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An incentive energy-efficient routing for data gathering in wireless cooperative networks

Yindong Zhang · Liusheng Huang · Hongli Xu · Zhenguo Yang

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Abstract Because of energy-constraint, it is an attractive problem to select energy-efficient paths from source nodes to sink for data gathering in wireless ad hoc networks. Cooperative communication is a promising mechanism to reduce transmit energy in such kind of case. One of the fundamental assumptions for cooperative communication is that each node should be unselfish, responsible, and willing to forwarding data he has received. However, in energy-constrained environment, because of limited energy, each node hates participating in data transmission without any incentive and tries to avoid forwarding data (this behavior is selfish). In this paper, a utility function is proposed to stimulate nodes to behave unselfishly. We prove that it is a Nash Equilibrium when nodes work in an unselfish manner. Also, we show that the selection of forwarding nodes and relay nodes for data transmission is a NP-hard problem even when nodes behave unselfishly. A heuristic algorithm (Algorithm for Node Selection Problem, ANSP) is provided to solve this selection problem. We also prove the convergence of this algorithm. The analysis shows that this algorithm can reach the approximate performance ratio of $2 \cdot (1+\alpha)$, where α is the maximal ratio of two power consumptions on two adjacent links in the network. The numerical results show that in a 100 node network, if nodes behave unselfishly, they will obtain a better utility, and more energy will be saved. The average saved energy when each node takes a selfish behavior, is 52.5% less than the average when nodes behave in an unselfish manner.

Keywords Cooperative communication · Utility function · Nash Equilibrium · NP-hard

Y. Zhang (✉) · L. Huang · H. Xu · Z. Yang
Department of Computer Science & Technology, University of Science & Technology of China, Hefei, China
e-mail: zhangyd@mail.ustc.edu.cn

L. Huang
e-mail: lshuang@ustc.edu.cn

H. Xu
e-mail: xuhongli@ustc.edu.cn

Z. Yang
e-mail: zgyang@mail.ustc.edu.cn

Y. Zhang · L. Huang · H. Xu · Z. Yang
Suzhou Institute for Advanced Study, University of Science & Technology of China, Suzhou, China

1 Introduction

With the development of micro-electronic technologies, nodes in wireless ad hoc networks become much smarter than before. However, this causes that each node has limited resources, such as energy, memory.

The mechanism, cooperative communication [8, 16], has attracted more and more attentions recently for its impressive advantages, for example, energy-efficiency. Compared with traditional communication, the most important advantage of cooperative communication is that, energy can be saved greatly with the help of cooperative nodes. We will show this advantage by an example. In Fig. 1, the Arabic numerals beside edges represent the power consumptions on these links. We assume that node i wants to send data to node j. If node i uses conventional communication for this transmission, the power consumption is 50. But if we use cooperative communication with the help of k, the power consumption for one transmission is only 25. Node i consumes 12.5 and k costs 12.5, respectively. We will show how to calculate the power consumption of cooperative transmission in Sect. 3.1. In this scenario, compared with traditional

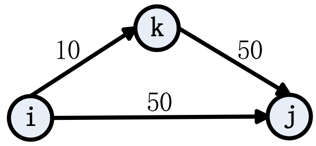


Fig. 1 Cooperative communication

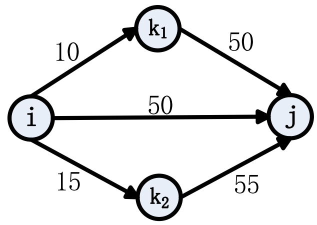


Fig. 2 Relay node selection in cooperative communication

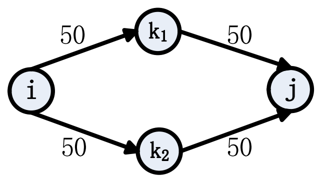


Fig. 3 Forwarding node selection in data transmission

communication, cooperative communication reduces transmit energy to 50%. We can say that cooperative communication is much more energy-efficient than conventional communication.

One of the fundamental assumptions for cooperative communication is that all nodes behave unselfishly. However, in resource-constrained environment, each node tries to avoid forwarding data for other nodes, so that he can use all his remaining energy to serve himself. This stimulates all nodes to behave in a selfish manner. For example, in Fig. 2, power consumption on $i-k_1$, $i-k_2$, $i-j$, k_1-j, and k_2-j is 10, 15, 50, 50 and 55, respectively. Because of limited power, node k_1 and k_2 are not willing to relay data for node i. When node i queries power consumptions on k_1-j and k_2-j, either of k_1 and k_2 behaves in a selfish manner. They cheat and report false values back to i. We assume that these two false values are 80 and 60, respectively. If k_1 is the relay node, the power consumption for transmission from i to j is 30.8. On the other hand, if k_2 is selected as the relay node, the power consumption is 27.3. So based on this computation, we choose k_2 as the relay node. But in fact, the power consumption for transmission between i and j is only 25 if nodes behave unselfishly and we choose k_1 as the relay. In this case, node i avoids to be a rely successfully. However, this leads to a higher power consumption for data transmission. So it is very important to ensure that each node behaves in an unselfish manner and reports replies honestly when we select cooperative nodes.

In Fig. 3, the Arabic numerals beside edges represent the power consumptions on these links. Obviously, the power consumption is 100 if we try to send data from i to j. But if k_2 acts in a selfish mode and reports a false power consumption, 65 on k_2-j, the "best" solution is $\langle i, k_1, j\rangle$, which is actually not the best.

We can see that either in cooperative node selection or forwarding node selection, selfish behaviors will lead to a poor performance. So how to ensure that each node behaves in an unselfish manner is not only an interesting problem, but also an important one.

