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POTENTIALS IN MODIFIED AdS_5 SPACES WITH A MODERATE INCREASE IN ENTROPY

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We investigate the relation between dilaton potentials and the b-factors of modified anti-de Sitter spaces and obtain the explicit form of dilaton potentials corresponding to b-factors that lead to a satisfactory description of the particle creation multiplicity in the framework of the holographic description.

Keywords: anti-de Sitter space, black hole, heavy-ion collision, particle creation multiplicity, membrane collision, potential for background metric, dilaton, phantom

1. Introduction

In [1], we considered the formation of black holes during a collision of domains [2], [3] in modified spaces with b-factors of the forms $b = (L/z)^a$, $b = e^{-z/R}$, and $b = (L/z)e^{-z^2/R^2}$ [4]. According to the holographic approach, formation of a quark–gluon plasma in the four-dimensional space corresponds to creation of a black hole in the dual five-dimensional space. In this case, the multiplicity of particle creation in heavy-ion collisions is proportional to the entropy or the trapped surface area of the black hole in the auxiliary space. It is known from experimental data [5] that the particle creation multiplicity is well approximated by a power-law function of the form $s_{NN}^{0.15}$ for energies from 10^2 GeV to 10^4 GeV. In the case of the AdS_5 space, the particle creation multiplicity is proportional to $s_{NN}^{1/3}$ [6]. To reproduce the experimental dependence in the holographic approach, Kiritsis and Taliotis [4] proposed modifying the AdS_5 space by introducing the b-factors previously considered in [7]–[9].

The experimentally known proportionality of the particle creation multiplicity to $s_{NN}^{0.15}$ is well modeled in the formation of black holes by colliding domains in a modified AdS_5 space with power-law b-factors with $a \approx 0.47$ [1]. But a similar proportionality can also be obtained with other b-factors. Here, we carefully consider the formation of black holes during the collision of domains in a space with a modernized mixed b-factor of the form $b = (L/z)^a e^{-z^2/R^2}$ yielding the same experimental dependence with logarithmic corrections at $a \approx 0.5$.

Until now, the question of the explicit dependence of potentials on fields in modified spaces with the considered b-factors have been studied only in the asymptotic approximation. The dependence of potentials on the b-factor and its derivatives was considered in [9]. Here, we consider the explicit analytic dependence of potentials on fields for power-law and exponential b-factors. We establish that the field is a phantom field for the modified space with a power-law b-factor with $a < 1$ and an ordinary field in the case of an exponential b-factor. In modified spaces with mixed b-factors $b = (L/z)^a e^{-z^2/R^2}$ with $a < 1$, a switch of the regime from phantom to ordinary occurs at the point z_0 ($\Phi(z_0) = 0$). We present the explicit dependence

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of potentials on fields in the cases of the exponential and the power-law b-factors. This dependence in the case of mixed b-factors can only be constructed numerically.

This work has the following structure. In Sec. 2, we discuss the relation between potentials and fields in modified spaces. In Secs. 2.2 and 2.3, we discuss particular cases, namely, the relations between potentials and fields in modified spaces with the power-law and the modernized mixed b-factors. In Sec. 3, we introduce a shock wave. In Sec. 4, based on a modified holographic model with the modernized mixed b-factor, we consider the possibility of modeling the particle creation multiplicity in the quark–gluon plasma obtained in heavy-ion collisions.

2. Potentials for the background metric

2.1. General case. We consider the action of five-dimensional gravity coupled to a scalar dilaton field:

$$S_5 = S_R + S_\Phi, \tag{1}$$

where S_R is the Einstein–Hilbert action with a negative cosmological constant,

$$S_R = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[R + \frac{d(d-1)}{L^2}\right]dx^5,$$

$d+1 = D = 5$, and S_Φ is the dilaton action,

$$S_\Phi = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[-\frac{4}{3}(\partial\Phi)^2 + V(\Phi)\right]dx^5.$$

Here, L is the characteristic dimensional parameter, and G_5 is the five-dimensional gravitational constant. Assuming that background metric has the form

$$ds^2 = b^2(z)(dz^2 + dx^i\,dx^i - dx^+ dx^-), \quad i = 1, 2, \tag{2}$$

we consider gravitational equations for several types of b-factors [4]. In this case, the Einstein equation reduces to two independent relations [7], [8]

$$\begin{aligned} &\frac{3b''}{b} + \frac{2}{3}(\Phi')^2 - \frac{b^2}{2}V(\Phi) - \frac{6b^2}{L^2} = 0, \\ &\frac{6(b')^2}{b^2} - \frac{2}{3}(\Phi')^2 - \frac{b^2}{2}V(\Phi) - \frac{6b^2}{L^2} = 0, \end{aligned} \tag{3}$$

where $b = b(z)$, $b' = \partial_z b$, and the dilaton field depends only on z, $\Phi = \Phi(z)$.

The scalar field equation has the form

$$\frac{1}{b^5}\frac{\partial}{\partial z}\left(b^3\frac{\partial}{\partial z}\right)\Phi + \frac{3}{8}\frac{\partial V(\Phi)}{\partial\Phi} = 0. \tag{4}$$

It is obvious from Eqs. (3) that the dilaton field and its potential can be expressed in terms of the b-factor and its derivatives [9]:

$$\begin{aligned} \Phi' &= \pm\frac{3}{2}\sqrt{\left(\frac{2(b')^2}{b^2} - \frac{b''}{b}\right)}, \\ V(\Phi(z)) &= \frac{3}{b^2}\left(\frac{b''}{b} + \frac{2(b')^2}{b^2} - \frac{4b^2}{L^2}\right). \end{aligned} \tag{5}$$

We note that these expressions ensure that Eq. (4) is satisfied. Expressions (5) do not give the explicit dependence of the potential on the field $V = V(\Phi)$. The explicit form of $V = V(\Phi)$ can be obtained by the superpotential method. To use this method, it is convenient to rewrite (3) and the metric in the coordinates[1]

$$u = \int b(z)\,dz. \tag{6}$$

As a result, metric (2) becomes

$$ds^2 = du^2 + e^{2A(u)}(dx^i\,dx^i - dx^+\,dx^-), \tag{7}$$

where $A(u) = \log(b(u))$. The potential is related to the superpotential W:

$$V(\Phi) = -\frac{4}{3}\left(\frac{dW}{d\Phi}\right)^2 + \frac{64}{27}W^2 - \frac{12}{L^2}. \tag{8}$$

