = 3$ to $N = 5$, C_{avg} value increases by 2.6% for a single antenna case, using $k = 1 + n$ fusion rule, at $\lambda = 6$, $p_e = 0.2$, and $\bar{\gamma} = 10$ dB. Finally, throughput value also depends upon the multiple antennas that are used at each CR. As the number of antennas at each CR increases from $M = 1$ to $M = 2$, C_{avg} value increases by 2.4% at $\lambda = 6$, $N = 4$, $p_e = 0.2$, and $\bar{\gamma} = 10$ dB. The above graph is simulated for following network parameters $\bar{\gamma} = 10$ dB, $p_e = 0$ and 0.2, $k = 1 + n$ ($n = 2$) fusion rule, $C_s = 10$, $\check{C}_s = 5$, $C_p = 20$, $\check{C}_p = 10$, $M = 1$ and 2 and $N = 3, 4$, and 5.

Figure 17 is drawn between C_{avg} versus λ using $k = N - n$ fusion rule at FC. The performance comparison between perfect (i.e. error free) R-channel and imperfect R-channel is provided. It can be observed from Fig. 17 that the throughput value is maximum with error free R-channel and performance decreases as the error rate increases in R-channel. As p_e value increases from $p_e = 0$ to $p_e = 0.2$, C_{avg} value decreases by 5.7% at $M = 2$ (multiple antennas case), $N = 3$, $\bar{\gamma} =$

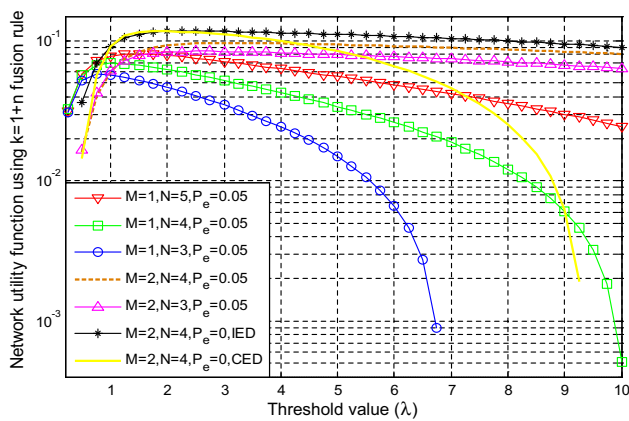


Fig. 18 NUF versus λ using $k = 1 + n$ rule at FC

10 dB, and $\lambda = 6$. C_{avg} value decreases with the cooperation of multiple numbers of SUs using $k = N - n$ fusion rule. As N value increases from $N = 3$ to $N = 5$, C_{avg} value decreases by 1.6% for single antenna case, using $k = N - n$ fusion rule at FC, at $\lambda = 10$, $p_e = 0.2$, $\bar{\gamma} = 10$ dB. Finally, as the number of antennas at each CR decreases from $M = 2$ to $M = 1$, C_{avg} value decreases by 1.7% at $N = 4$, $p_e = 0.2$, $\lambda = 8.25$, and $\bar{\gamma} = 10$ dB. The above mentioned simulation parameters (in Fig. 16) are used in this simulations also except $k = N - n$ fusion rule at FC.

The effect on C_{avg} performance using $k = 1 + n$ rule and $k = N - n$ fusion rules at FC is that, C_{avg} value increases with the cooperation of CR users using $k = 1 + n$ rule and it decreases using $k = N - n$ fusion rule at FC.

Figure 18 is drawn between network utility function (NUF) and detection threshold (λ) using $k = 1 + n$ fusion rules at FC. NUF performance is evaluated using an IED scheme in the proposed CSS network over Weibull fading channel with an error rate (p_e) in R-channel. Network utility factor decreases as the error rate increases in the R-channel and this value is maximum with error free channel. As p_e value increases from $p_e = 0$ to $p_e = 0.05$, NUF value decreases by 16.3% at $\lambda = 4$, $M = 2$, $N = 4$, $p = 3$, and $\bar{\gamma} = 10$ dB. Using $k = 1 + n$ fusion rule at FC, as the cooperation among the CR users are increases, NUF value increases. If the number of SUs are increases from $N = 3$ to $N = 5$, NUF value increases by 159.4% at $M = 1$, $p_e = 0.05$, $\lambda = 4$, and $\bar{\gamma} = 10$ dB. Similarly, as the number of antennas at each CR increases, NUF value increases. As M value increases from $M = 1$ to $M = 2$, NUF value decreases by 167.2% at $N = 4$, $p_e = 0.05$, $\lambda = 4$, and $\bar{\gamma} = 10$ dB. Simulation parameters $P(H_1) = 0.3$, $P(H_0) = 0.7$, $\mu_1 = 0.2$, $\mu_2 = 0.8$, $\mu_3 = 0.005$, $M = 1$ and 2, $N = 3, 4$ and 5, $p_e = 0$ and 0.05, and $\bar{\gamma} = 10$ dB are used to evaluate these MATLAB simulations (Figs. 18 and 19).

