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# Maximum bandwidth routing and maximum flow routing in wireless mesh networks

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**Abstract** The emergence of nomadic multimedia applications, such as multimedia conferencing, distance learning, video phones, video/movie-on-demand, and education-on-demand, has recently generated much interest in multi-hop wireless mesh networks (WMNs) to support diverse Quality-of-Service (QoS). In the existing WMN QoS routing protocols, the methods of bandwidth calculation and allocation were proposed to determine routes with bandwidth guaranteed for QoS applications. This paper studies two NP-hard problems, the maximum bandwidth routing problem (abbreviated to MBRP) and the maximum flow routing problem (abbreviated to MFRP). Given a source node $s$ and a destination node $d$ in a multi-hop wireless mesh network, the MBRP is to determine an $s$-to-$d$ path that can carry a maximum amount of traffic from $s$ to $d$ and the MFRP is to determine the maximum flow from $s$ to $d$, both retaining the network bandwidth-satisfied. In this paper, heuristic algorithms for the two problems are proposed. Upper bounds on their optimal values are derived, and a lower bound is derived on the feasible value obtained for the MBRP. With the upper bound and the lower bound, an approximation ratio for the heuristic algorithm of the MBRP is obtained. The effectiveness of the heuristic algorithms is further verified by experiments. A generalized interference model is also discussed.

**Keywords** Approximation ratio · Heuristic algorithm · Optimization problem · QoS routing · Wireless mesh network

## 1 Introduction

Recently, multi-hop wireless mesh networks (WMNs) have received much attention, because they can serve as last-mile broadband Internet access networks in next-generation communication systems [5]. A multi-hop WMN is composed of a collection of stationary wireless nodes that maintain wireless radio connectivity to form a backbone for various applications to deliver data packets in a multi-hop manner. Some of the wireless nodes function as gateways that are directly connected to the Internet via wired lines for providing Internet services. Current wireless mesh networking technologies enable a cost-effective scalable deployment without wired lines for establishing a community network or a metropolitan network [8].

In order to alleviate network congestion and thus improve network utilization in a multi-hop WMN, a routing algorithm that is capable of balancing network load is required. Previously, there were some wireless ad hoc routing protocols [16, 22–24, 30–32], that could forward packets

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along shortest paths (i.e., with minimum numbers of hops). Although they behave well for best-effort traffic, they are likely to use the same set of hops to relay packets for the same source-destination pair. Therefore, packets may be forwarded along congested shortest paths, while longer, but not congested, paths may be waste [10].

To balance network load in wired networks, routing paths with maximum bandwidth should be considered, in addition to shortest paths. For example, given a source-destination pair, the routing protocols in [6, 12, 21, 26] determined the maximum bandwidth path among all shortest paths, whereas the routing protocol in [13] determined the maximum bandwidth path among all $k$th shortest and $(k+1)$th shortest paths for some $k$. On the other hand, the routing protocols in [9, 29] determined the shortest path among all maximum bandwidth paths. The routing protocol in [19] took resource fairness into account, while selecting a maximum bandwidth path. The routing protocols in [14, 17, 28] selected uncongested paths by the aid of maximum flow algorithms [4]. A comparative study of load-balancing routing protocols can be found in [20].

In order to develop a load-balancing routing protocol in a multi-hop WMN, a straightforward approach is to enhance the above routing protocols so that they can behave well also in a multi-hop WMN. However, the problem of determining a maximum bandwidth path and the problem of determining a maximum flow, both in a multi-hop WMN, are intractable, due to the spatial contention among wireless links [15, 18]. In a wired network, they can be easily solved in polynomial time [4, 29].

In this paper, we study the two problems in a multi-hop WMN. The first problem is referred to as the *maximum bandwidth routing problem* (abbreviated to MBRP), and the second problem is referred to as the *maximum flow routing problem* (abbreviated to MFRP). We use the IEEE 802.11 MAC [3] as the underlying MAC since it has been widely adopted in the wireless community and many commercial products. For example, both Motorola's MeshNetworks Enabled Architecture (MEA) technology [1] and Nortel's wireless mesh network solution [2] use the IEEE 802.11 MAC as the underlying MAC.

The MBRP and the MFRP are first expressed as two maximization problems. Then, upper bounds on their maximum values are derived, and heuristic algorithms for them are provided. A lower bound is also derived on the feasible values obtained for the MBRP, which represents the worst-case behavior. Consequently, an approximation ratio for the heuristic algorithm of the MBRP is obtained, i.e., the ratio of the corresponding upper bound to the lower bound. The effectiveness of the two heuristic algorithms is further verified by experiments. The upper bounds also serve as benchmarks for evaluating the quality of obtained feasible values.

Previously, the MBRP was formulated in [15] as an integer linear programming. However, the formulation did not take network bandwidth satisfaction (explained in Sect. 2) into account. As a consequence, bandwidth violation to some existing paths may happen after a newly admitted routing path starts transmission. The worst-case performance and the best-case performance for the MFRP were also analyzed in [15]. No solution method for solving the two problems was suggested in [15]. In [27], a heuristic solution method for the MBRP was proposed and its solutions were compared with shortest paths for performance evaluation.

The rest of this paper is organized as follows. In Sect. 2, a multi-hop WMN is represented as a directed graph, and the interference model under the IEEE 802.11 MAC is introduced. In Sect. 3, the MBRP and the MFRP are expressed as two maximization problems. Upper bounds on the maximum values are given in Sect. 4, and heuristic algorithms for them are proposed in Sect. 5. Experimental results for performance evaluation are shown in Sect. 6. Finally, in Sect. 7, this paper concludes with a discussion on a generalized interference model.

