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HOLOGRAPHIC ESTIMATION OF MULTIPLICITY AND THE COLLISION OF MEMBRANES IN MODIFIED $\mathbf{AdS_5}$ SPACES

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The quark–gluon plasma formed as a result of heavy-ion collisions is currently investigated actively both theoretically and experimentally. According to the holographic approach, forming a quark–gluon plasma in the four-dimensional world is associated with creating black holes in a five-dimensional anti-de Sitter (AdS) space. The multiplicity of particles produced in heavy-ion collisions is then determined by the entropy of the five-dimensional black hole, which is estimated by the area of the trapped surface. In this approach, we can model the dependence of the entropy on the energy of the colliding ions and thus the dependence of the multiplicity on the energy, and we can also compare the theoretical results with experimental data. To obtain a variety of model dependences on the energy, we consider the formation of black holes in modified AdS spaces, namely, in AdS spaces with different b factors. We find dynamics of the change of the trapped surface area depending on the energy for each investigated space.

Keywords: anti-de Sitter space, black hole, trapped surface, heavy-ion collision, particle creation multiplicity

1. Introduction

The AdS/CFT duality is a powerful method for studying quantum systems in situations where the ordinary perturbation theory is inapplicable [1]–[3]. The description of the quark–gluon plasma (QGP) formation in heavy-ion collisions using the idea of AdS/CFT duality has recently been actively developed [4]. The QGP formation process (thermalization) is then interpreted as a black hole formation process in an auxiliary five-dimensional anti-de Sitter (AdS_5) space. The formation of black holes in the AdS_5 space is considered both using the analysis of shock waves [5]–[13] and using the Vaidya metric (see [14]–[16] and the references therein). Such a method allows deriving the physical characteristics of the quantum four-dimensional system based on results obtained in the AdS_5 space for a classical system. In particular, the holographic estimation of the multiplicity of particle production in heavy-ion collisions and its dependence on energy are very interesting. The multiplicity is assumed to be determined by the entropy of the black hole created in the AdS_5 space. This hypothesis allows estimating the dependence of the multiplicity on the energy and comparing it with the experimental data already obtained [17].

The elementary dual models considered in [5]–[13] require modification [12] to describe the experimental data most precisely. The problem of black hole formation for modified models is rather complicated in the case of pointlike sources (see, e.g., [18]), and it is therefore interesting to use domain walls as a model of colliding ions. This approach was proposed in [9]. A set of problems associated with the infinite sizes of the domain walls then arises, but the regularization in which a finite wall size is introduced, as we showed

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in [13], allows using this method. Here, we show that using domain walls significantly simplifies the problem for modified models and then obtaining a nontrivial dependence of the multiplicity on the energy.

We consider heavy-ion collisions in a modified AdS space. The basis for the proposed modification is the introduction of b factors[1] of the power-law, exponential, and mixed types. Our main goal here is to obtain the dependence of the trapped surface area on the energy for different types of b factors.

2. Einstein equation in a dilaton field for the shock-wave metric

We consider the action of five-dimensional gravity coupled to a field and a pointlike source in the presence of a negative cosmological constant [5], [12]:

$$S_5 = S_R + S_\Phi + S_{\mathrm{st}}. \tag{1}$$

Here, S_R is the Einstein–Hilbert action with the relative cosmological constant taken into account,

$$S_R = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[R + \frac{d(d-1)}{L^2}\right]dx^5,$$

S_Φ is the dilaton action,

$$S_\Phi = -\frac{1}{16\pi G_5}\int \sqrt{-g}\left[-\frac{4}{3}(\partial\Phi_s)^2 + V(\Phi_s)\right]dx^5,$$

and S_{st} is the action of a pointlike source moving along a trajectory $x^\mu = x^\mu_*(\eta)$,

$$S_{\mathrm{st}} = \int\left[\frac{1}{2e}g_{\mu\nu}\frac{dx^\mu_*}{d\eta}\frac{dx^\nu_*}{d\eta} - \frac{e}{2}m^2\right]d\eta,$$

where $d+1 = D = 5$, m is the particle mass, η is an arbitrary world-line parameter, e^a_μ is the frame associated with the metric, $g_{\mu\nu} = e^a_\mu e_{\nu\,a}$, and e is the square root of its determinant $e = \sqrt{-g}$. The Einstein equations have the form

$$\left(R_{\mu\nu} - \frac{g_{\mu\nu}}{2}R\right) - \frac{g_{\mu\nu}}{2}\left(-\frac{4}{3}(\partial\Phi_s)^2 + V(\Phi_s)\right) - \frac{4}{3}\partial_\mu\Phi_s\,\partial_\nu\Phi_s - g_{\mu\nu}\frac{d(d-1)}{2L^2} = 8\pi G_5 J_{\mu\nu}, \tag{2}$$

where $(\partial\Phi_s)^2 = g^{\mu\nu}\partial_\mu\Phi_s\partial_\nu\Phi_s$ and the current in lightlike coordinates (x^+, x^-, x^i, x), $i = 1, 2$, is given by

$$J_{++} = \frac{E}{b^3(z)}\delta(x^1)\delta(x^2)\delta(z - z_*)\delta(x^+).$$

In what follows, we assume that the particle mass is zero, which allows treating only lightlike geodesics.