In this paper, we consider a wireless ad hoc network for many-to-one communication, where each node is energy-constrained and prefers to behave in a selfish manner (in this case, each node tries to avoid forwarding data). The sink gathers all data generated by source nodes. To reduce transmit energy depletion, we select forwarding nodes and relay nodes carefully to participate in data transmission, while all nodes do their best to avoid forwarding data for other nodes. The main contributions of our work are as follows. First, we pose this problem as a noncooperative game and use game-theoretic analysis to address it. Second, we propose a utility function to motivate nodes to act unselfishly. Based on the utility function, we prove that it is a Nash Equilibrium when each node behaves in an unselfish manner. Third, we show that our selection problem is NP-hard even when each node behaves unselfishly. To solve this problem, we provide a heuristic algorithm (Algorithm for Node Selection Problem, ANSP). Fourth, the convergence of this algorithm is proved. And, the analysis shows that this algorithm can reach the approximate performance ratio of $2\cdot(1+\alpha)$, where α is the maximal ratio of two power consumptions on two adjacent links in the network. The numerical results show that if nodes behave unselfishly, those nodes who participate in data transmission will obtain a higher utility, and more energy can be saved. The average saved energy when nodes take selfish behaviors is 52.5% less than the average when nodes behave unselfishly.

The rest of this paper is organized as follows. In Sect. 2, we investigate some related work about wireless selfish network and cooperative communication. After that, a brief overview on noncooperative strategic-form game theory is provided. In the same section, we also present our network model and our problem in detail. In the following section, we show our proof on Nash Equilibrium and NP-hardness. Next, we describe our algorithm in detail, prove the convergence of this algorithm, and analyze the approximate performance ratio. Section 6 provides our numerical results. Finally, in Sect. 7, we conclude this paper.

2 Related work

In wireless selfish ad hoc networks, lots of schemes have been proposed to select a proper path, which is used to help a multi-hop transmission. Also, many researchers focus on relay node selection when cooperative communication is considered. In this section, we will present a synopsis of related work.

Protocols in [1, 3, 9, 10, 14] focus on wireless selfish networks. These papers studied the problem of forwarding node selection in wireless selfish networks. Tansu Alpcan et al. presented a game-theoretic of distributed power control in CDMA wireless systems and addressed pricing and allocation of a single resource among several users. Authors in [3] introduced a new concept, Network Assisted Power Control (NAPC) that maximized utilities for users while maintaining equal signal-to-interference ratios for all users in a cellular network. Protocols in these two papers are unsuitable in a multi-hop network. In [9], with the help of reputation mechanism and utility functions based on game theoretical approach, selfish nodes were selected to help forwarding packets received from other nodes. Authors in [10] have investigated the underlying cooperation incentives of reputation system and price-based systems in selfish multi-hop wireless ad hoc networks. [14] provided a protocol to encourage nodes in hot areas to stay in the network and help others forwarding packets. This solution was based on the penalty and incentive mechanism, in which forwarding nodes are provided incentive and those non-cooperative nodes are punished to force them to participate the communications. Though researchers noticed selfish behaviors in wireless networks, unfortunately, they haven't taken cooperative communication into consideration in their protocols.

We notice that protocols mentioned above are only used to select some nodes to forward packets using traditional communication, and don't take cooperative communication into account. These schemes still use conventional communication for packet delivery, and couldn't solve the problem we proposed.

[16, 17] have studied the problem of relay node assignment in wireless cooperative networks. In this two literatures, several source-destination pairs select relay nodes from the same relay node set. One of assumptions is that all relay nodes are willing to participate in data transmission. This is not so practical in energy-constrained networks. In such kind of networks, each node avoids delivering packets for other nodes because of its limited energy. Many researchers have noticed this case, and lots of protocols [2, 6, 11–13, 15, 18, 19] were proposed to solve the problem of relay node selection in wireless cooperative networks, which consisted of selfish nodes. In [2], authors have proposed a general utility function, and with the help of this function, authors could find out the conditions under which user would get enough payoffs to cooperate with each other. Nevertheless, Chen et al. only focused on a two-user cooperations. In [6], two mechanisms were proposed to choose proper relays for cooperative communications in a 2-hop network. One of these mechanisms can be used to selected more than one relay. But both of two mechanisms are only designed for 2-hop networks. [11] analyzed the performance of cooperative Amplify-and-Forward in a selfish network where a source communicated with a destination with the help of multiple selfish relay nodes. Cho Yiu Ng et al. [13] have considered a situation in which there are M distinct nodes pair and each source node is selfish. Each source node divides its data into M sub-streams and sends one of these streams to its destination, and other streams are forwarded by remaining source nodes. For each source node, authors try to assign the rates between these source nodes, with the aim of minimizing the total transmission power. The network model in [11, 13] literature was a one-hop network, and didn't work well in a multi-hop network. In [12, 18], authors concentrated on a four node network. Marina [12] has studied a power allocation game in a four node relay network which consists of two source and two destination nodes. Both of source nodes use part of their power for their own transmission, and use the remaining part to help another source. Authors established an upper bound on the worst-case equilibrium efficiency. Authors in [18] show that under their network model, cooperative transmission with optimum resource allocation is a Nash Equilibrium, either in non-fading channels or fading channels. Neither of these two literatures provided a solution to solve the relay node selection for large scale network. Guopeng Zhang et al. [19] studied the problem of resource sharing between two selfish nodes in cooperative relay networks and showed how to allocate resources fairly to achieve a win-win strategy for both nodes. Authors focused on the optimal signal-to-noise ratio increase. In [15], each node decides whether and when transmitting data packets over a shared wireless channel. However, both of these two protocols didn't take energy efficiency into account. This makes that they couldn't work well in our case.

