Theoretical and Mathematical Physics, **173**(3): 1637–1643 (2012)

RENORMALIZATION GROUP IN A FERMIONIC HIERARCHICAL MODEL IN PROJECTIVE COORDINATES

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We study the renormalization group action in a fermionic hierarchical model in the space of coefficients determining the Grassmann-valued density of the free measure. This space is interpreted as the two-dimensional projective space. The renormalization group map is a homogeneous quadratic map and has a special geometric property that allows describing invariant sets and the global dynamics in the whole space.

Keywords: fermionic model, hierarchical lattice, renormalization group, projective space, invariant set, dynamics

1. Introduction

The fermionic model on the hierarchical lattice is specified by the Hamiltonian

$$H(\psi^*;\alpha) = \sum_{i,j\in\Lambda} d_n^{-\alpha}(i,j)[\bar\psi_1(i)\psi_1(j) + \bar\psi_2(i)\psi_2(j)] + \sum_{i\in\Lambda} L(\psi^*(i);r,g),$$

where

$$L(\psi^*(i);r,g) = r(\bar\psi_1(i)\psi_1(i) + \bar\psi_2(i)\psi_2(i)) + g\bar\psi_1(i)\psi_1(i)\bar\psi_2(i)\psi_2(i).$$

We recall that a hierarchical lattice Λ is defined as a set of integers $\mathbb{Z}$ with a hierarchical distance $d(i,j)$, $i,j\in\mathbb{N}$. The distance $d(i,j) = n^{s(i,j)}$ if $i\neq j$, where $s(i,j)$ is the minimum value of s for which there exists k such that $i\in V_{k,s}$, $j\in V_{k,s}$, where $V_{k,s} = \{j\colon j\in\mathbb{N},\ (k-1)n^s < j \le kn^s\}$ and n is the size of the elementary cell (a fixed natural number). Four-component spins $\psi^*(i) = (\bar\psi_1(i),\psi_1(i),\bar\psi_2(i),\psi_2(i))$, whose components are the generators of the Grassmann algebra, are located at the nodes of this lattice. In the case where $n = p^d$ with a prime p, the lattice Λ can be interpreted as the lattice of purely fractional d-dimensional p-adic vectors (see [1]–[3]) with the p-adic distance between them.

It is also convenient to use the concept of the Grassmann-valued "density" of the free measure $f(\psi^*) = e^{-L(\psi^*;r,g)}$ instead of the Lagrangian $L(\psi^*;r,g)$. In the general case, the "density" of the free measure is given by

$$f(\psi^*;c) = c_0 + c_1(\bar\psi_1\psi_1 + \bar\psi_2\psi_2) + c_2\bar\psi_1\psi_1\bar\psi_2\psi_2,$$

where $c = (c_0,c_1,c_2)\in\mathbb{R}^3$. If $c_0\neq 0$ (the regular case), then the coordinates r and g are related to c by the formulas

$$r(c) = -\frac{c_1}{c_0}, \qquad g(c) = \frac{c_1^2 - c_0c_2}{c_0^2}.$$

If $c_0 = 0$ (for instance, as in the case where the density is given by the Grassmann delta function $\delta(\psi^*) = \bar\psi_1\psi_1\bar\psi_2\psi_2$), then the exponential representation is impossible.

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Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 173, No. 3, pp. 355–362, December, 2012. Original article submitted April 13, 2012; revised May 10, 2012.

The triple (c_0, c_1, c_2) can be naturally treated as a point in the two-dimensional real projective space $\mathbb{R}P^2$ because two sets that differ by a nonzero factor represent the same Gibbs state.

