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EXTERNAL GRAVITATIONAL FIELD OF A NONSTATIC SPHERICALLY SYMMETRIC BODY IN THE RELATIVISTIC THEORY OF GRAVITATION

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We demonstrate that the external gravitational field of a nonstatic spherically symmetric body is static.

Keywords: Riemann metric tensor, graviton, Minkowski space metric, inertial frame

In general relativity (GR), which relates the gravitational field to the metric tensor of a Riemannian space, the Birkhoff theorem proved in a class of admissible functions claims that the external field of a nonstatic spherically symmetric body can be only static. In the relativistic theory of gravitation (RTG), the gravitational field $\phi^{\mu\nu}$ is a physical field in the Minkowski space; the source of this field is the energy–momentum tensor of all matter fields (including the gravitational field) preserved in the Minkowski space. Such an approach results in the effective Riemannian space and correspondingly in a system of equations different from that in GR. The problem of a nonstationary source therefore deserves a special consideration. We studied this problem in [1], where some mistakes were unfortunately made; correcting them is our aim here. Moreover, in contrast to [1], the presentation here has a general character.

The complete system of RTG equations is [2]

$$R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R + \frac{m^2}{2}\left[g^{\mu\nu} + \left(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = 8\pi T^{\mu\nu}, \tag{1}$$

$$D_\nu \tilde{g}^{\nu\mu} = \partial_\nu \tilde{g}^{\nu\mu} + \gamma^{\mu}_{\alpha\beta}\,\tilde{g}^{\alpha\beta} = 0. \tag{2}$$

We define the physical field $\phi^{\mu\nu}$ by the equality

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu},$$

where $\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$, $\tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\gamma^{\mu\nu}$, and $\tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\phi^{\mu\nu}$. This field manifests the same symmetries as the metric tensor $g^{\mu\nu}$ of the Riemannian space.

The coordinate system in RTG is governed by the metric tensor $\gamma_{\mu\nu}$ of the Minkowski space. A graviton rest mass enters Eq. (1). It is well known (see [3], [4]) that introducing the graviton rest mass into the linear equations of the gravitational field results in a discrepancy: in this case, explaining observational data for gravitational effects in the solar system requires that a spherically symmetric nonstatic source emit a negative-energy flow of scalar gravitons, which is inconsistent from the physical standpoint. This observation dictates that the graviton rest mass must be zero. But this is relevant to a linear theory of gravity; because the RTG is a nonlinear theory, a nonzero graviton rest mass does not entail such discrepancies in the RTG framework.

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We show that in the RTG, as well as in GR, an external gravitational field of a nonstationary spherically symmetric body remains static, i.e., such a nonstatic body does not emit gravitational waves. In the general case of a nonstatic spherically symmetric body, the effective Riemannian space interval is

$$ds^2 = B(\tau, r)\, d\tau^2 - D(\tau, r)\, dr^2 - 2E(\tau, r)\, dr\, d\tau - W^2(\tau, r)(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{3}$$

We seek an integrating factor η that makes the expression

$$dt = \eta(B\, d\tau - E\, dr) \tag{4}$$

a total differential. We note that there are infinitely many such integrating factors. From expression (4), we have

$$d\tau = \frac{1}{\eta B}\, dt + \frac{E}{B}\, dr, \tag{5}$$

whence there exists a function $\alpha(t, r)$ whose differential is

$$d\tau = \dot{\alpha}\, dt + \acute{\alpha}\, dr, \tag{6}$$

where

$$\dot{\alpha} = \frac{1}{\eta B}, \qquad \acute{\alpha} = \frac{E}{B}.$$

We note that transformation (5) does not affect the reference frame adopted in (3). We recall that all physically measurable quantities must be chronometrically invariant [5]. Substituting (5) in interval (3), we obtain

$$ds^2 = \frac{1}{\eta^2 B}\, dt^2 - dr^2\left(D + \frac{E^2}{B}\right) - W^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{7}$$

Introducing the notation

$$U = \frac{1}{\eta^2 B}, \qquad V = \left(D + \frac{E^2}{B}\right), \tag{8}$$

we rewrite the expression for ds^2 in the form

$$ds^2 = U(t, r)\, dt^2 - V(t, r)\, dr^2 - W^2(t, r)(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{9}$$

The functions U, V, and W^2 characterize the gravitational field of a nonstatic spherically symmetric body.

Therefore, because an integrating factor exists, the general form of the interval of the Riemannian space of a nonstatic spherically symmetric body has form (9).

For the problem under study determined by interval (9), the interval in the inertial frame in the Minkowski space has the form in spherical coordinates

$$d\sigma^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\phi^2). \tag{10}$$

Below, we find that the external gravitational field of form (9) generated by a *nonstatic* spherically symmetric source in inertial frame (10) can be only *static*, i.e., the metric coefficients U, V, and W are independent of the time t.