3 System model

In this section, we will describe our network model. Then, based on the network model, problem definition will be presented. Also, we will provide a brief overview on noncooperative strategic-form game theory, and our problem will be described as a normal-form game.

3.1 Network model

Here, we consider a wireless ad hoc network for many-to-one communication, where all nodes are uniformly distributed in a square region. All nodes have the same maximal power P_{max}. In our network model, we assume that a MAC protocol has already been implemented to disregard interference between transmissions. There are N nodes, M *Sources*, and a destination node, *Sink*. *Sink* gathers all data generated by *Sources*. Other nodes are ordinary nodes and *Sources* hope that these nodes can help relay data to *Sink*.

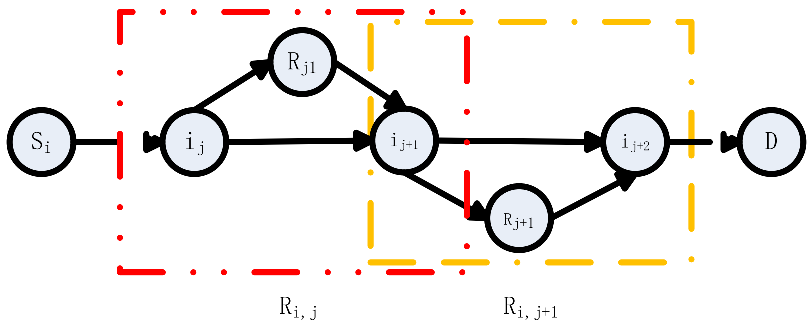


Fig. 4 An illustration of results for $Source_i$

However, because of energy constraint, these nodes behave in a selfish manner. This means that, to save its own energy, each node is not willing to forwarding any data they received.

In our network, we assume that two neighbor nodes can cooperate to transmit the same data to a single receiver. For example, in Fig. 1, when node i sends data to j, k can also receive such data correctly. So i and k can cooperate to send data to j successfully. From [7], we can see that the total transmitted power $P_{i,j,k}$ is

$$P_{i,j,k} = \frac{1}{\frac{1}{P_{i,j}} + \frac{1}{P_{k,j}}} \tag{1}$$

The power consumption P_i on i is:

$$P_i = \frac{1}{\frac{1}{P_{i,j}} + \frac{1}{P_{k,j}}} \cdot \frac{\frac{1}{P_{i,j}}}{\frac{1}{P_{i,j}} + \frac{1}{P_{k,j}}} \tag{2}$$

In the meantime, P_k indicates the power consumption on k,

$$P_k = \frac{1}{\frac{1}{P_{i,j}} + \frac{1}{P_{k,j}}} \cdot \frac{\frac{1}{P_{k,j}}}{\frac{1}{P_{i,j}} + \frac{1}{P_{k,j}}} \tag{3}$$

Each node knows nothing about the power consumptions for communications with its neighbors. If node u sends a message to query the power consumption between u and its neighbor t, t will send a reply message, which includes this power consumption. Here, node t is selfish and is not willing to help transmitting data. So he reports a false power consumption, which is no less than the real consumption. So that t may not be selected to participate the transmission and can save its energy.

We model this network as a graph $G = \{S, V, E, D\}$. S, D represents the *Source* node set and the *Destination*, respectively. $V = \{1, \ldots, n\}$ denotes the set of all other nodes and E is the set of communication links. For each link $\langle u, v\rangle$, we assume that node u knows nothing about how much power it should be used for data transmission to v when using traditional communication. In the following part, $\mathbb{N}_i$ represents the set of all node i's neighbors. For each link $\langle u, v\rangle$, we use $P_{u,v}$ to represent the real power consumption on this link. Because of selfishness, node u does not report the power consumption honestly, but a false value $P'_{u,v} \in [P_{u,v}, P_{max}]$.

3.2 Problem formulation

In this section, we will present our problem statement in detail.

Given a graph $G = \{S, V, E, D\}$, we try to find a node sequence $Q = \langle Q_1, \ldots, Q_M\rangle$ for all sources in S. Each node sequence Q_i consists of three parts: *Source* s_i, relay node sequences $\langle R_{i_1}, \ldots, R_{i_k}\rangle$, and D. R_{i_j} includes a forwarder F_{i_j}, a cooperative relay node CR_{i_j}, and a receiver RE_{i_j}. The receiver RE_{i_j} in R_{i_j} is the same node as the transmitter $T_{i_{j+1}}$ in $R_{i_{j+1}}$. For example, in Fig. 4, three parts of R_{i_j} are i_j (the forwarder), R_{j_1} (the cooperative relay node), and the receiver i_{j+1}. $R_{i,j}$, $R_{i,j+1}$ denotes R_{i_j}, $R_{i_{j+1}}$, respectively. $P_{F_{i_j}}$, $P_{CR_{i_j}}$ indicates the power consumption on $\langle F_{i_j}, RE_{i_j}\rangle$ and $\langle CR_{i_j}, RE_{i_j}\rangle$, respectively.

When we select forwarders and relays, we take energy-efficiency into account, and our aim is to:

$$Minimize: \quad \sum_{i=1}^{M} \sum_{R_{i_j} \in Q_i} (P_{F_{i_j}} + P_{CR_{i_j}}) \tag{4}$$

3.3 Game theory background

A strategic non-cooperative game [4] $\Gamma = \{N, A, u\}$ consists of three components:

1. Player set N: $N = \{1, 2, \ldots, n\}$, where n is the number of players in the game.
2. Action Set A: $a \in A = \times_{i=1}^{n} A_i$ is the space of all action vectors, where each component a_i of the vector a belongs to the set A_i, the set of actions of player i. Usually, we denote an action profile $a = \{a_i, a_{-i}\}$, where a_i is player i's action, and a_{-i} denotes the actions of the other $n-1$ players. Similarly, $A_{-i} = \times_{j \neq i} A_j$ is used to denote the set of action profiles for all players except i.
3. For each player $i \in N$, utility function $u_i : A \rightarrow \mathbb{R}$ figures his performances over the action profiles. $u =$

$(u_1, \ldots, u_n) : A \to \mathbb{R}^n$ presents the vector of such utility functions.