Figure 19 is drawn between NUF and λ using $k = N - n$ fusion rules at FC. The performance comparison between

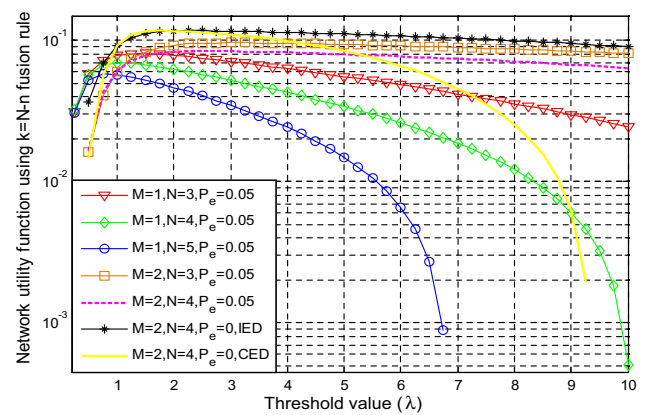


Fig. 19 NUF versus λ using $k = N - n$ rule at FC

error free and error rate present in R-channel is provided using the simulation. As p_e value increases from $p_e = 0$ to $p_e = 0.05$, NUF value decreases by 28.1% at $\lambda = 1$, $M = 2$, $N = 4$, $p = 3$, and $\bar{\gamma} = 10$ dB. Using $k = N - n$ fusion rule at FC, as the cooperation among the CR users are increases, NUF value decreases. If the number of SUs are increases from $N = 3$ to $N = 5$, NUF value decreases by 51.0% at $M = 1$, $p_e = 0.05$, $\lambda = 3$, and $\bar{\gamma} = 10$ dB. Similarly, the number of multiple antennas at each CR increases, it improves the NUF value. As M value increases from $M = 1$ to $M = 2$, NUF value decreases by 51.4% at $N = 3$, $p_e = 0.05$, $\lambda = 4$, and $\bar{\gamma} = 10$ dB. Finally, NUF value also depends upon arbitrary power of the received signal (p). From the graph we can able to say that NUF value improves with an IED scheme ($p = 3$) compared to CED scheme ($p = 2$), NUF value reduces with the reduction of p value.

7 Conclusion

In this paper, we have proposed the cooperative spectrum sensing (CSS) network which is equipped with multiple antennas at each cognitive radio (CR). An improved energy detector (IED) scheme is used as detection scheme and performance is accessed over Weibull fading channel. Selection combining (SC) scheme is used at each CR, it receives the binary decisions about the primary user (PU) from an IED technique using the multiple antennas and selects better detection value of PU. The sensing information about the PU is passed to the fusion center (FC) through reporting channel (R-channel). Final decision made at FC using different fusion rules (*OR-Rule*, *AND-Rule*, $k = 1 + n$, and $k = N - n$ rules). We have derived novel expressions for missed detection and probability of false alarm using the multiple antennas at each CR in the presence of Weibull fading channel. The performance is analyzed using complementary receiver operating characteristics (CROC) curves, total error rate, throughput

analysis, and network utility function using the proposed CSS network. Finally, we have derived closed form of expressions for network parameters such as optimum number of CRs, optimum arbitrary power of received signal, and optimum detection threshold at each CR to obtain the optimal performance of CSS network. Our simulation results are perfectly inaccordance with the analytical results. The simulation results are depends upon various values of network parameters such as average sensing channel SNR ($\bar{\gamma}$), multiple antennas (M) at each CR, detection threshold value (λ), arbitrary power of the received siganl (p), Weibull fading parameter (V), and number of CR users (N). The performance is evaluated considering single and multiple antennas at each CR in Weibull fading channel. Comparison between CED and IED schemes are also provided. Finally, we can conclude that detection performance is improved by considering multiple antennas at each CR using an IED scheme.

