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STRONG-COUPLING PHASES OF PLANAR $\mathcal{N}$=2* SUPER-YANG–MILLS THEORY

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The $\mathcal{N}$=2 theory (mass deformation of the $\mathcal{N}$=4 super-Yang–Mills theory) undergoes an infinite number of quantum phase transitions in the large-N limit. The phase structure and critical behavior can be analyzed using supersymmetric localization, which reduces the problem to an effective matrix model. We study this model in the strong-coupling phase.*

Keywords: supersymmetry, matrix model, $1/N$-expansion

1. Introduction

The maximally supersymmetric $\mathcal{N}$=4 Yang–Mills (SYM) theory is very rich mathematically, not least because it has a well-defined string description via the AdS/CFT correspondence. The closest relative of the $\mathcal{N}$=4 SYM theory with the conformal symmetry explicitly broken is the $\mathcal{N}$=2* theory. This theory obviously has fewer symmetries and more complicated dynamics, but some nonperturbative results do extend from $\mathcal{N}=4$ to $\mathcal{N}=2^*$. One of them is the exact computation of the path integral on S^4 by supersymmetric localization [1]. The path integral on S^4 reduces to a zero-dimensional matrix model in any gauge theory with the $\mathcal{N}$=2 supersymmetry. In the $\mathcal{N}$=4 case, this matrix model is just Gaussian [1]–[3].

The localization matrix model of the $\mathcal{N}$=2* SYM theory [1] is not Gaussian and has a very rich phase structure at large N (the regime where the gauge/string duality operates most simply) [4], [5]. The model is not completely solvable, except in some corners of the parameter space [5], [6]. In particular, its asymptotic strong-coupling solution is known and allows nontrivial tests of the holographic duality beyond AdS/CFT [6]. The free energy and the Wilson loop expectation values in the matrix model can be directly compared with the geometric data from the known supergravity dual of the $\mathcal{N}$=2* SYM theory [7] and its compactification on S^4 [8].

Another interesting regime is the decompactification limit. Compactification on S^4 can be viewed as a formal means for selecting the unique vacuum and making the path integral well-defined without imposing boundary conditions (this standpoint was articulated in [9]). The radius R of the four-sphere can then be viewed as an IR regulator, to be sent to infinity at the end of the calculation. Interestingly, the phase structure of the $\mathcal{N}$=2* SYM theory in an infinite volume is quite nontrivial. As the 't Hooft coupling changes from zero to infinity, the theory undergoes an infinite number of quantum phase transitions [4]. Phase transitions are a common feature of matrix models [10], [11], but those in the $\mathcal{N}$=2* theory are novel and have not been previously studied [5].

The exact solution of the localization matrix model in the weak-coupling phase was obtained in [4] following methods developed in [12], [13], while the strong-coupling phase has only been analyzed in an

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infinitesimal vicinity of the phase transition [5]. Our goal here is to study the strong-coupling phases (more precisely, phase) in more detail. A phase structure similar to that of the $\mathcal{N}$=2* SYM theory was found in three-dimensional massive theories amenable to localization [14], where the matrix model can be solved analytically [15]. We use a similar method in four dimensions. Although not leading to a full analytic solution, this method can be used to study the vacuum structure in detail across the entire phase diagram.

2. Localization matrix model

The field content of the $\mathcal{N}$=2* SYM theory consists of the gauge fields A_μ, their scalar superpartners Φ and Φ', the complex scalars Z and $\widetilde{Z}$ from the hypermultiplet, and four Majorana fermions. All fields are in the adjoint representation of the $SU(N)$ gauge group. The Lagrangian includes explicit mass terms for the complex scalars and their superpartners. When the mass deformation is switched off, the Lagrangian of the $\mathcal{N}$=2* theory becomes that of the $\mathcal{N}$=4 SYM theory.