The block-spin transformation of the Kadanoff–Wilson renormalization group (RG) is defined by the formula

$$r(\alpha)\psi^*(i) = n^{-\alpha/2} \sum_{j\in V_{i,1}} \psi^*(j).$$

The Gaussian part of the model Hamiltonian is invariant under the RG transformation, which reduces to the transformation $R(\alpha)$ of the coupling constants, $R(\alpha)(r,g) = (r', g')$, in the non-Gaussian part,

$$r' = \lambda\left(\frac{(r+1)^2 - g}{(r+1)^2 - g/n}(r+1) - 1\right), \qquad g' = \frac{\lambda^2}{n}\left(\frac{(r+1)^2 - g}{(r+1)^2 - g/n}\right)^2 g, \tag{1}$$

where $\lambda = n^{\alpha-1}$. The RG transformation in the space of the free measure "density" coefficients is also denoted by $R(\alpha)$: $R(\alpha)(c_0, c_1, c_2) = (c'_0, c'_1, c'_2)$,

$$\begin{aligned}
c'_0 &= n^2(c_2 - 2c_1 + c_0)^{n-2}\left((c_1 - c_0)^2 + \frac{1}{n}(c_0c_2 - c_1^2)\right),\\
c'_1 &= \lambda n^2(c_2 - 2c_1 + c_0)^{n-2}\left((c_1 - c_0)(c_2 - c_1) + \frac{1}{n}(c_0c_2 - c_1^2)\right),\\
c'_2 &= \lambda^2 n^2(c_2 - 2c_1 + c_0)^{n-2}\left((c_2 - c_1)^2 + \frac{1}{n}(c_0c_2 - c_1^2)\right).
\end{aligned} \tag{2}$$

If $c_2 - 2c_1 + c_0 \neq 0$, then we can omit this factor in formulas (2) because we consider the transformation in the projective space.

The RG transformation in the c space seems more aesthetic and allows visualizing the picture of the dynamics in the entire coupling-constant space because the projective space is compact and allows eliminating some singularities in the (r,g) coordinates. Indeed, map (1) is not defined at the points of the critical parabola $g = n(r+1)^2$, but formulas (2) of the RG transformation in the c space allow defining it. A point (r,g) of the critical parabola can be written in the c coordinates as $\bigl(1, -r, r^2 - n(r+1)^2\bigr)$, which becomes $(0, 1, (n-1)(r + (n+1)(n-1)^{-1}))$ under the transformation $R(\alpha)$ and thus leaves the plane (r,g). But the subsequent iteration of $R(\alpha)$ moves this point into the regular region. Simple calculations show that the nonregular region is mapped to the parabola $g = (1/n)(r+\lambda)^2$ under the RG transformation.

The transformation $R(\alpha)$ has the trivial (Gaussian) fixed point $(0,0)$ and two non-Gaussian fixed points in the (r,g) coordinates. In the c coordinates, the Gaussian point can be written as $(1,0,0)$, and one more fixed point $(0,0,1)$ can be seen, giving the Grassmann delta function $\delta(\psi^*) = \bar\psi_1\psi_1\bar\psi_2\psi_2$. We let δ denote this fixed point. In addition, the RG transformation has two regular fixed points that for $\alpha \neq 1$ are given in the (r,g) coordinates by the formulas

$$\begin{aligned}
r_+(\alpha) &= \frac{n^{1/2} - n^{\alpha-1}}{1 - n^{1/2}}, \qquad g_+(\alpha) = \frac{r_+(\alpha)(1 + r_+(\alpha))^2}{1 + r_+(\alpha) + n^{-1/2}}, \quad \alpha \neq \frac{1}{2},\\
r_-(\alpha) &= \frac{-n^{1/2} - n^{\alpha-1}}{1 + n^{1/2}}, \qquad g_-(\alpha) = \frac{r_-(\alpha)(1 + r_-(\alpha))^2}{1 + r_-(\alpha) - n^{-1/2}}.
\end{aligned}$$

At $\alpha = 1$, we have the entire line of fixed points $\{g = 0, r \neq -1\}$. The map $R(\alpha)$ itself, like the map from $\mathbb{R}P^2$ to $\mathbb{R}P^2$, is well defined everywhere except the point $(1,1,1)$ because $R(\alpha)(1,1,1) = (0,0,0)$. It is given by the coordinates $(-1,0)$ in the (r,g) plane. We call this point the singular point of the RG transformation.

