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Performance analysis of cellular networks with mobile relays under different modes

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Abstract With the popularity of smart phones and tablets, people make intensive use of these devices on public transport. The deployment of mobile relays on public transport may increase the quality of mobile services.The objective of this paper is to study the performance of cellular networks when mobile relays are deployed in public transport vehicles. We consider two modes: in the FDD mobile relay mode, the same spectrum is reused for all links while in the TDD/FDD hybrid mode, a small part of the spectrum is dedicated to the access link between the terminals inside a transport vehicle and the mobile relay. We provide a general analytical model for the two mobile relay modes by using the stochastic geometry approach. Key metrics like the CDF of the SINR and the CDF of the end-to-end rate are computed. Furthermore, the cell total average rate and the energy efficiency in different modes are evaluated. It has been found that penetration loss is a factor that determines how much gain mobile relay can bring. Numerical results show that when the ratio of vehicular UEs in the cell is 0.4 and the penetration loss is 20 dB, the FDD mobile relay mode and the TDD/FDD hybrid mobile relay mode can achieve +16.3, +29.1% cell rate gain respectively compared with the direct mode.

Keywords 4G network · Mobile relay · Mobility management · Stochastic geometry · Performance analysis

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1 Introduction

The use of wireless broadband services is rising significantly with the deployment of Long Term Evolution (LTE) networks and the generalisation of smart phones, tablet computers and other new mobile devices. People make intensive use of these devices to kill time when they are on public transport vehicles such as buses, trams, or trains. It is predicted that, by the year 2020, it will be fairly common to have up to 50 active vehicular User Equipment devices (UE) per bus and up to 300 active vehicular UEs per train [1].

The service quality on public transport vehicles is far from satisfactory. They are usually well shielded with coated windows, which leads to a rather high penetration loss between outdoor and in-vehicle [2]. For the 2 GHz band, the penetration loss can be as high as 25 dB, and even goes up to 35 dB in Shanghai high-speed magnetic levitation train [3]. Traditionally, the UEs inside the public transport system are connected to macro base stations via wireless links, in which the penetration loss severely attenuates the signal quality and decreases the achievable data rate.

Offering high-quality services to UE on public transport is becoming an important and challenging issue [4,5]. Several solutions have been discussed to address this issue, such as optimizing the deployment of macrocells, using layer-1 repeaters, or LTE backhaul plus on-board Wi-Fi access [6]. However, these solutions can only partially mitigate the problems. For example, deploying dedicated macrocells for vehicular UEs cannot eliminate signal loss caused by penetration loss, which is one of the major challenges faced by vehicular UEs. Layer-1 repeaters amplify signals in a given frequency band, but the Signal-to-Interference-plus-Noise Ratio (SINR) is not improved. Hence, they can only be deployed at places with good SINR. Using WiFi to provide Internet access to vehicular UEs is fairly common, but the quality of service and the security of the WiFi access are not guaranteed, and offering a seamless LTE-WiFi connectivity to users is still not possible.

Aware of the limitations of the above solutions, the Third Generation Partnership Project (3GPP) is investigating the deployment of mobile relays on public transport such as buses, trams, or trains [6]. When the users are grouped on a bus or a tram, this can be seen as a mobile hot spot. Wireless connectivity is provided to vehicular UE through the *access link*, which is defined as the link between vehicular UEs and the in-vehicle antenna of the mobile relay. The *backhaul link* is defined as the link between the outdoor antenna of the mobile relay and the donor base station. By using two separate transmission links, the penetration loss can be eliminated but interference may be increased because the number of transmitters is larger. One of the main issues related to mobile relay is how the spectrum is allocated on the different links. Using different carriers for the access link and for the backhaul link can avoid any interference between the two links and increase the data rate, but is a waste of precious radio resources. Using the same carriers for both links is more efficient, but would generate more interference and reduce the data rate. The question is thus to evaluate the capacity in Mbps of a cellular system with and without relays for different spectrum allocations.

1.1 Related works

Extensive work has been done on various aspects of fixed relay network, including downlink performance [7], uplink performance [8], coverage [9], resource allocation [10] and optimal relay placement [11]. However, the studies on fixed relay can not be directly applied to mobile relay scenarios: by definition, mobile relays can be anywhere in a cell. Furthermore, fixed relays do not compensate the signal loss caused by penetration loss, which is a major concern for vehicular UEs.

1.1.1 Mobile relay deployed on high speed trains

Most preliminary studies [12–18] focused on high speed train scenarios and showed capacity and coverage improvement through the deployment of mobile relays. In [12], the author investigated the capacity gain for high speed trains with relay assistance. The study is based on a simple system model without fast fading. In [14,15], the authors presented the throughput gain using coordinated and cooperative mobile relays on top of trains. The study in [19] discussed the resource allocation problem for high-speed railway communication. The authors formulated an optimization resource allocation problem and tried to maximize the number of vehicular UEs served. An algorithm based on the Hungarian method was proposed. In high-speed environment, the doppler frequency shift has significant effects. The authors in [16,17,20] addressed this issue. In [18], it was shown that by using predictor antennas mounted on top of vehicles, the reliability of the backhaul links can be enhanced.

The main issue related to mobile relays in trains is how to manage high speed. Due to the low density of trains, it is not worth studying the capacity of networks when the number of mobile relays increases.

1.1.2 Mobile relay deployed on buses

Several studies are related to the performance of mobile relays in buses. In [21], the authors presented a very simplified system model without shadowing fading and fast fading. In [22], the authors studied the performance of mobile relay assisted transmission and compared it with the transmission assisted by fixed relays. Then in [23], they continued to study the benefits of mobile relays from an energy efficiency point of view. However, the impact of the interferences from neighbor eNBs and other mobile relays was not investigated. In [24], the authors considered a two-cell system and studied the outage probability of a vehicular user served by mobile relay and compared it with optimized fixed relay assisted transmission. In [25], the authors evaluated different intercell interference coordination and multi-antenna interference suppression techniques for mobile relays but the study is based on simulations. In [26], the authors evaluated the performance of mobile relays but WiFi or Bluetooth are used on the access link. In [27], the authors did some field trial tests on uplink joint detection for mobile relays and compared the performance of different detection schemes.

The systems considered in the above mentioned studies were either a noise-limited system with one vehicular UE device or an interference-limited one but with only two cells. Most of the related works focused on regular network deployment (i.e., the hexagonal grid model) and evaluated the performance of mobile relays by system-level simulations. There is no system analysis with several cells, UEs and relays that is able to accurately capture the spatial irregularity and randomness of a real cellular network with mobile relays.

1.1.3 Stochastic analysis related to mobile relay

Stochastic geometry has recently been widely used as a tractable approach to model and quantify the key metrics (outage probability and rate) in wireless networks [28–31]. This approach has the advantage of being scalable to multiple layers of base stations and accurate to model location randomness. Additionally, powerful tools from stochastic geometry can be used to get closed-form expressions of the distributions of the SINR and the rate. Extensive studies have been made on various wireless networks using the stochastic geometry approach [28,32,33].

In [32], the authors developed a tractable and accurate model for a downlink heterogeneous cellular network consisting of K tiers of randomly located BSs and derived the average rate of the link between UE and the associated BS. In this model the authors considered a wired backhaul link, which is connected to a core network through fiber or cable line. But in a relay network, the backhaul link is wireless and it is usually the bottleneck. In [34], the authors proposed an analytical model to evaluate the energy efficiency of cooperative relay networks. Based on the spatial Poisson Point Process (PPP) the distribution of SINRs and mean achievable rates of both non-cooperative users and cooperative users were derived. In [34] a cooperative relaying technique was used as a complementary transmission for the UEs close to relay stations.

In [35,36], the authors studied the energy efficiency of a relay-assisted network with the assumption that an eNodeB only serves at most one UE or one relay at any time. How the bandwidth was shared among users within a cell was not considered. In [37], the energy efficiency of cooperative networks aided by the energy harvest technique was investigated through stochastic analysis. In [38], the energy efficiency of a relay-assisted cellular network was evaluated, but it only considered a single cell. To the best of our knowledge, none of the aforementioned studies used the stochastic geometry approach to investigate the benefits of mobile relays from the network point of view.