The $SU(N)$ symmetry of $\mathcal{N}$=2* theory is spontaneously broken by the expectation value of the scalar from the vector multiplet,

$$\langle\Phi\rangle = \mathrm{diag}(a_1,\ldots,a_N), \tag{2.1}$$

and almost all the fields obtain masses via the Higgs mechanism. Exceptions are gauge bosons of the unbroken $U(1)^{N-1}$. The masses of the components of the fields in the vector multiplet (i and j are the color indices) are $|a_i-a_j|$, while the hypermultiplet masses are $|a_i-a_j\pm M|$, where M is the mass parameter in the Lagrangian. The eigenvalues a_i and a_j separated by the distance M therefore form a resonance; there is a massless hypermultiplet associated with them. The phase transitions in the matrix model are due to precisely these resonances.

The path integral of the $\mathcal{N}$=2* theory on S^4 localizes to an eigenvalue integral over the Coulomb moduli a_i from (2.1) [1]:

$$Z = \int d^{N-1}a \prod_{i<j} \frac{(a_i-a_j)^2 H^2(a_i-a_j)}{H(a_i-a_j-M)H(a_i-a_j+M)} \exp\left[-\frac{8\pi^2 N}{\lambda}\sum_i a_i^2\right], \tag{2.2}$$

where

$$H(x) \equiv \prod_{n=1}^{\infty}\left(1+\frac{x^2}{n^2}\right)^n e^{-x^2/n}. \tag{2.3}$$

The exact partition function also contains an instanton contribution, but it is exponentially suppressed at large N [5] and can therefore be omitted. Localization allows calculating some special correlation functions, among which is the Wilson loop for the big circle of S^4 that maps to the exponential operator in the matrix model [1]:

$$W(C_{\mathrm{ircle}}) \equiv \left\langle \frac{1}{N}\,\mathrm{tr\,P}\exp\oint_{C_{\mathrm{ircle}}} ds\left(iA_\mu\dot{x}^\mu+\Phi|\dot{x}|\right)\right\rangle \overset{\mathrm{loc}}{=} \left\langle\frac{1}{N}\sum_i e^{2\pi a_i}\right\rangle. \tag{2.4}$$

In the leading planar approximation, the eigenvalue integral can be evaluated in the saddle-point approximation. After the eigenvalue density

$$\rho(x) = \left\langle \frac{1}{N}\sum_{i=1}^{N}\delta(x-a_i)\right\rangle \tag{2.5}$$

is introduced, the saddle-point equations become a singular integral equation:

$$⨍_{-\mu}^{\mu} dy\,\rho(y)\left(\frac{1}{x-y}-\mathcal{K}(x-y)+\frac{1}{2}\mathcal{K}(x-y+M)+\frac{1}{2}\mathcal{K}(x-y-M)\right) = \frac{8\pi^2}{\lambda}x, \tag{2.6}$$

where $\mathcal{K} = -H'/H$. This equation was studied in several papers [16], [4]–[6], [9], but its general analytic solution is unknown.

The saddle-point equation is somewhat simplified in the decompactification limit $R \to \infty$. In fact, the above equations are written in the dimensionless variables with $R = 1$. The dependence on R is recovered by rescaling $a_i \to a_i R$, $M \to MR$, $x \to xR$, and $y \to yR$. The argument of the kernel function $\mathcal{K}(Rx)$ becomes large as $R \to \infty$, and the function can be replaced with its asymptotic form at infinity: $\mathcal{K}(x) \to x \log x^2$. Differentiating the resulting equations twice, we obtain

$$\fint_{-\mu}^{\mu} dy\, \rho(y) \left(\frac{2}{x-y} - \frac{1}{x-y+M} - \frac{1}{x-y-M} \right) = 0. \tag{2.7}$$

This is the saddle-point of the decompactified theory that we study.

Wilson loop (2.4) in the decmpactification limit obeys the perimeter law:

$$\log W(C_{\text{ircle}}) = 2\pi R\mu. \tag{2.8}$$

The perimeter law actually applies to any large contour, not just to the big circle of S^4. This can be verified using holography. The proportionality coefficient of the perimeter law calculated holographically agrees with the solution of the matrix model in the strong-coupling limit [6].