Equations (1) for the problem governed by relations (9) and (10) yield equations for U, V, and W (see formulas (A.28), (A.29), and (A.31) in the appendix):

$$\begin{gathered}\frac{1}{W^2}-\frac{1}{2V}\frac{\partial}{\partial r}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial r}\right)-\frac{3}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2-\frac{\partial}{\partial r}\left(\frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\right)+\\
+\frac{1}{2UW^2}\frac{\partial W^2}{\partial t}\frac{\partial\log(VW)}{\partial t}+\frac{m^2}{2}\left[1-\frac{r^2}{W^2}+\frac{1}{2}\left(\frac{1}{U}-\frac{1}{V}\right)\right]=0,\\
\frac{1}{W^2}+\frac{1}{2U}\frac{\partial}{\partial t}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial t}\right)+\frac{3}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2+\frac{\partial}{\partial t}\left(\frac{1}{2UW^2}\frac{\partial W^2}{\partial t}\right)-\\
-\frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\frac{\partial\log(UW)}{\partial r}+\frac{m^2}{2}\left[1-\frac{r^2}{W^2}-\frac{1}{2}\left(\frac{1}{U}-\frac{1}{V}\right)\right]=0,\\
\frac{1}{W^2}\frac{\partial^2W^2}{\partial t\,\partial r}-\frac{1}{2W^4}\frac{\partial W^2}{\partial r}\frac{\partial W^2}{\partial t}-\frac{1}{2VW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial r}-\frac{1}{2UW^2}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial t}=0.\end{gathered}\tag{11}$$

With expressions (9) and (10) taken into account, Eqs. (2) become (see formulas (A.23) and (A.24) in the appendix)

$$W^2=\sqrt{\frac{U}{V}}\,q(r),\qquad \frac{\partial}{\partial r}\left(W^2\sqrt{\frac{U}{V}}\right)=2r\sqrt{UV},\tag{12}$$

where $q(r)$ is an arbitrary positive function.

Because the Hilbert causal principle implies that $U>0$, $V>0$, and $W^2>0$, in what follows, we use the convenient notation

$$U(t,r)=e^{\mu(t,r)},\qquad V(t,r)=e^{\nu(t,r)},\qquad W^2(t,r)=e^{\lambda(t,r)},\qquad q(r)=e^{\sigma(r)}.$$

In the variables μ, ν, λ, and σ, Eqs. (11) become

$$\begin{gathered}e^{-\lambda}-e^{-\nu}\left(\lambda''+\frac{3}{4}(\lambda')^2-\frac{1}{2}\lambda'\nu'\right)+\frac{1}{2}e^{-\mu}\dot{\lambda}\left(\dot{\nu}+\frac{1}{2}\dot{\lambda}\right)+\\
+\frac{m^2}{2}\left[1-r^2e^{-\lambda}+\frac{1}{2}(e^{-\mu}-e^{-\nu})\right]=0,\end{gathered}\tag{13}$$

$$\begin{gathered}e^{-\lambda}+e^{-\mu}\left(\ddot{\lambda}+\frac{3}{4}(\dot{\lambda})^2-\frac{1}{2}\dot{\lambda}\dot{\mu}\right)-\frac{1}{2}e^{-\nu}\lambda'\left(\mu'+\frac{1}{2}\lambda'\right)+\\
+\frac{m^2}{2}\left[1-r^2e^{-\lambda}-\frac{1}{2}(e^{-\mu}-e^{-\nu})\right]=0,\end{gathered}\tag{14}$$

$$\dot{\lambda}'+\frac{1}{2}\dot{\lambda}\lambda'-\frac{1}{2}\dot{\nu}\lambda'-\frac{1}{2}\dot{\lambda}\mu'=0,\tag{15}$$

where, for example, $\dot{\lambda}=\partial\lambda/\partial t$ and $\lambda'=\partial\lambda/\partial r$. Equations (12) become

$$\lambda-\frac{1}{2}(\mu-\nu)=\sigma(r),\tag{16}$$

$$\mu'-\nu'+\sigma'=2re^{\nu-\lambda}.\tag{17}$$

We introduce the notation

$$2\omega=\mu+\nu,\tag{18}$$

$$f=\lambda-\sigma\,(r).\tag{19}$$

In accordance with (16), we have

$$\mu - \nu = 2f. \tag{20}$$

From (18) and (20), we obtain

$$\mu = \omega + f, \qquad \nu = \omega - f. \tag{21}$$

We rewrite Eqs. (17) in terms of the functions ω, f and σ:

$$2f' + \sigma' = 2re^{\omega - 2f - \sigma}. \tag{22}$$

Differentiating Eq. (22) with respect to t, we obtain

$$2\dot{f}' = (2f' + \sigma')(\dot{\omega} - 2\dot{f}).$$

Substituting this expression in Eq. (15) and taking (19) and (21) into account, we obtain the inhomogeneous linear partial differential equation

$$\frac{\partial \omega}{\partial t} f' - \frac{\partial \omega}{\partial r} \dot{f} = 3\dot{f} f'. \tag{23}$$

The system of ordinary differential equations corresponding to Eq. (23) is

$$\frac{dt}{f'} = \frac{dr}{-\dot{f}} = \frac{d\omega}{3\dot{f} f'}.$$

Hence, we have

$$d\omega = 3\dot{f}\,dt, \qquad d\omega = -3f'\,dr.$$

Adding these equalities, we obtain

$$d\omega = \frac{3}{2}\frac{\partial f}{\partial t}\,dt - \frac{3}{2}\frac{\partial f}{\partial r}\,dr.$$

The total differential property implies that

$$\frac{\partial^2 f}{\partial t\,\partial r} = 0,$$

and this equality in turn implies that we can represent the function f in the form

$$f(t,r) = \psi(t) + \varphi(r). \tag{24}$$

The general solution of Eq. (23) is

$$\omega(t,r) = \frac{3}{2}(\psi(t) - \varphi(r)) + F(f), \tag{25}$$

where F is an arbitrary function.