The superpotential is related to the function A as

$$W(\Phi) = -\frac{9}{4}\frac{dA}{du}, \tag{9}$$

and the dilaton field is defined as

$$\Phi = \pm\frac{3}{2}\int\sqrt{-\frac{d^2A}{du^2}}du. \tag{10}$$

In the simplest case of a modified space with an exponential b-factor of the form $b = e^{-z/R}$ (we assume that $R \sim \Lambda_{\mathrm{QCD}}^{-1} \sim 1\,\mathrm{fm}$), using the superpotential method, we can obtain the dependence of the potential on the field:

$$V(\Phi) = -\frac{12}{L^2} + \frac{9}{R^2}e^{\pm 4(\Phi-\Phi_0)/3}, \tag{11}$$

where Φ_0 is an integration constant.

2.2. Power-law *b*-factor. We now consider a space with a power-law b-factor $b(z) = (L/z)^a$. If $a = 1$, then we have the AdS_5 space.

The potential and fields can be represented as functions of z. In the considered case, $\Phi = \Phi(z)$ is a single-valued function of z, and we can express the coordinate as a function of the field, $z = z(\Phi)$, and substitute z in the expression for the potential $V(z) = V(z(\Phi))$ to obtain

$$V(\Phi) = -\frac{12}{L^2} + \frac{3a(3a+1)}{L^{2a}}\exp\left(\pm\frac{4}{3}\sqrt{\frac{a-1}{a}}(\Phi-\Phi_0)\right). \tag{12}$$

The field Φ and potential $V(\Phi)$ are obviously real for $a > 1$.

For $a < 1$, we can consider the phantom field Φ_{p}, which corresponds to the action

$$S_{\Phi_{\mathrm{p}}} = -\frac{1}{16\pi G_5}\int\sqrt{-g}\left[\frac{4}{3}(\partial\Phi_{\mathrm{p}})^2 + \widetilde{V}(\Phi_{\mathrm{p}})\right]dx^5.$$

The phantom field Φ_{p} is related to the dilaton field Φ as $\Phi - \Phi_0 = i(\Phi_{\mathrm{p}} - \Phi_{\mathrm{p}0})$. For $a < 1$, the potential becomes

$$\widetilde{V}(\Phi_{\mathrm{p}}) = -\frac{12}{L^2} + \frac{3a(3a+1)}{L^{2a}}\exp\left(\pm\frac{4}{3}\sqrt{\frac{1-a}{a}}(\Phi_{\mathrm{p}}-\Phi_{\mathrm{p}0})\right).$$

For the phantom field, relations (5) are replaced with the analogous relations

$$\partial_z\Phi_{\mathrm{p}} = \pm\frac{3}{2}\sqrt{\left(\frac{b''}{b} - \frac{2(b')^2}{b^2}\right)}, \qquad \widetilde{V}(\Phi_{\mathrm{p}}(z)) = \frac{3}{b^2}\left(\frac{b''}{b} + \frac{2(b')^2}{b^2} - \frac{4b^2}{L^2}\right). \tag{13}$$

[1] Here, u is an analogue of the cosmic coordinate in the cosmological application of the superpotential method (see, e.g., [10]).

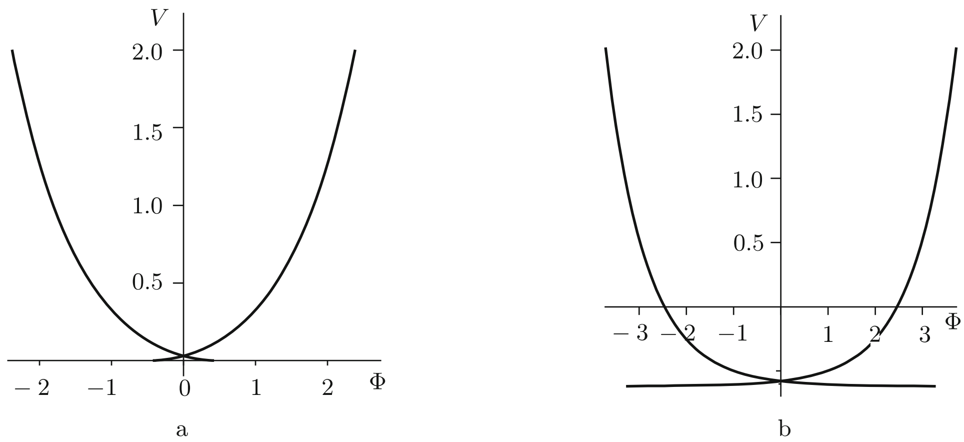

Fig. 1. The potentials corresponding to $b = (L/z)^a e^{-z^2/R^2}$: (a) at $a = 2$, $L = 4.4\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$ and (b) at $a = 1$, $L = 4.4\,\mathrm{fm}$, $R = 1\,\mathrm{fm}$, $\Phi_0 = -0.5$. The figures show the right and left branches corresponding to the signs + and − in (15).