1.2 Our contributions

Our objective is to determine whether the use of mobile relays in buses (or tramways) can increase the capacity and the energy efficiency of a cellular system. We assume an operator with a given spectrum and consider the different modes of bandwidth sharing between the access link and the backhaul link. We consider a first mode where the same spectrum is used for both links and a second mode where a part of the spectrum is dedicated to the access link (and not used on the backhaul link).

The main contributions of our paper are as follows. First, we use stochastic geometry to get a general analytical model and compute the Cumulative Distribution Function (CDF) of the SINR. In addition to our previous work [39], we consider different cases regarding the use of the spectrum on the access link.

Second, we compute the end-to-end user bit rate for the different types of UEs. Our model captures the following effects:

- For vehicular UE, the transmission is done in two steps. The bottleneck can be either on the access or on the backhaul link.
- When there is a strong interferer close to a bus, both the backhaul and the access links are impacted if the same frequency is used on both links. The rate of the backhaul link and the rate of the access link are thus correlated in that case.
- The bandwidth is shared among users within a cell or in other words between the backhaul links of relays and the ordinary UE. As a first-step analysis, we consider Round-Robin scheduling: the bandwidth is equally divided among all UE (vehicular and ordinary UE).

We also compute the cell capacity defined as the total average bit rate of all users and the energy efficiency of the network. These are the key performance indicators to evaluate the benefits of deploying mobile relays on a large scale.

The remainder of the paper is organized as follows: Sect. 2 describes the system model, including the different system modes, the network model and the propagation model. In Sect. 3, the CDF of the SINR on different links is computed. In Sect. 4, the CDF of the rate in different modes is derived. Then in Sect. 5, the power consumption model is defined and the analysis of energy efficiency is given. The numerical results are presented and analyzed in Sect. 6. Finally, concluding remarks are made in Sect. 7.

2 System model

2.1 System modes

We consider an operator which has $2W$ bandwidth (e.g. W for downlink and W for uplink) for the network. In this work we focus on the bit rates of mobile users inside or outside the buses for a simple scheduling policy (round robin). As schedulers typically operate on a 10-ms cycle, the speed of terminals or relays has no impact within a scheduling cycle (at 36 km/h, the distance travelled in 10 ms is 10 cm). We thus consider a snapshot of the system with a given distribution of users and buses. In the following, we consider three system modes and analyze their differences.

2.1.1 Direct mode

All types of UE (ordinary UE and vehicular UE) are directly associated with a base station. The link between UE and base station is defined as *the direct link*. There are no mobile relays in the network. For vehicular UEs, the signal is deteriorated by penetration loss. This mode is a baseline mode.

2.1.2 FDD mobile relay model

In the Frequency Division Duplex (FDD) mode, ordinary UE are associated with base stations, while vehicular UE are

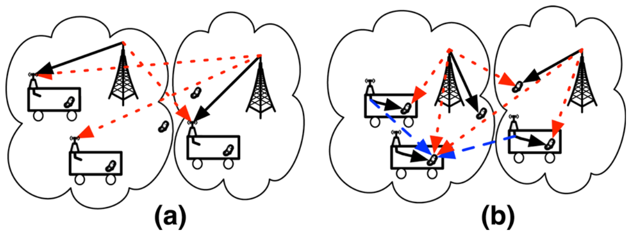

Fig. 1 Interference illustration in FDD mobile relay mode with frequency reuse (the power is represented as a *solid line*, interference is represented as a *dashed line*)

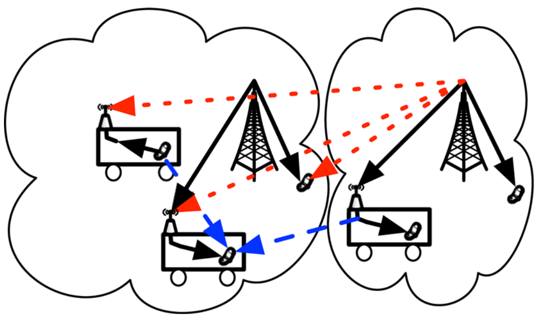

Fig. 2 Interference illustration in TDD/FDD hybrid mobile relay mode (the power is represented as a *solid line*, interference is represented as a *dashed line*)

associated with mobile relays installed on the bus. The backhaul links and the access links are both based on FDD with the same pair of frequencies. When a relay receives data from the donor base station, it cannot transmit data to vehicular UEs. Otherwise, it would create self-interference and degrade reception by the relay. However, when the relay transmits data to vehicular UE, the donor base station can transmit to ordinary UE. Full frequency reuse is applied among the direct links and the access links. Base stations and mobile relays transmit to their associated UE on the same spectrum simultaneously.

The interference on mobile relays comes from neighbor base stations. The interference on vehicular UE comes from both other relays and all base stations. The interference on ordinary UE comes from other base stations and all relays, as shown in Fig. 1.

2.1.3 TDD/FDD hybrid mobile relay mode

The Time Division Duplex / Frequency Division Duplex (TDD/FDD) hybrid mobile relay mode is based on the reservation for the access link of a portion $(1-\alpha)$ of the total bandwidth the operator has (i.e. $2W$). The bandwidth for the backhaul and for ordinary UE is thus lowered : as FDD is used, αW is available for downlink as well as for uplink. The access link of mobile relay operates on a dedicated bandwidth. The backhaul link and the access link can thus operate independently without additional complex mechanisms to avoid self-interference. The access link is used in TDD mode and thus, $(1-\alpha)2W$ is available. Let ρ be the parameter for downlink/uplink subframe configuration. The available resource for downlink on the access link is $\rho(1-\alpha)2W$.

The interference on vehicular UE comes from neighbor relays that transmit on the downlink and other vehicular UE that transmit on the uplink. The interference on ordinary UEs and mobile relays comes from neighbor base stations, as shown in Fig. 2.

The differences between these modes are shown in Table 1.

2.2 Network model

We consider transmissions on the downlink. Base stations, buses and ordinary UEs are spatially distributed as homogeneous PPP Π_s, Π_r and Π_o with respective intensities λ_s, λ_r and λ_o, as shown in Fig. 3. Each bus is equipped with one mobile relay. There are several vehicular UEs on each bus. Each vehicular UE is always attached to the mobile relay of the bus in which it is located. That relay is called the serving mobile relay. The number of vehicular UEs on each bus is assumed to follow a Poisson distribution with an average of $\overline{n_v}$.

The distance between vehicular UE and its serving mobile relay is relatively small compared to the distance to the base stations and between the relays. In other words, a bus and all vehicular UE on the bus can be seen as at the same location. Without loss of generality, we assume that the UE and the mobile relay under consideration are located at the origin. The other relays are distributed as a PPP.

2.3 Propagation model

The propagation loss is computed with the Okumura-Hata model. Both shadowing and fading are considered. Let y be the distance from a transceiver to a receiver, the received signal power p is:

$$p = rhky^{-\gamma}P = rhy^{-\gamma}\chi, \tag{1}$$

Table 1 The differences between different modes

	Penetration loss	Interference on the backhaul link	Interference on the access link
Direct mode	Yes	No backhaul link	No access link
FDD mobile relay mode	Eliminated	Neighbor BSs	All BSs and neighbor relays
TDD/FDD mobile relay mode	Eliminated	Neighbor BSs	Neighbor relays