Nash Equilibrium (NE) is the most prevalent and important equilibrium concept in non-cooperative strategic-form game theory. This solution concept is defined as a stable point because no player has any incentive to unilaterally change his action from it.

Definition 1 An action profile $a^* = (a_i^*, a_{-i}^*)$ is an NE if $\forall i \in N$ and $\forall a_i \in A_i$

$$u_i(a^*) \geq u_i(a_i, a_{-i}^*) \tag{5}$$

3.4 Game description

We now describe the node selection as a normal-form game.

1. Player set: All nodes, except sources and sink, are players, and the player set is composed of these selfish nodes. $N = \{1, \ldots, n\}$ represents the player set.
2. Action set: For each player i, its action set A_i is the same as any other player's. A selfish node can act as a free node, or a forwarder. Also, it can act as a cooperative relay node. But a player couldn't act as all these three roles at the same time. Sice selfish nodes hate helping data transmission, they don't report its power consumption truthfully. We define $A_i = \{P'_{i,u} | P'_{i,u} \in [P_{i,u}, P_{max}]$, for $\forall u \in \mathbb{N}_i\}$.
3. Utility function: If a selfish node participates in data transmission, it will get payoff to reward its help. Otherwise, the payoff is 0. However, does this node really benefit from its participation? If not so, the node prefer acting as a free node. We use a utility function to capture its real benefit.

$$u_i = \varphi_i - \chi_i \tag{6}$$

φ_i, χ_i indicates the payoff of node i, the cost of node i, respectively. If the selfish node acts as a cooperative relay node, we define its benefit as the saved energy caused by cooperative communication. We assume that node i is a relay node for $\langle u, v \rangle$, which satisfies:

$$u \in \mathbb{N}_i, \quad v \in \mathbb{N}_i, \quad u \in \mathbb{N}_v \tag{7}$$

The payoff and cost of this node can be expressed as:

$$\begin{aligned}\varphi_{i_1} &= \max\left\{P_{u,v} - \left(\frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \cdot \frac{\frac{1}{P_{i,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} + \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \cdot \frac{\frac{1}{P_{u,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}}\right)\right\} \\ &= \max\left\{P_{u,v} - \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}}\right\}, \\ &\quad \text{for all } \langle u, v \rangle \text{ defined by (7)}\end{aligned} \tag{8}$$

When $\langle u, v \rangle$ makes φ_{i_1} maximal, we define:

$$\chi_{i_1} = \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \cdot \frac{\frac{1}{P_{i,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \tag{9}$$

$P_{u,v}$ represents the power consumption caused by traditional communication. If the node acts as a forwarder, we assume that its receiver is v. The payoff and cost are:

$$\varphi_{i_2} = P_{max}, \tag{10}$$

$$\chi_{i_2} = P_{i,v} \tag{11}$$

Now, in conclusion,

$$u_i = \max\{\varphi_{i_1} - \chi_{i_1}, \varphi_{i_2} - \chi_{i_2}, 0\} \tag{12}$$

Node i decides its role in data transmission based on this utility. If $u_i = 0$, and i receives any data, he will just discard what he has received. Otherwise, he acts as a relay node or a forwarding node.

4 Nash equilibrium and NP-hardness

In this section, we will show that it is a NE (Nash Equilibrium) for all player nodes to truthfully report their power consumptions for communications with their neighbors. Then if each player behaves unselfishly, we prove that our problem is NP-hard.

4.1 Nash Equilibrium

Theorem 1 *Assuming that each player sends a measurement message, all neighbors who receive this message, will return back a reply which includes the power consumptions on these links. Then it is a Nash Equilibrium for all players to truthfully report these power consumptions.*

Proof We use $P_{u,v}$ to represent the power consumption on the link $\langle u, v \rangle$ caused by conventional communication. Without loss of generalization, we assume that for player i, $\langle u, v \rangle$ makes φ_{i_1} maximal.

1. We compute the utility of i by (8) (9):

   If the node player i uses the strategy which is reporting the power consumption between i and player v truthfully to the query node u, and other nodes use the best strategies, the utility is

$$u_i(a_i^*, a_{-i}^*) = P_{u,v} - \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} - \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \cdot \frac{\frac{1}{P_{i,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \tag{13}$$

Then if i reports a false power consumption between i and v, the utility of i is

$$u_i(a_i, a^*_{-i}) = P_{u,v} - \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v'}}} - \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P'_{i,v}}} \cdot \frac{\frac{1}{P'_{i,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P'_{i,v}}} \tag{14}$$

Here, the relation between $P'_{i,v}$ and $P_{i,v}$ is

$$P'_{i,v} \geq P_{i,v} \tag{15}$$

Now, we prove that $u_i(a^*_i, a^*_{-i}) \geq u_i(a_i, a^*_{-i})$.

$$\begin{aligned} u_i(a^*_i, a^*_{-i}) - u_i(a_i, a^*_{-i}) &= \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v'}}} \\ &+ \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P'_{i,v}}} \cdot \frac{\frac{1}{P'_{i,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P'_{i,v}}} \\ &- \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \\ &- \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \cdot \frac{\frac{1}{P_{i,v}}}{\frac{1}{P_{u,v}} + \frac{1}{P_{i,v}}} \\ &= \left(\frac{p'_{i,v} + P_{i,v}}{P_{u,v}} \cdot P_{i,v} \cdot P'_{i,v} + \frac{2}{\cdot P_{i,v} \cdot P'_{i,v}} \right) \\ &\quad \times (P'_{i,v} - P_{i,v}) \\ &\geq 0 \end{aligned} \tag{16}$$