2.1. Einstein equations. We assume that the metric has the shock-wave form [18]–[22]

$$ds^2 = b^2(z)\big(dz^2 + dx^i\,dx^i - dx^+\,dx^- + \phi(z, x^1, x^2)\delta(x^+)(dx^+)^2\big), \quad i = 1, 2, \tag{3}$$

and write the nonzero components of the Einstein tensor $G_{\mu\nu} = (R_{\mu\nu} - g_{\mu\nu}R/2)$ explicitly:[2]

$$G_{++} = -\frac{\delta(x^+)}{2}\left(\partial_i^2 + \partial_z^2 + \frac{3b'}{b}\partial_z - \frac{6b''}{b}\right)\phi(z, x^1, x^2), \qquad G_{+-} = -\frac{3b''}{2b},$$

$$G_{11} = \frac{3b''}{b}, \qquad G_{22} = \frac{3b''}{b}, \qquad G_{zz} = \frac{6(b')^2}{b^2},$$

[1] The b factors are called "wrapped factors" in the English literature.

[2] We recall that the presence of the δ-function in the shock-wave ansatz does not lead to difficulties related to the nonlinear nature of the Einstein equations, because it turns out that the δ-function enters the equation linearly [23].

where $b = b(z)$ and $b' = \partial_z b$.

We first consider the $+z$ component of Eq. (2). For metric (3), it is written as

$$\partial_{x^+}\Phi_s(x^+, z)\,\partial_z\Phi_s(x^+, z) = 0.$$

We assume that $\partial_z\Phi_s(x^+, z) \neq 0$. In this case, we obtain the independence of the field Φ_s from x^+. Considering the iz and $-z$ components of the Einstein tensor equation, we obtain the independence of the field Φ_s from x^i and x^-. Consequently, the dilaton field depends only on z: $\Phi_s = \Phi_s(z)$.

We examine the remaining components of Eq. (2). For the $++$ component, we have

$$-\frac{1}{2}\left(\partial_i^2 + \partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi + 3\left(\frac{b''}{b} - \frac{2b^2}{L^2}\right)\phi + \left(\frac{2}{3}(\Phi_s')^2 - \frac{b^2}{2}V(\Phi_s)\right)\phi = \frac{8\pi G_5 E}{b^3}\delta(x^1)\delta(x^2)\delta(z - z_*). \quad (4)$$

For the $+-$, (11), and (22) components, the Einstein equations reduce to

$$\frac{3b''}{b} + \frac{2}{3}(\Phi_s')^2 - \frac{b^2}{2}V(\Phi_s) - \frac{6b^2}{L^2} = 0, \quad (5)$$

and for the zz component, they reduce to

$$\frac{6(b')^2}{b^2} - \frac{2}{3}(\Phi_s')^2 - \frac{b^2}{2}V(\Phi_s) - \frac{6b^2}{L^2} = 0. \quad (6)$$

Using (5), we write (4) in the form

$$\left(\partial_{x^1}^2 + \partial_{x^2}^2 + \partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi(z, x_\perp) = -16\pi G_5\frac{E}{b^3}\delta(x^1)\delta(x^2)\delta(z_* - z). \quad (7)$$

It is hence clear that the dilaton field does not explicitly affect the shock-wave profile resulting from the source. But the dilaton field and its potential, as follows from (5) and (6), are related to the b factor:

$$V(\Phi_s) = \frac{3}{b^2}\left(\frac{b''}{b} + \frac{2(b')^2}{b^2} - \frac{4b^2}{L^2}\right), \quad (8)$$

$$\Phi_s' = \pm\frac{3}{2}\sqrt{\left(\frac{2(b')^2}{b^2} - \frac{b''}{b}\right)}. \quad (9)$$

The system of Einstein equations for metric (3) thus reduces to Eqs. (8) and (9), defining the relation of the field and the field potential to the b factor, and to differential equation (7) for the shock-wave profile.

2.2. Equation for the domain-wall profile. Here, we consider the Einstein equations for the shock wave resulting from mass uniformly distributed over the domain wall. The shock-wave motion generated by a point mass corresponds to Eq. (7). To obtain the Einstein equations for the shock waves generated by a domain wall, we consider the mass of a pointlike source averaged over the domain wall. Such an averaging method was proposed in [9], and we considered it in [13].