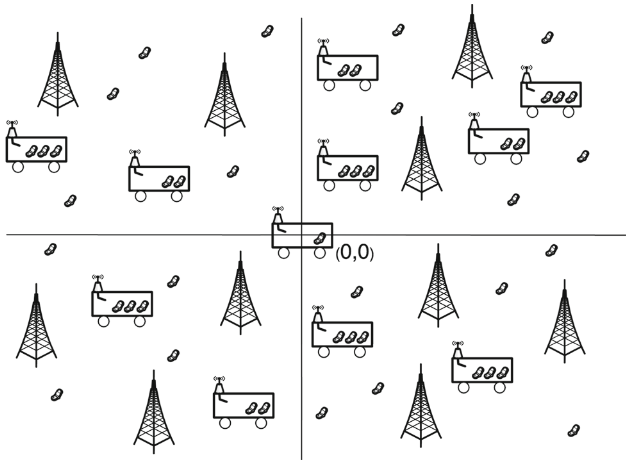

Fig. 3 Network Model

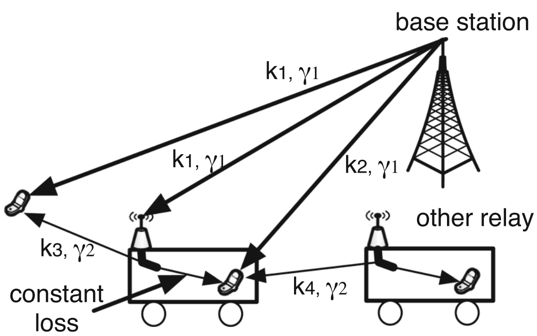

Fig. 4 Illustration of propagation loss on different links

where r and h are factors that take the fading effect and shadowing into account, k is a propagation factor, γ is the path loss exponent, P is the transmit power, χ is a reference power defined as kP. Variable χ takes both the transmit power and the specific propagation factor on the considered link into account. Note that χ is the power received at distance 1 ($y = 1$) when there is no fading ($r = 1$) and no shadowing ($h = 1$). We assume all base stations have the same transmit power P_S and all mobile relays have the same fixed transmit power P_R.

In the considered network, different links suffer different propagation loss (k, γ), as shown in Fig. 4. The direct link between ordinary UE and base station suffers outdoor path loss. The backhaul link also suffers outdoor path loss. The direct link between vehicular UE and base station suffers outdoor path loss and penetration loss. We assume free space propagation without shadowing on the access link between vehicular UE and its serving mobile relay. The received power on the access link when there is no fading is thus constant and denoted by p_0.

The fading effect is modeled by an exponential r.v. of mean 1. The shadowing effect is modeled by a log-normal random variable H. In other words, $10 \log H$ is a Gaussian r.v. with standard deviation σ. It has been proved that the effect of shadowing can be equivalently interpreted as a random displacement of the original location [40].

Lemma 1 *For a homogeneous PPP $\Phi \in \mathbb{R}^2$ with density λ, if each point $x \in \Phi$ is transformed to $x' \in \mathbb{R}^2$ such that $x' = h^{-\frac{1}{\gamma}} x$, where h are i.i.d (independent and identically distributed), such that $\mathbb{E}h^{\frac{2}{\gamma}} < \infty$, the new point process $\Phi^{(e)} \in \mathbb{R}^2$ defined by the transformed points x' is also a homogeneous PPP with density $\lambda^{(e)} = \lambda \mathbb{E}h^{\frac{2}{\gamma}}$.*

The details of the proof can be found in [41]. Using the transformed points, we can re-write (1) as

$$p = r\left(y'\right)^{-\gamma} \chi, \tag{2}$$

where y' is the distance of a point in the new PPP with density $\lambda \mathbb{E}h^{\frac{2}{\gamma}}$. Therefore, base stations, buses and ordinary UEs can be equivalently spatially distributed as PPPs Π'_s, Π'_r and Π'_o with intensities λ'_s, λ'_r and λ'_o, where $\lambda'_i = \lambda_i \mathbb{E}h^{\frac{2}{\gamma}}$, $i = \{s, r, o\}$ and $\gamma = \{\gamma_1, \gamma_2\}$.

In the following, we consider only the process $\Phi^{(e)}$ and the distances between points of this process. However, we use variable y instead of y' for the sake of simplicity.

2.4 Association policy

We assume that each ordinary UE and mobile relay is associated with the base station that gives the best signal without considering fast fading. In other words, UEs and relays are connected to the closest base station in process $\Phi^{(e)}$. Based on that, the association region of each base station is a Voronoi cell. Let s be the distance between ordinary UE/mobile relay and its associated base station. The probability density function (pdf) of s is thus $e^{-\lambda'_s \pi s^2} 2\pi \lambda'_s s$ [28]. For vehicular UE, its serving mobile relay is assumed to be known.

2.5 Distribution of UEs in the cell

Denote the area of a single Voronoi cell as $\mathscr{A}$. A simple approximation for the pdf of $\mathscr{A}$ has been proved accurate for practical purposes. The pdf of the area of a single Voronoi cell is given by [42]:

$$f(\mathscr{A}) = \frac{343}{15}\sqrt{\frac{7}{2\pi}}(\mathscr{A}\lambda'_s)^{\frac{5}{2}} \exp(-\frac{7}{2}\mathscr{A}\lambda'_s)\lambda'_s. \tag{3}$$

Denote the number of ordinary UEs in a cell as n_o, which is a Poisson random variable with mean $\lambda'_o \mathscr{A}$. Conditioned on $\mathscr{A}$, the probability generating function (PGF) of the n_o is given by [42]:

$$G_o(z) = \frac{343}{8}\sqrt{\frac{7}{2}}\left(\frac{7}{2} - \frac{\lambda'_o}{\lambda'_s}(z-1)\right)^{-\frac{7}{2}}. \tag{4}$$

The distribution of n_o in a cell is therefore given by the ith derivative of $G_o(z)$ denoted by $G_o^{(i)}(z)$:

$$\mathbb{P}\{n_o = i\} = \frac{G_o^{(i)}(0)}{i!},\ i \in \mathbb{N}. \tag{5}$$

Denote the number of buses in a cell as n_r, which is also a Poisson random variable with mean $\lambda'_r \mathscr{A}$. Similarly, $\mathbb{P}\{n_r = i\}$ can be obtained by considering $\frac{\lambda'_r}{\lambda'_s}$ in (4).

Denote the total number of vehicular UEs in a cell as n_{tv}. Denote the number of vehicular UEs in a bus under a certain mobile relay as n_v. Variable n_v follows a Poisson distribution with mean $\overline{n_v}$. Then the total number of vehicular UEs n_{tv} is the sum of n_r independent Poisson random variables: $n_{tv} = \sum_{i=1}^{n_r} n_v^i$. By conditioning on the number of buses n_r, we can write the distribution of n_{tv}:

$$\begin{aligned}\mathbb{P}\{n_{tv} = i\} &= \sum_{j=0}^{\infty} \mathbb{P}\{n_{tv} = i | n_r = j\}\mathbb{P}\{n_r = j\} \\ &= \sum_{j=0}^{\infty} \frac{e^{-j\overline{n_v}}(j\overline{n_v})^i}{i!}\mathbb{P}\{n_r = j\}.\end{aligned} \tag{6}$$

Denote the total number of UEs in a cell as n_u. We have $n_u = n_o + n_{tv}$. The distribution of n_u can be obtained by:

$$\begin{aligned}\mathbb{P}\{n_u = u\} &= \sum_{i=0}^{\infty} \mathbb{P}\{n_o = i\}\mathbb{P}\{n_{tv} = u - i\} \\ &= \sum_{i=0}^{\infty} \mathbb{P}\{n_o = i\} \sum_{j=0}^{\infty} \frac{e^{-j\overline{n_v}}(j\overline{n_v})^{u-i}}{(u-i)!}\mathbb{P}\{n_r = j\}.\end{aligned} \tag{7}$$

We note that $\mathbb{P}\{n_u = u\}$ is a function of λ'_s, λ'_r, λ'_o and $\overline{n_v}$. Ordinary UEs and buses (mobile relays) are independent processes, hence we assume that n_o and n_r are independent.

2.6 Scheduling and rate

We consider Round Robin scheduling [43]. The bandwidth is equally shared among all users within a cell. For an available bandwidth w, the downlink rate of a typical UE is:

$$\mathscr{R} = \frac{w}{n_u}\log_2(1 + S), \tag{8}$$

where S is the SINR. Note that n_u and S are in general correlated. For tractability, we assume random variables n_u and S are independent. Such an assumption does not compromise the accuracy of the analysis and has been justified by S. Dhillon et al in [44].