8 Appendix

8.1 Expression for threshold value

The closed form of expression for threshold value using the multiple antennas at each CR can be calculated with the help of Eq. (18). Applying *logarithm* on both sides to Eq. (18) then it reduces to:

$$\begin{aligned} \Rightarrow M \ln \left(1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) &= \ln(1 - P_f) \\ \Rightarrow \exp(\ln(1 - P_f)/M) &= 1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right), \\ \Rightarrow 1 - \exp(\ln(1 - P_f)/M) &= \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right), \\ \Rightarrow \ln(1 - \exp(\ln(1 - P_f)/M)) &= \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right), \end{aligned} \tag{63}$$

$$\lambda = \left(\frac{\sigma_n^2}{\Gamma(P)} (\ln(1/(1 - \exp(\ln(1 - P_f)/M))))^{1/C} \right)^{p/2} \tag{64}$$

8.2 Optimization of CSS network parameters

A: Optimization of threshold value using P_f and P_m expressions:

An optimum value of threshold (λ_{opt}) is required to decide the existance of PU with minimum threshold value. The closed

form of expression for optimum value of threshold for single antenna case can be calculated by differentiate Eqs. (9 and 17) w.r.t to λ :

$$\begin{aligned} \frac{\partial P_f}{\partial \lambda} + \frac{\partial P_m}{\partial \lambda} &= 0 \tag{65} \\ \Rightarrow - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \\ &\quad \frac{2\lambda^{(2/p)-1} \Gamma(P)}{p\sigma_n^2} + \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \\ &\quad C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{(1 + \gamma) \sigma_n^2} \right\}^{C-1} \frac{2\lambda^{(2/p)-1} \Gamma(P)}{p\sigma_n^2 (1 + \gamma)} = 0 \\ \Rightarrow \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \\ &= \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \left\{ \frac{1}{(1 + \gamma)} \right\}^C, \\ \Rightarrow \left(\left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) &= \left(\left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \\ &\quad + C \ln(1 + \gamma), \\ \Rightarrow \left(\left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right) \\ &= C \ln(1 + \gamma), \\ \Rightarrow \left(\left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) &= C \ln(1 + \gamma) / \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right), \end{aligned} \tag{66}$$

$$\lambda_{opt} = \left(\frac{\sigma_n^2}{\Gamma(P)} \left(C \ln(1 + \gamma) / \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \right)^{p/2} \tag{67}$$

B: Optimization of arbitrary power of received signal (p) using P_f and P_m expressions:

It is also necessary to optimize the arbitrary power of the received signal. The optimum value of p (p_{opt}) can be calculated by differentiating Eqs. (9) and (17) w.r.t p for a single antenna case:

$$\frac{\partial P_f}{\partial p} + \frac{\partial P_m}{\partial p} = 0 \tag{68}$$

$$\frac{\partial P_m}{\partial p} = -M \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) \right]^{M-1} \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^{C-1} \frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2(1+\gamma)} \quad (69)$$

$$\frac{\partial P_f}{\partial p} = M \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right]^{M-1} \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2} \quad (70)$$

adding Eqs. (69) and (70), then make equal to zero.

$$\begin{aligned} &\Rightarrow \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \\ &= \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) \left\{ \frac{1}{(1+\gamma)} \right\}^C, \\ &\Rightarrow \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \left(1 - \left(\frac{1}{(1+\gamma)} \right)^C \right) = C \log(1+\gamma), \\ P_{opt} &= \frac{2 \ln \lambda}{\ln \left(\left\{ \frac{C \ln(1+\gamma)}{\left[1 - \left(\frac{1}{1+\gamma} \right)^C \right]} \right\}^{1/C} \left(\frac{\sigma_n^2}{\Gamma(P)} \right) \right)} \quad (71) \end{aligned}$$

8.3 Optimization of network parameters using OR-Rule at FC

a: Optimization of number of CRs (N_{opt}):