With expressions (24) and (25) taken into account, Eq. (22) becomes

$$2\varphi' + \sigma' = 2r \exp\left[-\frac{1}{2}\psi(t) - \frac{7}{2}\varphi(r) + F(f) - \sigma\right]. \tag{26}$$

The left-hand side of this equation is independent of t; hence, its right-hand side must also be independent of t. This is possible if $\psi(t) = \text{const}$: the functions μ, ν, and λ then become independent of t. *In this case, the gravitational field of form* (9) *is static.* But we also have another possible solution with $F = f/2$. Then

$$\omega(t,r) = 2\psi(t) - \varphi(r),$$

and the functions μ, ν, and λ by (21) and (19) must be

$$\mu = 3\psi(t), \qquad \nu(t,r) = \psi(t) - 2\varphi(r), \qquad \lambda(t,r) = \psi(t) + \varphi(r) + \sigma(r) = f + \sigma. \tag{27}$$

We then obtain the interval in which the variables t and r are separated in the metric coefficients, and the metric coefficient U depends only on time. Equation (26) then becomes

$$2\varphi' + \sigma' = 2re^{-3\varphi(r)-\sigma(r)}.$$

We analyze this case using Eqs. (13) and (14). Their difference gives

$$e^{-\nu}\left[-\lambda'' - \frac{1}{2}(\lambda')^2 + \frac{1}{2}\lambda'(\mu' + \nu')\right] + \\ + e^{-\mu}\left[-\ddot{\lambda} - \frac{1}{2}(\dot{\lambda})^2 + \frac{1}{2}\dot{\lambda}(\dot{\mu} + \dot{\nu})\right] + \frac{m^2}{2}(e^{-\mu} - e^{-\nu}) = 0, \tag{28}$$

while their sum gives

$$2e^{-\lambda} - e^{-\nu}\left[\lambda'' + (\lambda')^2 - \frac{1}{2}\lambda'(\nu' - \mu')\right] + \\ + e^{-\mu}\left[\ddot{\lambda} + (\dot{\lambda})^2 - \frac{1}{2}\dot{\lambda}(\dot{\mu} - \dot{\nu})\right] + m^2(1 - r^2 e^{-\lambda}) = 0. \tag{29}$$

By (16), we have

$$(\dot{\lambda})^2 - \frac{1}{2}\dot{\lambda}(\dot{\mu} - \dot{\nu}) = 0,$$

which allows simplifying Eq. (29),

$$2e^{-\lambda} - e^{-\nu}\left[\lambda'' + (\lambda')^2 - \frac{1}{2}\lambda'(\nu' - \mu')\right] + e^{-\mu}\ddot{\lambda} + m^2(1 - r^2 e^{-\lambda}) = 0. \tag{30}$$

The variables t and r become separated in Eqs. (28) and (30),

$$-\varphi'' - \sigma'' - \frac{3}{2}\acute{\varphi}^2 - \frac{1}{2}\acute{\sigma}^2 - 2\acute{\varphi}\acute{\sigma} - \frac{m^2}{2} = ke^{-2\varphi}, \tag{31}$$

$$\ddot{\psi} - \frac{3}{2}\dot{\psi}^2 - \frac{m^2}{2} = ke^{2\psi}, \tag{32}$$

$$\varphi'' + \sigma'' + 2\acute{\varphi}^2 + \acute{\sigma}^2 + 3\acute{\varphi}\acute{\sigma} - 2e^{-3\varphi-\sigma}\left(1 - \frac{m^2 r^2}{2}\right) = pe^{-2\varphi}, \tag{33}$$

$$\ddot{\psi} + m^2 e^{3\psi} = pe^{2\psi}. \tag{34}$$

We note that the system of equations (22), (31), and (33) has the solution

$$\varphi = 0, \qquad \sigma = \log r^2, \qquad k = -\frac{m^2}{2}, \qquad p = m^2. \tag{35}$$

We now turn to Eqs. (32) and (34). We introduce the new variable $\psi(t) = \log a^2$. Equations (32) and (34) then become

$$2a\ddot{a} - 8\dot{a}^2 - \frac{m^2}{2}a^2 = ka^6, \tag{36}$$

$$2a\ddot{a} - 2\dot{a}^2 + m^2 a^8 = pa^6. \tag{37}$$

Hence, we have

$$\dot{a}^2 = -\frac{1}{12}[m^2a^2 + 2m^2a^8 + 2(k-p)a^6]. \tag{38}$$

Differentiating, we obtain

$$\ddot{a} = -\frac{1}{24}[2m^2a + 16m^2a^7 + 12(k-p)a^5]. \tag{39}$$

Substituting (38) and (39) in (36), we obtain the relation between the variable separation constants,

$$p = -2k. \tag{40}$$

Introducing the new time variable $d\tau = a^3\,dt$, we transform Eq. (38) to the form

$$\frac{1}{a^2}\left(\frac{da}{d\tau}\right)^2 = \frac{m^2}{12}\left(-\frac{1}{a^6} - 2 + \frac{3}{\beta^4a^2}\right), \tag{41}$$

where we use the notation

$$|k| = \frac{m^2}{2\beta^4}. \tag{42}$$

Equation (39) then becomes

$$\frac{1}{a}\frac{d^2a}{d\tau^2} = -\frac{m^2}{6}\left(1 - \frac{1}{a^6}\right). \tag{43}$$

Equation (41) admits a nontrivial solution for $\beta < 1$. This solution oscillates, and the scale factor a then changes between its limit values, $a_{\min} \le a \le a_{\max}$. Because the acceleration must be negative at the turn point $a_{\max}$, we obtain the inequality $a_{\max} > 1$ from (43).