2.2.1. Mass distribution over the domain wall. To derive the equations of the domain wall, we use the expression for the induced metric over the wall surface:

$$h_{\alpha\beta} = \frac{\partial x^\mu}{\partial\sigma^\alpha}\frac{\partial x^\nu}{\partial\sigma^\beta}g_{\mu\nu} = b^2\delta_{\alpha\beta}. \quad (10)$$

We integrate (7) over $x_\perp = (x_1, x_2)$. According to (10), we have

$$\int \sqrt{h}\, dx_\perp = \int b^2\, dx_\perp,$$

and hence

$$\int b^2 \left(\partial_z^2 + \partial_{x^1}^2 + \partial_{x^2}^2 + \frac{3b'}{b}\partial_z\right)\phi(z, x_\perp)\, dx_\perp = -16\pi G_5 b^2 \frac{E}{b^3}\delta(z_* - z).$$

Assuming that the derivatives of $\phi(z, x_\perp)$ with respect to the transverse variables $x_\perp$ decrease at $\pm\infty$, we obtain the equation of motion for the membrane wall:

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^{\mathrm{W}}(z) = -16\pi G_5 \frac{E}{b^3}\delta(z_* - z),$$

where

$$\phi^{\mathrm{W}}(z) = \int \phi(z, x_\perp)\, dx_\perp. \tag{11}$$

2.2.2. Mass distribution over the finite region. We can assume that the size of the moving domain is finite and average the mass over the finite surface in (7). We consider a wave profile resulting from the mass uniformly distributed over the surface perpendicular to the direction of motion. Therefore, wave profile (11) depends on the coordinate along which the motion occurs, and the equation of the wave profile becomes

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^{\omega}(z) = -16\pi G_5 \frac{E}{b^3}\delta(x_\perp)\delta(z_* - z).$$

The assumption that the domain is a disk of radius L allows transforming the equation into the form

$$\left(\partial_z^2 + \frac{3b'}{b}\partial_z\right)\phi^{\omega}(z) = -16\pi G_5 \frac{E^*}{b^3}\delta(z_* - z), \qquad \text{where } E^* = \frac{E}{L^2}.$$

This shows that the cases of the mass distribution over finite and infinite surfaces are equivalent, i.e., the profiles differ by a constant factor corresponding to the size of the finite object $\phi^{\omega}(z) = \phi^{\mathrm{W}}(z)/L^2$.

2.3. Condition for the trapped surface formation. In the case $b = L/z$, the conditions on the boundary points z_A and z_B of the trapped surface were obtained in [9], [13]:

$$(\partial_z \phi^{\omega})\big|_{z=z_A} = 2, \qquad (\partial_z \phi^{\omega})\big|_{z=z_B} = -2, \tag{12}$$

where $z_A < z_* < z_B$ is assumed. Obviously, expressions (12) lead to the condition[3] $(\partial_z \phi^{\omega})^2\big|_{\mathrm{TS}} = 4$. Because $\phi^W(z) = L^2\phi^{\omega}(z)$, the condition on the boundary of the trapped surface formed by the collision of two infinite domain walls is written as

$$(\partial_z \phi^{\mathrm{W}})^2\big|_{\mathrm{TS}} = 4L^4.$$

It is therefore obvious that the values of boundary points of a quasi-trapped surface are independent of whether the mass is distributed over a finite or infinite surface.

[3] We note that the condition $(\partial_z \phi^{\omega})^2\big|_{\mathrm{TS}} = 8$ was used in [12]. The difference in the boundary conditions is associated with the choice of the shock-wave metric in [12] in the form

$$ds^2 = b^2\{dz^2 + dx^i dx^i - 2dx^+ dx^- + \phi(z, x^1, x^2)(dx^+)^2\}.$$

3. Domain walls with b factors

In this paper, we generalize the approach proposed in [9] to spaces with b factors other than $b = L/z$. In this section, we consider three types of b factors [12]:

$$b = \left(\frac{L}{z}\right)^a, \quad a > 0,$$

$$b = \mathrm{e}^{-z/R}, \qquad R = \Lambda^{-1}_{\mathrm{QCD}} = 1\,\mathrm{fm},$$

$$b = \frac{L}{r}\,\mathrm{e}^{-z^2/R^2}.$$

3.1. Power-law b factor. We note that a power-law factor of the form $b = (L/(z-z_0))^a$ was used in [12]. If $a = 1$ and $b = L/(z-z_0)$, then the Einstein equation becomes

$$\left(\partial_z^2 - \frac{3}{z-z_0}\partial_z\right)\phi^\omega(z) = -16\pi G_5 E^* \frac{(z-z_0)^3}{L^3}\delta(z-z_*). \tag{13}$$

Because z_0 is a constant, replacing z with $z - z_0$ reduces Eq. (13) to the equation considered in [9]. For definiteness in what follows, we assume that $z_0 = 0$ in the power-law b factors.

The equation of the domain-wall profile in the space with the power-law factor $b = (L/z)^a$ is written as

$$\left(\partial_z^2 - \frac{3a}{z}\partial_z\right)\phi^\omega(z) = -16\pi G_5 \left(\frac{z}{L}\right)^{3a} E^*\delta(z-z_*). \tag{14}$$

We consider this equation separately before and after the collision. The boundary points of the quasi-trapped surface of the black hole are denoted by z_A and z_B, $z_A < z_* < z_B$. The solution of (14) is written as

$$\phi^\omega(z) = \phi^\omega_A\Theta(z_* - z) + \phi^\omega_B\Theta(z - z_*), \tag{15}$$

where

$$\phi^\omega_A(z) = C_0 z_A z_B \left(\left(\frac{z_*}{z_B}\right)^{3a+1} - 1\right)\left(\left(\frac{z}{z_A}\right)^{3a+1} - 1\right),$$

$$\phi^\omega_B(z) = C_0 z_A z_B \left(\left(\frac{z_*}{z_A}\right)^{3a+1} - 1\right)\left(\left(\frac{z}{z_B}\right)^{3a+1} - 1\right),$$

$$C_0 = -\frac{16\pi G_5 E z_A^{3a} z_B^{3a}}{(1+3a)L^{3a+2}(z_B^{3a+1} - z_A^{3a+1})}.$$