3 SINR distribution on different links

A key performance indicator for wireless systems is the SINR level at the receiver. In this section, we first derive the CDF of the SINR in direct mode. The computation in direct mode is similar to the computation provided in [28–30]. For the sake of completeness and clarity, we give the main steps here. Then we derive the CDF of SINR on the backhaul link and the access link, which are assumed to be independent in this section. We assume full load conditions: all base stations always transmit at maximum power. Hence, the SINR distribution of UEs does not depend on the number of UEs in a cell.

3.1 Direct mode

All UEs in this mode are associated with the nearest base station. The interference comes from other base stations. Let S^D be the SINR of UE in direct mode, which is defined by:

$$S^D = \frac{r_{y_c}\chi {y_c}^{-\gamma_1}}{N_u + I_{BS}}, \tag{9}$$

where y_c is the closest distance to the serving base station, γ_1 is the path loss exponent on the direct link, N_u is the noise power of UE, and $I_{BS} = \sum_{y_i \neq y_c} r_i \chi {y_i}^{-\gamma_1}$ is the cumulative interference from other base stations.

Conditioned on the event that the distance between the considered UE and the served base station is y_c, the CDF of the SINR on the direct link is:

$$\mathbb{P}(S^D \le T) = 1 - \mathbb{E}_{y_c}[\mathbb{P}(S^D > T | y_c)], \tag{10}$$

where T denotes a certain SINR threshold. Using the fact that $r \sim \exp(1)$, the conditional probability can be expressed as:

$$\begin{aligned}\mathbb{P}(S^D > T | y_c) &= \mathbb{E}_{I_{BS}}[\mathbb{P}(r_{y_c} \ge \frac{T y_c^{\gamma_1}(N_u + I_{BS})}{\chi} | y_c, I_{BS})] \\ &= \mathbb{E}_{I_{BS}}\left(e^{-\frac{T y_c^{\gamma_1}(N_u + I_{BS})}{\chi}} | y_c\right) \\ &= e^{-\frac{T y_c^{\gamma_1} N_u}{\chi}} \mathscr{L}_{I_{BS}}\left(\frac{T y_c^{\gamma_1}}{\chi}\right),\end{aligned} \tag{11}$$

where $\mathscr{L}_{I_{BS}}$ is:

$$\begin{aligned}\mathscr{L}_{I_{BS}}(u) &= \mathbb{E}_{\Pi'_s, r}\left[\exp\left(-u \sum_{y \neq y_c} r_i \chi {y_i}^{-\gamma_1}\right)\right] \\ &= \mathbb{E}_{\Pi'_s}\left[\prod_{y_i \neq y_c} \mathbb{E}_r[\exp(-u r_i \chi {y_i}^{-\gamma_1})]\right],\end{aligned} \tag{12}$$

where i refers to neighbor base station i.

As $\mathscr{L}_{I_{BS}}$ is expressed as a probability generating functional (PGFL) [45], we have:

$$\mathscr{L}_{I_{BS}}(u) = \exp\left(-2\pi\lambda_s' \int_{y_c}^{\infty} (1 - \mathbb{E}_r[e^{-ur_i\chi y_i^{-\gamma_1}}]) s \mathrm{d}s\right) = \exp\left(-2\pi\lambda_s' \int_{y_c}^{\infty} (1 - \frac{1}{1+u\chi s^{-\gamma_1}}) s \mathrm{d}s\right). \quad (13)$$

Note that the integration limits are from y_c to ∞, as the UE is connected to the nearest base station and all the interfering base stations are further away.

The CDF of the SINR averaged over the plane is:

$$\mathbb{P}(S^D \le T) = 1 - \int_{\mathbb{R}^+} e^{-\frac{T y_c^{\gamma_1} N_1}{\chi}} \mathscr{L}_I(\frac{T y_c^{\gamma_1}}{\chi}) f(y_c) \mathrm{d}y_c, \quad (14)$$

where $f(y_c) = e^{-\lambda_s' \pi y_c^2} 2\pi\lambda_s' y_c$.

Therefore, for ordinary UE and vehicular UE, the CDF of the SINR is obtained by applying $\chi_1 = k_1 P_S$ and $\chi_2 = k_2 P_S$ into (14) respectively.

3.2 FDD mobile relay mode

3.2.1 Ordinary UE

In this mode, the interference for ordinary UE not only comes from neighbor base stations, but also from mobile relays due to frequency reuse. Let S_o^F be the SINR of ordinary UE in this mode, which is defined by:

$$S_o^F = \frac{r_{y_c}\chi_1 y_c^{-\gamma_1}}{N_u + I_{BS} + I_R}, \quad (15)$$

where the cumulative interference from mobile relays $I_R = \sum r_i \chi_3 z^{-\gamma_2}$ with $\chi_3 = k_3 P_R$. By using a similar approach in Sect. 3.1, the CDF of the SINR for ordinary UE in this mode is:

$$\mathbb{P}(S_o^F \le T) = 1 - \int_{\mathbb{R}^+} e^{-\frac{T y_c^{\gamma_1} N_u}{\chi_1}} \mathscr{L}_{I_{BS}}\left(\frac{T y_c^{\gamma_1}}{\chi_1}\right) \mathscr{L}_{I_R}\left(\frac{T y_c^{\gamma_1}}{\chi_1}\right) f(y_c) \mathrm{d}y_c, \quad (16)$$

where the Laplace transform $\mathscr{L}_{I_R}(u)$ is given by:

$$\mathscr{L}_{I_R}(u) = \mathbb{E}_{\Pi_r', r}\left[\exp\left(-u \sum r_i \chi_3 z_i^{-\gamma_2}\right)\right] = \exp\left(-2\pi\lambda_r' \int_0^{\infty} (1 - \mathbb{E}_r[e^{-ur_i\chi_3 z_i^{-\gamma_2}}]) s \mathrm{d}s\right), \quad (17)$$

where the integration limits are from 0 to ∞, as all mobile relays are interferers for ordinary UE.

3.2.2 Mobile relay

Mobile relay is associated with the closest base station, as for ordinary UE. Let the CDF of the SINR for mobile relay be S_r^F, which can be obtained by applying $\chi_1 = k_1 P_S$ and the noise power of mobile relay N_r into (14).

3.2.3 Vehicular UE

For vehicular UE, the interference comes from all base stations (including all neighbor base stations and the base station that serves the relay), and other relays. Let S_v^F be the SINR on the access link, the CDF of the SINR is defined by:

$$\mathbb{P}(S_v^F \le T) = 1 - \mathbb{P}\left(\frac{r_0 p_0}{N_u + I_{allBS} + I_R} > T\right) = 1 - \mathbb{E}_{I_{allBS}, I_R}\left[\mathbb{P}(r_0 \ge \frac{T(N_u + I_{allBS} + I_R)}{p_0} | I_{allBS}, I_R)\right] = 1 - \exp\left(-\frac{T N_u}{p_0}\right) \mathscr{L}_{I_{allBS}}\left(\frac{T}{p_0}\right) \mathscr{L}_{I_R}\left(\frac{T}{p_0}\right), \quad (18)$$

where $I_{allBS} = \sum r_i \chi_2 y_i^{-\gamma_1}$. The Laplace transform $\mathscr{L}_{I_{allBS}}(u)$ can be computed by using (13), but the integration limits are from 0 to ∞, as the donor base station is also one source of interference. The Laplace transform $\mathscr{L}_{I_R}(u)$ is computed by using (17) with $\chi_4 = k_4 P_R$ instead of χ_3.

3.3 TDD/FDD hybrid mobile relay mode

3.3.1 Ordinary UE

The CDF of the SINR for ordinary UE in this mode S_o^T is the same as the case in direct mode.

3.3.2 Mobile relay

The CDF of the SINR for mobile relay in this mode S_r^T is the same as the case in FDD mobile relay mode.

3.3.3 Vehicular UE

In this mode mobile relay operates in TDD on the access link. Each mobile relay is an independent TDD system. The interferences come from either neighbor relays that are transmitting on the downlink or vehicular UEs that are transmitting on the uplink. The interference signals suffer from path loss and two times penetration loss. For vehicular UE, the distance to the serving mobile relay is relative small compared with the distance to other mobile relays or other vehicular UE, the interference from other vehicular UE on the uplink can be approximated by the interference from other relays. So the interferences on the access link in this mode are assumed to come from all neighbor relays.

