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CLASSICAL EUCLIDEAN WORMHOLE SOLUTIONS IN THE PALATINI $f(\widetilde{R})$ COSMOLOGY

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We study the classical Euclidean wormholes in the context of extended theories of gravity. Without loss of generality, we use the dynamical equivalence between $f(\widetilde{R})$ gravity and scalar–tensor theories to construct a pointlike Lagrangian in the flat Friedmann–Robertson–Walker space–time. We first show the dynamical equivalence between the Palatini $f(\widetilde{R})$ gravity and the Brans–Dicke theory with a self-interaction potential and then show the dynamical equivalence between the Brans–Dicke theory with a self-interaction potential and the minimally coupled O'Hanlon theory. We show the existence of new Euclidean wormhole solutions for this O'Hanlon theory; in a special case, we find the corresponding form of $f(\widetilde{R})$ that has a wormhole solution. For small values of the Ricci scalar, this $f(\widetilde{R})$ agrees with the wormhole solution obtained for the higher-order gravity theory $\widetilde{R}+\epsilon\widetilde{R}^2$, $\epsilon<0$.

Keywords: Euclidean wormhole, $f(R)$ cosmology, scalar–tensor theory

1. Introduction

There are two kinds of classical wormholes: Lorentzian and Euclidean. Lorentzian wormholes are known as the vacuum solutions of the Lorentzian Einstein field equations, such as Schwarzschild wormholes or Einstein–Rosen bridges. These are *real bridges* between different areas of space–time. According to Visser's definition of Lorentzian wormholes [1], if a Minkowski space–time contains a compact region Ω, if the topology of Ω is of the form $\Omega \sim R \times \Sigma$, where Σ is a three-manifold of a nontrivial topology whose boundary has a topology of the form $d\Sigma \sim S^2$, and, furthermore, if the hypersurfaces Σ are all spacelike, then the region Ω contains a quasipermanent intrauniverse wormhole.

On the other hand, Euclidean wormholes have been studied mainly as instantons, namely, solutions of the classical Euclidean Einstein field equations. Euclidean wormholes are usually regarded as Euclidean metrics that consist of two asymptotically flat regions connected by a narrow throat (handle). In general, such wormholes can represent *quantum tunneling* between different areas of space–time having generally different topologies. They are possibly useful for understanding black hole evaporation [2], for allowing nonlocal connections that could determine fundamental constants, and for zeroing the cosmological constant Λ [3]–[5]. They are even considered as an alternative to the Higgs mechanism [6]. Consequently, such solutions are particularly important in studying quantum aspects of gravity. The reason classical wormholes may exist is related to an implication of a theorem of Cheeger and Glommol [7] stating that a *necessary* (but not sufficient) condition for a classical wormhole to exist is that the eigenvalues of the Ricci tensor be negative *somewhere* on the manifold [8].

Lorentzian wormholes have been extensively studied in modified theories of gravity, such as Brans–Dicke gravity [9]–[11], modified teleparallel gravity [12], and $f(R)$ gravity [9]. But studying Euclidean

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wormholes in modified gravities has not been received much attention, so far as we know. Deciding to fill this knowledge gap, we study Euclidean wormholes in $f(R)$ gravity.

There are two formalisms for studying $f(R)$ gravity: metric $f(R)$ gravity and Palatini $f(R)$ gravity [13]–[16]. The first is the standard metric formalism, in which the field equations are derived by varying the action with respect to the metric tensor. The second is the Palatini formalism, in which the metric and connection are treated as independent variables in the action variation. These two formalisms generally yield different field equations for a nonlinear action, but it is well known that the solutions of Palatini $f(R)$ gravity represent a subclass of solutions of metric $f(R)$ gravity [17]–[19]. In fact, each Palatini model of gravity has its purely metric counterpart leading to fourth-order field equations.