In [4]–[9], we studied the RG dynamics in the (r, g) coordinates, although we used the c coordinates to obtain the global picture. Here, we describe the invariant sets and the RG dynamics in the c space. Many proofs then become clearer and simpler because of the special geometric property of the transformation $R(\alpha)$ expressed in Lemma 1. Hereafter, we assume that $\alpha > 1$. In this case, the fixed point δ is the only attracting fixed point. We note that the fermionic hierarchical model is interesting because it admits an exact RG analysis and is an anticommuting analogue of the bosonic ϕ^4 theory, where the RG is studied only in the perturbation theory framework in the neighborhood of the Gaussian point.

2. Invariant sets and RG dynamics in the projective space

To describe the global flow of the RG dynamics, we use the (r, g) coordinates and the projective c coordinates simultaneously. More precisely, we consider the realization of the projective c space in the form of the hemisphere $S = \{(c_0, c_1, c_2)\colon c_0^2+c_1^2+c_2^2 = 1,\ c_0 \ge 0\}$, where the opposite points of the boundary circle $c_1^2 + c_2^2 = 1$ are identified. To obtain the flat (two-dimensional) picture, we use the orthogonal projection S on the disc $D = \{(c_1, c_2)\colon c_1^2 + c_2^2 \le 1\}$. The regular point (r, g) then corresponds to $\big(c_1(r, g), c_2(r, g)\big)$, where

$$c_1(r,g) = -\frac{r}{\sqrt{1+r^2+(r^2-g)^2}}, \qquad c_2(r,g) = -\frac{r^2-g}{\sqrt{1+r^2+(r^2-g)^2}}.$$

We note that the points $\big(c_1(r, g), c_2(r, g)\big)$ belong to the interior of the disc D. The trivial fixed point $r = 0$, $g = 0$ is also represented by the point $(0, 0)$ in the c coordinates. The fixed point given by the delta function in the c coordinates is determined by the point $(0, 1)$. The line $g = 0$ in the c space is described by the curve $l_0 = \{(c_1(r, 0), c_2(r, 0));\ r \in \mathbb{R}\}$. We note that $\big(c_1(r, 0), c_2(r, 0)\big) \to (0, 1)$ as $r \to \pm\infty$. Completing the curve l_0 with the limit point $(0, 1)$, we obtain a closed curve l in the c space. The lower half-plane $\{(r, g)\colon g < 0\}$ is given in the c space by the region D_1 bounded by the curve l.

We consider the parametric family of curves in the (r, g) space given by functions of the form

$$g_1(r; a, b) = \frac{r-a}{r-b}(r+1)^2.$$

Lemma 1. *The equality*

$$R(\alpha)(r, g_1(r; a, b)) = (r', g_\lambda(r'; a', b'))$$

holds, where

$$r' = \lambda\frac{a-b-1+n^{-1}}{1-n^{-1}}\cdot\frac{r-a_1}{r-b_1},$$

$$a_1 = \frac{(n^{-1}-1)a}{a-b-1+n^{-1}}, \qquad b_1 = \frac{b-n^{-1}a}{1-n^{-1}},$$

$$g_\lambda(r'; a', b') = \frac{r'-a'}{r'-b'}(r'+\lambda)^2,$$

$$a' = \lambda a, \qquad b' = \lambda(n(b+1)-1), \qquad \lambda = n^{\alpha-1}.$$

Proof. The lemma is proved by direct calculation.

It is easy to see that

$$c_1(r, g_1(r; a, b)) \to c_2^0(a, b) = -\frac{1}{\sqrt{1+(a-b-2)^2}},$$

$$c_2(r, g_1(r; a, b)) \to c_2^0(a, b) = \frac{a-b-2}{\sqrt{1+(a-b-2)^2}}$$

as $r \to \pm\infty$. The points $\bigl(c_1^0(a,b), c_2^0(a,b)\bigr)$ belong to the circle $c_1^2 + c_2^2 = 1$, and we compactify the curves

$$\{c_1(r, g_1(r; a, b)), c_2(r, g_1(r; a, b));\ r \in \mathbb{R}\},$$

adding the new limit points $\bigl(c_1^0(a,b), c_2^0(a,b)\bigr)$ to them.