2.3. Modernized mixed *b*-factor. We consider a space with the modernized mixed b-factor

$$b(z) = \left(\frac{L}{z}\right)^a e^{-z^2/R^2}. \tag{14}$$

The superpotential method is inapplicable in this case because z cannot be represented explicitly in terms of the u variable and $b(u)$ cannot represented explicitly. For b-factor (14), we express $\partial_z \Phi(z)$ and $V(\Phi(z))$ using (5) as

$$\partial_z \Phi(z) = \pm \frac{3}{2}\frac{\xi}{R^2 z}, \tag{15}$$

where

$$\xi = \sqrt{\zeta}, \qquad \zeta = 4z^4 + 2R^2(2a+1)z^2 + aR^4(a-1),$$

and

$$V(z) = -\frac{12}{L^2} + \frac{3(L/z)^{-2a}(aR^4(3a+1) + 2z^2R^2(6a-1) + 12z^4)e^{2z^2/R^2}}{z^2R^4}. \tag{16}$$

Integrating (15), we obtain

$$\Phi_\pm = \pm\left(\frac{3}{4}\frac{\xi}{R^2} + \frac{3}{8}(2a+1)\log\frac{2\xi + (2a+1)R^2 + 4z^2}{2\mu_0^2} - \right.$$
$$\left. - \frac{3}{4}\sqrt{a(a-1)}\log\frac{2R^2(a(a-1)R^2 + (2a+1)z^2 + \xi\sqrt{a(a-1)})}{\mu_0^2 z^2}\right) + \Phi_{0\pm}, \tag{17}$$

where μ_0 is an arbitrary dimensional constant (for definiteness, $\mu_0 = 1\,\mathrm{fm}$) and the arbitrariness in choosing the constant is included in $\Phi_{\pm 0}$.

If $a > 1$, then expression (17) is well defined. As $z \to \infty$, we obtain $\Phi \sim 3z^2/2R^2$ and $V \sim \Phi^{a+1}e^{4\Phi/3}$. The potential $V(\Phi)$ is shown in Fig. 1 at $a = 1$ and $a = 2$.

If $a < 1$, then the function $\zeta(z)$ has a positive root, $\zeta(z_0) = 0$, $\zeta(z) > 0$ for $z > z_0$, and $\zeta(z) < 0$ for $0 < z < z_0$.

The function $\zeta(z)$ is shown in Fig. 2. We obtain $z_0 \approx 0.249\,\mathrm{fm}$ for $a = 1/3$ and $z_0 \approx 0.243\,\mathrm{fm}$ for $a = 1/2$. In the neighborhood of $z = z_0$, the function $\Phi(z)$ has the form

$$\Phi(z) \sim (z^2 - z_0^2)^{3/2}(A + O(z^2 - z_0^2)), \qquad A = \pm\frac{2(8a+1)^{1/4}}{\sqrt{z_0}}. \tag{18}$$

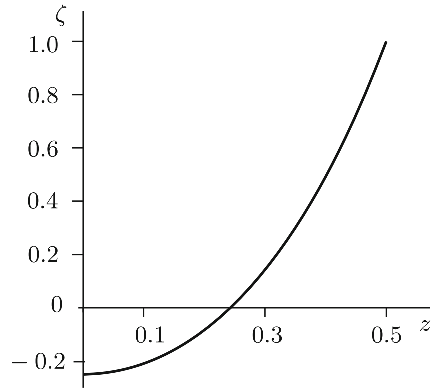

Fig. 2. The function $\zeta = \zeta(z)$ at $a = 1/2$.

The field $\Phi(z)$ is real for $z > z_0$,

$$\Phi_{\mathrm{s}\pm} = \frac{3}{4R^2}\biggl(\sqrt{\zeta} - \sqrt{-(a^2-a)R^4}\arctan\biggl(\frac{(a^2-a)R^4 + (2a+1)R^2z^2}{\sqrt{-\zeta(a^2-a)R^4}}\biggr) + \\ + \frac{1}{2}(2a+1)R^2\log\frac{a(4\sqrt{\zeta} + 8z^2 + 2(2a+1)R^2)}{\mu_0^2}\biggr) + \Phi_{\mathrm{s}0\pm},$$

and imaginary for $z < z_0$, $\Phi_{\mathrm{s}\pm} - \Phi_{\mathrm{s}0\pm} = i(\Phi_{\mathrm{p}\pm} - \Phi_{\mathrm{p}0\pm})$,

$$\Phi_{\mathrm{p}\pm} = \pm\frac{3}{4R^2}\biggl(\sqrt{aR^4(1-a)}\,\times \\ \times\log\biggl(\frac{2\sqrt{aR^4(1-a)}\sqrt{-\zeta} + 2R^2[(a-a^2)R^2 - 2z^2(a+1/2)]}{\mu_0^2z^2}\biggr) - \\ - \biggl(\biggl(a + \frac{1}{2}\biggr)R^2\arctan\biggl(\frac{(2a+1)R^2 + 4z^2}{2\mu_0^2\sqrt{-\zeta}}\biggr) - \frac{\sqrt{-\zeta}}{\mu_0^2}\biggr)\biggr) + \Phi_{p0\pm}.$$

A convenient choice of constants is $\Phi_{\mathrm{s}\pm}(z_0) = \Phi_{\mathrm{p}\pm}(z_0) = 0$. The scalar field can be represented as $\Phi = \Phi_{\mathrm{s}}\Theta(z-z_0) + i\Phi_{\mathrm{p}}\Theta(z_0-z)$. We hence have a theory with an alternating sign of the kinetic term for $z < z_0$.

The potential can be represented parametrically as a function of the real component Φ_{s} for $z > z_0$ and as a function of the imaginary component Φ_{p} for $z < z_0$. Such a potential is shown graphically in Fig. 3.

3. Shock wave

In this section, we make several comments on our previous consideration [1]. To take the shock wave into account, in (1), we add the action of a pointlike source moving along the trajectory $x^\mu = x_*^\mu(\eta)$,

$$S_{\mathrm{st}} = \int\biggl[\frac{1}{2e}g_{\mu\nu}\frac{dx_*^\mu}{d\eta}\frac{dx_*^\nu}{d\eta} - \frac{e}{2}m^2\biggr]d\eta,$$

where m is the particle mass and η is a worldline parameter. Further, we assume that the particle mass m is zero, which allows treating only lightlike geodesics, e_μ^a is the frame associated with a metric $g_{\mu\nu} = e_\mu^a e_{\nu a}$, $e = \sqrt{-g}$, and g is the determinant of the metric. We assume that the metric has the shock wave form [11]–[18]

$$ds^2 = b^2(z)\bigl(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi(z,x^1,x^2)\delta(x^+)(dx^+)^2\bigr), \quad i = 1,2. \tag{19}$$