We recently considered four types of actions: metric Jordan, Palatini–Jordan, metric Einstein, and Palatini–Einstein [20]. The symmetry between Jordan and Einstein frames is a conformal symmetry that corresponds to kinetic terms appearing or vanishing. On the other hand, the transition from the metric Jordan action to the Palatini–Jordan action requires that the kinetic term appear, while the transition from the metric Einstein action to the Palatini–Einstein action requires that the kinetic term vanish. In both transitions from metric Jordan to metric Einstein and to Palatini–Jordan actions, the kinetic term appears; in both transitions from metric Einstein and from Palatini–Jordan to Palatini–Einstein actions, the kinetic term vanishes.

The Jordan and Einstein frames are dynamically equivalent from the conformal symmetry standpoint. Although the metric and Palatini formalisms are also connected through a conformal transformation, they do not seem to be dynamically equivalent. The metric Jordan action differs from the Palatini–Jordan action by a dynamical *advanced* kinetic term. Similarly, the metric Einstein action differs from the Palatini–Einstein action by a dynamical *retarded* kinetic term. But the Palatini–Jordan action when reduced to the Palatini–Einstein action takes the same form as the metric Jordan action, namely, it becomes an O'Hanlon-type action, where the dynamics is completely determined by the self-interaction potential. On the other hand, the metric Einstein and Palatini–Jordan actions represent the same dynamical features in that both have a dynamical kinetic term plus a potential.

In summary, for each map between the Jordan and Einstein frames, there exists a corresponding map between the Palatini and metric formalisms. Similarly, for each map between the two O'Hanlon-type actions, namely, the metric Jordan and Palatini–Einstein actions, there exists a corresponding map between the Palatini–Jordan and metric Einstein actions. Therefore, the apparent differences between the Palatini and metric formalisms strictly depend on the representation, while the number of degrees of freedom is preserved. This means that the dynamical contents of the two formalisms are identical.

In fact, the Palatini and metric formalisms become nonequivalent on-shell in the presence of matter [19]. But an advantage of the Palatini formulation is based on the second-order field equations, which are easier to solve. In this sense, the Palatini formalism is easier to handle and simpler to analyze than the corresponding metric formalism [19]. Moreover, if we regard the Einstein frame as the physical frame, then the Palatini formalism is more convenient than the metric formalism in the present study of Euclidean wormholes because unlike the metric formalism, whose action becomes the scalar–tensor theory in the Jordan frame, the action in the Palatini formalism can be cast in the form of the Einstein frame. Based on the above discussion, we study the Euclidean wormholes in the Palatini $f(R)$ gravity.

Our approach here is similar in spirit to the approach used in [21]–[26]. We use the dynamical equivalence between the Palatini $f(\widetilde{R})$ gravity [27] and scalar–tensor theories to construct a pointlike Lagrangian in the flat Friedmann–Robertson–Walker (FRW) universe [28], [29]. In doing so, we briefly review the well-known dynamical equivalence between the Palatini $f(\widetilde{R})$ gravity and the Brans–Dicke theory with a self-interaction potential [30]–[32]. We then study the well-known dynamical equivalence between the Brans–Dicke theory with a self-interaction potential and the minimally coupled O'Hanlon theory, where the dynamics is completely determined by the self-interaction potential. Finally, we show the existence of new Euclidean wormhole solutions for this O'Hanlon theory and obtain the possible corresponding forms of $f(\widetilde{R})$ in the Palatini formalism.