Let S_v^T be the SINR on the access link, the CDF of S_v^T is thus defined by:

$$\begin{aligned}\mathbb{P}(S_v^T \le T) &= 1 - \mathbb{P}\left(\frac{r_0 p_0}{N_u + I_R} > T\right)\\ &= 1 - \mathbb{E}_{I_R}\big[\mathbb{P}(r_0 \ge \frac{T(N_u + I_R)}{p_0}|I_R)\big]\\ &= 1 - \exp\left(-\frac{TN_u}{p_0}\right)\mathscr{L}_{I_R}\left(\frac{T}{p_0}\right),\end{aligned} \quad (19)$$

where $I_R = \sum r\chi_4 z^{-\gamma_2}$ with $\chi_4 = k_4 P_R$. The Laplace transform $\mathscr{L}_{I_R}(u)$ is computed by using (17).

4 Distribution of the rate

In this section, we turn our attention to the CDF of the rate of a UE. We first compute the rate in direct mode, then the computation of the end-to-end rate for vehicular UE in mobile relay modes is given. As the spectrum in a cell is shared between all users, the user bit rate depends on the number of UEs in a cell.

4.1 Direct mode

For total available bandwidth W Hz, the downlink rate in bps of a UE in direct mode is:

$$\mathscr{R}^D = \frac{W}{n_u}\log_2(1 + S^D), \quad (20)$$

where n_u is defined as the number of UEs in a cell. For a given value of n_u the CDF of the rate $F_{\mathscr{R}^D}$ is thus given by:

$$F_{\mathscr{R}^D}(t) = \mathbb{P}(\mathscr{R}^D \le t) = \mathbb{P}(S^D \le 2^{\frac{n_u}{W}t} - 1), \quad (21)$$

where t denotes a certain rate. The CDF of S^D can be obtained from Eq. (14).

4.2 FDD mobile relay mode

In this mode, the resource of the base station is shared between ordinary UEs and mobile relays. A mobile relay first receives data from base station, then the mobile relay transmits to vehicular UEs. The resource of a mobile relay is shared among n_v UEs that are associated with it.

4.2.1 Ordinary UE

Denote the rate of an ordinary UE in this mode as $\mathscr{R}_o^F$, denote the CDF of the rate as $F_{\mathscr{R}_o^F}(t)$. Similarly, $F_{\mathscr{R}_o^F}(t)$ can be obtained by using Eq. (21).

4.2.2 Vehicular UE

Assume the backhaul link and access link are equally separated in the time domain. The downlink end-to-end rate in bps of vehicular UE is:

$$\mathscr{R}_v^F = \frac{1}{2}\min\left\{\frac{W}{n_u}\log_2(1 + S_r^F), \frac{W}{n_v}\log_2(1 + S_v^F)\right\}, \quad (22)$$

here n_v is the number of vehicular UEs associated with a certain mobile relay. The $1/2$ factor is due to the 2-phase operation of the relay transmission. The backhaul and access links are both interfered by neighbor base stations, there is a dependency between two links. To be accurate, we first compute the CCDF (Complementary Cumulative Distribution Function) of $\mathscr{R}_v^F$ conditioned on the closest base station y_c that the mobile relay is associated to:

$$\begin{aligned}&\mathbb{P}(\mathscr{R}_v^F > t|y_c)\\ &= \mathbb{P}\left(\frac{1}{2}\min\{\frac{W}{n_u}\log_2(1 + S_r^F), \frac{W}{n_v}\log_2(1 + S_v^F)\} > t|y_c\right)\\ &= \mathbb{P}(S_r^F > 2^{2tn_u/W} - 1, S_v^F > 2^{2tn_v/W} - 1|y_c)\\ &= \mathbb{P}\left(\frac{r_1\chi_1 y_c^{-\gamma_1}}{N_r + I_{BS}} > g_1(t), \frac{r_0 p_0}{N_u + I_R + I_{allBS}} > g_2(t)|y_c\right),\end{aligned} \quad (23)$$

where $g_1(t) = 2^{2tn_u/W} - 1$ and $g_2(t) = 2^{2tn_v/W} - 1$. The interference from all base stations on the access link can be seen as the interference from other base stations on the backhaul link plus the interference from donor base station, with additional penetration loss $L = \frac{k_2}{k_1}$, which means $I_{allBS} = L(I_{BS} + r_{y_c}\chi_1 y_c^{-\gamma_1})$. Equation (23) can be written as:

$$\begin{aligned}&\mathbb{P}(\mathscr{R}_v^F > t|y_c) = \mathbb{P}(r_1 > \frac{g_1(t)(N_r + I_{BS})}{\chi_1 y_c^{-\gamma_1}},\\ &r_0 > \frac{g_2(t)(N_u + I_R + L(I_{BS} + r_{y_c}\chi_1 y_c^{-\gamma_1}))}{p_0}|y_c).\end{aligned} \quad (24)$$

Let $f_{I_{BS}}$ and f_{I_R} denote the pdf of I_{BS} and I_R, respectively. Using the fact $r_i \sim \exp(1)$, $i = \{0, 1, y_c\}$, the conditional CCDF of $\mathscr{R}_v^F$ can be expressed as:

$$\begin{aligned}&\mathbb{P}(\mathscr{R}_v^F > t|y_c) = \mathbb{E}[e^{-\frac{g_1(t)(N_r + I_{BS})}{\chi_1 y_c^{-\gamma_1}}}\\ &, e^{-\frac{g_2(t)(N_u + I_R + L(I_{BS} + r_{y_c}\chi_1 y_c^{-\gamma_1}))}{p_0}}|y_c, I_{BS}, I_R, r_{y_c})],\end{aligned} \quad (25)$$

which can be further computed by using triple points:

$$\begin{aligned}
&\mathbb{P}(\mathscr{R}_v^F > t|y_c)\\
&= \int_{x=0}^{\infty} f_{I_{BS}}(x) \exp\left[-\frac{g_1(t)(N_r+x)}{\chi_1 y_c^{-\gamma_1}}\right] \int_{y=0}^{\infty} e^{-y} \int_{z=0}^{\infty} f_{I_R}(z)\\
&\quad \times \exp\left[-\frac{g_2(t)(N_u+z+L(x+y\chi_1 y_c^{-\gamma_1}))}{p_0}\right] \mathrm{d}z\mathrm{d}y\mathrm{d}x\\
&= \exp\left[-\frac{g_1(t)N_r}{\chi_1 y_c^{-\gamma_1}} - \frac{g_2(t)N_u}{p_0}\right] \mathscr{L}_{I_{BS}}\left(\frac{g_1(t)}{\chi_1 y_c^{-\gamma_1}} + \frac{g_2(t)L}{p_0}\right)\\
&\quad \times \mathscr{L}_{I_R}\left(\frac{g_2(t)}{p_0}\right) \frac{1}{1+\frac{L\chi_1 y_c^{-\gamma_1}}{p_0}}. \qquad (26)
\end{aligned}$$

By averaging on all y_c, the CDF of the end-to-end rate of vehicular UE in FDD mobile relay mode is:

$$F_{\mathscr{R}_v^F}(t) = 1 - \int_{\mathbb{R}^+} \mathbb{P}(\mathscr{R}_v^F \geq t|y_c) 2\pi\lambda_s' y_c e^{-\lambda_s' \pi y_c^2} \mathrm{d}y_c. \quad (27)$$

4.3 TDD/FDD hybrid mobile relay mode

4.3.1 Ordinary UE

In this mode, note that the available bandwidth for ordinary UE is αW. Similarly, as in Sect. 4.1, the downlink rate in bps of an ordinary UE can be obtained.