## 2 Network model and interference model

A multi-hop WMN is conveniently expressed as a directed graph $G(N, L)$, where $N$ denotes the set of wireless nodes and $L$ denotes the set of wireless links. There is a link $(i, j) \in L$ if node $j$ is within the transmission range of node $i$. Notice that $L$ is not symmetric, i.e., $(i, j) \in L$ does not necessarily imply $(j, i) \in L$. Two distinct links, say $(u_1, v_1)$ and $(u_2, v_2)$, are *one-hop neighboring* if they have a common end node, and *two-hop neighboring* if they have no common end node, but $u_2$ or $v_2$ is within the transmission range of $u_1$ or $v_1$. We use $L^1_{(i,j)}$ ($L^2_{(i,j)}$) to denote the subset of $L$ whose each link is one-hop (two-hop) neighboring to $(i, j)$. Notice that $L^1_{(i,j)} = L^1_{(j,i)}$ and $L^2_{(i,j)} = L^2_{(j,i)}$.

By $F$ we denote a matrix of size $|N| \times |N|$, where each entry $F(i, j)$ ($\geq 0$) represents the amount (in terms of units per second) of traffic requested to be transmitted over the link $(i, j)$. If $(i, j) \notin L$, set $F(i, j) = 0$. A link $(i, j) \in L$ is *active* if $F(i, j) > 0$, and *inactive* if $F(i, j) = 0$. We use $A$ ($\subseteq L$) to denote the set of all active links in $G$, and $A^1_{(i,j)}$ ($A^2_{(i,j)}$) to denote the subset of $A$ whose each link is one-hop (two-hop) neighboring to $(i, j)$. Notice that $A^1_{(i,j)} = A^1_{(j,i)}$ and $A^2_{(i,j)} = A^2_{(j,i)}$. The capacity of link $(i, j) \in L$ is denoted by $c_{(i,j)}$ (in terms of units per second).

Throughout this paper, we use $s$ and $d$ to denote a pair of source node and destination node in $G$. Let $S$ ($D$) denote the set of links $(i, j) \in L$ with $i = s$ or $j = s$ ($i = d$ or $j = d$). Also, let $L^1_S$ ($L^1_D$) denote the subset of $L$ whose each link is one-hop neighboring to some link in $S$ ($D$). For each path $P = \{(x_1, x_2), (x_2, x_3), \ldots, (x_n, x_{n+1})\}$ in $G$, we define $A_P$ to be the subset of $A$ whose each link is one-hop or two-hop

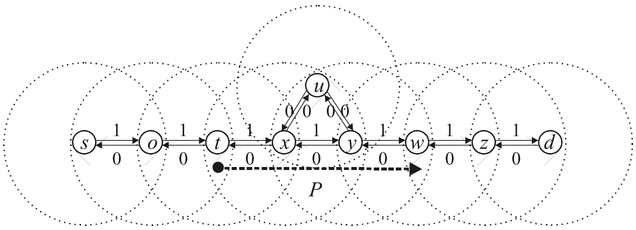


**Fig. 1** An illustrative example

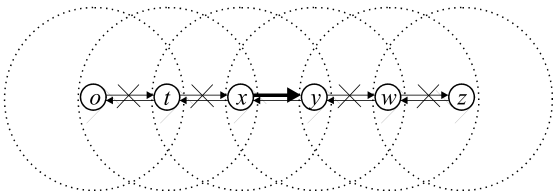


**Fig. 2** Spatial contention in a wireless chain network

neighboring to some link in $P$, but is not contained in $P$, i.e.,

$$A_P = \bigcup_{(i,j)\in P} (A^1_{(i,j)} \cup A^2_{(i,j)}) - P.$$

Refer to Fig. 1 for an illustrative example. The nonnegative numbers represent the values of $F(i,j)$s, e.g., $F(x,y)=1, F(y,x)=0$, and $F(y,w)=1$. We have $L^1_{(x,y)} = \{(y,x),(t,x),(x,t),(y,w),(w,y),(x,u),(u,x),(y,u),(u,y)\}$, $L^2_{(x,y)} = \{(o,t),(t,o),(w,z),(z,w)\}$, $A = \{(s,o),(o,t),(t,x),(x,y),(y,w),(w,z),(z,d)\}$, $A^1_{(x,y)} = \{(t,x),(y,w)\}$, $A^2_{(x,y)} = \{(o,t),(w,z)\}$, $S=\{(s,o),(o,s)\}$, $D=\{(z,d),(d,z)\}$, $L^1_S = \{(o,t),(t,o)\}$, and $L^1_D = \{(w,z),(z,w)\}$. If $P=\{(t,x),(x,y),(y,w)\}$, then $A_P = \{(s,o),(o,t),(w,z),(z,d)\}$.

The interference in a multi-hop WMN relies on the underlying MAC layer. The underlying MAC we adopt is the IEEE 802.11 MAC whose interference model is explained with the example of Fig. 2. Suppose that there is ongoing transmission over link $(x,y)$. Then, transmission over one-hop neighboring links (i.e., $(y,x),(t,x),(x,t),(y,w)$ and $(w,y)$) and two-hop neighboring links (i.e., $(o,t),(w,z)$, $(z,w)$ and $(t,o)$) of $(x,y)$ should not be initiated, in order to avoid interference. Transmission over any of $(o,t),(w,z)$ and one-hop neighboring links of $(x,y)$ will cause collision. Although transmission over $(z,w)$ and $(t,o)$ does not cause collision, it prevents the transmission of the ACK (CTS) packet generated by the two-way (four-way) handshaking. More detailed description about the interference model of the IEEE 802.11 MAC can be found in [7].