2. Dynamical equivalence between the Palatini $f(\widetilde{R})$ gravity and minimally coupled O'Hanlon theory

The action in the Palatini $f(\widetilde{R})$ gravity formalism has the form

$$S = \frac{1}{2k}\int d^4x\,\sqrt{-g}f(\widetilde{R}), \tag{1}$$

where $f(\widetilde{R})$ is a function of $\widetilde{R} = g^{\mu\nu}R_{\mu\nu}(\widetilde{\Gamma})$ and $\widetilde{\Gamma}^{\lambda}_{\mu\nu}$ is the connection. This action depends on two dynamical variables, namely, the metric and connection. Variation of Eq. (1) with respect to the metric leads to

$$f'(\widetilde{R})\widetilde{R} - \frac{1}{2}f(\widetilde{R})g_{\mu\nu} = 0, \tag{2}$$

where $f'(\widetilde{R}) = df/d\widetilde{R}$. The trace of Eq. (2) is

$$f'(\widetilde{R})\widetilde{R} - 2f(\widetilde{R}) = 0, \tag{3}$$

and the variation of Eq. (1) with respect to the connection gives

$$(\sqrt{-g}f'(\widetilde{R})g^{\mu\nu})_{;\lambda} = 0, \tag{4}$$

where $(\,\cdot\,)_;$ denotes the covariant derivative. Therefore, the connection is compatible with the new metric $h_{\mu\nu} = f'(\widetilde{R})g_{\mu\nu}$, and we obtain

$$\widetilde{R} = R + \frac{3}{2f'(\widetilde{R})}\partial_\lambda f'(\widetilde{R})\partial^\lambda f'(\widetilde{R}) - \frac{3}{f'(\widetilde{R})}\Box f'(\widetilde{R}), \tag{5}$$

where R is a Ricci scalar constructed from the Levi-Civita connection $g_{\mu\nu}$. It can be easily verified that action (1) is dynamically equivalent to [28][1]

$$S = \frac{1}{2k}\int d^4x\,\sqrt{-g}\left(\Phi R + \frac{3}{2\Phi}\Phi_{;\mu}\Phi^{;\mu} - V(\Phi)\right), \tag{6}$$

where $\Phi = f'(\widetilde{R})$, $V(\Phi) = \chi(\Phi)\Phi - f(\chi(\Phi))$, and $\widetilde{R} = \chi(\Phi)$. This is the well-known action of the Brans–Dicke theory with the coupling parameter equal to $-3/2$. With the redefinition $\Phi \equiv \varphi^2$, action (6) becomes

$$S = \frac{1}{2k}\int d^4x\,\sqrt{-g}(\varphi^2 R + 6\varphi_{;\mu}\varphi^{;\mu} - V(\varphi)). \tag{7}$$

This action is dynamically equivalent to

$$S = \frac{1}{2k}\int d^4x\,\sqrt{-g}\left(F(\varphi)R + \frac{1}{2}\varphi_{;\mu}\varphi^{;\mu} - U(\varphi)\right), \tag{8}$$

[1]Using a general theory with a divergence-free current, we can also demonstrate the equivalence between action (6) in the Palatini formalism and the action

$$S = \frac{1}{2k}\int d^4x\,\sqrt{-g}(\Phi R - V(\Phi))$$

in the metric formalism of $f(R)$ gravity [32]. In fact, considering a simple divergence theory and a suitably defined current in terms of Φ, we can generalize the conformal equivalence between the metric and Palatini formalisms by an equation for the conservation of this current.

where $F(\varphi) = \varphi^2/12$ and $U(\varphi) = V(\varphi)/12$. A conformal transformation of the type [30], [31]

$$\bar{g}_{\mu\nu} = e^{2\sigma} g_{\mu\nu} \tag{9}$$

results in the Lagrangian density in the Einstein frame

$$\sqrt{-g}\left(FR + \frac{1}{2}\varphi_{;\mu}\varphi^{;\mu} - U\right) = \sqrt{-\bar{g}}\left(\frac{1}{2}\overline{R} + 3\Box_{\overline{\Gamma}}\sigma + \frac{3F_\varphi^2 - F}{4F^2}\varphi_{;\alpha}\varphi_{;}^{\alpha} - \frac{U}{4F^2}\right). \tag{10}$$