The boundary conditions on the density in the decompactification limit differ from those usually applied in matrix models [17]:

$$\rho(x) \sim \frac{\text{const}}{\sqrt{\mu \mp x}}, \quad x \to \pm\mu. \tag{2.9}$$

The density should be unit-normalized, which is the condition that usually fixes the endpoints of the eigenvalue distribution in terms of the coupling constant. In our case, the density satisfies homogeneous equation (2.7), and the normalization condition does not impose additional constraints. Instead, the boundary of the interval μ is determined by the integral form of (2.7) obtained from (2.6) by differentiating once. Written at $x = 0$, this condition is

$$\int_{-\mu}^{\mu} dy\, \rho(y) \log \frac{|M^2 - y^2|}{y^2} = \frac{8\pi^2}{\lambda}. \tag{2.10}$$

It is in fact more convenient to regard M and μ (instead of M and λ) as independent variables. The saddle-point equation in form (2.7) has a unique normalized solution for fixed M and μ. The coupling constant, as a function of M and μ (more precisely, their dimensionless ratio), is then determined by auxiliary condition (2.10).

The structure of the solution of (2.7) depends crucially on the relation between the width of the eigenvalue distribution 2μ and the mass M. If $M > 2\mu$, then Eq. (2.7) is of the Hilbert type with a single pole in the integration domain) and can be solved in terms of elliptic integrals [4] using the method developed in [12], [13]. This solution has a singularity at $\mu = M/2$, which signals a transition to a new phase. In terms of the 't Hooft constant, the weak-coupling solution exists for $\lambda < \lambda_c$ with $\lambda_c \simeq 35.425$ [4]. We are interested in the structure of the eigenvalue density for $\lambda > \lambda_c$.

The structure of the density in the infinitesimal vicinity of the phase transition was analyzed in [5]. It was found that the density develops two cusps at $x = \pm(M - \mu)$, which are the resonance images of the endpoint positions. The cusps have a lambdalike shape such that the density approaches a finite limit from the inside of the distribution and has an inverse-square-root singularity on the outside of the cusp. It was also observed that the density can be written in terms of a single auxiliary function that has no singularities at the resonance points. In the next section, we generalize these arguments to the case of a finite distance from the phase transition point. We then find a more general ansatz valid in any phase with an arbitrary number of resonances.

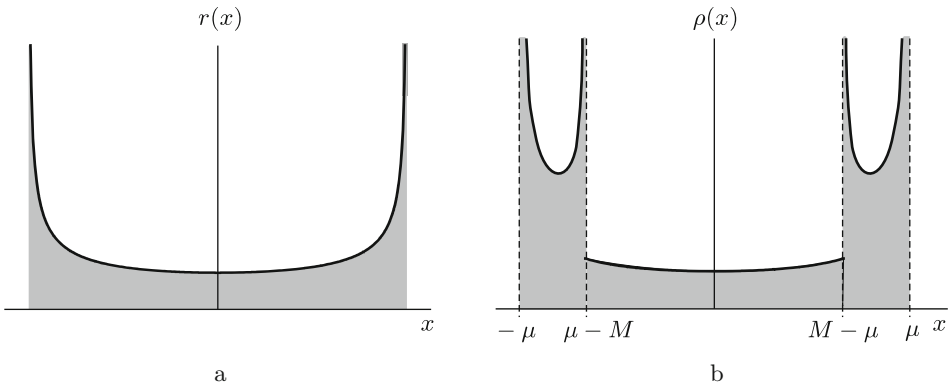

Fig. 1. Nonlocal map (3.1) induces additional singularities inside the interval $(-\mu,\mu)$: (a) the auxiliary function $r(x)$ and (b) its image $\rho(x)$.