We note that the function $g_1(r; a, b)$ is undefined at the point $r = b$, but because

$$c_1(r, g_1(r; a, b)) \to 0, \qquad c_2(r, g_1(r; a, b)) \to 1$$

as $r \to b$, we can define the curves at the point $r = b$ by $c_1(b, g_1(b; a, b)) = 0$ and $c_2(b, g_1(b; a, b)) = 1$.

We choose the parameter b such that $b = b' = \lambda(n(b+1) - 1)$ and let $b_0 = -(\lambda n - \lambda)/(\lambda n - 1)$ be the solution of this equation. We consider the regions

$$G_+(a) = \{(r, g)\colon r \geq a,\ 0 \leq g \leq g_1(r; a, b_0)\},$$

$$G_-(a) = \{(r, g)\colon r \leq a,\ 0 \leq g \leq g_1(r; a, b_0)\}$$

in the (r, g) space. We also consider the curves

$$\gamma_+(a) = \{r, g_1(r; a, b_0)\colon r \geq a\}, \qquad \gamma_-(a) = \{r, g_1(r; a, b_0)\colon r \leq a\}.$$

We note that each point in $G_+(a)$ or $G_-(a)$ respectively belongs to a curve $\gamma_+(\tilde{a})$ or $\gamma_-(\tilde{a})$, where $\tilde{a} \geq a$ or $\tilde{a} \leq a$. Let $T_+^0(a)$, $T_-^0(a)$, $l_+^0(a)$, and $l_-^0(a)$ denote the images of $G_+(a)$, $G_-(a)$, $\gamma_+(a)$, and $\gamma_-(a)$ in the c space. We add the limit point $\bigl(c_1^0(a, b_0), c_2^0(a, b_0)\bigr)$ to the curves $l_+^0(a)$ and $l_-^0(a)$ and let $l_+(a)$ and $l_-(a)$ denote the obtained curves. We also add the limit points $\bigl(c_1^0(\tilde{a}, b_0), c_2^0(\tilde{a}, b_0)\bigr)$, $\tilde{a} \geq a$, and $\bigl(c_1^0(\tilde{a}, b_0), c_2^0(\tilde{a}, b_0)\bigr)$, $\tilde{a} \leq a$, to the respective regions $T_+^0(a)$ and $T_-^0(a)$ and let $T_+(a)$ and $T_-(a)$ denote the obtained regions.

The region $T_+(a)$ is the curvilinear triangle in the c space bounded by the three curvilinear segments $l_+(a)$, the segment of the disc D_1 boundary $\{(c_1(r, 0), c_2(r, 0))\colon r \geq a\}$, where

$$c_1(r, 0) = -\frac{r}{\sqrt{1 + r^2 + r^4}}, \qquad c_2(r, 0) = \frac{r^2}{\sqrt{1 + r^2 + r^4}},$$

and the segment of the disc D boundary $\{(c_1(\tilde{a}), c_2(\tilde{a}))\colon \tilde{a} \geq a\}$, where

$$c_1(\tilde{a}) = -\frac{1}{\sqrt{1 + (\tilde{a} - b_0 - 2)^2}}, \qquad c_2(\tilde{a}) = \frac{\tilde{a} - b_0 - 2}{\sqrt{1 + (\tilde{a} - b_0 - 2)^2}}.$$

The last two segments meet at the vertex $(0, 1)$ because $\bigl(c_1(\tilde{a}), c_2(\tilde{a})\bigr) \to (0, 1)$ as $\tilde{a} \to +\infty$. Analogously, $T_-(a)$ is the curvilinear triangle bounded by the segment $l_-(a)$, the segment of the D_1 boundary $\bigl\{t(c_1(r, 0), c_2(r, 0))\colon r \leq a\bigr\}$, and the segment of the D boundary $\bigl\{(c_1(\tilde{a}), c_2(\tilde{a}))\colon \tilde{a} \leq a\bigr\}$.