Introducing a new scalar field $\bar{\varphi}$ and the potential $\overline{U}$ defined by

$$\bar{\varphi}_{;\alpha} = \sqrt{\frac{3F_\varphi^2 - F}{4F^2}}\varphi_{;\alpha}, \qquad \overline{U}(\bar{\varphi}(\varphi)) = \frac{U}{4F^2}, \tag{11}$$

we obtain [30]

$$\sqrt{-g}\left(FR + \frac{1}{2}g^{\mu\nu}\varphi_{;\mu}\varphi^{;\mu} - U\right) = \sqrt{-\bar{g}}\left(\frac{1}{2}\overline{R} + \frac{1}{2}\bar{\varphi}_{;\alpha}\bar{\varphi}_{;}^{\alpha} - \overline{U}\right). \tag{12}$$

If we set $F(\varphi) = \varphi^2/12$ in the first definition in (11), then we obtain $\bar{\varphi}_{;\alpha} = 0$, which leads to the action

$$\mathcal{S} = \int d^4x \sqrt{-\bar{g}}\left(\frac{1}{2}\overline{R} - \overline{U}\right), \tag{13}$$

which is known as the O'Hanlon action in the Einstein frame, where the dynamics is completely determined by the self-interaction potential [33].

3. Classical Euclidean wormholes in O'Hanlon theory

In this section, we seek wormhole solutions in system (13) of the minimally coupled scalar field with the Lagrangian density

$$\mathcal{L} = \frac{1}{2}\overline{R} - \overline{U}, \tag{14}$$

where $U(\bar{\varphi})$ is a self-interaction potential. We do not specify the form of the potential in advance. By analyzing the field equations and the corresponding wormhole solutions, we can use the conformal equivalence discussed in the preceding section to go in the opposite direction and obtain the corresponding wormhole solutions in Palatini $f(R)$ gravity.

The Einstein equations of motion are obtained as

$$\overline{R}_{\mu\nu} = \overline{T}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\overline{T}, \tag{15}$$

where the energy–momentum tensor and its trace are given by

$$\overline{T}_{\mu\nu} = \bar{g}_{\mu\nu}\overline{U}(\bar{\varphi}), \qquad \overline{T} = 4\overline{U}(\bar{\varphi}). \tag{16}$$

Substituting (16) in Eq. (15), we obtain

$$\overline{R}_{\mu\nu} = -\bar{g}_{\mu\nu}\overline{U}(\bar{\varphi}). \tag{17}$$

It can be seen that for a positive-definite Euclidean metric $\bar{g}_{\mu\nu}$, the Ricci tensor $\overline{R}_{\mu\nu}$ has negative eigenvalues if and only if the potential $\overline{U}(\bar{\varphi})$ is positive. Consequently, wormhole solutions can exist in this system if and only if the two conditions

$$\overline{U}(\bar{\varphi}) > 0, \qquad -\bar{g}_{\mu\nu}\overline{U}(\bar{\varphi}) < 0 \tag{18}$$

are satisfied. For the flat FRW universe, the Euclidean metric $\bar{g}_{\mu\nu}$ is written as

$$dS^2 = dt^2 + a^2(t)\, d^2\Omega_3, \tag{19}$$

where $d^2\Omega_3$ is the line element on the three-sphere. The Euclidean field equation for a is obtained as

$$\dot{a}^2 = 1 - a^2\overline{U}(\bar{\varphi}), \tag{20}$$

where the dot marks the derivative with respect to time. We now seek wormhole solutions of Eq. (20). It is generally believed that a wormhole has two asymptotically flat regions connected by a throat at which $\dot{a} = 0$ and is described by an expression of the form

$$\dot{a}^2 = 1 - \frac{C}{a^n}, \tag{21}$$

where C is a positive constant. To have an asymptotic Euclidean wormhole, $\dot{a}^2$ must remain positive at large a, and this requires $n > 0$. Comparing Eqs. (20) and (21) shows that we can choose a suitable form of the potential $\overline{U}(\bar{\varphi})$ such that Eq. (20) represents a wormhole. The existence of wormholes for the O'Hanlon theory is thus established.