3. Two-resonance phase

We first assume that $2\mu > M$ but $\mu < M$ and there are hence exactly two resonances, at $x = M-\mu$ and $x = -M+\mu$. For the density, we take the following ansatz,[1] which generalizes the structure found in the vicinity of the phase transition at $2\mu - M \ll M$,

$$\frac{1}{C}\rho(x) = \begin{cases} r(x), & x \in (-M+\mu, M+\mu), \\ \dfrac{4}{3}r(x) + \dfrac{2}{3}r(x-M), & x \in (M-\mu,\mu), \\ \dfrac{4}{3}r(x) + \dfrac{2}{3}r(x+M), & x \in (-\mu,-M+\mu). \end{cases} \tag{3.1}$$

The overall factor C is introduced for later convenience and is eventually fixed by the normalization condition. It is important that this ansatz is nonlocal. Because it is nonlocal, the endpoint singularities in $r(x)$ induce discontinuities of the density in the middle of the eigenvalue interval. Even if $r(x)$ itself is a smooth function between $-\mu$ and μ, the density $\rho(x)$ has singularities at $M-\mu$ and $-M+\mu$. These singularities arise as images of the endpoints under the map $x \to x \pm M$, and are associated with resonances on nearly massless hypermultiplets.

The coefficients in (3.1) are chosen such that after this ansatz is substituted in integral equation (2.7), the resonance terms (i.e., those with poles at $y = x \pm M$) cancel between different parts of the equation. Collecting the remaining contributions, we obtain an integral equation of the Hilbert type for $r(x)$:

$$\begin{aligned} \fint_{-\mu}^{\mu} dy\, r(y)\frac{2}{x-y} = {} & \int_{-M+\mu}^{M-\mu} dy\, r(y)\left(\frac{1}{x-y-M} + \frac{1}{x-y+M}\right) + \\ & + \frac{2}{3}\int_{M-\mu}^{\mu} dy\, r(y)\left(\frac{2}{x-y-M} + \frac{1}{x-y+2M}\right) + \\ & + \frac{2}{3}\int_{-\mu}^{M+\mu} dy\, r(y)\left(\frac{2}{x-y+M} + \frac{1}{x-y-2M}\right). \end{aligned} \tag{3.2}$$

All three integrals in the right-hand side are proper and consequently define a smooth function on the interval $(-\mu,\mu)$. It follows from the basic theory of singular integral equations that the solution with the

[1] This ansatz was initially suggested to us by D. Volin.

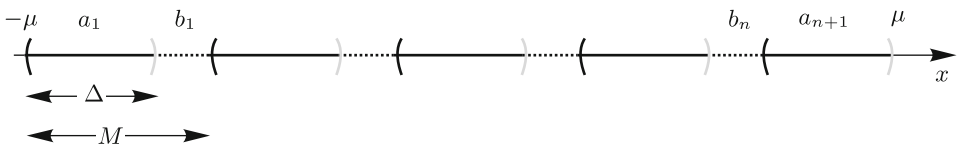

Fig. 2. The resonance points divide the eigenvalue interval into subintervals a_k and b_k on which the density is nonsingular.

boundary conditions

$$r(x) \to \frac{1}{\sqrt{\mu \mp x}}, \quad x \to \pm\mu, \tag{3.3}$$

exists, is unique, and has no singularities in the integration domain. The function $r(x)$ therefore has the shape shown in Fig. 1a.

From ansatz (3.1), we then find that the density behaves as

$$\rho(x) \to \frac{4C}{3\sqrt{\mu \mp x}}, \quad x \to \pm\mu, \tag{3.4}$$

at the interval boundaries. As x approaches one of the resonance points from the outside, the density diverges as

$$\rho(x) \to \frac{2C}{3\sqrt{\pm x - M + \mu}}, \quad x \to \pm M \mp \mu \pm 0. \tag{3.5}$$

On the inside, the density approaches a finite value $Cr(M-\mu)$. This structure is illustrated in Fig. 1b.

Finally, the normalization constant C is determined by the condition

$$C\biggl(\int_{-\mu}^{\mu} + \int_{-\mu}^{-M+\mu} + \int_{M-\mu}^{\mu}\biggr)\, dx\, r(x) = 1. \tag{3.6}$$

4. General structure

As μ increases, more resonances appear, which leads to secondary transitions each time 2μ becomes equal to an integer multiple of M. There hence exist infinitely many phases, and each phase is characterized by a different number of spikes in the eigenvalue density. The nth phase has $2n$ spikes. To enumerate the different phases, it is convenient to introduce the variables

$$n = \biggl[\frac{2\mu}{M}\biggr], \qquad \Delta = \biggl\{\frac{2\mu}{M}\biggr\}, \qquad 2\mu = nM + \Delta, \tag{4.1}$$

where $[\,\cdot\,]$ and $\{\,\cdot\,\}$ denote the respective integer and fractional parts.