4.3.2 Vehicular UE

In this mode, the backhaul link and the access link operate independently at the same time. The downlink end-to-end rate in bps of a vehicular UE is:

$$\mathscr{R}_v^T = \min\left\{\frac{\alpha W}{n_u}\log_2(1+S_r^T), \rho\frac{2W(1-\alpha)}{n_v}\log_2(1+S_v^T)\right\}, \quad (28)$$

where ρ is the downlink/uplink configuration on the access link. The CCDF of $\mathscr{R}_v^T$ is given by:

$$\begin{aligned}
&\mathbb{P}(\mathscr{R}_v^T > t)\\
&= \mathbb{P}(\min\left\{\frac{\alpha W}{n_u}\log_2(1+S_r^T), \rho\frac{2W(1-\alpha)}{n_v}\log_2(1+S_v^T)\right\} > t)\\
&= \mathbb{P}\left(\frac{\alpha W}{n_u}\log_2(1+S_r^T) > t\right) \mathbb{P}\left(\frac{\rho 2W(1-\alpha)}{n_v}\log_2(1+S_v^T) > t\right)\\
&= \mathbb{P}(S_r^T > 2^{\frac{n_u t}{\alpha W}} - 1)\mathbb{P}(S_v^T > 2^{\frac{n_v t}{\rho 2W(1-\alpha)}} - 1), \qquad (29)
\end{aligned}$$

where the CCDF of S_r^T and S_v^T can be similarly computed according to Sect.3.3 with (19).

5 Energy efficiency analysis

In this section, a suitable power consumption model is first defined, then the energy efficiency expression is given for all modes.

5.1 Power consumption model

The total power consumption can be separated into two fundamental parts. The first one describes the static power consumption, which includes signal processing overhead, battery backup, cooling power consumption, etc. The second part represents the output power [46,47]. As a result, for base station and mobile relay, the total power consumption can be given by:

$$P_{S\_tot} = P_{S\_0} + \tau_S P_S \quad (30)$$
$$P_{R\_tot} = P_{R\_0} + \tau_R P_R, \quad (31)$$

where $1/\tau_S, 1/\tau_R$ denote the efficiency of the power amplifier for base stations and mobile relays, P_S and P_R denote the transmit power of base stations and mobile relays, P_{S_0} and P_{R_0} are the static power consumption for base stations and mobile relays, respectively.

5.2 Energy efficiency function

We define the energy efficiency for a mobile relay network as:

$$\eta = \frac{\epsilon}{W(P_{S\_tot} + P_{R\_ave})} \text{ (bps/Hz/W)}, \quad (32)$$

where ϵ denotes the cell total average rate, W is the system bandwidth and P_{R_ave} denotes the average power consumption of mobile relays in a cell.

5.3 Direct mode

The number of UEs n_u in a cell is a random variable. For a given number of ordinary UEs $n_o = i$ and vehicular UEs $n_{tv} = j$, the cell total average rate conditioned on i and j is:

$$\begin{aligned}
\epsilon_D^{i,j} &= \mathbb{E}\left[\sum_i \mathscr{R}_o^D + \sum_j \mathscr{R}_v^D\right] = i\mathbb{E}[\mathscr{R}_o^D] + j\mathbb{E}[\mathscr{R}_v^D]\\
&= i\int_0^{\infty} \mathbb{P}(\mathscr{R}_o^D > x)\mathrm{d}x + j\int_0^{\infty} \mathbb{P}(\mathscr{R}_v^D > x)\mathrm{d}x, \qquad (33)
\end{aligned}$$

where the CCDF of $\mathscr{R}_o^D$ and $\mathscr{R}_v^D$ can be computed with the CDF given by Eq. (21). By averaging all possible i and j, the cell's average rate in direct mode is:

$$\epsilon_D = \sum_i \sum_j \epsilon_D^{i,j} \mathbb{P}(n_o = i)\mathbb{P}(n_{tv} = j), \quad (34)$$

where $\mathbb{P}\{n_o = i\}, \mathbb{P}\{n_{tv} = j\}$ can be computed from Eqs. (5) and (6). The cell energy efficiency in direct mode is thus:

$$\eta_D = \frac{\epsilon_D}{W P_{S\_tot}}. \quad (35)$$

5.4 FDD mobile relay mode

In this mode, the end-to-end rate of vehicular UE $\mathscr{R}_v^F$ depends on the number of vehicular UEs associated to a certain mobile relay $n_v = k$. For a given number of ordinary UEs i, vehicular UE j and k, the conditional cell total average rate is given by:

$$\epsilon_F^{i,j,k} = \mathbb{E}\left[\sum_i \mathscr{R}_o^F + \sum_{j,k} \mathscr{R}_v^F\right], \quad (36)$$

where the CCDF of $\mathscr{R}_o^F$ and $\mathscr{R}_v^F$ can be computed from Sect.4.2. By averaging all i, j, k, the cell's average rate is:

$$\epsilon_F = \sum_i \sum_{j,k} \epsilon_F^{i,j,k} \mathbb{P}(n_o = i)\mathbb{P}(n_{tv} = j, n_v = k), \quad (37)$$

where the joint probability $\mathbb{P}(n_{tv} = j, n_v = k)$ can be easily obtained since the number of vehicular UEs in other buses $j - k$ is still a sum of independent Poisson variables. Since mobile relays only transmit in the access phase in this mode, the average power consumption of a mobile relay in the cell is:

$$P_{R\_ave}^F = \sum_i i(P_{R\_0} + \frac{1}{2}\tau_r P_R)\mathbb{P}\{n_r = i\}. \quad (38)$$

The cell energy efficiency in this mode is thus:

$$\eta_F = \frac{\epsilon_F}{W(P_{S\_tot} + P_{R\_ave}^F)}. \quad (39)$$

5.5 TDD/FDD hybrid mobile relay mode

Similarly, based on the CCDF of $\mathscr{R}_o^T$ and $\mathscr{R}_v^T$, which can be computed in Sect. 4.3, the cell's average rate in this mode is:

$$\epsilon_T = \sum_i \sum_{j,k} \epsilon_T^{i,j,k} \mathbb{P}(n_o = i)\mathbb{P}(n_{tv} = j, n_v = k). \quad (40)$$

In this mode, mobile relays can receive and transmit simultaneously. The average power consumption of mobile relay in the cell is:

$$P_{R\_ave}^T = \sum_i i(P_{R\_0} + \tau_R P_R)\mathbb{P}\{n_r = i\}. \quad (41)$$

The cell energy efficiency in TDD/FDD hybrid mobile relay mode is thus:

$$\eta_T = \frac{\epsilon_T}{W(P_{S\_tot} + P_{R\_ave}^T)}. \quad (42)$$

6 Performance and discussion

We study the performance of a cellular network without and with mobile relays for different configurations. The default parameters are given in Table 2. The path loss exponent γ_1 and γ_2 are set at 4, for which simplification of calculations is possible. The power model parameters are obtained from [46]. Considering the bus is normally no longer than 20 meters, the distance between a vehicular UE and its serving mobile relay is set at 20 m, the transmit power of mobile relay is set at 0 dBm [48]. In TDD/FDD hybrid mobile relay mode, the access link operates in TDD, downlink/uplink configuration ρ is set at 0.5 (half of the resource is allocated to downlink transmission on the access link).

6.1 CDF of SINR in direct mode

Firstly, we compare analytical results with Monte-Carlo simulations. The main objective is to assess the validation of our proposed model. The CDF of the SINR of an ordinary UE and a vehicular UE in direct mode are displayed in Fig. 5, which shows that the simulation results of the PPP model are well consistent with the analytical results. We also compare the analytical results with the simulation results of the hexagonal model. For a fair comparison, the area of a hexagonal cell is set to be $1/\lambda_s$. As we can see, the CDF of the PPP model is always higher than that of the hexagonal model. This is because in the PPP model there is substantial interference generated by nearby base stations. Thus the PPP model gives pessimistic results compared with the results of the hexagonal model.

At high SINR, the outage probability can be approximated by the CDF of the SINR. We can see for a given SINR threshold, vehicular UEs always have higher outage probability than ordinary UE. This is because they suffer penetration loss, which deteriorates the signal and must be taken into account.