We now seek the corresponding wormholes in the Palatini $f(\widetilde{R})$ theory. In doing so, we can rewrite the potential in the form

$$\overline{U}(\bar{\varphi}) = \frac{U}{4F^2} = 3\frac{V(\varphi)}{\varphi^4} = 3\frac{\widetilde{R}\Phi - f(\widetilde{R})}{\Phi^2} = 3\frac{\widetilde{R}f' - f(\widetilde{R})}{f'^2}. \tag{22}$$

For this potential to determine a wormhole after it is substituted in Eq. (20), we should take the equation

$$\frac{\widetilde{R}f' - f(\widetilde{R})}{f'^2} = \frac{C}{a^{n+2}}. \tag{23}$$

We can try to rewrite this equation as a first-order differential equation for f as a function of a or $\widetilde{R}$. In both cases, we must express the Ricci scalar $\widetilde{R}$ in terms of the scale factor a. Using metric (19), the scalar curvature is obtained as

$$\widetilde{R} = 6\left[\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right]. \tag{24}$$

If we are to regard (21) as the wormhole solution, then the Ricci scalar should take the form

$$\widetilde{R} = 6\left[\frac{C}{a^{n+2}}\left(\frac{n}{2} - 1\right) + \frac{1}{a^2}\right], \tag{25}$$

where the values of n and C can fix the sign of the Ricci scalar.

It is a valuable exercise to obtain wormhole solutions in the $f(\widetilde{R})$ theory whose defining equations are the same as the well-known wormhole solutions in the conventional Einstein–Hilbert theory. For simplicity, we first take $n = 2$. This case corresponds to the typical known wormhole of a conformal scalar field coupled to the Einstein–Hilbert action [34]–[36]. The Ricci scalar and f' then become

$$\widetilde{R} = \frac{6}{a^2}, \tag{26}$$

$$f' = \frac{df}{d\widetilde{R}} = \frac{df}{da}\frac{da}{d\widetilde{R}} = -\frac{a^3}{12}\frac{df}{da}. \tag{27}$$

Substituting Eqs. (26) and (27) in (23), we obtain the first-order differential equation

$$a^6\left(\frac{df}{da}\right)^2 + Aa^5\left(\frac{df}{da}\right) + 2Aa^4 f = 0. \tag{28}$$

We can also use (28) and its derivative d/da to obtain the second-order differential equation

$$Aa^5\frac{d^2f}{da^2} + 2a^6\frac{df}{da}\frac{d^2f}{da^2} + 2a^5\left(\frac{df}{da}\right)^2 + 3Aa^4\frac{df}{da} = 0, \tag{29}$$

where $A = 72/C$. Alternatively, we can use (26) in Eq. (23) to express a in terms of $\widetilde{R}$. Doing so, we obtain the first-order differential equation

$$\widetilde{R}^2\left(\frac{df}{d\widetilde{R}}\right)^2 - \widetilde{R}\left(\frac{df}{d\widetilde{R}}\right) + f = 0. \tag{30}$$

Similarly, using the derivative $d/d\widetilde{R}$ of Eq. (30) we obtain the second-order differential equation for f in terms of $\widetilde{R}$

$$\frac{d^2f}{d\widetilde{R}^2}\left(1 - 2\widetilde{R}\frac{df}{d\widetilde{R}}\right) - 2\left(\frac{df}{d\widetilde{R}}\right)^2 = 0. \tag{31}$$