According to the discussion above, not only the bandwidth consumption of an active link, but the bandwidth consumption of its active one-hop neighboring links and its active two-hop neighboring links should be considered, in order not to violate its link capacity. An active link $(i,j)$ is said to be *bandwidth-satisfied* if $F(i,j) + \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j') \le c_{(i,j)}$, i.e., the total bandwidth consumption of $(i,j)$, its active one-hop neighboring links and its active two-hop neighboring links does not exceed its link capacity. Further, $G$ is *bandwidth-satisfied* if all its active links are bandwidth-satisfied. Notice that when a bandwidth-satisfied link is mentioned, an active link is meant.

Suppose that $G$ is bandwidth-satisfied and $P$ is a path in $G$. There is an interesting problem which asks the maximal amount of traffic so that when it is injected into $P$, $G$ remains bandwidth-satisfied. Notice that $G$ is bandwidth-satisfied if and only if each link $(i,j)\in P\cup A_P$ is bandwidth-satisfied. The problem can be easily solved in $O(|P\cup A_P|)$ time, by examining each link in $P\cup A_P$.

## 3 MBRP and MFRP

Given a bandwidth-satisfied $G$, the matrix $F$, a pair of source node $s$ and destination node $d$ and the capacities of all links of $G$, the MBRP is to determine the $s$-to-$d$ path, among all $s$-to-$d$ paths, that can carry a maximum amount of traffic from $s$ to $d$, while retaining $G$ bandwidth-satisfied. On the other hand, the MFRP is to determine a set of (not necessarily disjoint) $s$-to-$d$ paths that can carry a maximum amount of traffic from $s$ to $d$, while retaining $G$ bandwidth-satisfied. Equivalently, the MFRP is to find the maximum flow from $s$ to $d$ in $G$, subject to the constraint that $G$ is bandwidth-satisfied. Both the MBRP and the MFRP are NP-hard, which was shown in [18] and [15], respectively. Their intractability is mainly due to the spatial contention among the links of $G$.

Let $X$ denote a matrix of size $|N|\times|N|$, where each entry $X(i,j)\ge 0$ is a variable representing an amount of traffic that can be added to link $(i,j)$ without causing bandwidth violation of $G$. Also let $b$ denote a variable representing an amount of traffic that can be emitted from $s$ and received by $d$, also without causing bandwidth violation of $G$. The MFRP can be formulated as a nonlinear programming as follows:

Maximize $b$

subject to

$$\sum_{j\in N} X(s,j) - \sum_{j\in N} X(j,s) = b; \tag{1}$$

$$\sum_{j\in N} X(j,d) - \sum_{j\in N} X(d,j) = b; \tag{2}$$

$$\sum_{j\in N} X(i,j) - \sum_{j\in N} X(j,i) = 0, \quad \forall i\in N-\{s,d\}; \tag{3}$$

$$(F(i,j)+X(i,j)) \times \Bigg[(F(i,j)+X(i,j)) + \sum_{(i',j')\in L^1_{(i,j)}\cup L^2_{(i,j)}} (F(i',j')+X(i',j'))\Bigg] \le (F(i,j)+X(i,j)) \times c_{(i,j)}, \quad \forall (i,j)\in L; \tag{4}$$

$$b \ge 0,\ X(i,j) \ge 0, \quad \forall (i,j)\in L. \tag{5}$$

Constraints (1)–(3) ensure that all nodes in $G$ obey the flow conservation rule. Constraint (4) ensures that all active links (i.e., those $(i,j)$s with $F(i,j) > 0$) and the links that are used to carry the flow from $s$ to $d$ (i.e., those $(i,j)$s with $X(i,j) > 0$) are bandwidth-satisfied. Notice that constraint (4) is also satisfied with the links $(i,j)$ having $F(i,j) = 0$ and $X(i,j) = 0$, because the multiplier $F(i,j) + X(i,j) = 0$. If constraint (4) is satisfied with all links of $G$, then $G$ is bandwidth-satisfied. Constraint (5) requires that only the nonnegative amount of traffic is considered.

In case all links of $G$ are active, the multiplier of constraint (4) is positive and thus can be removed. Then, a linear programming can result, which is polynomial-time solvable.

## 4 Two upper bounds

In this section, two upper bounds are derived on the optimal values of the MBRP and the MFRP, respectively. For a maximization problem, an upper bound is usually adopted as a benchmark for the effectiveness of a heuristic solution method.

### 4.1 An upper bound for the MBRP

Throughout this section, we assume that the distance (in terms of the number of hops) between $s$ and $d$ is greater than or equal to four.

**Lemma 1** *The maximal amount of traffic that can be carried by an s-to-d path without causing bandwidth violation of G does not exceed* $\min\{(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j'))/3 \mid (i,j)$ *is the first link or last link of the s-to-d path*$\}$.