By virtue of expressions (27), the interval in our case becomes

$$ds^2 = a^2[a^4\,dt^2 - e^{-2\varphi}\,dr^2 - e^{\varphi+\sigma}(d\theta^2 + \sin^2\theta\,d\phi^2)]. \tag{44}$$

We note that for interval (44), harmonic equation (12) is independent of the time variable in the domain outside the body, where it has the form

$$\frac{d}{dr}(e^{2\varphi+\sigma}) = 2re^{-\varphi}. \tag{45}$$

In the expression in square brackets in (44), the three-dimensional spatial part is determined only by the functions $\varphi(r)$ and $\sigma(r)$. Because interval (44) expresses the external gravitational field of a nonstatic spherically symmetric body in the absence of a wave process and energy flow, the physical solution for the functions φ and σ at spatial infinity where the static field is also absent must result in the Euclidean geometry written in spherical coordinates, i.e., we must have the limits

$$\lim_{r\to\infty}\varphi(r) = 0, \qquad \lim_{r\to\infty}e^{\sigma(r)} = r^2. \tag{46}$$

These two conditions are related by Eq. (45). We write Eqs. (31) and (33) in a more compact form, expressing them in terms of components of the curvature tensor of the three-dimensional space. They take the form (see formulas (A.39) and (A.40) in the appendix)

$$
\begin{aligned}
R^2_{121} &= \frac{1}{2}e^{-2\varphi}\left(k + \frac{m^2}{2}e^{2\varphi}\right), \\
R^1_{212} + R^3_{232} &= e^{\varphi+\sigma}\left(k + \frac{m^2 r^2}{2}e^{-(\varphi+\sigma)}\right).
\end{aligned}
\tag{47}
$$

Because the curvature tensor components vanish at spatial infinity, expressions (47) with (46) taken into account result in the variable separation parameter

$$
k = -\frac{m^2}{2}. \tag{48}
$$

From (42), we then have $\beta = 1$. But in accordance with this equality, Eq. (41) admits only a trivial solution $a = 1$, and we again obtain a static solution.

We therefore have the general conclusion: for a *nonstatic* spherically symmetric source in the inertial frame, the metric coefficients of the external *gravitational field* interval can only be *static*. This source hence does not emit gravitational waves [2]–[4], [6]–[9].

Appendix

The interval of the effective Riemannian space–time for a nonstatic spherically symmetric body has general form (9). The nonzero coefficients of the connection corresponding to interval (9) are

$$
\begin{aligned}
&\Gamma^0_{00} = \frac{1}{2U}\frac{\partial U}{\partial t}, \qquad \Gamma^0_{01} = \frac{1}{2U}\frac{\partial U}{\partial r}, \qquad \Gamma^0_{11} = \frac{1}{2U}\frac{\partial V}{\partial t}, \qquad \Gamma^0_{22} = \frac{1}{2U}\frac{\partial W^2}{\partial t}, \\
&\Gamma^0_{33} = \frac{1}{2U}\frac{\partial W^2}{\partial t}\sin^2\theta, \qquad \Gamma^1_{00} = \frac{1}{2V}\frac{\partial U}{\partial r}, \qquad \Gamma^1_{01} = \frac{1}{2V}\frac{\partial V}{\partial t}, \qquad \Gamma^1_{11} = \frac{1}{2V}\frac{\partial V}{\partial r}, \\
&\Gamma^1_{22} = -\frac{1}{2V}\frac{\partial W^2}{\partial r}, \qquad \Gamma^1_{33} = -\frac{1}{2V}\frac{\partial W^2}{\partial r}\sin^2\theta, \qquad \Gamma^2_{02} = \frac{1}{2W^2}\frac{\partial W^2}{\partial t}, \\
&\Gamma^2_{12} = \frac{1}{2W^2}\frac{\partial W^2}{\partial r}, \qquad \Gamma^2_{33} = -\sin\theta\cos\theta, \qquad \Gamma^3_{03} = \frac{1}{2W^2}\frac{\partial W^2}{\partial t}, \\
&\Gamma^3_{13} = \frac{1}{2W^2}\frac{\partial W^2}{\partial r}, \qquad \Gamma^3_{23} = \cot\theta.
\end{aligned}
\tag{A.1}
$$

The Minkowski space interval in the inertial system in spherical coordinates has form (10), and the nonzero coefficients of the connection are

$$
\gamma^1_{22} = -r, \qquad \gamma^1_{33} = -r\sin^2\theta, \qquad \gamma^2_{12} = \gamma^3_{13} = \frac{1}{r}, \qquad \gamma^2_{33} = -\sin\theta\cos\theta, \qquad \gamma^3_{23} = \cot\theta. \tag{A.2}
$$

Using the representation

$$
R_{\sigma\nu} = \partial_\lambda \Gamma^\lambda_{\sigma\nu} - \partial_\nu \Gamma^\lambda_{\sigma\lambda} + \Gamma^\tau_{\sigma\nu}\Gamma^\lambda_{\tau\lambda} - \Gamma^\tau_{\sigma\lambda}\Gamma^\lambda_{\nu\tau} \tag{A.3}
$$

for the Ricci tensor with (A.1) taken into account, we find that the nonzero components of $R_{\sigma\nu}$ are