We consider the images of the introduced regions $G_+(a)$, $G_-(a)$, $T_+(a)$, and $T_-(a)$ under the RG transformation $R(\alpha)$. Let

$$G'_+(a) = R(\alpha) G_+(a), \qquad G'_-(a) = R(\alpha) G_-(a),$$

$$T'_+(a) = R(\alpha) T_+(a), \qquad T'_-(a) = R(\alpha) T_-(a).$$

Theorem 1. *If $a \geq -(1-n^{-1})/(\lambda n - 1)$, then*

$$G'_+(a) = \{(r,g)\colon \lambda a \leq r < r_1(a;\lambda),\ 0 \leq g < g_1(r;\lambda a, b_0)\} \cup$$
$$\cup \left\{(r,g)\colon r_1(a;\lambda) \leq r,\ 0 \leq g < \frac{1}{n}(r+\lambda)^2\right\}.$$

If $a \leq b_0 = -\lambda(n-1)/(\lambda n - 1)$, then

$$G'_-(a) = \{(r,g)\colon r_1(a;\lambda) < r \leq \lambda a,\ 0 \leq g < g_1(r;\lambda a, b_0)\} \cup$$
$$\cup \left\{(r,g)\colon r \leq r_1(a;\lambda),\ 0 \leq g < \frac{1}{n}(r+\lambda)^2\right\},$$

where

$$r_1(a;\lambda) = \lambda\left(\left(1-\frac{1}{n}\right)^{-1} a + (\lambda n - 1)^{-1}\right). \tag{3}$$

Proof. It follows from Lemma 1 that

$$R(\alpha)(r, g_1(r;a,b_0)) = (r', g_\lambda(r';a',b_0)),$$

where

$$r' = \lambda\left(a\left(1-\frac{1}{n}\right)^{-1} + (\lambda n - 1)^{-1}\right)\frac{r-a_1}{r-b_1},$$
$$a_1 = -\frac{(1-n^{-1})a}{a+(1-n^{-1})/(\lambda n - 1)}, \qquad b_1 = -\frac{(\lambda n - \lambda)/(\lambda n - 1) + n^{-1}a}{1-n^{-1}}, \tag{4}$$
$$g_\lambda(r';(a'_1,b_0)) = g_\lambda(r';(\lambda a, b_0)) = \frac{r'-\lambda a}{r'-b_0}(r'+\lambda)^2.$$

It follows from Eqs. (1) and (2) that $a_1 > b_1$ if $a > -(1-n^{-1})/(\lambda n - 1)$ and that r' increases monotonically with r for $r > b_1$. If $a < -\lambda(n-1)/(\lambda n - 1)$, then $b_1 > a_1$, and r' decreases monotonically with r for $r < b_1$.

Let $a > 0$. Then each curve $\gamma_+(a) = \{(r, g_1(r;a,b_0))\colon r \geq a\}$ is mapped into $\gamma'_+(a) = \{(r, g_\lambda(r;\lambda a, b_0))\colon \lambda a \leq r < r_1(a;\lambda)\}$, where $r_1(a;\lambda)$ is given by (1). We note that $g_\lambda(r_1(a;\lambda);\lambda a, b_0) = n^{-1}(r_1(a;\lambda)+\lambda)^2$. Any point of the region $G_+(a)$ belongs to $\gamma_+(\tilde{a})$ for some $\tilde{a} \geq a$, and $\gamma_+(\tilde{a}) \subset G_+(a)$ for all $\tilde{a} \geq a$. It hence follows that the family of curves $\gamma'_+(\tilde{a})$, $\tilde{a} \geq a$, covers the set $G'_+(a)$.