For profile (15), the formation conditions for the quasi-trapped surface at the boundary points $z = z_A$ and $z = z_B$ are

$$\begin{aligned}
\frac{8\pi G_5 E z_A^{3a}(1 - z_B^{3a+1}/z_*{}^{3a+1})}{L^{3a+2}(z_B^{3a+1}/z_*{}^{3a+1} - z_A^{3a+1}/z_*{}^{3a+1})} &= -1,\\
\frac{8\pi G_5 E z_B^{3a}(1 - z_A^{3a+1}/z_*{}^{3a+1})}{L^{3a+2}(z_B^{3a+1}/z_*{}^{3a+1} - z_A^{3a+1}/z_*{}^{3a+1})} &= 1.
\end{aligned} \tag{16}$$

The collision point z_* can be not fixed but found from the system regarded as a system of equations for z_* and z_A with a given z_B:

$$z_A = \left(\frac{z_B^{3a}}{-1 + z_B^{3a}C^2}\right)^{1/3a}, \qquad z_* = \left(\frac{z_A^{3a} z_B^{3a}(z_B + z_A)}{z_A^{3a} + z_B^{3a}}\right)^{1/(3a+1)}, \tag{17}$$

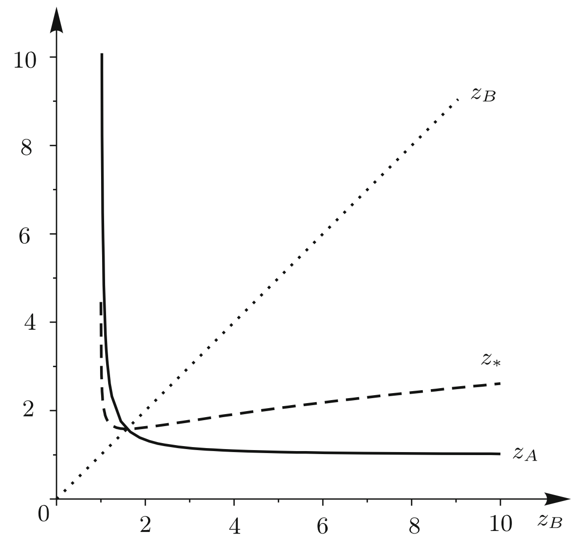

Fig. 1. The solution of system of equations (16) for a given z_B: the unit of length in Figs. 1–4 is the femptometer.

where $C^2 = 8\pi G_5 E/L^{3a+2}$. The solution of this system is shown in Fig. 1.

For $z_B^{3a} C^2 \gg 1$, we consider $z_A \ll z_* \ll z_B$. Based on (17), we have the approximation

$$z_A \sim \left(\frac{1}{C^2}\right)^{1/3a}, \qquad z_* \sim \left(\frac{z_B}{C^2}\right)^{1/(3a+1)}.$$

The trapped surface area is calculated as

$$S_{\text{trap}} = \frac{1}{2G_5} \int_C \sqrt{\det |g_{\text{AdS}_3}|}\, dz\, d^2 x_\perp,$$

where $\det |g_{\text{AdS}_3}|$ is the metric determinant of the three-dimensional AdS_3 space. In what follows, we calculate the relative area s of the quasi-trapped surface defined by

$$s = \frac{S_{\text{trap}}}{\int d^2 x_\perp} = \frac{1}{2G_5} \int_{z_A}^{z_B} b^3\, dz.$$

In the considered case where $b(z) = (L/z)^a$, the formula for the relative area of the quasi-trapped surface becomes

$$s = \frac{1}{2G_5(3a-1)} \left(z_A \left(\frac{L}{z_A}\right)^{3a} - z_B \left(\frac{L}{z_B}\right)^{3a} \right),$$

and s determines the relative entropy. With the assumption $3a > 1$ and the used approximation, it is clear from this expression that the relative area of the trapped surface tends to its maximum value at infinite z_B:

$$s|_{z_B \to \infty} = \frac{L^{3a}}{2G_5(3a-1)} z_A^{1-3a} = \frac{L}{2G_5} \left(\frac{8\pi G_5}{L^2}\right)^{(3a-1)/3a} E^{(3a-1)/3a}. \tag{18}$$

We thus find that for $a > 1/3$, the entropy S increases as $E^{(3a-1)/3a}$.