The spikes are located at the resonance points $-\mu + kM$ and $\mu - kM = -\mu + (n-k)M + \Delta$, which divide the interval $(-\mu, \mu)$ into $2n{+}1$ subintervals a_k and b_k, as shown in Fig. 2:

$$\begin{aligned} a_k &= \bigl(-\mu + (k-1)M, -\mu + (k-1)M + \Delta\bigr), \quad k = 1, \ldots, n+1, \\ b_k &= (-\mu + (k-1)M + \Delta, -\mu + kM), \qquad\qquad k = 1, \ldots, n. \end{aligned} \tag{4.2}$$

Our basic assumption is that the density can be obtained from a unique analytic function $r(x)$ by applying an appropriate shift operator. We hence choose an ansatz generalizing (3.1) to the case of an

arbitrary number of subintervals:

$$\rho(x)=\begin{cases}\displaystyle\sum_{l=-k+1}^{n+1-k} A_{k+l,l}\,r(x+lM), & x\in a_k,\\ \displaystyle\sum_{l=-k+1}^{n-k} B_{k+l,l}\,r(x+lM), & x\in b_k,\end{cases}\tag{4.3}$$

where A_{mp} and B_{mp} are numerical coefficients to be determined. Substituting this ansatz in saddle-point equation (2.7), we obtain a linear combination of integrals of the forms

$$I_{mp}=\fint_{a_k}\frac{dy\,r(y)}{x-y+pM},\qquad J_{mp}=\fint_{b_k}\frac{dy\,r(y)}{x-y+pM},\tag{4.4}$$

namely,

$$\begin{aligned}\fint_{-\mu}^{\mu} dy\,\rho(y)\biggl(\frac{2}{x-y}-\frac{1}{x-y+M}-\frac{1}{x-y-M}\biggr)&=\\ =\sum_{m=1}^{n+1}\sum_{l=m-n-1}^{m-1} A_{ml}(2I_{ml}-I_{m,l+1}-I_{m,l+1})&+\\ +\sum_{m=1}^{n}\sum_{l=m-n}^{m-1} B_{ml}(2J_{ml}-J_{m,l+1}-J_{m,l+1}).&\end{aligned}$$

This results in a singular integral equation for $r(x)$. Our goal is to ensure that this equation is of the Hilbert type by adjusting the coefficients A_{mp} and B_{mp}. For this, we must eliminate all integrals that have singular kernels except the unshifted Hilbert kernel $1/(x-y)$.

The dangerous integrals are I_{mp} with $m-n-1\le p\le m-1$, $p\ne 0$, and J_{mp} with $m-n\le p\le m-1$, $p\ne 0$. Imposing the condition that the coefficients of dangerous integrals cancel and normalizing the coefficient of the Hilbert kernel to 2, we obtain a system of linear equations for A_{mp} and B_{mp}:

$$\begin{aligned}&2A_{mp}-A_{m,p+1}-A_{m,p-1}=2\delta_{p0},\quad p=m-n-1,\dots,m-1,\\ &2B_{mp}-B_{m,p+1}-B_{m,p-1}=2\delta_{p0},\quad p=m-n,\dots,m-1.\end{aligned}\tag{4.5}$$

Solving this system, we obtain

$$A_{mp}=\begin{cases}\dfrac{2(n+2-m)(m-p)}{n+2}, & p\ge 0,\\ \dfrac{2m(n+2-m+p)}{n+2}, & p\le 0,\end{cases}\tag{4.6}$$

and

$$B_{mp}=\begin{cases}\dfrac{2(n+1-m)(m-p)}{n+1}, & p\ge 0,\\ \dfrac{2m(n+1-m+p)}{n+1}, & p\le 0.\end{cases}\tag{4.7}$$

It is easy to verify that we reproduce ansatz (3.1) at $n=1$.