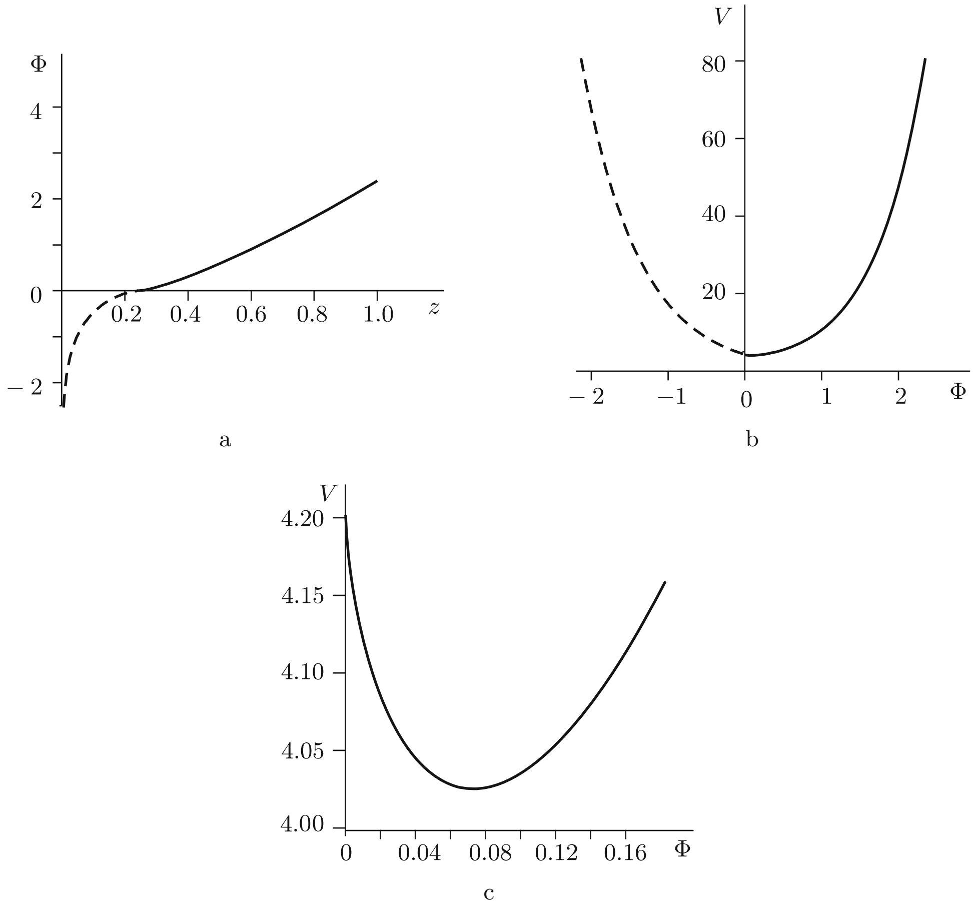

Fig. 3. (a) The phantom field $\Phi_{\rm p}$ (dashed line) and the dilaton field $\Phi_{\rm s}$ (solid line) as functions of z. (b) The dependence of the potential V on the dilaton and the phantom fields. (c) The potential V as a function of the dilaton field for small Φ (enlarged lower part of the curve in Fig. 3b). The plots correspond to $a = 1/2$, $L = 4.4\,\text{fm}$, $R = 1\,\text{fm}$, and the sign + in (15).

The shock-wave metric solves the Einstein equation

$$\left(R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R\right) - \frac{g_{\mu\nu}}{2}\left(-\frac{4}{3}(\partial\Phi)^2 + V(\Phi)\right) - \frac{4}{3}\partial_\mu\Phi\,\partial_\nu\Phi - g_{\mu\nu}\frac{d(d-1)}{2L^2} = 8\pi G_5 J_{\mu\nu}, \tag{20}$$

where $(\partial\Phi)^2 = g^{\mu\nu}\,\partial_\mu\Phi\,\partial_\nu\Phi$ and $J_{\mu\nu}$ is the current given by the expression [6]

$$J_{\mu\nu} = \frac{1}{\sqrt{-g}}\int e\,p_\mu p_\nu \delta(x^\mu - x_*^\mu)\,d\eta, \tag{21}$$

where $p_\mu = e^{-1}g_{\mu\nu}(dx^\nu/d\eta)$ is the conjugate momentum. The nonzero component of the current in lightlike coordinates (x^+, x^-, x^i, z), $i = 1, 2$, is written as

$$J_{++} = \frac{E}{b^3(z)}\delta(x^1)\delta(x^2)\delta(z - z_*)\delta(x^+),$$

where E corresponds to the ion collision energy. Compared with metric (2), metric (19) has the added profile of the shock wave $\phi(z, x_\perp)$, which with (20) taken into account solves the equation

$$\left(\partial_{x^1}^2 + \partial_{x^2}^2 + \partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi(z, x_\perp) = -16\pi G_5\frac{E}{b^3}\delta(x^1)\delta(x^2)\delta(z - z_*). \tag{22}$$

It is clear from (22) that the dilaton field influences the wave profile resulting from the source not directly but only through the b-factor.

4. Holographic simulation of experimental multiplicity curves using the modernized *b*-factor

The domain wave profile equation in the space with a modernized mixed b-factor $b = (L/z)^a e^{-z^2/R^2}$ can be written as [1], [19]

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^\omega(z) = -\frac{16\pi G_5 E}{L^2}\frac{\delta(z-z_*)}{b^3(z)}, \tag{23}$$

where

$$\phi^\omega(z) = \frac{1}{L^2}\int \phi(z, x_\perp)\, dx_\perp$$

and L is the domain radius. Equation (23) is considered separately before and after the collision point. The boundary points of the trapped surface are denoted by z_a and z_b, $z_a < z_* < z_b$, where z_* is the collision point. We represent the solution of (23) in the form

$$\phi^\omega(z) = \phi_a \Theta(z_* - z) + \phi_b \Theta(z - z_*), \tag{24}$$

where

$$\phi_a = C_a \int_{z_a}^{z} b^{-3}\, dz, \qquad \phi_b = C_b \int_{z_b}^{z} b^{-3}\, dz.$$