*Proof* Exclusive of $(i,j)$, there is one link (one link or more) in the $s$-to-$d$ path that is one-hop (two-hop) neighboring to $(i,j)$. That is, at least two links in the $s$-to-$d$ path may interfere with $(i,j)$. Suppose that $t$ is the amount of traffic carried by the $s$-to-$d$ path. If $(i,j)$ is bandwidth-satisfied, then

$$3t + F(i,j) + \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j') \le c_{(i,j)},$$

from which

$$t \le \Bigg(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j')\Bigg)\Big/3.$$

□

The following two lemmas can be proved similarly.

**Lemma 2** *The maximal amount of traffic that can be carried by an s-to-d path without causing bandwidth violation of G does not exceed* $\min\{(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j'))/4 \mid (i,j) \in L^1_S \cup L^1_D$ *is a link of the s-to-d path*$\}$.

**Lemma 3** *The maximal amount of traffic that can be carried by an s-to-d path without causing bandwidth violation of G does not exceed* $\min\{(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j'))/5 \mid (i,j) \in L - (S \cup D) - (L^1_S \cup L^1_D)$ *is a link of the s-to-d path*$\}$.

If the distance between $s$ and $d$ is smaller than four, then the maximal amount of traffic carried by an $s$-to-$d$ path without causing bandwidth violation of $G$ can be computed similarly. As an immediate consequence of Lemmas 1 to 3, there are upper bounds, denoted by $u_{(i,j)}$s, on the maximal amount of traffic carried by an $s$-to-$d$ path without causing bandwidth violation of $G$, where $(i,j)$s are the links of the $s$-to-$d$ path. Notice that $u_{(i,j)}$s are also upper bounds on the optimal value of the MBRP, if the optimal solution to the MBRP contains those $(i,j)$s. An upper bound on the optimal value of the MBRP can be obtained as follows.

First, arrange all links $(i,j)$ of $G$ into a nondecreasing sequence of $u_{(i,j)}$s. Then, remove $(i,j)$s sequentially, starting from the beginning of the sequence, until a link is encountered whose removal causes that $d$ is not reachable from $s$. We denote the link by $(i^+, j^+)$. Since the optimal solution to the MBRP must contain at least one of those removed links and $(i^+, j^+)$, $u_{(i^+,j^+)}$ is an upper bound on the optimal value of the MBRP.

### 4.2 An upper bound for the MFRP

If the constraint (4) of the nonlinear programming in Sect. 3 is relaxed to

$$(F(i,j)+X(i,j)) + \sum_{(i',j')\in L^1_{(i,j)}\cup L^2_{(i,j)}} (F(i',j')+X(i',j')) \le c_{(i,j)}, \quad \forall (i,j)\in A,$$

then a linear programming results. The optimal value of the linear programming is an upper bound on the optimal value of the nonlinear programming (i.e., the MFRP).

## 5 Heuristic algorithms

In this section, heuristic algorithms for the MBRP and the MFRP are presented.

### 5.1 A heuristic algorithm for the MBRP

Let $N_{(i,j)} \subseteq N$ be the set of the end nodes of those links in $\{(i,j)\} \cup L^1_{(i,j)} \cup L^2_{(i,j)}$.

**Lemma 4** *Suppose that* $(i,j)$ *is a link of* $G$ *and* $P_{(i,j)}$ *is an* $s$*-to-*$d$ *path that contains one or more links in* $\{(i,j)\} \cup L^1_{(i,j)} \cup L^2_{(i,j)}$. *If the amount of traffic carried by* $P_{(i,j)}$ *is smaller than or equal to* $(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j'))/(|N_{(i,j)}|-1)$, *then* $(i,j)$ *must be bandwidth-satisfied.*

*Proof* Suppose that the amount of traffic carried by $P_{(i,j)}$ is $t$, and

$$t \le \left(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j')\right) /(|N_{(i,j)}|-1).$$

If $(i,j)$ is not bandwidth-satisfied, then

$$nt + F(i,j) + \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j') > c_{(i,j)}$$

(or equivalently,

$$t > \left(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j')\right)/n,$$

where $n$ is the number of links in $\{(i,j)\} \cup L^1_{(i,j)} \cup L^2_{(i,j)}$ that are contained in $P_{(i,j)}$.

It is not difficult to see that $n$ is not greater than $|N_{(i,j)} - 1|$. Hence,

$$t > \left(c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j')\right) /(|N_{(i,j)}|-1),$$

which is a contradiction. □

For each link $(i,j)$ of $G$, let $l_{(i,j)} = (c_{(i,j)} - F(i,j) - \sum_{(i',j')\in A^1_{(i,j)}\cup A^2_{(i,j)}} F(i',j'))/(|N_{(i,j)}|-1)$. According to Lemma 4, if an $s$-to-$d$ path contains one or more links in $\{(i,j)\} \cup L^1_{(i,j)} \cup L^2_{(i,j)}$, then the maximal amount of traffic that it can carry without causing bandwidth violation of $(i,j)$ is at least $l_{(i,j)}$. The following procedure can provide a heuristic solution to the MBRP.

Step 1. Arrange all links $(i,j)$ of $G$ into a nondecreasing sequence of $l_{(i,j)}$s. Denote the sequence by $Q = \{(i_1,j_1),(i_2,j_2),\ldots,(i_{|L|},j_{|L|})\}$. Set $k=1$ and $G' = G$.
Step 2. **while** $d$ is reachable from $s$ in $G' - \{(i_k,j_k)\} \cup L^1_{(i_k,j_k)} \cup L^2_{(i_k,j_k)}$ **do**
begin
$G' = G' - \{(i_k,j_k)\} \cup L^1_{(i_k,j_k)} \cup L^2_{(i_k,j_k)}$;
$k = k+1$;
end.
Step 3. Return a shortest $s$-to-$d$ path in $G'$.