$$R_{00} = -\frac{1}{2V}\frac{\partial^2 V}{\partial t^2} + \frac{1}{2V}\frac{\partial^2 U}{\partial r^2} + \frac{1}{4V^2}\left(\frac{\partial V}{\partial t}\right)^2 - \frac{1}{4UV}\left(\frac{\partial U}{\partial r}\right)^2 - $$
$$- \frac{1}{4V^2}\frac{\partial U}{\partial r}\frac{\partial V}{\partial r} + \frac{1}{4UV}\frac{\partial V}{\partial t}\frac{\partial U}{\partial t} - \frac{1}{W^2}\frac{\partial^2 W^2}{\partial t^2} + \frac{1}{2W^4}\left(\frac{\partial W^2}{\partial t}\right)^2 +$$
$$+ \frac{1}{2UW^2}\frac{\partial U}{\partial t}\frac{\partial W^2}{\partial t} + \frac{1}{2VW^2}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial r}, \tag{A.4}$$

$$R_{11} = \frac{1}{2U}\frac{\partial^2 V}{\partial t^2} - \frac{1}{4U^2}\frac{\partial U}{\partial t}\frac{\partial V}{\partial t} - \frac{1}{2U}\frac{\partial^2 U}{\partial r^2} + \frac{1}{4U^2}\left(\frac{\partial U}{\partial r}\right)^2 -$$
$$- \frac{1}{W^2}\frac{\partial^2 W^2}{\partial r^2} + \frac{1}{2W^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{2UW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t} + \frac{1}{4UV}\frac{\partial V}{\partial r}\frac{\partial U}{\partial r} +$$
$$+ \frac{1}{2VW^2}\frac{\partial V}{\partial r}\frac{\partial W^2}{\partial r} - \frac{1}{4UV}\left(\frac{\partial V}{\partial t}\right)^2, \tag{A.5}$$

$$R_{22} = -\frac{1}{2V}\frac{\partial^2 W^2}{\partial r^2} + \frac{1}{4UV}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t} + \frac{1}{4V^2}\frac{\partial V}{\partial r}\frac{\partial W^2}{\partial r} +$$
$$+ \frac{1}{2U}\frac{\partial^2 W^2}{\partial t^2} - \frac{1}{4U^2}\frac{\partial U}{\partial t}\frac{\partial W^2}{\partial t} - \frac{1}{4UV}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial r} + 1, \tag{A.6}$$

$$R_{33} = R_{22}\sin^2\theta, \tag{A.7}$$

$$R_{01} = -\frac{1}{W^2}\frac{\partial^2 W^2}{\partial r\,\partial t} + \frac{1}{2W^4}\frac{\partial W^2}{\partial t}\frac{\partial W^2}{\partial r} + \frac{1}{2UW^2}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial t} +$$
$$+ \frac{1}{2VW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial r}, \tag{A.8}$$

and the scalar curvature $R = g^{\sigma\nu}R_{\sigma\nu}$ has the form

$$R = \frac{1}{UV}\frac{\partial^2 U}{\partial r^2} - \frac{1}{2V^2U}\frac{\partial V}{\partial r}\frac{\partial U}{\partial r} - \frac{1}{UV}\frac{\partial^2 V}{\partial t^2} + \frac{1}{2UV^2}\left(\frac{\partial V}{\partial t}\right)^2 -$$
$$- \frac{2}{UW^2}\frac{\partial^2 W^2}{\partial t^2} + \frac{1}{2UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 + \frac{1}{2VU^2}\frac{\partial U}{\partial t}\frac{\partial V}{\partial t} - \frac{1}{2VU^2}\left(\frac{\partial U}{\partial r}\right)^2 +$$
$$+ \frac{2}{VW^2}\frac{\partial^2 W^2}{\partial r^2} - \frac{1}{2VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{U^2W^2}\frac{\partial U}{\partial t}\frac{\partial W^2}{\partial t} -$$
$$- \frac{1}{V^2W^2}\frac{\partial V}{\partial r}\frac{\partial W^2}{\partial r} + \frac{1}{UVW^2}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial r} - \frac{1}{UVW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t} - \frac{2}{W^2}. \tag{A.9}$$

We next find the components of the tensors

$$R_{\sigma\nu} - \frac{1}{2}g_{\sigma\nu}R, \qquad R^{\mu}_{\nu} - \frac{1}{2}\delta^{\mu}_{\nu}R. \tag{A.10}$$

Substituting (A.4) and (A.9) in the identity

$$R_{00} - \frac{1}{2}g_{00}R \equiv R_{00} - \frac{1}{2}UR,$$

we obtain

$$R_{00} - \frac{1}{2}UR = \frac{U}{W^2} + \frac{U}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{U}{2W^2V^2}\frac{\partial V}{\partial r}\frac{\partial W^2}{\partial r} + \\ + \frac{1}{2VW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t} + \frac{1}{4W^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{U}{VW^2}\frac{\partial^2 W^2}{\partial r^2}. \tag{A.11}$$

Because

$$R_0^0 - \frac{1}{2}R = \frac{1}{U}\left(R_{00} - \frac{1}{2}UR\right), \tag{A.12}$$

we obtain the expression for the left-hand side of (A.12):

$$R_0^0 - \frac{1}{2}R = \frac{1}{W^2} + \frac{1}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{2V^2W^2}\frac{\partial V}{\partial r}\frac{\partial W^2}{\partial r} + \\ + \frac{1}{2UVW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t} + \frac{1}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{1}{W^2V}\frac{\partial^2 W^2}{\partial r^2}. \tag{A.13}$$

Analogously, substituting (A.5) and (A.9) in the identity

$$R_{11} - \frac{1}{2}g_{11}R \equiv R_{11} + \frac{1}{2}VR,$$

we obtain

$$R_{11} + \frac{1}{2}VR = -V\left[\frac{1}{W^2} + \frac{1}{UW^2}\frac{\partial^2 W^2}{\partial t^2} - \frac{1}{2UW^2V}\frac{\partial W^2}{\partial r}\frac{\partial U}{\partial r} + \right. \\ \left. + \frac{1}{2U^2W^2}\frac{\partial W^2}{\partial t}\frac{\partial U}{\partial t} - \frac{1}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{1}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2\right]. \tag{A.14}$$