We substitute parameters and variables with the dimension of length in formula (18) using the relation $1\,\text{GeV} = 5\,\text{fm}^{-1}$ and choose the parameters G_5 and L based on phenomenological reasons [5]: $G_5 = L^3/1.9$

and $L = 4.4\,\mathrm{fm}$. We here assume that we consider collisions of lead ions. The multiplicity of particles produced in heavy-ion collisions (PbPb and AuAu collisions) depends on energy as $s_{NN}^{0.15}$ according to the experimental data [17] in the range from 10 to 10^3 GeV. Therefore, $a \approx 0.47$. For $a = 0.47$, we have

$$\frac{L}{2G_5}\left(\frac{8\pi G_5}{L^2}\right)^{(3a-1)/3a} \approx 0.16\,\mathrm{fm}^{-1.85}.$$

3.2. Factor of the form $b = \mathrm{e}^{-z/R}$. The equation of the domain-wall wave profile in a space with an exponential b factor $b = \mathrm{e}^{-z/R}$ is written as

$$\left(\partial_z^2 - \frac{3}{R}\partial_z\right)\phi^\omega(z) = -16\pi G_5 E^* \mathrm{e}^{3z/R}\delta(z - z_*),$$

and we construct the solution in the form (in what follows, we write the subscripts a and b instead of A and B in the notation for the boundary points and other variables)

$$\phi^\omega(z) = \phi_a(z)\Theta(z_* - z) + \phi_b(z)\Theta(z - z_*),$$

where

$$\phi_a = C_a\frac{R}{3}\mathrm{e}^{3z/R} + \widetilde{C}_a, \qquad \phi_b = C_b\frac{R}{3}\mathrm{e}^{3z/R} + \widetilde{C}_b,$$

$$C_a = -\frac{16\pi G_5 E^*\left(\mathrm{e}^{3z^*/R} - \mathrm{e}^{3z_b/R}\right)}{\mathrm{e}^{3z_b/R} - \mathrm{e}^{3z_a/R}}, \qquad C_b = -\frac{16\pi G_5 E^*\left(\mathrm{e}^{3z^*/R} - \mathrm{e}^{3z_a/R}\right)}{\mathrm{e}^{3z_b/R} - \mathrm{e}^{3z_a/R}},$$

$$\widetilde{C}_a = -C_a\frac{R}{3}\mathrm{e}^{3z_a/R}, \qquad \widetilde{C}_b = -C_b\frac{R}{3}\mathrm{e}^{3z_b/R}.$$

The conditions at the boundaries of the trapped surface are

$$\begin{gathered}\frac{8\pi G_5 E}{L^2}\frac{(\mathrm{e}^{3z^*/R} - \mathrm{e}^{3z_b/R})\mathrm{e}^{3z_a/R}}{\mathrm{e}^{3z_b/R} - \mathrm{e}^{3z_a/R}} = -1, \\ \frac{8\pi G_5 E}{L^2}\frac{(\mathrm{e}^{3z^*/R} - \mathrm{e}^{3z_a/R})\mathrm{e}^{3z_b/R}}{\mathrm{e}^{3z_b/R} - \mathrm{e}^{3z_a/R}} = 1.\end{gathered} \tag{19}$$

We analyze these conditions. We set $Z_0 = \mathrm{e}^{3z^*/R}$, $Z_a = \mathrm{e}^{3z_a/R}$, and $Z_b = \mathrm{e}^{3z_b/R}$ and substitute these values in conditions (19). We obtain the equations

$$\frac{8\pi G_5 E}{L^2}\frac{(Z_0 - Z_b)Z_a}{Z_b - Z_a} = -1, \qquad \frac{8\pi G_5 E}{L^2}\frac{(Z_0 - Z_a)Z_b}{Z_b - Z_a} = 1.$$

As in the previous case, we consider the equations for Z_a and Z_0 with a fixed Z_b. This system has the trivial solution $Z_a = Z_b = Z_0$ and the solution

$$Z_a = \frac{L^2}{16\pi G_5 E}\frac{Z_b}{Z_b - L^2/16\pi G_5 E}, \qquad Z_0 = \frac{L^2}{8\pi G_5 E}. \tag{20}$$

This solution is shown in Fig. 2 at two values of the parameter Z_0:

$$\frac{L^2}{16\pi G_5 E} = 1, \qquad \frac{L^2}{16\pi G_5 E} = \frac{1}{2}.$$

The relative area of the quasi-trapped surface is given by

$$s = \frac{3}{2RG_5}\left(\frac{1}{\mathrm{e}^{3z_a/R}} - \frac{1}{\mathrm{e}^{3z_b/R}}\right) = \frac{3}{2RG_5}\left(\frac{1}{Z_a} - \frac{1}{Z_b}\right). \tag{21}$$

The maximum entropy is attained for $Z_b \gg 1$. In this approximation,

$$Z_a \sim \frac{L^2}{16\pi G_5 E}, \qquad s \sim \frac{24\pi E}{RL^2}, \tag{22}$$

which gives a linear dependence of the entropy on the energy.