2. We compute the utility of i by (10) (11):

$$u_i(a^*_i, a^*_{-i}) = P_{max} - P_{u,v}$$
$$u_i(a_i, a^*_{-i}) = P_{max} - P'_{u,v}$$

Now, we show that $u_i(a^*_i, a^*_{-i}) \geq u_i(a_i, a^*_{-i})$.

$$\begin{aligned} u_i(a^*_i, a^*_{-i}) - u_i(a_i, a^*_{-i}) &= (P_{max} - P_{u,v}) - (P_{max} - P'_{u,v}) \\ &= P'_{u,v} - P_{u,v} \\ &\geq 0 \end{aligned} \tag{17}$$

From above analysis, we can conclude that $u_i(a^*_i, a^*_{-i}) \geq u_i(a_i, a^*_{-i})$. This means that a^* is a Nash Equilibrium. □

In the above, we have shown that it is a Nash Equilibrium for all players to truthfully report the power consumptions between this node and its neighbors.

4.2 NP-hard proof

Before we prove our problem (Node Selection Problem, NSP) is NP-hard, we first show that when $|S| = 1$, this problem is still NP-hard.

Here, we provide a known NP-hard problem [5] as follows:

Definition 2 SWCP (Shortest Weight-Constraint Path)
Given a graph $G(V, E)$, each edge is assigned two parameters, called length l and weight w. There are two nodes named source s and destination d. We want to find a shortest path from s to d in graph G, with the constraint that the total weight on this path is no more than W.

Lemma 1 *With the assumption that $|S| = 1$, if each node reports the power consumption for communication links with their neighbors, our problem is NP-hard. We call this reduced problem OneNSP.*

Proof We prove this by showing that: (1) SWCP can be reduced to OneNSP in polynomial time, (2) OneNSP $\notin$ NP.

1. SWCP can be reduced to OneNSP in polynomial time. Given a SWCP instance $G(V, E)$, we construct OneNSP as follows: for each edge $\langle u, v \rangle$, we add a new node w and the power cost on $\langle w, v \rangle$ is $\min\{P_{v,k} | k \in \mathbb{N}_v\} - \epsilon$. Here, $\epsilon < 0$. $P_{k,v}$ indicates the power consumption on communication link $\langle k, v \rangle$. $P_{u,v}$ is assigned a new value $P'_{u,v}$. There is a constraint on $P_{k,v}$ and $P'_{u,v}$, which is

$$\frac{1}{\frac{1}{P_{k,v}} + \frac{1}{P'_{u,v}}} = P_{u,v} \tag{18}$$

We argue that if the optimal solution for OneNSP is available, we can get the optimal solution for SWCP. We assume that $\langle R_{1_1}, \ldots, R_{1_k} \rangle$ is the best solution. For $\forall R_{1_j}$, CR_{1_j} is the added node. This is because for any node T in $\mathbb{N}_{RE_{1_j}}$,

$$\frac{1}{\frac{1}{P_{F_{1_j}, RE_{1_j}}} + \frac{1}{P_{CR_{1_j}, RE_{1_j}}}} < \frac{1}{\frac{1}{P_{F_{1_j}, RE_{1_j}}} + \frac{1}{P_{T, RE_{1_j}}}} \tag{19}$$

We can draw a conclusion that $\langle s_1, F_{1_1}, \ldots, F_{1_k}, D \rangle$ is the optimal solution for SWCP. If we can find a better solution $\langle s_1, F'_{1_1}, \ldots, F'_{1_k}, D \rangle$, we add new nodes into this node sequence, we will get a better solution for OneNSP. This results in a conflict.
2. If $\langle R_{1_1}, \ldots, R_{1_k} \rangle$ is the solution we have found, we will verify whether this is optimal. In the worst case, it will take $O(n \cdot 2^n)$ for this verification. This can not terminate in polynomial time.

We can draw a conclusion that OneNSP is NP-hard. □

With the help of this lemma, we can prove that NSP is a NP-hard problem.

Theorem 2 *If each node can report the power consumptions for communication links with their neighbors, our problem (NSP) is NP-hard.*

Proof Assuming that we have already found the optimal solution for NSP, the solution for each source node s_i in NSP is just the solution for OneNSP. If we can find a better solution in OneNSP for s_i, this solution can be used to take the place of the solution in NSP. That means, we can find a better solution for NSP. This is a contradiction.

If we can solve NSP in polynomial time, we can get the optimal solution for OneNSP in polynomial time too. This is inconstant with Lemma 1.

So, we conclude that NSP is a NP-hard problem. □

5 Solution for NSP

Since the problem NSP is NP-hard, we propose a heuristic algorithm (Algorithm for Node Selection Problem, ANSP) in this section. In essential, ANSP is a greedy algorithm. It consists of three phases: the initialization phase, the adaption phase, and the update phase. In this section, we also prove the convergence of ANSP, and analysis the performance of our algorithm.

5.1 Algorithm description

Now, we show our algorithm in detail, which consists of three phases: the initialization phase, the adaption phase, and the update phase. We will present them in the following part.