The constants C_a and C_b are defined by

$$C_a = \frac{16\pi G_5 E}{L^2}\int_{z_b}^{z_*} b^{-3}\, dz \left(\int_{z_b}^{z_a} b^{-3}\, dz\right)^{-1}, \qquad C_b = \frac{16\pi G_5 E}{L^2}\int_{z_a}^{z_*} b^{-3}\, dz \left(\int_{z_b}^{z_a} b^{-3}\, dz\right)^{-1}.$$

The condition for the trapped surface formation [1] leads to additional requirements on the domain wave,

$$\partial_z \phi^\omega(z)\big|_{z=z_a} = 2, \qquad \partial_z \phi^\omega(z)\big|_{z=z_b} = -2.$$

These conditions yield

$$\begin{aligned} \frac{8\pi G_5 E}{L^2} b^{-3}(z_a) \int_{z_b}^{z_*} b^{-3}\, dz \left(\int_{z_b}^{z_a} b^{-3}\, dz\right)^{-1} &= 1, \\ \frac{8\pi G_5 E}{L^2} b^{-3}(z_b) \int_{z_a}^{z_*} b^{-3}\, dz \left(\int_{z_b}^{z_a} b^{-3}\, dz\right)^{-1} &= -1. \end{aligned} \tag{25}$$

Using the notation

$$\int_{z_i}^{z_j} b^{-3}\, dz = F(z_j) - F(z_i)$$

in Eqs. (25), we obtain the respective relations between the points z_*, z_a, z_b and z_a, z_b

$$F(z_*) = \frac{b^{-3}(z_b)F(z_a) + b^{-3}(z_a)F(z_b)}{b^{-3}(z_a) + b^{-3}(z_b)}, \tag{26}$$

$$b^{-3}(z_a) = \frac{b^{-3}(z_b)}{(8\pi G_5 E/L^2) b^{-3}(z_b) - 1}. \tag{27}$$

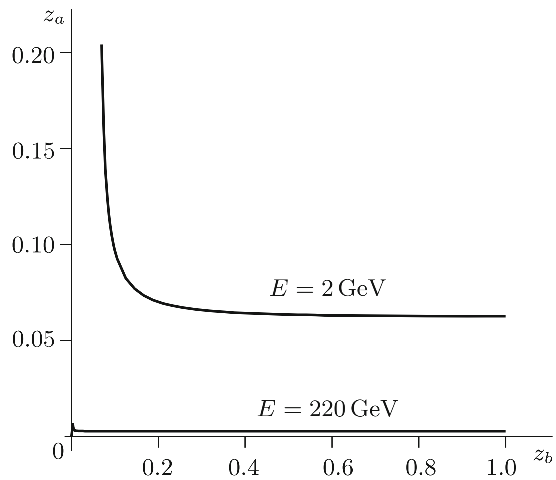

Fig. 4. The dependence of z_a on z_b corresponding to $a = 1/2$ at the energies $E = 2\,\mathrm{GeV}$ and $E = 220\,\mathrm{GeV}$.

For considering the case of the b-factor $b(z) = (L/z)^a e^{-z^2/R^2}$, we can write expression (27) as

$$\left(\frac{z_a}{L}\right)^{3a} e^{3z_a^2/R^2} = \frac{(z_b/L)^{3a} e^{3z_b^2/R^2}}{(8\pi G_5 E/L^2)(z_b/L)^{3a} e^{3z_b^2/R^2} - 1}. \tag{28}$$

The solution of (28) is

$$z_a = R\sqrt{\frac{a}{2}\mathrm{W}\left(\frac{2L^2}{aR^2}\left(\frac{(z_b/L)^{3a} e^{3z_b^2/R^2}}{(8\pi G_5 E/L^2)(z_b/L)^{3a} e^{3z_b^2/R^2} - 1}\right)^{2/3a}\right)}, \tag{29}$$

where $\mathrm{W}(x)$ is the Lambert W-function.

The obtained expression has the simplest case for $a = 1/3$, namely,

$$z_a = \frac{R}{\sqrt{6}}\sqrt{\mathrm{W}\left(\frac{6L^2}{R^2}\left(\frac{(z_b/L) e^{3z_b^2/R^2}}{(8\pi G_5 E/L^2)(z_b/L) e^{3z_b^2/R^2} - 1}\right)^{2}\right)}. \tag{30}$$

The dependence of z_a on z_b is shown in Fig. 4 at $a = 1/2$ and fixed energies. For sufficiently large z_b, expression (28) can be simplified and tends to the form

$$\left(\frac{z_a}{L}\right)^{3a} e^{3z_a^2/R^2} \underset{z_b\to\infty}{\longrightarrow} \frac{L^2}{8\pi G_5 E}, \tag{31}$$

whence we obtain

$$z_a \underset{z_b\to\infty}{\longrightarrow} R\sqrt{\frac{a}{2}}\sqrt{\mathrm{W}\left(\frac{2L^2}{aR^2}\left(\frac{L^2}{8\pi G_5 E}\right)^{2/3a}\right)}. \tag{32}$$

Substituting (27) in (26), we obtain the equation

$$\frac{8\pi G_5 E}{L^2} b^{-3}(z_b) = \frac{\Gamma((3a+1)/2, -3z_b^2/R^2) - \Gamma((3a+1)/2, -3z_a^2/R^2)}{\Gamma((3a+1)/2, -3z_*^2/R^2) - \Gamma((3a+1)/2, -3z_a^2/R^2)}, \tag{33}$$

where $\Gamma(A, X)$ is the incomplete gamma function. For $a = 1/3$, we write (33) as

$$\frac{8\pi G_5 E}{L^2} b^{-3}(z_b) = \frac{e^{3zb^2/R^2} - e^{3za^2/R^2}}{e^{3z_*^2/R^2} - e^{3za^2/R^2}}. \tag{34}$$

Using the representation of the incomplete gamma function

$$\Gamma(A,X)=\Gamma(A)-\frac{X^A{}_1F_1(A,A+1,-X)}{A}, \tag{35}$$

where ${}_1F_1$ is the confluent hypergeometric function, we write (33) as

$$\frac{8\pi G_5E}{L^2}b^{-3}(z_b)=\frac{z_a^{(3a+1)/2}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_a^2}{R^2}\right)-z_b^{(3a+1)/2}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_b^2}{R^2}\right)}{z_a^{(3a+1)/2}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_a^2}{R^2}\right)-z_*^{(3a+1)/2}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_*^2}{R^2}\right)}. \tag{36}$$