6.2 Data rates of ordinary UEs

The CDF of the rate of ordinary UE is illustrated in Fig. 6. Ordinary UEss have the same CDF of rates in direct mode and in FDD mobile relay mode. This means that the interferences

Table 2 Model parameters

Total system bandwidth	$W = 10$ MHz
Base station intensity	$\lambda_s = 1$ BS/km^2
Bus intensity (default value)	$\lambda_r = 5$ buses/km^2
Ordinary UE intensity (default value)	$\lambda_o = 20$ UEs/km^2
Average number of UEs in a bus (default value)	$\overline{n_v} = 4$
Base station transmit power	$P_S = 4$ W (36 dBm)
Base station static power consumption	$P_{S\_0} = 130$ W
Base station power slope	$\tau_S = 4.7$
Mobile relay transmit power	$P_R = 1$ mW (0 dBm)
Mobile relay static power consumption	$P_{R\_0} = 4.8$ W
Mobile relay power slope	$\tau_R = 8.0$
Bus penetration loss	$L = 20$ dB
Propagation factor for ordinary UEs/mobile relays	$k_1 = 10^{-14.2}$
Propagation factor for vehicular UEs	$k_2 = 10^{-14.2-L/10}$
Path loss exponent	$\gamma_1 = \gamma_2 = 4$
Shadowing	$\sigma = 6$ dB
Noise power spectral density	-174 dBm/Hz
Mobile relay noise figure	5 dB
UE noise figure	9 dB
Distance between mobile relay and vehicular UE	20 m
Frequency division parameter in TDD/FDD mode	$\alpha = 0.9$
Downlink/uplink configuration in TDD/FDD mode	$\rho = 0.5$

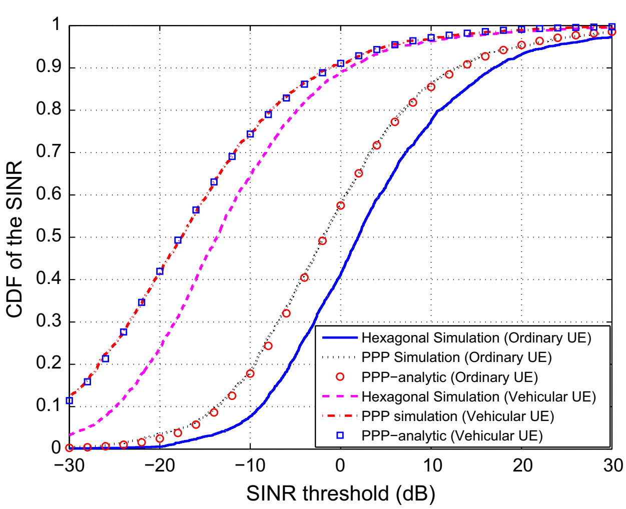

Fig. 5 CDF of SINR in direct mode

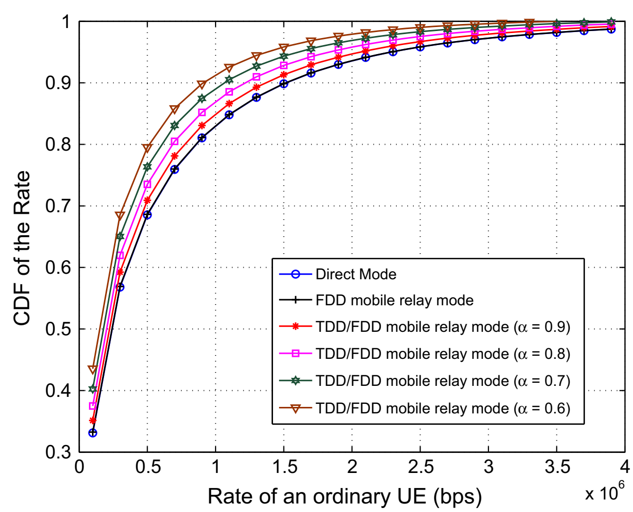

Fig. 6 CDF of the rate of an ordinary UE

are mainly due to neighbor base stations. The interferences from mobile relays in FDD mobile relay mode are negligible due to low transmit power of mobile relay and large penetration loss (20 dB in this plot).

We also study the impact of parameter α on ordinary UEs in TDD/FDD hybrid mobile relay mode. The rates of ordinary UEs becomes smaller when α decreases, since more bandwidth is dedicated to vehicular UEs for the access link. So the dedicated bandwidth to vehicular UEs should be carefully set in order to guarantee the rate of ordinary UE. Vehicular UEs and their serving mobile relays are close to each other and the access link has a high SINR. As shown in 6.3, a small dedicated bandwidth is enough to guarantee the high speed transmission on the access link ($\alpha = 0.9$). The rate of ordinary UE is thus slightly decreased by the reservation of a part of the bandwidth specifically for the access link.

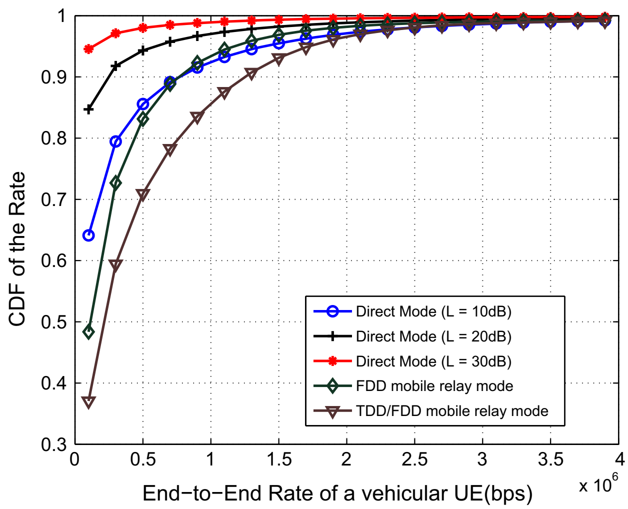

Fig. 7 CDF of the end-to-end rate of a vehicular UE

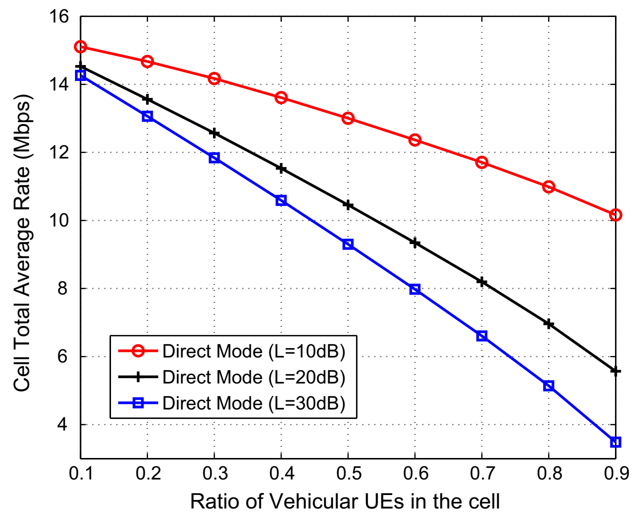

Fig. 8 Cell total average rate for different penetration losses in direct mode

6.3 End-to-end data rates of vehicular UEs

The CDF of the end-to-end rate of vehicular UE is illustrated in Fig. 7. As we can see, with the increase of penetration loss, the end-to-end rate of vehicular UE in direct mode becomes significantly worse. If the penetration loss is large, the mobile relay modes can both have better rate. But if the penetration loss is small, for example $L = 10$ dB, FDD mobile relay mode cannot always bring data rate gain to all vehicular UEs due to the 2-phase transmission with relays. The backhaul link and the access link cannot operate simultaneously in this mode.

A large value of α is used in this plot: $\alpha = 0.9$. TDD/FDD hybrid mobile relay mode achieves the best rate for vehicular UEs. This is because the backhaul link and the access link can operate simultaneously in this mode, the data can be transmitted to vehicular UE all the time. Considering the impact of α on ordinary UE as discussed before, parameter α is set to be 0.9 in default. Finding the optimal value for α under certain parameters isbeyond the scope of this paper.

6.4 Impact of the penetration loss

The cell total average rate in direct mode under different penetration losses is illustrated in Fig. 8. It can be seen that with the increase of the ratio of vehicular UE in the cell, the cell total average rate in direct mode decreases, this is because more and more vehicular UE suffers penetration loss. This confirms the interest of mobile relays.