1. **Initialization phase**
   Each node broadcasts a probe message to its neighbors at the maximal power level. If node v receives a probe message from node u, it calculates the power consumption on link $\langle u, v\rangle$. We use $P_{u,v}$ to represent the real power consumption. After this, v sends an ACK message back to u, which includes node u, $P_{u,v}$. Then, we start a BFS (Breadth-First Search) search. In the end of this operation, all nodes know their minimum hop counts to the sink D (Algorithm 1).
2. **Adaption phase**
   In this phase, a path from s_i to D is generated. When we select a receiver for node u, we compute $Val = P_{max} - P_{u,v}$. The node j who have the maximal *val* will be selected. In the following, for $\forall w \in \mathbb{N}_v \cap \mathbb{N}_u$, compute the utility using (8) (9) and choose the node k as the cooperative relay node, who has the maximal utility. We mark j and k black to indicate that they have already been chosen to help data transmission. Next, j acts as a forwarder, and we find a proper receiver for him. We repeat these operations until the receiver is D (Algorithm 2).
3. **Update phase**
   We have already selected nodes for data transmission between s_i and D. These black nodes can not be used in other paths. We remove these nodes and edges which are related to these node from the graph G. We will run our algorithm on the updated graph. Eventually, we will find a solution for NSP by repeating Phase 2 and Phase 3. Algorithm 3 describes this phase.

Algorithm 1 Initialization phase

For $\forall$ node $u \in V$, broadcast a request message at its maximal power level
For any node v, who receives probe messages from any other node u, calculate the power consumption on $\langle u, v\rangle$
send the ACK message back to u
Tree = BFS(G, D)

Algorithm 2 Adaption phase

$u = s_i$, max $= 0$, *utility* $= 0$, $Q_i = \phi$, $\mathbb{N}_0 = \phi$, $j = 1$, color all nodes white
while $u \neq D$ **do**
  $F_{i_j} = u$
  for $\forall v \in \mathbb{N}_u$, $H(v) < H(F_{i_j})$ **do**
    if v is white, max $< P_{max} - P_{F_{i_j},v}$ **then**
      max $= P_{max} - P_{F_{i_j},v}$
      u = v
    end if
  end for
  color u black, $RE_{i_j} = u$
  $\mathbb{N}_0 = \mathbb{N}_{F_{i_j}} \cap \mathbb{N}_u$
  for $\forall w \in \mathbb{N}_0$ **do**
    send $P_{w,u}$ to F_{i_j}
    if $Utility(F_{i_j}, w, u) > utility$ **then**
      $R_{i_j} = \{F_{i_j}, w, u\}$
      relay $= w$
      utility = Utility(F_{i_j}, w, u)
    end if
  end for
  color *relay* black, $CR_{i_j} = relay$
  j++
end while

5.2 Convergence of the ANSP

Now, we analyze the convergence of the ANSP.

Theorem 3 *The ANSP converges to an equilibrium solution.*

Proof Let us define utility of the ith node $u_i^{(n)}$ at any iteration (n). $u_i^{(n+1)}$ is the utility at next iteration. In Algorithm 2,

Algorithm 3 Update phase

1: $G = G(V - Q_i)$
2: **for** $\forall s_i \in S$ **do**
3:   **if** s_i is not black **then**
4:     Adaption Phase
5:     Update Phase
6:   **end if**
7: **end for**

Line 5 and Line 14 indicate that the Inequality (20) holds after each iteration.

$$u_i^{(n)} \le u_i^{(n+1)} \quad (20)$$

For all nodes in the network, (20) always holds at any time. Hence, (20) is a contraction mapping, which leads to the global convergence of the ANSP. □

5.3 Approximate performance ratio

In the following part, we will analyze the performance of ANSP. First, we prove the following lemma.

Lemma 2 *MST (Minimum Spanning Tree) can achieve the approximate performance ratio of* $2 \cdot (1+\alpha)$ *for problem NSP, where* α *is the maximal ratio of two power consumptions on two adjacent links in this network.*

Proof We assume that Q is the optimal solution for NSP. We construct another topology Q' based on this optimal solution. In this case, each node reports the power consumption honestly. For $\langle u, t, v\rangle \in Q$, if $t = 0$, we add $\langle u, v\rangle$ to Q'. Otherwise, $\min\{\langle t, v\rangle + \langle u, t\rangle, \langle u, v\rangle\}$ are added to Q'. Here, $\min\{\langle t, v\rangle + \langle u, t\rangle, \langle u, v\rangle\}$ represents the link which has a smaller power consumption in $\{\langle t, v\rangle + \langle u, t\rangle, \langle u, v\rangle\}$.

$$P_{u,v,t} = \frac{1}{\frac{1}{P_{u,v}} + \frac{1}{P_{t,v}}} \ge \frac{1}{2} \cdot \min\{P_{u,v}, P_{t,v}\} \quad (21)$$

Assuming that

$$\alpha = \max\left\{\frac{P_{u,t}}{P_{u,v}},\ \text{for any node } u, \text{ node } t, v \text{ are two different neighbors of } u\right\} \quad (22)$$

If we use conventional transmission scheme, the minimum power consumption is

$$P'_{u,v,t} = \min\{P_{u,v}, P_{u,t} + P_{t,v}\} = \min\{P_{u,v}, (1+\alpha_1) \cdot P_{t,v}\}, \quad \text{here, } \alpha_1 = \frac{P_{u,t}}{P_{t,v}} \quad (23)$$

So,

$$P_{u,v,t} \ge \frac{1}{2} \cdot \min\{P_{u,v}, P_{t,v}\} \ge \frac{1}{2 \cdot (1+\alpha_1)} \cdot P'_{u,v,t} \ge \frac{1}{2 \cdot (1+\alpha)} \cdot P'_{u,v,t} \quad (24)$$

Now, we can conclude that MST can reach the approximate performance ratio of $2 \cdot (1+\alpha)$ for NSP. □

With the help of this lemma, we analyze the performance of NASP.