Using the series expansion of the confluent hypergeometric function

$${}_1F_1(\alpha,\gamma,z)=\sum_{k=0}^{\infty}\frac{(\alpha)_k}{(\gamma)_k}\frac{z^k}{k!}, \tag{37}$$

where $(\alpha)_k$ and $(\gamma)_k$ are Pochhammer symbols, we simplify formula (36):

$$\frac{8\pi G_5E}{L^2}\left(\frac{z_b}{L}\right)^{3a}e^{3z_b^2/R^2}=\sum_{k=0}^{\infty}\frac{(3z_a^2/R^2)^{k+(3a+1)/2}-(3z_b^2/R^2)^{k+(3a+1)/2}}{(3z_a^2/R^2)^{k+(3a+1)/2}-(3z_*^2/R^2)^{k+(3a+1)/2}}. \tag{38}$$

We note that condition (38) agrees with the inequality $z_a<z_*<z_b$.

For an arbitrary a, relation (26) becomes

$$z_*^{2a}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_*^2}{R^2}\right)=$$
$$=\frac{z_a^{2a}z_b^{2a}\left(z_b^{a}e^{3z_b^2/R^2}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_a^2}{R^2}\right)+z_a^{a}e^{3z_a^2/R^2}{}_1F_1\left(\frac{3a+1}{2},\frac{3(a+1)}{2},\frac{3z_b^2}{R^2}\right)\right)}{z_a^{3a}e^{3z_a^2/R^2}+z_b^{3a}e^{3z_B^2/R^2}}.$$

This equation is rather difficult to solve analytically, but expression (26) can be simplified in the case $a=1/3$,

$$e^{3z_*^2/R^2}=\frac{z_a+z_b}{z_ae^{-3z_b^2/R^2}+z_be^{-3z_a^2/R^2}}, \tag{39}$$

and has the analytic solution

$$z_*=\sqrt{\frac{R^2}{3}\log\left(\frac{z_a+z_b}{z_ae^{-3z_b^2/R^2}+z_be^{-3z_a^2/R^2}}\right)}.$$

The functions ϕ_a and ϕ_b can be represented as

$$\begin{aligned}\phi_a&=\frac{16\pi G_5E}{L^2}\int_{z_b}^{z_*}b^{-3}\,dz\cdot\int_{z_a}^{z}b^{-3}\,dz\left(\int_{z_b}^{z_a}b^{-3}\,dz\right)^{-1},\\ \phi_b&=\frac{16\pi G_5E}{L^2}\int_{z_a}^{z_*}b^{-3}\,dz\cdot\int_{z_b}^{z}b^{-3}\,dz\left(\int_{z_b}^{z_a}b^{-3}\,dz\right)^{-1}.\end{aligned} \tag{40}$$

Using property (35), we obtain

$$\int b^{-3}\,dz=\frac{z(L/z)^{-3a}{}_1F_1((3a+1)/2,3(a+1)/2,3z^2/R^2)}{3a+1}+C.$$

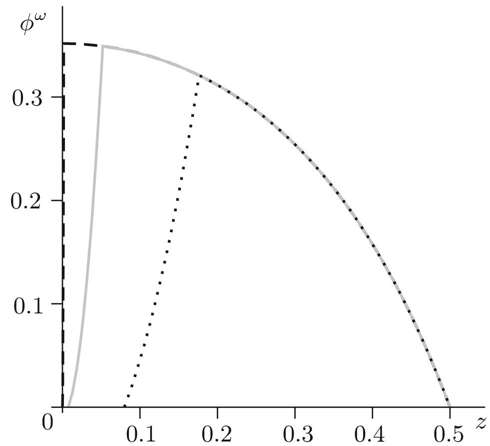

Fig. 5. The profile ϕ^ω corresponding to the mixed b-factor $b(z) = (L/z)^a e^{-z^2/R^2}$ at $z_b = 0.5\,\mathrm{fm}$, $a = 1/3$, $R = 1\,\mathrm{fm}$ for $E = 0.2\,\mathrm{GeV}$ (dotted line), $E = 2\,\mathrm{GeV}$ (solid line), and $E = 2\,\mathrm{TeV}$ (dashed line).

Introducing the new notation

$$\Upsilon(z) = z\left(\frac{L}{z}\right)^{-3a} {}_1F_1\left(\frac{3a+1}{2}, \frac{3(a+1)}{2}, \frac{3z^2}{R^2}\right), \tag{41}$$

we obtain representations of expressions (40):

$$\begin{aligned} \phi_a &= \frac{16\pi G_5 E}{(3a+1)L^2} \frac{(\Upsilon(z_b) - \Upsilon(z_*))(\Upsilon(z) - \Upsilon(z_a))}{\Upsilon(z_b) - \Upsilon(z_a)}, \\ \phi_b &= \frac{16\pi G_5 E}{(3a+1)L^2} \frac{(\Upsilon(z_*) - \Upsilon(z_a))(\Upsilon(z_b) - \Upsilon(z))}{\Upsilon(z_b) - \Upsilon(z_a)}. \end{aligned} \tag{42}$$

Expression (41) at $a = 1/3$ becomes

$$\Upsilon(z) = \frac{R^2}{3L}(e^{3z^2/R^2} - 1). \tag{43}$$

The functions ϕ_a and ϕ_b can now be represented as

$$\begin{aligned} \phi_a &= \frac{8}{3} \frac{\pi G_5 E R^2}{L^3} \frac{(e^{3z_b^2/R^2} - e^{3z_*^2/R^2})(e^{3z^2/R^2} - e^{3z_a^2/R^2})}{e^{3z_b^2/R^2} - e^{3z_a^2/R^2}}, \\ \phi_b &= \frac{8}{3} \frac{\pi G_5 E R^2}{L^3} \frac{(e^{3z_*^2/R^2} - e^{3z_a^2/R^2})(e^{3z_b^2/R^2} - e^{3z^2/R^2})}{e^{3z_b^2/R^2} - e^{3z_a^2/R^2}}. \end{aligned} \tag{44}$$

Function (24) constructed with ϕ_a and ϕ_b for $a = 1/3$, $G_5 = L^3/1.9$, $L = 4.4\,\mathrm{fm}$, and $z_b = 0.5\,\mathrm{fm}$ is shown in Fig. 5.