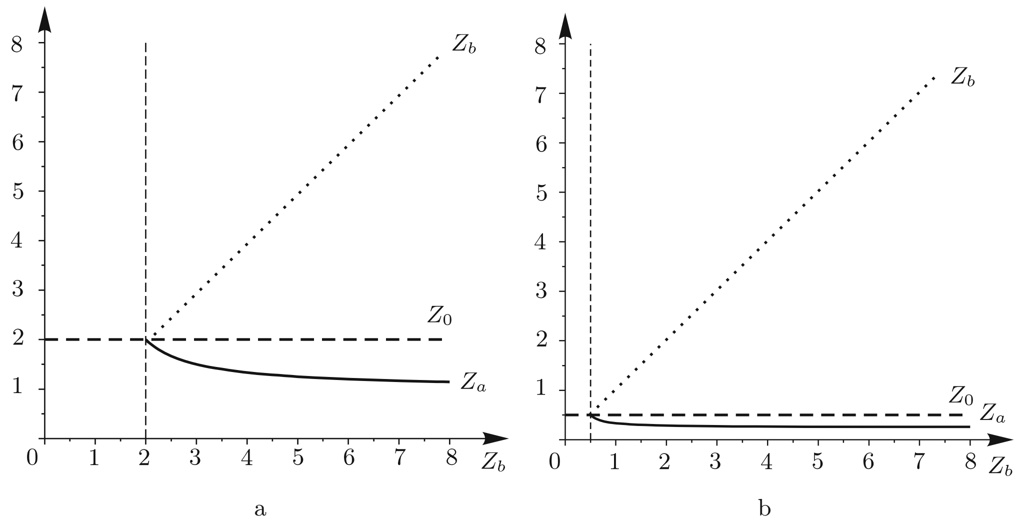

Fig. 2. Solution (20) at (a) $Z_0 = 2$ and (b) $Z_0 = 1$: the vertical dashed line separates the region where $Z_b > Z_0$.

3.3. Mixed factor of the form $b = (L/z)e^{-z^2/R^2}$. The equation of the profile corresponding to domain-wall motion in a space with a b factor of the form $b = (L/z)e^{-z^2/R^2}$ is written as

$$\left(\partial_z^2 - 3\left(\frac{1}{z} + \frac{2z}{R^2}\right)\partial_z\right)\phi^\omega = -16\pi G_5 E^* \left(\frac{z}{L}\right)^3 e^{3z^2/R^2}\delta(z - z_*).$$

We consider the solution of the obtained equation:

$$\phi^\omega = \phi_a^\omega \Theta(z_* - z) + \phi_b^\omega \Theta(z - z_*), \tag{23}$$

where

$$\phi_a^\omega = -C_a(R^2 - 3z_a^2)e^{3z_a^2/R^2} + C_a(R^2 - 3z^2)e^{3z^2/R^2},$$

$$\phi_b^\omega = -C_b(R^2 - 3z_b^2)e^{3z_b^2/R^2} + C_b(R^2 - 3z^2)e^{3z^2/R^2},$$

and the factors C_a and C_b are given by

$$\begin{aligned} C_a &= C_0\big(-(R^2 - 3z_b^2)e^{3z_b^2/R^2} + (R^2 - 3z_*^2)e^{3z_*^2/R^2}\big), \\ C_b &= C_0\big(-(R^2 - 3z_a^2)e^{3z_a^2/R^2} + (R^2 - 3z_*^2)e^{3z_*^2/R^2}\big), \end{aligned} \tag{24}$$

where

$$C_0 = \frac{8\pi G_5 E^* R^2}{9L^3\big((R^2 - 3z_b^2)e^{3z_b^2/R^2} - (R^2 - 3z_a^2)e^{3z_a^2/R^2}\big)}. \tag{25}$$

We differentiate each of the two terms in the right-hand sides of solution (23):

$$\frac{d\phi_a^\omega(z)}{dz} = -C_a \frac{18z^3}{R^2} e^{3z^2/R^2}, \qquad \frac{d\phi_b^\omega(z)}{dz} = -C_b \frac{18z^3}{R^2} e^{3z^2/R^2}.$$

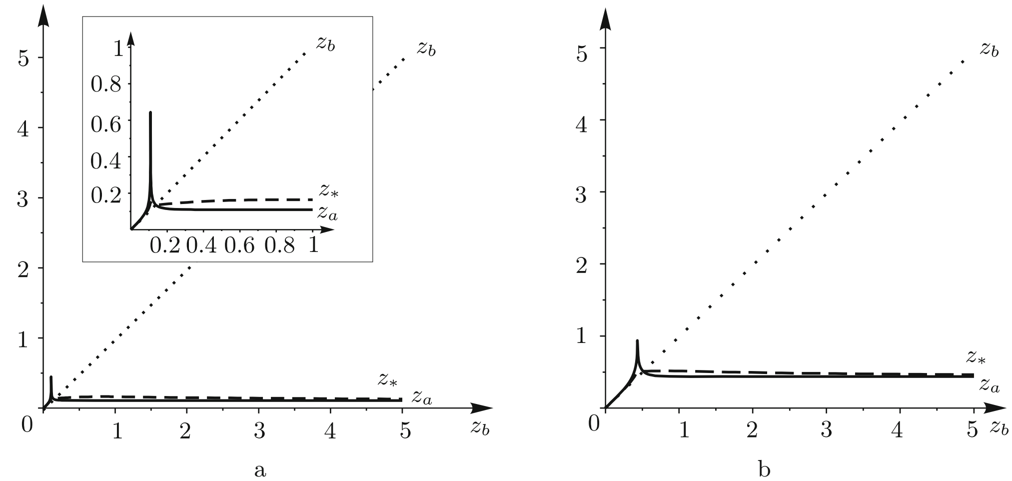

Fig. 3. The dependence of z_a and z_* on z_b at energies (a) $E = 220\,\mathrm{GeV}$ and (b) $E = 2\,\mathrm{GeV}$: the region of the plot corresponding to $z_b < 1$ is shown at $E = 220\,\mathrm{GeV}$ in the inset.