6.5 Cell total average rate

We consider a given total number of UEs and different proportions of vehicular UE. The total intensity of UE is constant

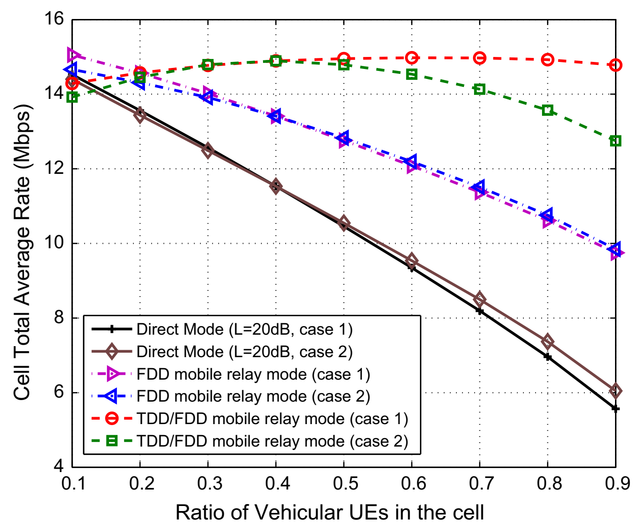

Fig. 9 Cell total average rate against the ratio of vehicular UEs

or in other words $\lambda_t = \lambda_o + \lambda_r \overline{n_v}$ is constant. We make the ratio of vehicular UE $\upsilon = \frac{\lambda_r \overline{n_v}}{\lambda_t}$ vary from 0.1 to 0.9. There are two cases. In case 1, we consider that the average number of UEs on a bus $\overline{n_v}$ is fixed and make υ vary (hence, λ_o and λ_r are variables). In case 2, we consider that the intensity of buses λ_r is fixed and make υ vary (hence, λ_o and $\overline{n_v}$ are variables). In both cases, we consider the same density of UE but different distribution patterns of vehicular UE. The cell total average rate under different modes against the ratio of vehicular UE is illustrated in Fig. 9.

As we can see, with the increase of the ratio of vehicular UEs in the cell, both FDD and TDD/FDD relay modes perform better than the direct mode. For instance, when the ratio of vehicular UEs is 0.4, the FDD and TDD/FDD relay modes can respectively achieve about +16.3 and +29.1% cell rate gain compared to the direct mode. Furthermore, the cell total average rate in FDD mobile relay mode decreases with the increase of vehicular UEs in the cell while it is rather stable in the TDD/FDD hybrid mode. This is because more UEs are served through two phase transmission in time where only half of the time is fully effective in FDD mobile relay mode.

The TDD/FDD relay mode has the best performance among the three modes when the ratio of vehicular UEs is larger than 0.1. This is because although ordinary UE has lower rates in this mode, the end-to-end rate gain of vehicular UE can compensate for the rate loss of ordinary UE. From the capacity point of view, the cell can thus achieve a higher total cell rate. But when the cell has far more ordinary UEss than vehicular UEs, the rate loss of ordinary UE in TDD/FDD hybrid mobile relay mode cannot be compensated for by the rate gain of vehicular UE. This is why the cell total average rate of TDD/FDD hybrid mobile relay mode is lower when the ratio of vehicular UE is 0.1. It is not worth using a dedicated bandwidth for the access link when there are a very small number of vehicular UEs in the cell. Furthermore, it is very interesting to see that the cell average rate does not make much difference between case 1 and case 2 in direct mode and FDD mobile relay mode, but the cell average rate in case 2 of TDD/FDD hybrid mobile relay mode decreases when the ratio of vehicular UEs increases above 0.5. This is because in case 2, there are more and more vehicular UEs in a bus, so the access link becomes a bottle neck if the dedicated bandwidth for the access link remains the same.

6.6 Influence of the number of buses

In the previous section, we consider that the total number of UEs is constant, now we investigate only the influence of the number of buses. We consider the case at rush hour, the intensity of buses increases but we assume that the intensity of ordinary UEs remains the same. The global load thus increases. The result is shown in Fig. 10.

As we can see, with the increase in the number of buses in the cell, there are more and more vehicular UEs that need to be served. Since more and more UE suffers penetration loss, the cell total average rate in direct mode decreases, while it increases in the FDD relay mode. This is because in FDD mobile relay mode the rate gain brought about by the elimination of penetration loss outweighs the transmission loss of two separate links. Of the three modes, the TDD/FDD hybrid mobile relay mode still has the best and most stable cell total average rate in this case.

6.7 Energy efficiency

Fig. 11 presents energy efficiency against the ratio of vehicular UE in the cell in case 1 (constant number of users in a bus) and 2 (constant number of buses). Since the distribution patterns of vehicular UE mainly affect the TDD/FDD relay mode, the two cases are shown only for this mode. When the penetration loss is high, mobile relay modes can achieve better energy efficiency compared with the direct mode because the data rate gain brought about by mobile relays exceeds the extra power consumption of the relays. For instance, when $L = 20$ dB and the ratio of vehicular UE is 0.5, the FDD and TDD/FDD modes can respectively achieve +5.7 and +24.2% energy efficiency gain compared with the direct mode. But when the penetration loss is small ($L = 10$ dB) and the ratio of vehicular UEs is smaller than 0.5, the direct mode is better from the energy efficiency point of view. In this case the data rate gain brought about by mobile relay is cancelled out by the extra power consumption of mobile relays.

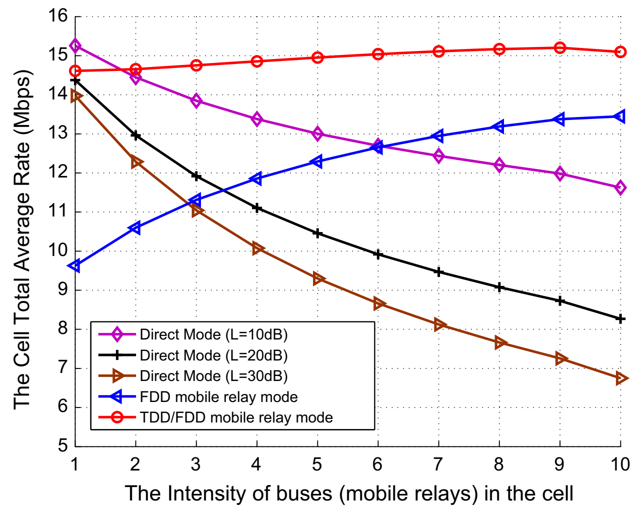

Fig. 10 Cell total average rate against the intensity of buses

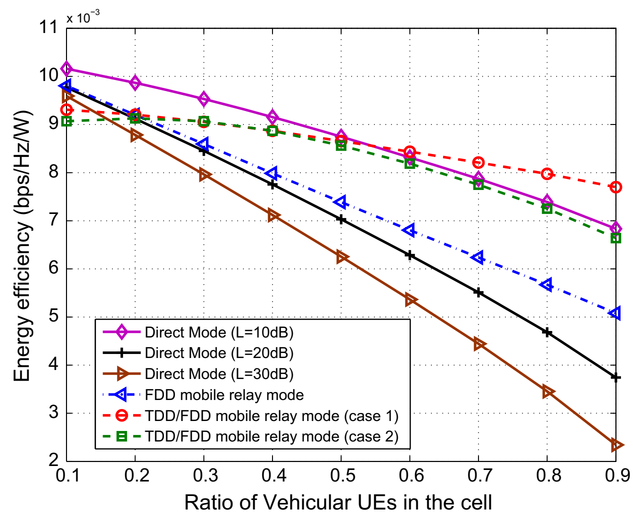

Fig. 11 Energy efficiency against the ratio of vehicular UEs

7 Conclusion

This paper studied the capacity of a cellular network with mobile relays on public transport. We considered different modes of bandwidth-sharing between the access link and the backhaul link and provided an analytical model using stochastic geometry. The analysis considers the different types of interferences. The correlation of the backhaul link and the access link in FDD mobile relay mode is taken into account when deriving the CDF of the end-to-end rate. The cell total rate and energy efficiency were evaluated as a function of the ratio of users on buses. Dedicating a part of the spectrum for the access link inside buses and using TDD for this increases the total average rate, as soon as the proportion of vehicular UE is higher than 10%. One important property of TDD/FDD systems is that the total rate only varies slightly when the proportion of vehicular UE increases.

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