But because

$$R_1^1 - \frac{1}{2}R = -\frac{1}{V}\left(R_{11} + \frac{1}{2}VR\right),$$

we have

$$R_1^1 - \frac{1}{2}R = \frac{1}{W^2} + \frac{1}{UW^2}\frac{\partial^2 W^2}{\partial t^2} - \frac{1}{2UVW^2}\frac{\partial W^2}{\partial r}\frac{\partial U}{\partial r} - \\ - \frac{1}{2U^2W^2}\frac{\partial W^2}{\partial t}\frac{\partial U}{\partial t} - \frac{1}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{1}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2. \tag{A.15}$$

We can also find that for

$$R_{22} + \frac{1}{2}W^2R$$

after substituting (A.6) and (A.9), we obtain the expression

$$R_{22} + \frac{1}{2}W^2R = -W^2\left[\frac{1}{2UW^2}\frac{\partial^2 W^2}{\partial t^2} + \frac{1}{2UV}\left(\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 U}{\partial r^2}\right) - \frac{1}{2VW^2}\frac{\partial^2 W^2}{\partial r^2}\right] - \\ - \frac{W^2}{4}\left[\frac{1}{VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{V^2W^2}\frac{\partial W^2}{\partial r}\frac{\partial V}{\partial r} - \frac{1}{VU^2}\left(\frac{\partial V}{\partial t}\frac{\partial U}{\partial t} - \left(\frac{\partial U}{\partial r}\right)^2\right) + \right. \\ + \frac{1}{UV^2}\left(\frac{\partial U}{\partial r}\frac{\partial V}{\partial r} - \left(\frac{\partial V}{\partial t}\right)^2\right) - \frac{1}{U^2W^2}\frac{\partial W^2}{\partial t}\frac{\partial U}{\partial t} - \frac{1}{UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \\ \left. - \frac{1}{UVW^2}\left(\frac{\partial W^2}{\partial r}\frac{\partial U}{\partial r} - \frac{\partial W^2}{\partial t}\frac{\partial V}{\partial t}\right)\right], \tag{A.16}$$

and for

$$R_2^2 - \frac{1}{2}R = -\frac{1}{W^2}\left(R_{22} + \frac{1}{2}W^2 R\right),$$

we obtain the expression

$$\begin{aligned} R_2^2 - \frac{1}{2}R = \frac{1}{2}\left[\frac{1}{UW^2}\frac{\partial^2 W^2}{\partial t^2} + \frac{1}{UV}\left(\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 U}{\partial r^2}\right) - \frac{1}{VW^2}\frac{\partial^2 W^2}{\partial r^2}\right] + \\ + \frac{1}{4}\left[\frac{1}{VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{V^2W^2}\frac{\partial W^2}{\partial r}\frac{\partial V}{\partial r} - \frac{1}{VU^2}\left(\frac{\partial V}{\partial t}\frac{\partial U}{\partial t} - \left(\frac{\partial U}{\partial r}\right)^2\right) + \right. \\ + \frac{1}{UV^2}\left(\frac{\partial U}{\partial r}\frac{\partial V}{\partial r} - \left(\frac{\partial V}{\partial t}\right)^2\right) - \frac{1}{U^2W^2}\frac{\partial W^2}{\partial t}\frac{\partial U}{\partial t} - \\ \left. - \frac{1}{UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{1}{UVW^2}\left(\frac{\partial W^2}{\partial r}\frac{\partial U}{\partial r} - \frac{\partial W^2}{\partial t}\frac{\partial V}{\partial t}\right)\right]. \end{aligned} \tag{A.17}$$

Finally, by virtue of (A.8), we obtain the expression for R_1^0:

$$\begin{aligned} R_1^0 = \frac{1}{U}R_{01} = -\frac{1}{U}\left(\frac{1}{W^2}\frac{\partial^2 W^2}{\partial t\,\partial r} - \frac{1}{2W^4}\frac{\partial W^2}{\partial t}\frac{\partial W^2}{\partial r} - \right. \\ \left. - \frac{1}{2UW^2}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial t} - \frac{1}{2VW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial r}\right). \end{aligned} \tag{A.18}$$

Introducing the notation

$$M_\nu^\mu = \frac{m^2}{2}\left(\delta_\nu^\mu + g^{\mu\alpha}\gamma_{\alpha\nu} - \frac{1}{2}\delta_\nu^\mu g^{\alpha\beta}\gamma_{\alpha\beta}\right), \tag{A.19}$$

we find that because

$$g^{00} = \frac{1}{U}, \qquad g^{11} = -\frac{1}{V}, \qquad g^{22} = -\frac{1}{W^2}, \qquad g^{33} = -\frac{1}{W^2\sin^2\theta}, \tag{A.20}$$

the nonzero components of M_ν^μ are

$$\begin{aligned} M_0^0 &= \frac{m^2}{2}\left[1 - \frac{r^2}{W^2} + \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right], \\ M_1^1 &= \frac{m^2}{2}\left[1 - \frac{r^2}{W^2} - \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right], \\ M_2^2 = M_3^3 &= \frac{m^2}{2}\left[1 - \frac{1}{2}\left(\frac{1}{U} + \frac{1}{V}\right)\right]. \end{aligned} \tag{A.21}$$