The closed form of expression for N_{opt} value for OR-Rule can be calculated using Eq. (33) as follows:

$$\text{Total error rate: } Z(N) = Q_m + Q_f$$

$$\begin{aligned} \Delta Z(N) &= Z(N+1) - Z(N) \\ &\Rightarrow 1 - [(1 - P_f)]^{N+1} - 1 + [(1 - P_f)]^N \\ &\quad + [P_m]^{N+1} - [P_m]^N = 0, \end{aligned}$$

$$\begin{aligned} &\Rightarrow [P_m]^{N+1} - [(1 - P_f)]^{N+1} - [P_m]^N \\ &\quad + [(1 - P_f)]^N = 0, \\ &\Rightarrow 1 - ((1 - P_f)/P_m)^{N+1} - (1/(P_m)) \\ &\quad + \frac{1}{(P_m)} ((1 - P_f)/P_m)^N = 0, \\ &\Rightarrow 1 - (1/(P_m)) = ((1 - P_f)/P_m)^{N+1} \\ &\quad - \frac{1}{(P_m)} ((1 - P_f)/P_m)^N, \quad (72) \end{aligned}$$

after simplification, the above expression reduces to

$$\Rightarrow ((1 - P_m)/P_f) = ((1 - P_f)/P_m)^N,$$

applying *logarithm* on both sides, finally, the closed form of expression for N_{opt} using OR-Rule at FC is

$$N_{opt} = \left\lceil \frac{\ln \left(\frac{1-P_m}{P_f} \right)}{\ln \left(\frac{1-P_f}{P_m} \right)} \right\rceil \quad (73)$$

b: Optimization of threshold value (λ_{opt})

The closed form of expression for λ_{opt} value can be calculated by differentiating Eq. (33) w.r.t to λ , and equating to zero.

$$\frac{\partial Q_m}{\partial \lambda} + \frac{\partial Q_f}{\partial \lambda} = 0 \quad (74)$$

$$\Rightarrow N(1 - P_f)^{N-1} \frac{\partial P_f}{\partial \lambda} + N(P_m)^{N-1} \frac{\partial P_m}{\partial \lambda} = 0 \quad (75)$$

For a single antenna case ($M = 1$), $\partial P_f/\partial \lambda$ and $\partial P_m/\partial \lambda$ expressions are given in Eq. (24), substituting Eqs. (24) in (75):

$$\begin{aligned} &\Rightarrow (1 - P_f)^{N-1} \left(- \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) \\ &\quad C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \frac{2\lambda^{(2/p)-1} \Gamma(P)}{p\sigma_n^2} \\ &\quad + N(P_m)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) \right) \\ &\quad C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{(1+\gamma)\sigma_n^2} \right\}^{C-1} \frac{2\lambda^{(2/p)-1} \Gamma(P)}{p\sigma_n^2(1+\gamma)} = 0, \end{aligned}$$

after some algebraic simplifications, above equation reduces to

$$\begin{aligned} &\Rightarrow (1 - P_f)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) \\ &= (P_m)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \right) \left(\frac{1}{1 + \gamma} \right)^C, \end{aligned}$$

applying *logarithm* on both sides and solving algebraic expressions, the above expression reduces to

$$\begin{aligned} \lambda_{opt} &= \left(\frac{\sigma_n^2}{\Gamma(P)} \left((N - 1) \left(\ln \left(\frac{1 - P_f}{P_m} \right) \right) \right. \right. \\ &\quad \left. \left. + C \ln(1 + \gamma) \right) / \left(1 - \left\{ \frac{1}{(1 + \gamma)} \right\}^C \right) \right)^{1/C} \end{aligned} \tag{76}$$

c: Optimization of arbitrary power of the received signal (p_{opt})

The closed form of expression for p_{opt} can be calculated by differentiating Eq. (33) w.r.t to p ,

$$\frac{\partial Q_m}{\partial p} + \frac{\partial Q_f}{\partial p} = 0 \tag{77}$$

$$\Rightarrow (1 - P_f)^{N-1} \frac{\partial P_f}{\partial p} + (P_m)^{N-1} \frac{\partial P_m}{\partial p} = 0 \tag{78}$$