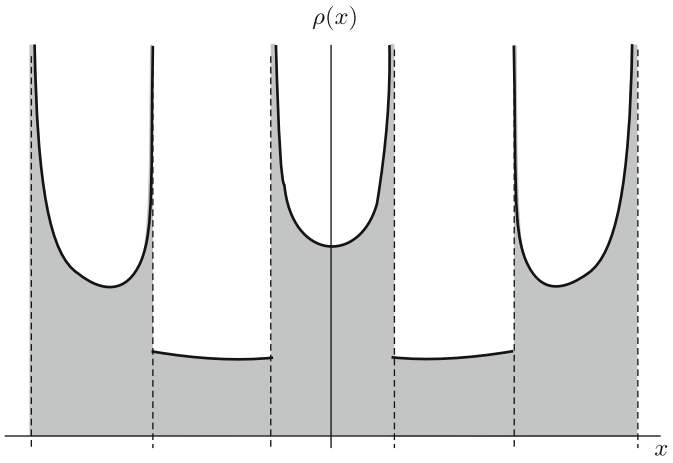

Fig. 3. The eigenvalue density for $n = 2$.

The resulting integral equation for $r(x)$ has the form

$$\fint_{-\mu}^{\mu} dy\, r(y) \frac{2}{x-y} = \frac{1}{n+2} \sum_{m=1}^{n+1} \int_{a_m} dy\, r(y) \left[\frac{m}{x-y-(n+2-m)M} + \frac{n+2-m}{x-y+mM} \right] + \\ + \frac{1}{n+1} \sum_{m=1}^{n} \int_{b_m} dy\, r(y) \left[\frac{m}{x-y-(n+1-m)M} + \frac{n+1-m}{x-y+mM} \right]. \tag{4.8}$$

The right-hand side is indeed an analytic function on $(-\mu, \mu)$, and the solution of this equation with asymptotic form (3.3) hence exists and is unique. The analogue of Eq. (3.6) is now

$$C \left[\sum_{m=1}^{n+1} m(n+2-m) \int_{a_m} + \sum_{m=1}^{n} m(n+1-m) \int_{b_m} \right] dx\, r(x) = 1. \tag{4.9}$$

Although we do not know how to solve for $r(x)$ with $n > 0$, the qualitative structure of the density and salient features of the critical behavior are mostly independent of the detailed form of this function and can be inferred from map (4.3) and explicit solutions (4.6) and (4.7) for the coefficients A_{mp} and B_{mp}. We can therefore study the critical behavior without knowing the explicit form of $r(x)$. It can be easily seen that the density is nonsingular on all the b intervals and has singularities at both ends of the a intervals, namely,

$$\begin{aligned} \rho(x) &\simeq \frac{2C(n+1-l)}{(n+2)\sqrt{\varepsilon}}, \qquad && x = -\mu + lM + \varepsilon, \\ \rho(x) &\simeq \frac{2Cl}{(n+2)\sqrt{\varepsilon}}, && x = -\mu + lM + \Delta - \varepsilon, \end{aligned} \tag{4.10}$$

as $\varepsilon \to 0$, which is shown in Fig. 3.

In the phase with odd n (see Fig. 1b), the a intervals grow and the b integrals shrink as μ increases. If n is even, as in Fig. 3, then the a intervals shrink and the b intervals grow as μ increases. There are consequently two types of phase transitions, associated with either a or b intervals shrinking to zero size, as shown in Fig. 4.

The coupling constant increases as μ increases, and when the coupling becomes large, the width of the eigenvalue distribution is parametrically larger than the mass scale: $\mu \gg M$. The density has a very