The procedure iteratively removes from $L$ the link $(i,j)$ with the least $l_{(i,j)}$, its one-hop neighboring links and its two-hop neighboring links until $d$ is not reachable from $s$. Then, a shortest $s$-to-$d$ path is preferred, because it is most likely that a shorter $s$-to-$d$ path may carry a larger amount of traffic from $s$ to $d$, while retaining $G$ bandwidth-satisfied. We use $P$ to denote the shortest $s$-to-$d$ path and $b_P$ to denote the maximal amount of traffic that $P$ can carry without causing bandwidth violation of $G$. Recall that $b_P$ can be easily determined by examining the links in $P \cup A_P$ (see Sect. 2).

Suppose that $(i^-,j^-)$ is the link that ends the execution of the while-loop (i.e., Step 2). As a consequence of the procedure, we have $l_{(i,j)} \ge l_{(i^-,j^-)}$ for each link $(i,j) \in P \cup A_P$. Hence, $b_P \ge l_{(i^-,j^-)}$, meaning that $l_{(i^-,j^-)}$ is a lower bound on $b_P$. Recall that $u_{(i^+,j^+)}$ is an upper bound on the optimal value of the MBRP (see Sect. 4). Let $b^*$ denote the optimal value of the MBRP. Then, $\frac{b^*}{b_P} \le \frac{u_{(i^+,j^+)}}{l_{(i^-,j^-)}}$.

### 5.2 A heuristic algorithm for the MFRP

If the constraint (4) of the nonlinear programming in Sect. 3 is changed to

$$(F(i,j)+X(i,j)) + \sum_{(i',j')\in L^1_{(i,j)}\cup L^2_{(i,j)}} (F(i',j')+X(i',j')) \le c_{(i,j)}, \quad \forall (i,j) \in L,$$

then a linear programming results whose optimal value is smaller than or equal to the optimal value of the nonlinear programming. Hence, the heuristic algorithm for the MFRP is expressed by the linear programming, which can be solved in polynomial time.

## 6 Experimental results

Two network topologies, grid and random graph, were considered in the experiments. The grid was of size $10 \times 10$, and one of its farthest pairs of nodes (i.e., the two end nodes

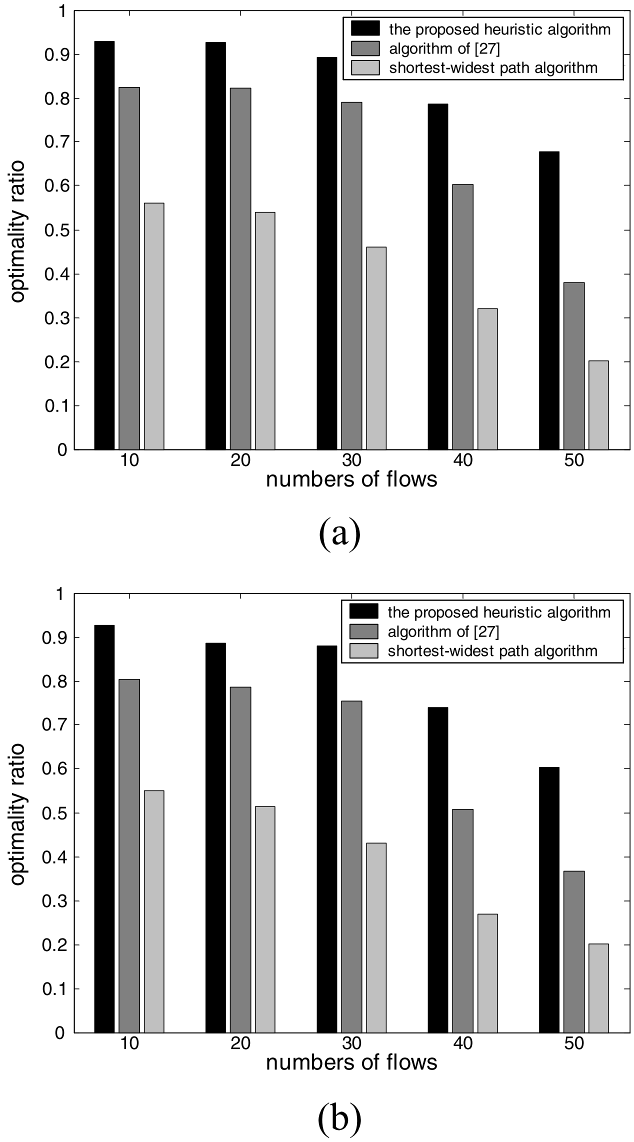


**Fig. 3** Optimality ratios of feasible values to optimal values with respect to the MBRP. (**a**) The grid network. (**b**) The random network

of a diagonal) were selected as the source node and destination node. The random graph contained 100 nodes that were distributed over a $600 \times 600$ (m$^2$) square. Each node of the graph had the same transmission range of 70 meters, and $(i, j)$ was a link of the graph if and only if nodes $i$, $j$ were within the transmission range of each other. The source node and destination node were randomly selected from the 100 nodes, and their distance was restricted to be four or more. We assumed that each link in the network was bi-directional, i.e., $(i, j) \in L$ implying $(j, i) \in L$, and had the same capacity of 54 (units/s).