Taking equalities (A.20) and the equality

$$\sqrt{-g} = \sqrt{UV}W^2\sin\theta$$

into account, for the tensor density components $\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$, we obtain

$$\begin{aligned} &\tilde{g}^{00} = \sqrt{\frac{V}{U}}\,W^2\sin\theta, \qquad \tilde{g}^{11} = -\sqrt{\frac{U}{V}}\,W^2\sin\theta, \qquad \tilde{g}^{22} = -\sqrt{UV}\sin\theta, \\ &\tilde{g}^{33} = -\sqrt{UV}\frac{1}{\sin\theta}, \qquad \tilde{g}^{\mu\nu} = 0 \quad \text{for } \mu \neq \nu. \end{aligned} \tag{A.22}$$

Using (A.2) and (A.22) in Eq. (2), we obtain

$$\frac{\partial}{\partial t}\left(\sqrt{\frac{V}{U}}W^2\right) = 0 \tag{A.23}$$

and

$$\frac{\partial}{\partial r}\left(\sqrt{\frac{U}{V}}W^2\right) = 2r\sqrt{UV}. \tag{A.24}$$

It is obvious from (A.23) that

$$\sqrt{\frac{V}{U}}W^2 = q(r), \tag{A.25}$$

where $q(r)$ is an arbitrary positive function depending only on r.

We now rewrite (A.13) and (A.15). We write four terms of (A.13) in the form

$$\frac{1}{W^2} - \frac{1}{W^2V}\frac{\partial^2 W^2}{\partial r^2} + \frac{1}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{2W^2V^2}\frac{\partial V}{\partial r}\frac{\partial W^2}{\partial r} =$$
$$= \frac{1}{W^2} - \frac{1}{2V}\frac{\partial}{\partial r}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial r}\right) - \frac{3}{4}\frac{1}{VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 - \frac{\partial}{\partial r}\left(\frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\right),$$

and the two remaining terms in the form

$$\frac{1}{2UVW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t} + \frac{1}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 = \frac{1}{2UW^2}\frac{\partial W^2}{\partial t}\frac{\partial}{\partial t}\log(VW).$$

We then have

$$R_0^0 - \frac{1}{2}R = \frac{1}{W^2} - \frac{1}{2V}\frac{\partial}{\partial r}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial r}\right) - \frac{3}{4}\frac{1}{VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 -$$
$$- \frac{\partial}{\partial r}\left(\frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\right) + \frac{1}{2UW^2}\frac{\partial W^2}{\partial t}\frac{\partial}{\partial t}\log(VW). \tag{A.26}$$

Analogously, we write four terms of (A.15) in the form

$$\frac{1}{W^2} + \frac{1}{UW^2}\frac{\partial^2 W^2}{\partial t^2} - \frac{1}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{1}{2W^2U^2}\frac{\partial W^2}{\partial t}\frac{\partial U}{\partial t} =$$
$$= \frac{1}{W^2} + \frac{1}{2U}\frac{\partial}{\partial t}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial t}\right) + \frac{3}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 + \frac{\partial}{\partial t}\left(\frac{1}{2W^2U}\frac{\partial W^2}{\partial t}\right)$$

and the remaining two terms in the form

$$-\frac{1}{2UVW^2}\frac{\partial W^2}{\partial r}\frac{\partial U}{\partial r} - \frac{1}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 = -\frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\frac{\partial}{\partial r}\log(UW).$$

Hence,

$$R_1^1 - \frac{1}{2}R = \frac{1}{W^2} + \frac{1}{2U}\frac{\partial}{\partial t}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial t}\right) + \frac{3}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 +$$
$$+ \frac{\partial}{\partial t}\left(\frac{1}{2W^2U}\frac{\partial W^2}{\partial t}\right) - \frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\frac{\partial}{\partial r}\log(UW). \tag{A.27}$$

Taking expressions (A.17), (A.21), (A.26), and (A.27) into account, for the system of equations

$$R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R + M^\mu_\nu = \varkappa T^\mu_\nu,$$

we obtain

$$\begin{aligned} R^0_0 - \frac{1}{2}R + M^0_0 = {} & \frac{1}{W^2} - \frac{1}{2V}\frac{\partial}{\partial r}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial r}\right) - \frac{3}{4VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 - \\ & - \frac{\partial}{\partial r}\left(\frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\right) + \frac{1}{2UW^2}\frac{\partial W^2}{\partial t}\frac{\partial}{\partial t}\log(VW) + \\ & + \frac{m^2}{2}\left[1 - \frac{r^2}{W^2} + \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = \varkappa T^0_0, \end{aligned} \tag{A.28}$$

$$\begin{aligned} R^1_1 - \frac{1}{2}R + M^1_1 = {} & \frac{1}{W^2} + \frac{1}{2U}\frac{\partial}{\partial t}\left(\frac{1}{W^2}\frac{\partial W^2}{\partial t}\right) + \frac{3}{4UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 + \\ & + \frac{\partial}{\partial t}\left(\frac{1}{2UW^2}\frac{\partial W^2}{\partial t}\right) - \frac{1}{2VW^2}\frac{\partial W^2}{\partial r}\frac{\partial}{\partial r}\log(UW) + \\ & + \frac{m^2}{2}\left[1 - \frac{r^2}{W^2} - \frac{1}{2}\left(\frac{1}{U} - \frac{1}{V}\right)\right] = \varkappa T^1_1, \end{aligned} \tag{A.29}$$