Because we are interested in the explicit function $f(\widetilde{R})$, we consider Eq. (31), whose solution can be obtained either in the parametric forms

$$\begin{aligned} f(T) &= \frac{1}{4} - \frac{1}{4}\log T^2 - \frac{C}{2}\log T - \frac{1}{4}C^2, \\ R(T) &= \frac{1}{2T}[1 - \operatorname{sgn}(\log T + C)\log T - \operatorname{sgn}(\log T + C)C] \end{aligned} \tag{32}$$

and

$$\begin{aligned} f(T) &= \frac{1}{4} - \frac{1}{4}\log T^2 - \frac{C}{2}\log T - \frac{1}{4}C^2, \\ R(T) &= \frac{1}{2T}[1 + \operatorname{sgn}(\log T + C)\log T + \operatorname{sgn}(\log T + C)C], \end{aligned} \tag{33}$$

where C is a constant, or in the explicit form[2]

$$f(\widetilde{R}) = -\frac{1}{4}[\mathrm{LambertW}(-2C_1\widetilde{R})]^2 - \frac{1}{2}\,\mathrm{LambertW}(-2C_1\widetilde{R}) + C_2, \tag{34}$$

where C_1 and C_2 are constants. For small values of the Ricci scalar $\widetilde{R}$, we obtain the form modulo the constant C_1

$$f(\widetilde{R}) \simeq \widetilde{R} - C_1\widetilde{R}^2 + \frac{C_2}{C_1}. \tag{35}$$

We note that the above solution, modulo the constant term C_2/C_1, is the wormhole found in [37] in the higher-order gravity theory $\widetilde{R} + \epsilon\widetilde{R}^2$, $\epsilon < 0$, for the closed FRW universe. This interesting agreement confirms the correctness of the general form $f(\widetilde{R})$ given by Eq. (34), for which we expect to find classical Euclidean wormholes.

[2] The Lambert function LambertW satisfies $\mathrm{LambertW}(x)\exp(\mathrm{LambertW}(x)) = x$ and has an infinite number of branches for each (nonzero) value of x: exactly one of these branches is analytic at zero. We have $\mathrm{LambertW}(x) \simeq x$ for small x.

Further, we can take $n = 4$, which corresponds to the *axion* field as the matter source coupled to the Einstein–Hilbert action that leads to the Giddings–Strominger wormhole [8]. The Ricci scalar and f' then become

$$\widetilde{R} = 6\left[\frac{C}{a^6} + \frac{1}{a^2}\right], \tag{36}$$

$$f' = \frac{df}{d\widetilde{R}} = \frac{df}{da}\frac{da}{d\widetilde{R}} = -\frac{1}{6}\frac{df}{da}\left[\frac{a^7}{6C + 2a^4}\right]. \tag{37}$$

Substituting (36) and (37) in Eq. (23), we obtain the first-order differential equation

$$Aa^8\left(\frac{df}{da}\right)^2 + a(C + a^4)(6C + 2a^4)\frac{df}{da} + (6C + 2a^4)^2 f = 0. \tag{38}$$

We can also use (38) and its derivative d/da to obtain the second-order differential equation

$$a(C + a^4)(6C + 2a^4)\frac{d^2 f}{da^2} + 2Aa^8\frac{df}{da}\frac{d^2 f}{da^2} + 8Aa^7\left(\frac{df}{da}\right)^2 + \\ + (42C^2 + 64Ca^4 + 22a^8)\frac{df}{da} + a^3(96C + 32a^4)f = 0, \tag{39}$$

where $A = 36/C$. Unfortunately, neither the first-order nor the second-order differential equation gives exact solutions (at least using the available mathematical software such as Maple).

Alternatively, if we wish to use Eq. (36) to express a in terms of $\widetilde{R}$ and construct a differential equation like Eq. (30) or (31), then we obtain more complicated differential equations with no exact solution, and we therefore do not consider this case.