In [27], there was another heuristic algorithm for the MBRP, which had the same time complexity as the method proposed in Sect. 5.1. Figure 3 showed the *optimality ratios* of the feasible values, which were obtained by the proposed heuristic algorithm and the algorithm of [27], to the optimal values versus different numbers of flows generated in the grid network and random network. Since computing the optimal values was very time-consuming, they were replaced in the experiments with the upper bounds obtained in Sect. 4.1. In addition, Fig. 3 also showed the optimality ratio of the feasible values of shortest-widest path algorithm [29] to the upper bounds. The shortest-widest path algorithm is to determine a widest path (i.e., a path with maximum bottleneck bandwidth), where the widest path does not consider the wireless interference into consideration. When there is more than one widest path, it chooses the one with minimum hop-count. Notice that in all experiments, we generated each flow on a randomly selected link and assumed the amount of traffic to be uniformly distributed between 1 (units/s) and 7 (units/s).

It was observed from Fig. 3 that the proposed heuristic algorithm outperformed the algorithm of [27] and shortest-widest path algorithm everywhere in grid network and random network. Since shortest-widest path algorithm does not consider the wireless interference, its optimality ratio is relatively smaller than the proposed heuristic algorithm and the algorithm of [27]. Figure 3 also reveals that the performance degraded as the number of flows increased. The grid network had better performance than the random network, because there were more paths from the source node to the destination node in the grid network than in the random network. This caused that the grid network had a higher probability of generating a higher-bandwidth path.

Figure 4 showed the *connection blocking ratios*, i.e., the ratio of the number of blocked connections to the total number of connection requests, of the proposed heuristic algorithm, the algorithm of [27] and the shortest-widest path algorithm. It was observed from Fig. 4 that the connection blocking ratio of the proposed heuristic algorithm outperformed than the algorithm of [27] and shortest-widest path algorithm everywhere in grid network and random network. Also, the shortest-widest path algorithm is worse than the proposed heuristic algorithm and the algorithm of [27].

Figure 5 showed the approximation ratios for the proposed heuristic algorithm with respect to the MBRP, where the approximation ratios were defined as $\frac{u_{(i^+,j^+)}}{l_{(i^-,j^-)}}$. The random network had higher approximation ratios than the grid network, because there were some denser regions in the random network. It was of high probability that the link $(i^-, j^-)$ was located in one denser region, which caused a lower value of $l_{(i^-,j^-)}$.

Previously, no heuristic algorithm for the MFRP was provided. Figure 6 showed the optimality ratios of the feasible values, which were obtained by the method proposed in Sect. 5.2, to the optimal values with respect to the MFRP. To save excessive computation time, we replaced the optimal values with the upper bounds obtained in Sect. 4.2.