$$\begin{aligned} R^2_2 - \frac{1}{2}R + M^2_2 = {} & \frac{1}{2}\left[\frac{1}{UW^2}\frac{\partial^2 W^2}{\partial t^2} + \frac{1}{UV}\left(\frac{\partial^2 V}{\partial t^2} - \frac{\partial^2 U}{\partial r^2}\right) - \frac{1}{VW^2}\frac{\partial^2 W^2}{\partial r^2}\right] + \\ & + \frac{1}{4}\left[\frac{1}{VW^4}\left(\frac{\partial W^2}{\partial r}\right)^2 + \frac{1}{V^2W^2}\frac{\partial W^2}{\partial r}\frac{\partial V}{\partial r} - \frac{1}{VU^2}\left(\frac{\partial V}{\partial t}\frac{\partial U}{\partial t} - \left(\frac{\partial U}{\partial r}\right)^2\right) + \right. \\ & + \frac{1}{UV^2}\left(\frac{\partial U}{\partial r}\frac{\partial V}{\partial r} - \left(\frac{\partial V}{\partial t}\right)^2\right) - \frac{1}{UVW^2}\left(\frac{\partial W^2}{\partial r}\frac{\partial U}{\partial r} - \frac{\partial V}{\partial t}\frac{\partial W^2}{\partial t}\right) - \\ & \left. - \frac{1}{UW^4}\left(\frac{\partial W^2}{\partial t}\right)^2 - \frac{1}{U^2W^2}\frac{\partial W^2}{\partial t}\frac{\partial U}{\partial t}\right] + \frac{m^2}{2}\left[1 - \frac{1}{2}\left(\frac{1}{U} + \frac{1}{V}\right)\right] = \varkappa T^2_2, \end{aligned} \tag{A.30}$$

$$-R^0_1 = \frac{1}{UW^2}\frac{\partial^2 W^2}{\partial t\,\partial r} - \frac{1}{2W^4U}\frac{\partial W^2}{\partial r}\frac{\partial W^2}{\partial t} - \frac{1}{2UVW^2}\frac{\partial V}{\partial t}\frac{\partial W^2}{\partial r} - \frac{1}{2U^2W^2}\frac{\partial U}{\partial r}\frac{\partial W^2}{\partial t} = -\varkappa T^0_1. \tag{A.31}$$

According to (44), the square of the length element of the spatial component of the interval ds^2 is

$$dl^2 = e^{-2\varphi}\,dr^2 + e^{\varphi+\sigma}(d\theta^2 + \sin^2\theta\,d\phi^2).$$

We let η_{mn} denote the corresponding metric coefficients, i.e.,

$$\eta_{11} = e^{-2\varphi(r)}, \qquad \eta_{22} = e^{\varphi(r)+\sigma(r)}, \qquad \eta_{33} = \eta_{22}\sin^2\theta, \qquad \eta_{mn} = 0 \quad \text{for } m \neq n.$$

Using these expressions, we can find that the nonzero components of the connection

$$\eta^l_{mn} = \frac{1}{2}\eta^{lk}(\partial_m\eta_{kn} + \partial_n\eta_{km} - \partial_k\eta_{mn})$$

are

$$\begin{aligned} &\eta^1_{11} = -\varphi'(r), \qquad \eta^1_{22} = -\frac{1}{2}(\varphi' + \sigma')e^{3\varphi+\sigma}, \qquad \eta^1_{33} = \eta^1_{22}\sin^2\theta, \\ &\eta^2_{12} = \eta^3_{13} = \frac{1}{2}(\varphi' + \sigma'), \qquad \eta^2_{33} = -\sin\theta\cos\theta, \qquad \eta^3_{23} = \cot\theta. \end{aligned} \tag{A.32}$$

It is obvious that

$$\eta^p_{1p} = \sigma', \qquad \eta^p_{2p} = \cot\theta, \qquad \eta^p_{3p} = 0. \tag{A.33}$$

For the components of the curvature tensor of the three-dimensional space

$$R^k_{mnp} = \partial_n \eta^k_{mp} - \partial_p \eta^k_{mn} + \eta^e_{mp}\eta^k_{en} - \eta^e_{mn}\eta^k_{ep}, \tag{A.34}$$

we obtain the expressions

$$R^1_{212} = -\frac{1}{2}e^{3\varphi+\sigma}\left[\varphi'' + \sigma'' + \frac{1}{2}(\varphi' + \sigma')(3\varphi' + \sigma')\right], \tag{A.35}$$

$$R^1_{313} = R^1_{212}\sin^2\theta, \tag{A.36}$$

$$R^2_{121} = -\frac{1}{2}\left[\varphi'' + \sigma'' + \frac{1}{2}(\varphi' + \sigma')(3\varphi' + \sigma')\right], \tag{A.37}$$

$$R^2_{323} = \left[1 - \frac{1}{4}(\varphi' + \sigma')^2 e^{3\varphi+\sigma}\right]\sin^2\theta, \qquad R^3_{131} = R^2_{121},$$

$$R^3_{232} = 1 - \frac{1}{4}(\varphi' + \sigma')^2 e^{3\varphi+\sigma}. \tag{A.38}$$

With relation (A.37) taken into account in Eq. (31), we obtain

$$R^2_{121} = \frac{1}{2}e^{-2\varphi}\left(k + \frac{m^2}{2}e^{2\varphi}\right). \tag{A.39}$$

Analogously, adding (A.35) and (A.38) and taking (40) into account, we can write Eq. (33) in the form

$$R^1_{212} + R^3_{232} = e^{\varphi+\sigma}\left(k + \frac{m^2 r^2}{2}e^{-(\varphi+\sigma)}\right). \tag{A.40}$$

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