4. Conclusion

Every Euclidean wormhole solution is especially important from macroscopic and microscopic standpoints. In particular, very small Euclidean wormholes are studied as instantons, namely, the saddle points in Euclidean path integrals. They can therefore be used for a semiclassical treatment in the dilute wormhole approximation, where the interaction between the large-scale ends of wormholes is neglected. On the other hand, because the black holes evaporate in theories with reasonable matter contents, new wormhole solutions may provide new contributions for black hole evaporation. Similarly, new wormhole solutions are assumed to play their own important roles in the vanishing of the cosmological constant. Taking the importance of new wormhole solutions into account and motivated by the existence of Euclidean wormhole solutions for some higher-order gravity theories, we have studied the classical Euclidean wormhole solutions for modified general $f(\widetilde{R})$ theories of gravity in the Palatini formalism. We used a well-known dynamical equivalence between the $f(\widetilde{R})$ gravity and the minimally coupled O'Hanlon theory. We showed the existence of new Euclidean wormhole solutions for this O'Hanlon theory; in a special case, we obtained the corresponding (wormhole) form of $f(\widetilde{R})$ which for small $\widetilde{R}$ agrees with the wormhole solution obtained for the higher-order gravity theory $\widetilde{R} + \epsilon\widetilde{R}^2$, $\epsilon < 0$.

In general, such wormholes in $f(\widetilde{R})$ gravity can represent the same characteristic features as studied in general relativity. They can be used in several problems: (a) to describe the quantum amplitude for tunneling between different areas of space–time, (b) to realize black hole evaporation, (c) to allow nonlocal connections in determining fundamental constants, (d) to zero the cosmological constant, and (e) to treat the dilute wormhole approximation semiclassically. Moreover, such wormholes can be used to understand whether they possibly affect the dynamical feature of $f(\widetilde{R})$ gravity that effectively corresponds to the problem of dynamical dark energy.

As previously mentioned, it is well known that the metric and Palatini theories of extended gravity become nonequivalent on-shell in the presence of matter, but the solutions of the Palatini $f(\widetilde{R})$ gravity represent a subclass of solutions of the metric $f(R)$ gravity in the pure $f(R)$ theory [17]–[19]. Also, it can be shown using divergence-free currents that the metric and Palatini formalisms in pure $f(R)$ gravity are equivalent [32]. As a result, because we have considered only the purely gravitational case, the wormhole solutions in the present paper can be used to obtain the corresponding solutions of metric $f(R)$ gravity. In other words, the results reported here can be framed in a more general context where the Palatini and metric approaches for extended theories of gravity are considered [38].

The solutions here were obtained for the isotropic FRW cosmology with one scale factor representing the wormhole throat. The extension to other anisotropic Bianchi cosmological models with three different scale factors is also an interesting activity. For this, the physical interpretation of wormhole solutions with more than one throat must first be established. Then such possible wormhole-like solutions can be sought in anisotropic Bianchi cosmological models.