Hence, the conditions for forming the quasi-trapped surface at the boundary points become

$$C_a \frac{9z_a^3}{R^2} e^{3z_a^2/R^2} = -1, \qquad C_b \frac{9z_b^3}{R^2} e^{3z_b^2/R^2} = 1. \tag{26}$$

Considering this system with C_a and C_b defined by formulas (24) and (25), we obtain a system of two equations for the three unknowns z_a, z_b, and z^*. We assume that z_a and z^* are unknown and z_b is given.

From system of equations (26), we can obtain the relations

$$z_a^3 e^{3z_a^2/R^2} = \frac{L^3 z_b^3 e^{3z_b^2/R^2}}{8\pi G_5 E^* z_b^3 e^{3z_b^2/R^2} - L^3},$$

$$(R^2 - 3z_*^2) e^{3z_*^2/R^2} = \frac{(z_a^3 R^2 - 3z_a^3 z_b^2 - 3z_b^3 z_a^2 + z_b^3 R^2) e^{3z_a^2/R^2} e^{3z_b^2/R^2}}{z_a^3 e^{3z_a^2/R^2} + z_b^3 e^{3z_b^2/R^2}}.$$

We consider only energies satisfying $8\pi G_5 E^* z_b^3 e^{3z_b^2/R^2} > L^3$. For the further analysis of the quasi-trapped surface formation, we must obtain the solution of system (26) in an explicit form. A nontrivial solution of the system (we recall that z_a, z^*, and z_b are positive) has the form

$$z_a = \frac{R}{\sqrt{2}} \sqrt{W_{\mathrm{A}}},$$
$$z_* = \frac{R}{\sqrt{3}} \sqrt{1 + W\left(\frac{-(z_a^3 R^2 - 3z_a^3 z_b^2 - 3z_b^3 z_a^2 + z_b^3 R^2) e^{3(z_a^2 + z_b^2)/R^2}}{R^2 (z_a^3 e^{3z_a^2/R^2} + z_b^3 e^{3z_b^2/R^2}) e}\right)}, \tag{27}$$

where

$$W_{\mathrm{A}} = W\left(2\left(\frac{(L^3 z_b^3/R^3) e^{3z_b^2/R^2}}{8\pi G_5 E^* z_b^3 e^{3z_b^2/R^2} - L^3}\right)^{2/3}\right) \tag{28}$$

and $W(z)$ is Lambert W-function.

The dependences of z_a and z_* on z_b defined by formulas (27) and (28) are shown in Figs. 3a and 3b at the respective energy values $E = 220\,\mathrm{GeV}$ and $E = 2\,\mathrm{GeV}$. We see that z_a tends to its the lowest value at

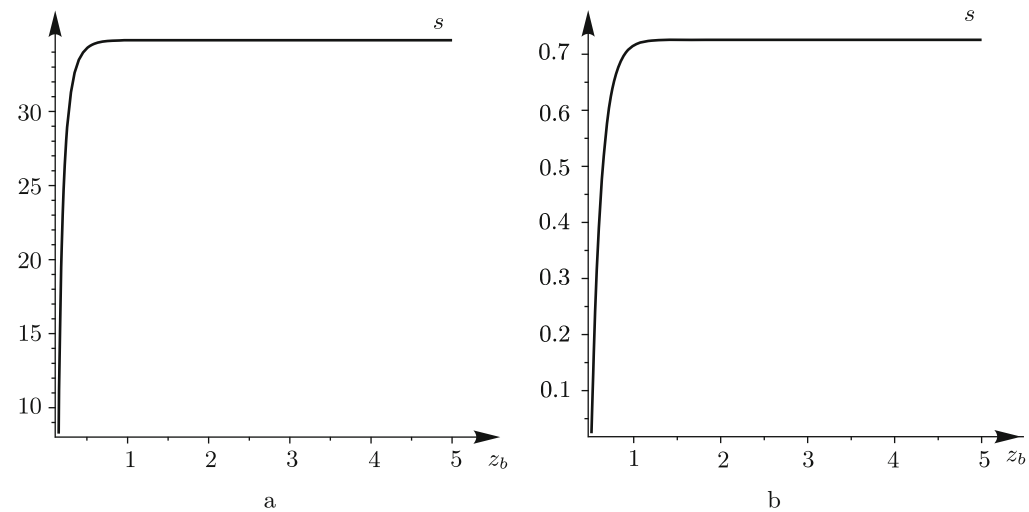

Fig. 4. The dependence of s on z_b at (a) $E = 220\,\mathrm{GeV}$ and (b) $E = 2\,\mathrm{GeV}$.