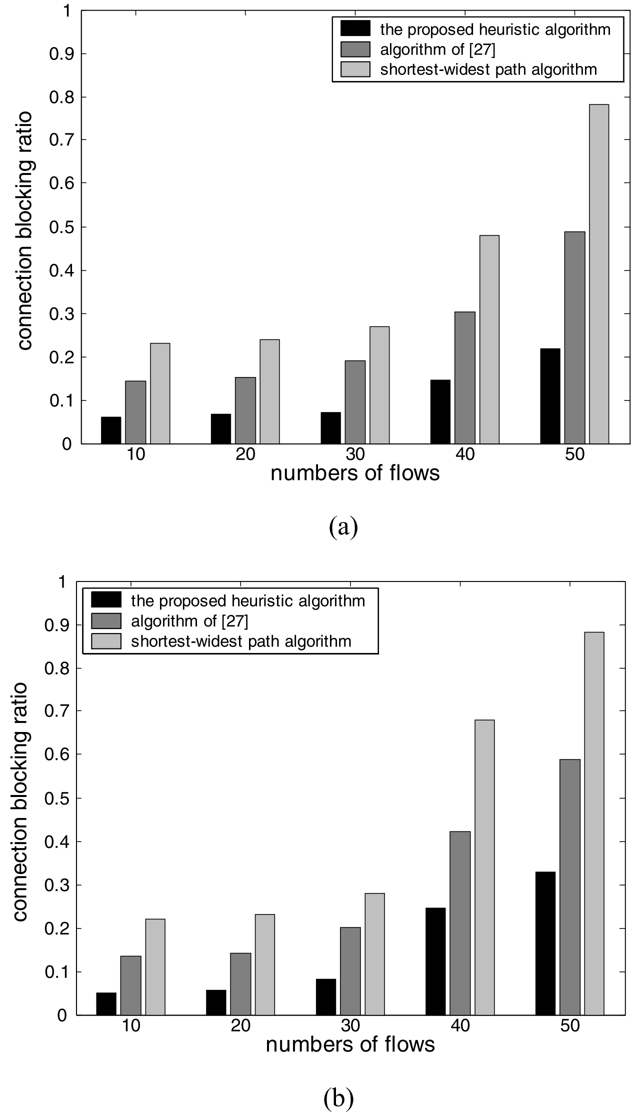


**Fig. 4** Connection blocking ratios with respect to the MBRP. (**a**) The grid network. (**b**) The random network

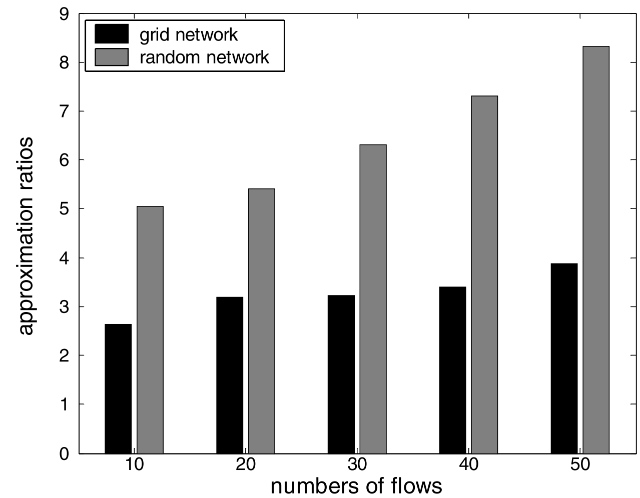


**Fig. 5** Approximation ratios for the proposed heuristic algorithm with respect to the MBRP

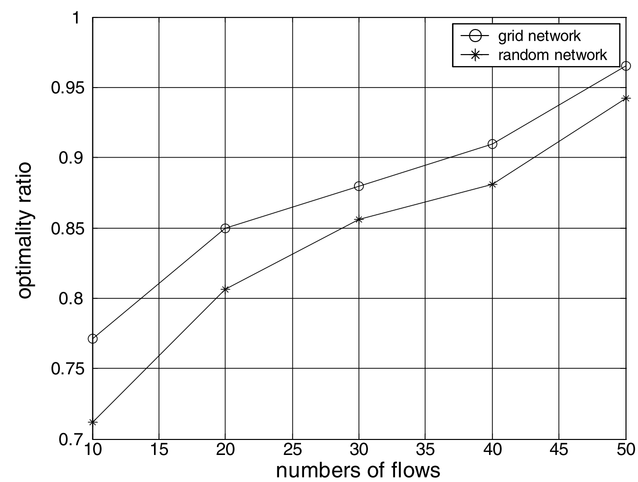


**Fig. 6** Optimality ratios of feasible values to optimal values with respect to the MFRP

With the same arguments for Fig. 5, the grid network had better performance than the random network. Besides, as explained below, the performance of both networks became better when the number of flows increased.

Recall that the proposed heuristic algorithm for the MFRP solved the linear programming of Sect. 5.2, which resulted by removing the multiplier $F(i,j) + X(i,j)$ of the constraint (4) in the nonlinear programming of Sect. 3. In fact, for each active link $(i,j)$, the multiplier was positive and thus could be removed from the nonlinear programming. When the number of flows increased, more links in the network were activated and their corresponding multipliers in the nonlinear programming could be removed. Hence, the feasible region of the nonlinear programming shrank toward the feasible region of the linear programming. That is, the optimal value of the former was closer to the optimal value of the latter.

Figure 7 further showed the bandwidth improvement made by the MFRP over the MBRP for both networks. When the traffic load increased, more paths generated by the MFRP might have congested links, which could reduce the bandwidth improvement.

## 7 Discussion and conclusion

In this paper, we studied two NP-hard problems: the MBRP and the MFRP. Given a pair of source node $s$ and destination node $d$ in a multi-hop wireless mesh network, the MBRP is to determine an $s$-to-$d$ path that could carry a maximum amount of traffic from $s$ to $d$, while retaining the network

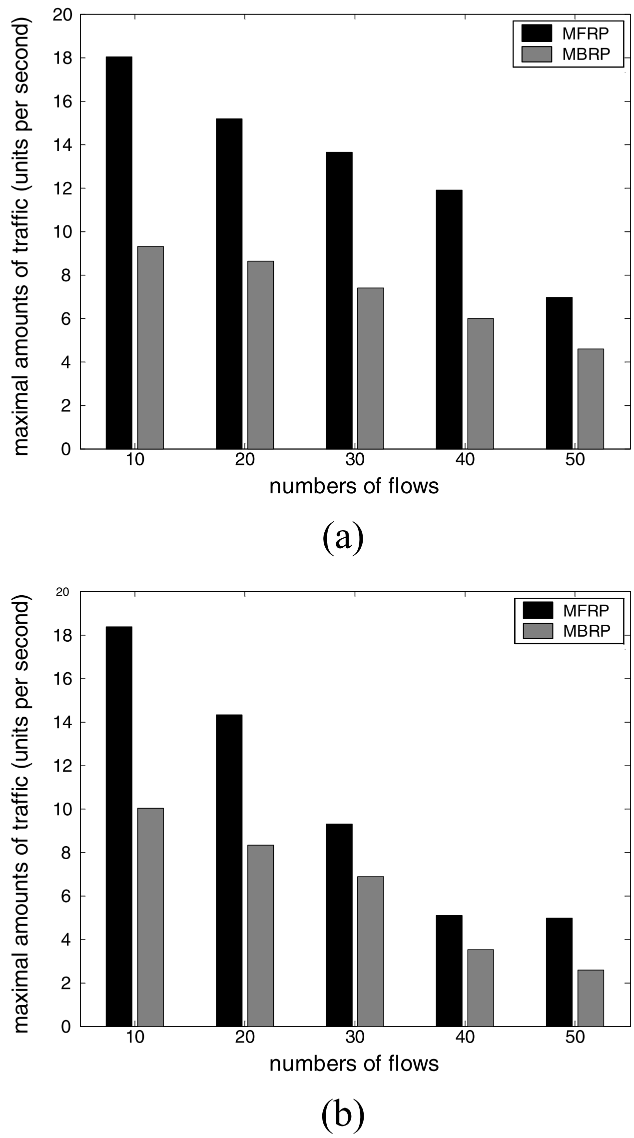


**Fig. 7** Maximal amounts of traffic obtained by the proposed heuristic algorithms for the MFRP and the MBRP. (**a**) The grid network. (**b**) The random network

bandwidth-satisfied. On the other hand, the MFRP is to determine the maximum flow from $s$ to $d$ in the network, also retaining the network bandwidth-satisfied.