The condition $z_a < z_b$ is satisfied if the estimate

$$E > \frac{L^3}{4\pi G_5 z_b e^{3z_b^2/R^2}}$$

holds. A trapped surface is not created at the energy

$$E = \frac{L^3}{4\pi G_5 z_b e^{3z_b^2/R^2}}, \tag{45}$$

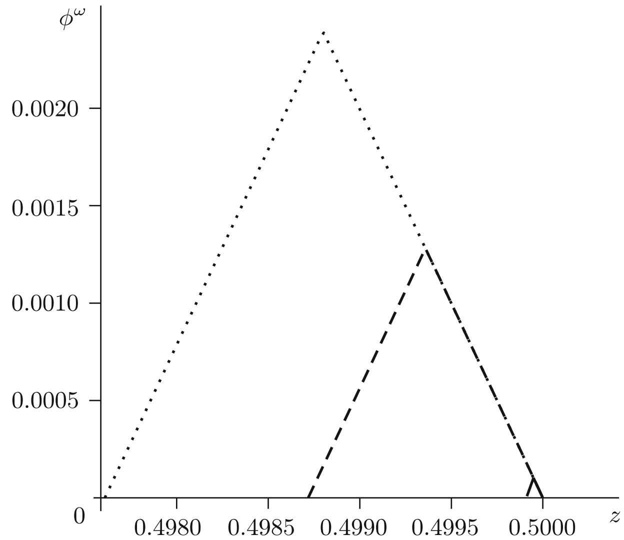

Fig. 6. The profile ϕ^ω corresponding to the mixed b-factor $b(z) = (L/z)^a e^{-z^2/R^2}$ for $z_b = 0.5$ fm, $a = 1/3$, $R = 1$ fm, $E = 28.74$ MeV (dotted line), $E = 28.66$ MeV (dashed line), and $E = 28.57$ MeV (solid line).

which is approximately 28.568 MeV for the parameter values $G_5 = L^3/1.9$, $L = 4.4$ fm, $a = 1/3$, and $z_b = 0.5$ fm (Fig. 6).

The black hole entropy is proportional to the trapped surface area. We previously used this relation to calculate the relative entropy in modified spaces with b-factors [1]. We now consider the relative entropy in a space with a b-factor of the form $b(z) = (L/z)^a e^{-z^2/R^2}$. In this case, the relative entropy is

$$s = \frac{\Xi(z_a) - \Xi(z_b)}{2G_5 \cdot 3(3\,a-1)(a-1)}, \tag{46}$$

where

$$\Xi(z) = \left(\frac{L}{z}\right)^{3a} z e^{-3z^2/2R^2} \times \\ \times \left(2\left(\frac{3z^2}{R^2}\right)^{(3a-1)/4} \mathbf{M}\left(\frac{-3a+1}{4}, \frac{3(-a+1)}{4}, \frac{3z^2}{R^2}\right) + 3(1-a)e^{-3z^2/2R^2}\right),$$

and $\mathbf{M}(\mu,\nu,z) = e^{-z/2} z^{1/2+\nu} {}_1F_1(1/2+\nu-\mu, 1+2\nu, z)$ is the Whittaker function, $a \neq 1/3$, $a \neq 1$.

We show the dependence of s on z_b at the energies 2 GeV and 220 Gev in Fig. 7.

The relative entropy has the maximum value at infinite z_b:

$$s \underset{z_b \to \infty}{\longrightarrow} \frac{(L/z_a)^{3a} z_a e^{-3z_a^2/2R^2}}{6G_5 \cdot (3a-1)(1-a)} \times \\ \times \left(2\left(\frac{3z_a^2}{R^2}\right)^{(3a-1)/4} \mathbf{M}\left(\frac{-3a+1}{4}, \frac{3(-a+1)}{4}, \frac{3z_a^2}{R^2}\right) + 3(1-a)e^{-3z_a^2/2R^2}\right), \tag{47}$$

where $a > 1/3$, $a \neq 1$, and z_a is defined in (32). The behavior of relative entropy (47) at $a = 1/2$ is shown in Fig. 8, and the corresponding approximation is shown in Fig. 9.

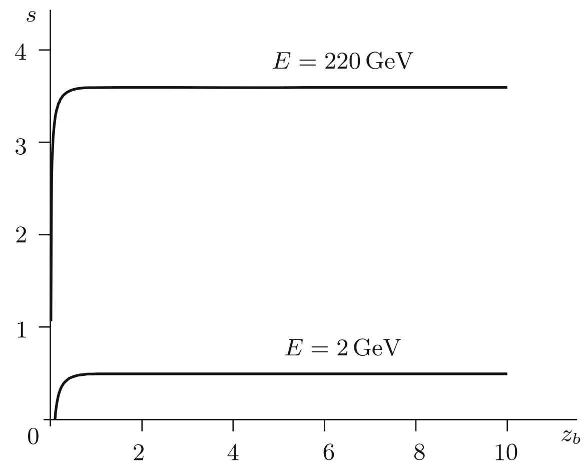

Fig. 7. The dependence of the relative entropy on z_b at $a = 1/2$ and the energies $2\,\mathrm{GeV}$ and $220\,\mathrm{GeV}$.

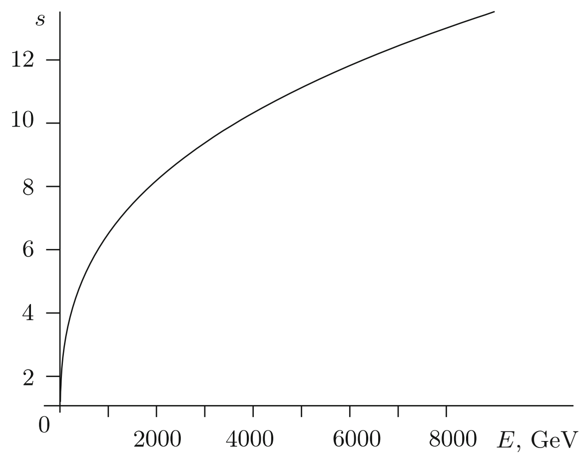

Fig. 8. The dependence of the maximum relative entropy on energy at $a = 1/2$.