For a single antenna case ($M = 1$), $\partial P_f/\partial p$ and $\partial P_m/\partial p$ expressions are given in Eqs. (28) and (29), substituting these equations in Eq. (78):

$$\begin{aligned} &\Rightarrow N (1 - P_f)^{N-1} \left(-M \left[1 - \exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right] \right)^{M-1} \\ &\quad \exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^C \right) C \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2} \\ &+ N (P_m)^{N-1} \left(-M \left[1 - \exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \right] \right)^{M-1} \\ &\quad \exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) C \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^{C-1} \frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2 (1 + \gamma)} = 0, \end{aligned}$$

after some algebraic simplifications, above equation reduces to

$$\begin{aligned} &\Rightarrow (1 - P_f)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) \\ &= (P_m)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \right) \left(\frac{1}{1 + \gamma} \right)^C, \end{aligned}$$

applying *logarithm* on both sides and solving algebraic expressions, the above expression reduces to

$$\Rightarrow \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} = \frac{(N - 1) \left(\ln \left(\frac{1 - P_f}{P_m} \right) \right) + C \ln(1 + \gamma)}{\left(1 - \left(\frac{1}{1 + \gamma} \right)^C \right)},$$

apply *logarithm* on both sides, after simplification, the closed form of expression for p_{opt} is:

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left(\frac{(N-1) \left(\ln \left(\frac{1 - P_f}{P_m} \right) \right) + C \ln(1 + \gamma)}{\left(1 - \left(\frac{1}{1 + \gamma} \right)^C \right)} \right)^{1/C} \frac{\sigma_n^2}{\Gamma(P)} \right)} \tag{79}$$

8.4 Optimization of network parameters using AND-Rule at FC

a: Optimization of threshold value (λ_{opt})

The closed form of expression for λ_{opt} value can be calculated using Eqs. (24), (35), (38) and (39):

$$\begin{aligned} &\Rightarrow N (P_f)^{N-1} \left(- \exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) \\ &\quad C \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \frac{2 \lambda^{(2/p)-1} \Gamma(P)}{p \sigma_n^2} \\ &+ N (1 - P_m)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \right) \\ &\quad C \left\{ \frac{\lambda^2/p \Gamma(P)}{(1 + \gamma) \sigma_n^2} \right\}^{C-1} \frac{2 \lambda^{(2/p)-1} \Gamma(P)}{p \sigma_n^2 (1 + \gamma)} = 0 \end{aligned} \tag{80}$$

after some algebraic simplifications, above equation reduces to

$$\begin{aligned} &\Rightarrow (P_f)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) \\ &= (1 - P_m)^{N-1} \left(\exp \left(- \left\{ \frac{\lambda^2/p \Gamma(P)}{\sigma_n^2 (1 + \gamma)} \right\}^C \right) \right) \left(\frac{1}{1 + \gamma} \right)^C, \end{aligned}$$

applying *logarithm* on both sides and solving algebraic expressions, the above expression reduces to

$$\lambda_{opt} = \left(\frac{\sigma_n^2}{\Gamma(P)} \left((N-1) \left(\ln \left(\frac{P_f}{1-P_m} \right) \right) + C \ln(1+\gamma) \right) \right)^{1/C} \left(1 - \left\{ \frac{1}{1+\gamma} \right\}^C \right)^{1/C} \tag{81}$$

b: Optimization of arbitrary power of the received signal (p_{opt})

The closed form of expression for p_{opt} value can be calculated using Eqs. (35), (42), (43), (28) and (29):

$$\begin{aligned} \Rightarrow N(P_f)^{N-1} & \left(-M \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right] \right)^{M-1} \\ & \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^{C-1} \frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2} \right) \\ + N(1-P_m)^{N-1} & \left(-M \left[1 - \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) \right] \right)^{M-1} \\ & \exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) C \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^{C-1} \frac{2 \lambda^{(2/p)} \log \lambda \Gamma(P)}{p^2 \sigma_n^2(1+\gamma)} \right) = 0 \end{aligned} \tag{82}$$

after some algebraic simplifications, above equation reduces to

$$\begin{aligned} \Rightarrow (P_f)^{N-1} & \left(\exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2} \right\}^C \right) \right) \\ = (1-P_m)^{N-1} & \left(\exp \left(- \left\{ \frac{\lambda^{2/p} \Gamma(P)}{\sigma_n^2(1+\gamma)} \right\}^C \right) \right) \left(\frac{1}{1+\gamma} \right)^C, \end{aligned}$$

applying *logarithm* on both sides and solving algebraic expressions, the above expression reduces to

$$p_{opt} = \frac{2 \ln \lambda}{\ln \left(\left(\frac{(N-1) \left(\ln \left(\frac{P_f}{1-P_m} \right) \right) + C \ln(1+\gamma)}{\left(1 - \left(\frac{1}{1+\gamma} \right)^C \right)} \right)^{1/C} \frac{\sigma_n^2}{\Gamma(P)} \right)} \tag{83}$$

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