We first derived upper bounds on the optimal values of the two problems, and then proposed two heuristic algorithms that could generate feasible solutions to them. For the MBRP, a lower bound on the obtained feasible values was also derived; hence the ratio of the upper bound to the lower bound could serve as a worst-case bound on the ratio of the optimal value to the obtained feasible value. That is, an approximation ratio was derived for the heuristic algorithm of the MBRP.

Experiments were carried out to evaluate the effectiveness of heuristic algorithms. Two kinds of networks, i.e., grid networks and random networks, were considered in the experiments. Since it is time-consuming to compute the optimal values, they were replaced with the derived upper bounds. The following were observed from the experimental results.

- For the MBRP, the proposed heuristic algorithm outperforms the heuristic algorithm of [27] and the shortest-widest path algorithm for both grid networks and random networks.
- For the MBRP, approximation ratios obtained for grid networks are smaller than those obtained for random networks.
- Both heuristic algorithms behave better for grid networks than for random networks.
- For both grid networks and random networks, the heuristic algorithm of the MFRP generated greater feasible values than the heuristic algorithm of the MBRP.

The interference model described in Sect. 2 implicitly assumed that all nodes have equal transmission range. Moreover, the interference range was smaller than or equal to the transmission range. The interference model of [11] assumed that the interference range relied on the received power level at a receiver, which could successfully receive packets if the signal-to-interference (SIR) ratio exceeded a threshold. Thus, the interference range might be larger than the transmission range.

Suppose that there is an ongoing transmission over a link $(t, r)$, where $t$ is the transmitter and $r$ is the receiver, and at the same time $r$ is interfered with another transmitter $t'$. Let $d_{t,r}$ ($d_{t',r}$) denote the geodesic distance between $t$ ($t'$) and $r$. In an open space environment, the path loss of a signal is usually estimated according to the two-ray ground model [25], in which $r$ can successfully receive the signal from $t$ provided the following inequality holds:

$$\mathrm{SIR} = (d_{t',r}/d_{t,r})^4 \geq \mathrm{SIRT}. \tag{6}$$

The variable SIRT above represents the threshold to SIR, and it is usually set to 10 for an open space environment. Thus, from (6), we have

$$d_{t',r} \geq 1.78 \times d_{t,r}, \tag{7}$$

where $1.78 \times d_{t,r}$ is the interference range of $r$. In other words, $r$ can successfully receive the signal from $t$ provided $t'$ is at least $1.78 \times d_{t,r}$ distant from $r$. Notice that the interference range of $r$ is at most 1.78 larger than its transmission range.

As described in Sect. 2, when the interference range of $t$ (and $r$) is smaller than or equal to the transmission range of $t$ (and $r$), transmission over one-hop neighboring links and two-hop neighboring links of $(t, r)$ should be avoided, in order to guarantee a successful transmission over $(t, r)$. When the interference range of $t$ (and $r$) exceeds the transmission

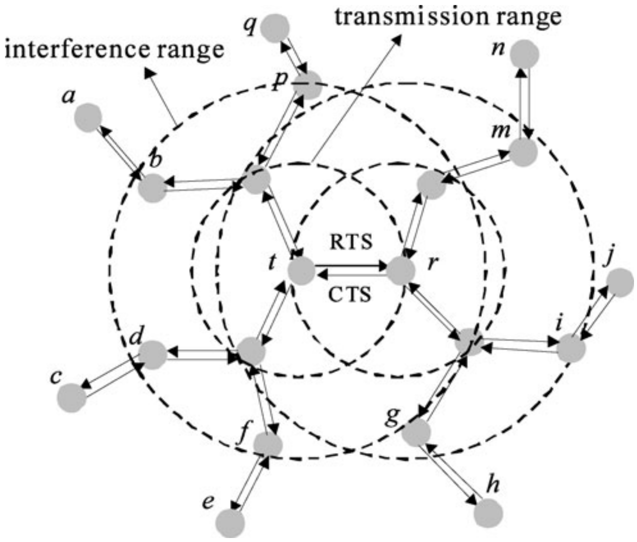


**Fig. 8** The situation when the interference range is larger than the transmission range

range of $t$ (and $r$), transmission over three-hop neighboring links of $(t, r)$ should be also avoided. Refer to Fig. 8 for an example, where smaller (larger) circles denote the transmission (interference) ranges of $t$ and $r$. If transmission over three-hop neighboring links, i.e., $(a, b)$, $(b, a)$, $(c, d)$, $(d, c)$, $(e, f)$, $(f, e)$, $(g, h)$, $(h, g)$, $(i, j)$, $(j, i)$, $(m, n)$, $(n, m)$, $(p, q)$ and $(q, p)$, of $(t, r)$ is initiated, then $r$ can not successfully receive the signal from $t$.

The interference model should be modified as follows. First, each receiver can determine its interference range according to (7). If the interference range is smaller than or equal to (larger than) the transmission range, then a link is bandwidth-satisfied provided the total bandwidth consumption of all links within two hops (three hops) from it does not exceed its capacity. The lower bounds, upper bounds and heuristic algorithms with respect to the MBRP and the MFRP can be modified accordingly.

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