In the case $a = 1/3$, the relative entropy can be represented as

$$s = \frac{L}{4G_5}\left(\mathrm{Ei}\left(1, \frac{3z_a^2}{R^2}\right) - \mathrm{Ei}\left(1, \frac{3z_b^2}{R^2}\right)\right), \tag{48}$$

and

$$s \underset{z_b\to\infty}{\longrightarrow} \frac{L}{4G_5}\mathrm{Ei}\left(1, \frac{3z_a^2}{R^2}\right) = \frac{L}{4G_5}\mathrm{Ei}\left(1, \frac{1}{2}\mathrm{W}\left(\frac{6L^2}{R^2}\left(\frac{L^2}{8\pi G_5 E}\right)^2\right)\right). \tag{49}$$

For $a = 1/3$, the entropy S therefore changes as

$$S \sim \mathrm{Ei}\left(1, \frac{1}{2}\mathrm{W}\left(\frac{6L^2}{R^2}\left(\frac{L^2}{8\pi G_5 E}\right)^2\right)\right),$$

where $\mathrm{Ei}(1, x)$ is the integral exponent.

5. Conclusion

In the cases that well described experiment ($a < 1$), we found that the scalar field is a phantom in a space with a power-law b-factor and is a phantom in the interval $z < z_0$ and a dilaton in the interval

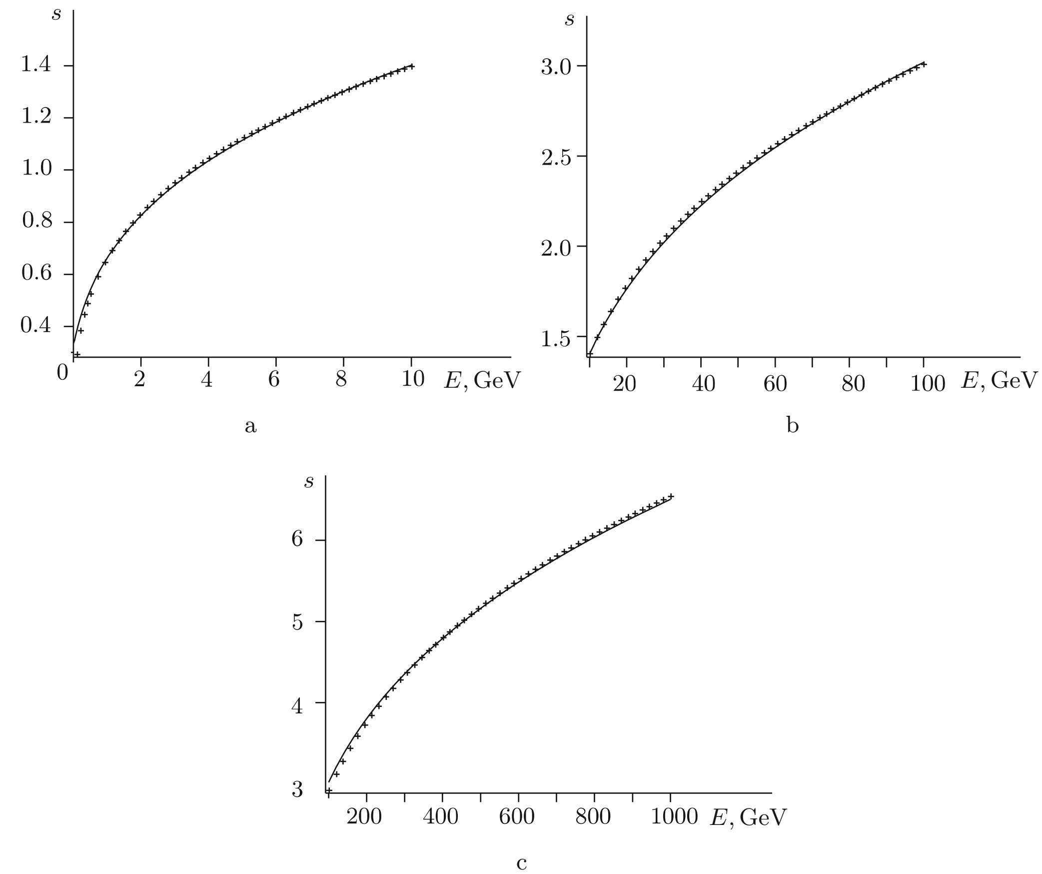

Fig. 9. The dependence of the maximum relative entropy on energy (solid line) and its approximation (crosses) at the parameter value $a = 1/2$: (a) approximation $(E^{0.3}(57-29.75(\log(E+100)))-7)/2G_5$ in the energy interval $0 < E < 10$ GeV, (b) approximation $(E^{0.3}(61-45.05(\log(E+100)))-24)/2G_5$ in the energy interval $10 < E < 100$ GeV, and (c) approximation $(E^{0.3}(81-5.95\log(E+100))-67)/2G_5$ in the energy interval $10^2 < E < 10^3$ GeV.

$z > z_0$ in a space with a mixed b-factor of the form $b = (L/z)^a e^{-z^2/R^2}$. We investigated the possibility of forming a trapped surface during domain collisions in a modified AdS_5 space with the modernized mixed b-factor. We analyzed the dependence of entropy on the energy of colliding heavy ions in the space with the modernized mixed b-factor using the condition for forming a trapped surface. Based on the AdS/CFT duality, our results allow modeling the dependence of the multiplicity of particle creation on the energy of the colliding heavy ions. The results can be used to compare with the experimental curves for the particle creation multiplicity in heavy-ion collisions. In the future, using the model with the explicit form of the potential taken into account, we plan to study other physical properties of quark–gluon plasma such as the spectrum, the temperature dependence of string breaking between quarks, etc. [18], [20].

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