Theorem 4 *ANSP algorithm can reach the approximate performance ratio of* $2 \cdot (1+\alpha)$, *where* α *is the maximal ratio of two power consumption on two adjacent links in this network.*

Proof From the description of ANSP, we can see that MST is the foundation of ANSP. ANSP first runs a BFS search. Then in the rest of ANSP, all operations is designed on improving the MST. So ANSP can reach the approximate performance ratio of $2 \cdot (1+\alpha)$, where α is the maximal ratio of two power consumptions on two adjacent links in this network. □

6 Numerical results

In this section, we evaluate the performance of ANSP. First, we will show that our protocol indeed prevent nodes from the selfish behaviors. Second, energy-efficiency will also be simulated. Our network consists of 100 nodes, and from these nodes, 5 nodes are selected randomly to act as sources. All nodes are uniformly distributed in a fixed-size square area, 100×100. The destination D is placed in the center of this area. The communication range of each node is the same, 20. All nodes have the same maximum power level, 1000. The power consumption between two neighbor nodes is $2.5L^2$, where L is the Euclidean distance between two neighbors. Each node can take a selfish behavior by reporting a false power consumption between two adjacent nodes. Meanwhile, we run our algorithm 100 times.

6.1 Cheating behavior and node utility

Based on our assumptions, nodes have two types of behaviors: unselfish (honest) behavior and selfish (cheating) behavior. For link $\langle u, v\rangle$, when node v takes an unselfish behavior, it reports a power consumption between u and v honestly. Oppositely, when node v takes a cheating behavior, it reports a false power consumption between u and v, which

Fig. 5 Utility of forwarder

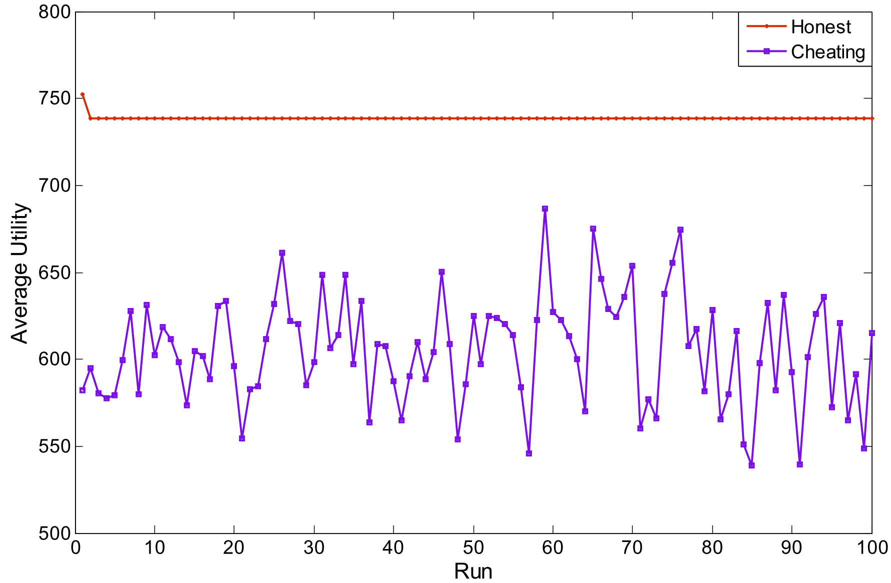
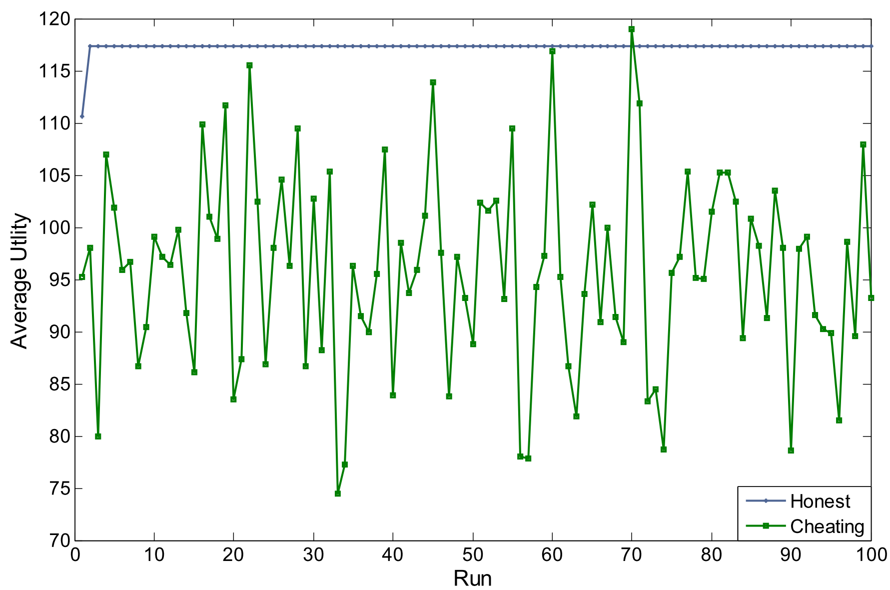


Fig. 6 Utility of cooperative nodes



is larger than $P_{u,v}$ and less than P_{max}. Our algorithm is repeated 100 times.

First, in Fig. 5, we show the average utility of all forwarders in each round. Here, forwarders means F_{i_j} defined in Sect. 3.2. We can observe that when nodes take selfish behaviors, the average utility in each round is different from each other. This is because the power consumption reported by each node is a random value. Nevertheless, each of these utility is less than the utility obtained when nodes behave unselfishly. When nodes act unselfishly, the average utility of forwarders is almost the same. In a sense, this reflects the convergence of our algorithm.

Figure 6 illustrates the utility of cooperative nodes. We can easily draw a conclusion that the utility obtained by honest report is more than that obtained by cheating report. In Fig. 6, there is a round in which the utility obtained by cheating report is close to that obtained by honest report. This is because most of nodes have reported the power consumptions which are close to the real power consumptions. Meanwhile, in the 70th round, the average utility obtained by cheating reports is larger than that obtained by honest reports.