infinitely large z_b. In this limit, z_a and z_* are given by

$$z_a|_{z_b\to\infty} = \frac{R}{\sqrt{2}}\sqrt{W_{\mathrm{AM}}}, \qquad z_*|_{z_b\to\infty} = \frac{R}{\sqrt{3}}\sqrt{1 + W\left(\frac{(3z_a^2 - R^2)\mathrm{e}^{3z_a^2/R^2}}{\mathrm{e}R^2}\right)}, \tag{29}$$

where

$$W_{\mathrm{AM}} = W\left(\frac{L^2}{2(\pi G_5 E^*)^{2/3} R^2}\right).$$

In general, the relative area of the trapped surface depends on the energy and on z_b as

$$s = \frac{L^3}{2G_5}\left(-\frac{1}{2z_b^2\mathrm{e}^{3z_b^2/R^2}} + \frac{1}{2z_a^2\mathrm{e}^{3z_a^2/R^2}} + \frac{3\,\mathrm{Ei}(1, 3z_b^2/R^2)}{2R^2} - \frac{3\,\mathrm{Ei}(1, 3z_a^2/R^2)}{2R^2}\right),$$

where z_a depends on z_b according to formulas (27) and (28) and $\mathrm{Ei}(1, x)$ is the exponential integral. The dependence of the relative area of the trapped surface is shown at the energies $E = 220\,\mathrm{GeV}$ and $E = 2\,\mathrm{GeV}$ in Fig. 4. It can be seen that the maximum value of the trapped surface area s is attained at infinite z_b:

$$s|_{z_b\to\infty} = \frac{3}{4}\frac{L^3}{G_5 R^2}\left(-\,\mathrm{Ei}\left(1, \frac{3z_a^2}{R^2}\right) + \frac{1}{3}\frac{R^2}{z_a^2\mathrm{e}^{3z_a^2/R^2}}\right),$$

where z_a is given by (29). We note that the expression in the parentheses is always positive.

Figure 5 shows the dependence of the relative area of the trapped surface on the energy (at low energies in Fig. 5a and high energies in Fig. 5b). Figure 5b also shows the function $E^{2/3}(1 + 0.007\log E) - 3$ approximating the obtained dependence at $10\,\mathrm{GeV} \lesssim E < 1\,\mathrm{TeV}$.

4. Conclusion

We have investigated the possibility of black hole formation in the domain-wall collisions in AdS_5 spaces with b factors. We considered several types of b factors: power-law, exponential, and mixed. We analyzed the dependence of entropy on the energy of colliding ions in the spaces with b factors based on the analysis of the conditions for forming the trapped surfaces. With the AdS/CFT duality taken into account, the obtained results allow modeling the dependence of the multiplicity of produced particles on the energy

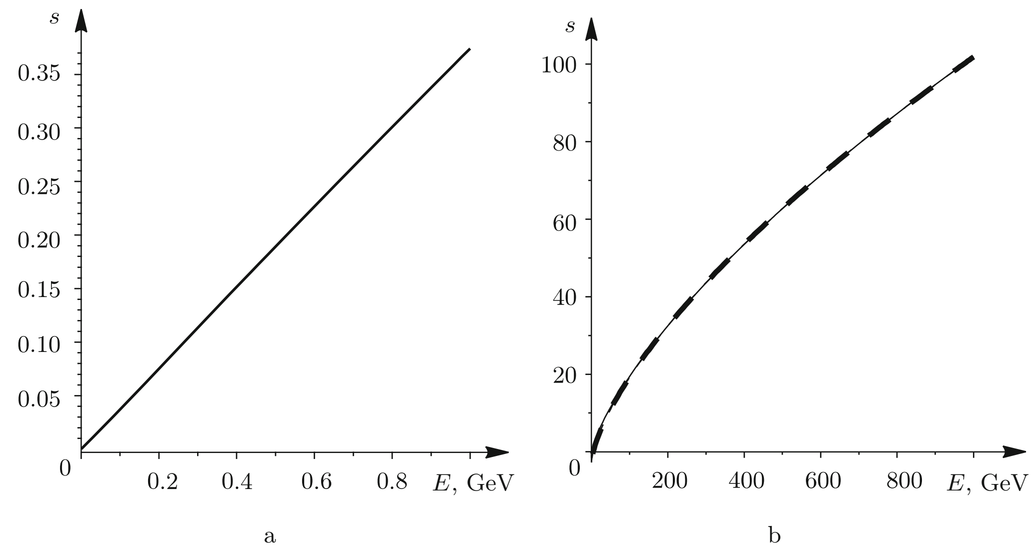

Fig. 5. The dependence of the relative area of the trapped surface on the energy at (a) low energies and (b) high energies: the function approximating the calculated dependence $E^{2/3}(1+0.007\log E)-3$ is shown with bold dashes.

of the colliding heavy ions. We note that the results for the power-law factors agree with the conclusions in the previously examined cases of central collisions of pointlike sources. The exponential factors of the collision domains do not lead to additional logarithms, which arise in the case of central collisions of pointlike sources [12] in the presence of exponential b factors; nevertheless, additional logarithms appear when the mixed factor is considered. The derived results can be used to compare with the experimental curves for the multiplicity of particle formation in heavy-ion collisions.

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