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REFERENCES

1. M. Visser, *Lorentzian Wormholes: From Einstein to Hawking*, AIP, Melville, N.Y. (1995).
2. S. W. Hawking, *Phys. Rev. D*, **37**, 904–910 (1988).
3. S. Coleman, *Nucl. Phys. B*, **307**, 867–882 (1988).
4. S. Coleman, *Nucl. Phys. B*, **310**, 643–668 (1988).
5. I. Klebanov, L. Susskind, and T. Banks, *Nucl. Phys. B*, **317**, 665–692 (1989).
6. S. Mignemi and I. Moss, *Phys. Rev. D.*, **48**, 3725–3730 (1993).
7. J. Cheeger and D. Grommol, *Ann. of Math.* (2), **96**, 413–443 (1972).
8. S. Giddings and A. Strominger, *Nucl. Phys. B*, **306**, 890–907 (1988).
9. F. S. N. Lobo, and M. A. Oliveira, *Phys. Rev. D*, **80**, 104012 (2009); arXiv:0909.5539v2 [gr-qc] (2009).
10. A. G. Agnese and M. La Camera, *Phys. Rev. D*, **51**, 2011–2013 (1995).
11. K. K. Nandi, B. Bhattacharjee, S. M. K. Alam, and J. Evans, *Phys. Rev. D*, **57**, 823–828 (1998).
12. C. G. Boehmer, T. Harko, and F. S. N. Lobo, *Phys. Rev. D*, **85**, 044033 (2012); arXiv:1110.5756v2 [gr-qc] (2011).
13. S. Capozziello and M. Francaviglia, *Gen. Relativity Gravitation*, **40**, 357–420 (2008); arXiv:0706.1146v2 [astro-ph] (2007).
14. S. Capozziello, S. Nojiri, and S. D. Odintsov, *Phys. Lett. B*, **634**, 93–100 (2006); arXiv:hep-th/0512118v3 (2005).
15. S. Nojiri and S. D. Odintsov, *Int. J. Geom. Meth. Mod. Phys.*, **4**, 115–145 (2007).
16. S. Nojiri and S. D. Odintsov, *Phys. Rev. D*, **74**, 086005 (2006); arXiv:hep-th/0608008v3 (2006).
17. G. Allemandi, A. Borowiec, and M. Francaviglia, *Phys. Rev. D*, **70**, 043524 (2004).
18. G. Allemandi, M. Capone, S. Capozziello, and M. Francaviglia, *Gen. Relativity Gravitation*, **38**, 33–60 (2006); arXiv:hep-th/0409198v1 (2004).
19. G. Allemandi, A. Borowiec, M. Francaviglia, and S. D. Odintsov, *Phys. Rev. D*, **72**, 063505 (2005); arXiv:gr-qc/0504057v2 (2005).
20. S. Capozziello, F. Darabi, and D. Vernieri, *Modern Phys. Lett. A*, **25**, 3279–3289 (2010); arXiv:1009.2580v1 [gr-qc] (2010).

21. H. Fukutaka, K. Ghoroku, and K. Tanaka, *Phys. Lett. B*, **222**, 191–194 (1989).
22. F. S. Accetta, A. Chodos, and B. Shao, *Nucl. Phys. B*, **333**, 221–252 (1990).
23. P. F. González-Diaz, *Phys. Lett. B*, **233**, 85–88 (1989).
24. P. F. González-Diaz, *Phys. Lett. B*, **247**, 251–256 (1990).
25. O. Bertolami, *Phys. Lett. B*, **234**, 258–264 (1990).
26. D. H. Coule and K. Maeda, *Class. Q. Grav.*, **7**, 955–963 (1990).
27. H. A. Buchdahl, *Mon. Not. Roy. Astron. Soc.*, **150**, 1 (1970).
28. T. P. Sotiriou, *Class. Q. Grav.*, **23**, 5117–5128 (2006); arXiv:gr-qc/0604028v2 (2006).
29. M. Roshan and F. Shojai, *Phys. Lett. B*, **668**, 238–240 (2008).
30. S. Capoziello, R. de Ritis, and A. A. Marino, *Class. Q. Grav.*, **14**, 3243–3258 (1997).
31. S. Capoziello, R. de Ritis, and A. A. Marino, *Class. Q. Grav.*, **14**, 3259–3268 (1998).
32. S. Capozziello, F. Darabi, and D. Vernieri, *Modern Phys. Lett. A*, **26**, 65–72 (2011); arXiv:1006.0454v1 [gr-qc] (2010).
33. J. O'Hanlon, *Phys. Rev. Lett.*, **29**, 137–138 (1972).
34. J. J. Halliwell and R. Laflamme, *Class. Q. Grav.*, **6**, 1839–1846 (1989).
35. Y. Verbin and A. Davidson, *Nucl. Phys. B*, **339**, 545–559 (1990).
36. D. N. Page, *J. Math. Phys.*, **32**, 3427–3438 (1991).
37. H. Fukutaka, K. Ghoroku, and K. Tanaka, *Phys. Lett. B*, **222**, 191–194 (1989).
38. S. Capozziello and M. De Laurentis, *Phys. Rep.*, **509**, 167–321 (2011); arXiv:1108.6266v2 [gr-qc] (2011).