From Figs. 5 and 6, we notice that in the first several runs, the average utility of forwarders, the average utility of cooperative nodes varies, respectively. When nodes behave unselfishly, the average utility of forwarders decreases and the average utility of cooperative nodes increases. Now, we will present the reason why the average utility of forwarders decreases. Node u acted as a free node. But in the next round, u was selected as a forwarder, and the utility of u was smaller

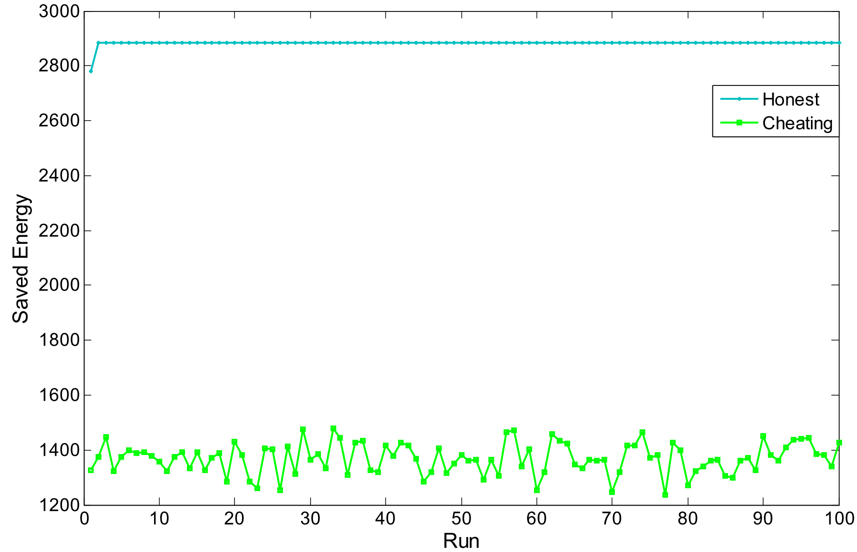


Fig. 7 Energy efficiency of honest reporting

than the average utility of the forwarders, who were selected in the former round. So if we calculated the average utility of forwarders after this round, the new average decreased. In the meantime, when we added u as a forwarder, a cooperative node v might be selected. The utility of this node was larger than the average utility of relays. When this rounder ended, the average utility of cooperative nodes increased. However, from this two figures, we can see that, several runs later, the average utility of forwarders and the average utility of relays hold the lines. This reflects the convergence of ANSP in a sense.

6.2 Cheating behavior and energy-efficiency

We also compare energy-efficiency induced by selfish behaviors with energy-efficiency caused by honest reports. In these two different strategies, cooperative communication is used to save transmit energy.

In Fig. 7, we can see that we save much more energy if nodes truthfully report their power consumptions on their related links. This is reasonable. For example, on link $\langle u, v\rangle$, if node v reports a larger power consumption on this link than its real power consumption, node u will use the false power to transmit data. This leads to unnecessary cost. So honest reports can save much more energy than cheating behaviors. Based on our results, we can get that, the average of saved energy in 100 runs is 2884 when nodes report honestly, and the average is 136.94 when nodes take selfish behaviors. The later is 52.5% less than the former.

7 Conclusion

In this paper, we consider a wireless ad hoc network for many-to-one communication, where each node is energy-constrained and prefers to behave in a selfish manner (in this case, each node tries to avoid forwarding data). To reduce transmit energy depletion, we select forwarding nodes and relay nodes carefully. We pose this problem as a non-cooperative game and use game-theoretic analysis to address it. Based on the utility function we proposed, we prove that it is a Nash Equilibrium when each node behave unselfishly. Meanwhile, we show that the problem is NP-hard. To solve this problem, a heuristic algorithm (Algorithm for Node Selection Problem, ANSP) is provided. We also prove the convergence of this algorithm. The analysis shows that this algorithm can reach the approximate performance ratio of $2 \cdot (1 + \alpha)$, where α is the maximal ratio of two power consumptions on two adjacent links in the network. The numerical results show that if nodes behave unselfishly, these nodes will obtain a better utility, and we can save more energy. The average saved energy when nodes take selfish behaviors is 52.5% less than the average when nodes behave in an unselfish manner.

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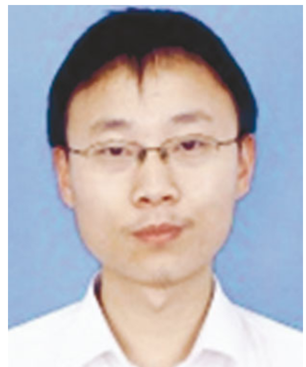
Yindong Zhang received his B.S. in Computer Science from the University of Science and Technology of China in 2003. He is currently a doctor-candidate in Computer Science at the University of Science and Technology of China. He will receive the doctor degree in 2012. His research interests are wireless sensor networks, and cooperative communications.

Liusheng Huang is currently a professor and Ph.D. supervisor of the School of Computer Science and Technology at the University of Science and Technology of China. He has published 6 books and more than 90 papers. His research interests are wireless sensor networks, information security, distributed computing and high performance algorithms.



Hongli Xu received his Ph.D. degree in Computer Science from the University of Science and Technology of China in 2007. Currently, he is a post-doctor in the School of Computer Science and Technology at the University of Science and Technology of China. His research interests are wireless sensor networks and cooperative communications.

Zhenguo Yang is currently a Ph.D. candidate in the Department of Computer Science and Technology, University of Science and Technology of China. He will receive his Ph.D. degree in 2012. His research interests include wireless sensor networks and delay tolerant networks.