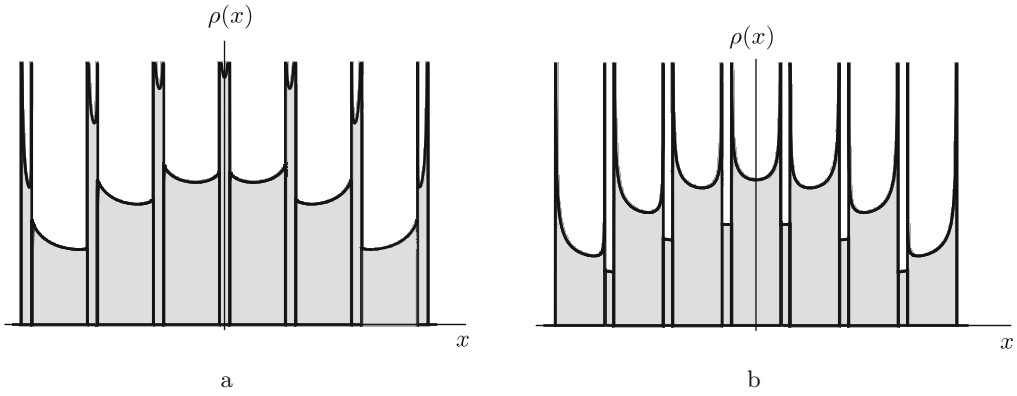

Fig. 4. Two types of critical behavior in the $\mathcal{N}$=2* SYM theory: (a) with the a intervals shrinking to zero size and (b) with the b intervals shrinking.

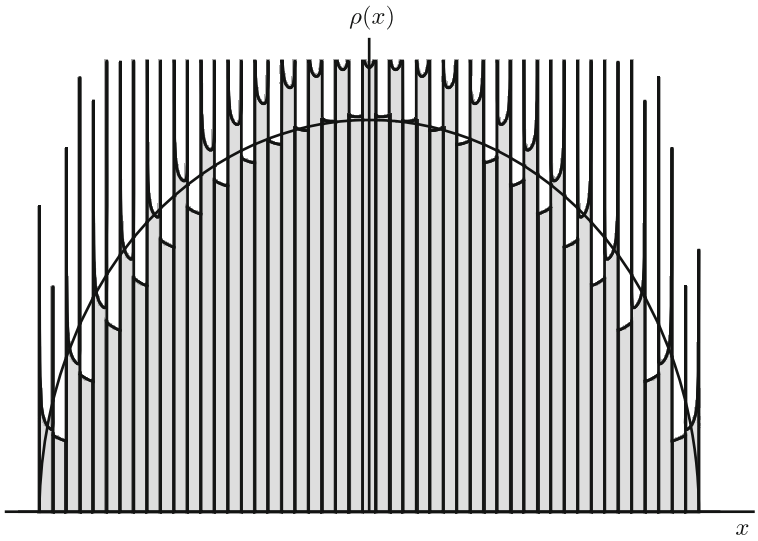

Fig. 5. The eigenvalue density in the strong-coupling limit: at small scales, the density has a complicated spiky structure with a large number of singularities, but the average density is well approximated by the Wigner semicircle, shown here by the dashed line.

irregular spiky shape in this limit. On the other hand, irregularities arise at small scales of the order of M. When averaged over a sufficiently large interval, the density should approach the smooth strong-coupling solution, which is just a simple Wigner semicircle [6]:[2]

$$\rho_\infty(x) = \frac{2\sqrt{\mu^2 - x^2}}{\pi\mu^2}. \tag{4.11}$$

That this is indeed the case can be seen in Fig. 5.

5. Conclusions

The $\mathcal{N}$=2* SYM theory has a rich phase structure, which can be studied using supersymmetric localization. Although it is currently unknown how to solve the localization matrix model exactly in the

[2]An alternative derivation of the semicirle law that does not rely on S^4 localization was given in [18].

strong-coupling phase, we can obtain a sufficiently detailed picture of the eigenvalue density. The critical behavior is associated with mutations of the singularities of the density occurring as the width of the eigenvalue distribution increases.

The description of such behavior in the framework of the holographic duality seems most interesting. Because the majority of the phase transitions happen at the strong-coupling domain, the quantum regime of the dual string theory might not be needed for this. In principle, the eigenvalue density can be investigated more or less directly in the holographic framework by placing a D-brane in the dual geometry [19], [20]. But a classical D-brane is sensitive to the average density and apparently cannot resolve the complicated singular structure at small scales [19], [6]. A string description of the phase transitions observed in the matrix model remains an open problem.

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