Let $a < b_0 = -\lambda(n-1)/(\lambda n - 1)$. Then the curve $\gamma_-(a) = \{(r, g_1(r;a,b_0))\colon r \leq a\}$ is mapped into $\gamma'_-(a) = \{(r, g_\lambda(r;\lambda a, b_0))\colon r_1(a;\lambda) < r \leq \lambda a\}$, where $r_1(a;\lambda)$ is also given by (1). The family of curves $\gamma_-(\tilde{a})$, $\tilde{a} \leq a$, covers the set $G_-(a)$. Consequently, the family $\gamma'_-(\tilde{a})$, $\tilde{a} \leq a$, covers the set $G'_-(a)$. The statement of the theorem hence follows.

Theorem 2. *If $a \geq 4$, then*

$$G'_+(a) \subset G_+\left(\frac{\lambda+1}{2}a\right) \subset G_+(a).$$

If $a \leq -4$, then

$$G'_-(a) \subset G_-\left(\frac{\lambda+1}{2}a\right) \subset G_-(a).$$

All points belonging to $G_+(a)$, $a \geq 4$, or $G_-(a)$, $a \leq -4$, tend to δ, the fixed point $(0,1)$, under iterations of the RG transformation.

Proof. Let $a \geq 4$. We show that the curve $\gamma'_+(a)$ belongs to the set $G_+((\lambda+1)a/2)$. Because $\gamma'_+(a)$ is part of the curve given by the equation $g = g_\lambda(r; \lambda a, b_0)$, it suffices to show that

$$g_1\left(r; \frac{\lambda+1}{2}a, b_0\right) \geq g_\lambda(r; \lambda a, b_0). \tag{5}$$

Direct calculation shows that

$$g_1\left(r; \frac{\lambda+1}{2}a, b_0\right) - g_\lambda(r; \lambda a, b_0) =$$

$$= \frac{\lambda-1}{r-b_0}\left[\frac{a-4}{2}r^2 + (2\lambda+1)\left(a - \frac{\lambda+1}{2\lambda+1}\right)r + a\left(\lambda^2+\lambda+\frac{1}{2}\right)\right]. \tag{6}$$

It is hence easy to see that inequality (5) holds for $a \geq 4$ if $r \geq \lambda a$.

Let $a \leq -4$. We note that $r - b_0 < 0$ for $r \leq a$. We use Eq. (6). To prove inequality (5), it suffices to show that

$$\frac{a-4}{2}r^2(2\lambda+1)\left(a - \frac{\lambda+1}{2\lambda+1}\right)r < 0 \tag{7}$$

because $a(\lambda^2+\lambda+1/2) < 0$ for negative a. Inequality (5) holds if

$$\frac{a-4}{2}r + (2\lambda+1)a - (\lambda+1) > 0$$

for $r < \lambda a$. Taking $a \leq -4$, $r < \lambda a$, and $\lambda > 1$ into account, we obtain

$$\frac{a-4}{2}r + (2\lambda+1)a - (\lambda+1) \geq \frac{a-4}{2}\lambda a + (2\lambda+1)a - (\lambda+1) = \frac{a^2\lambda}{2} + a - \lambda - 1 \geq$$

$$\geq -\frac{4a\lambda}{2} + a - \lambda - 1 = a(1-2\lambda) - \lambda - 1 >$$

$$> 4(2\lambda-1) - \lambda - 1 = 7\lambda - 5 > 0.$$

It hence follows that

$$g_1\left(r; \frac{\lambda+1}{2}a, b_0\right) \geq \min\left(g_\lambda(r; \lambda a, b_0), \frac{1}{n}(r+\lambda)^2\right)$$

for $a \geq 4$ and $r \geq \lambda a$ and for $a \leq -4$ and $r \leq \lambda a$. It follows from Theorem 1 with $(\lambda+1)/2 > 1$ taken into account that

$$G'_+(a) \subset G_+\left(\frac{\lambda+1}{2}a\right) \subset G_+(a) \quad \text{for } a \geq 4,$$

$$G'_-(a) \subset G_-\left(\frac{\lambda+1}{2}a\right) \subset G_-(a) \quad \text{for } a \leq -4.$$

Let $R^n(\alpha)(r,g) = (r^{(n)}, g^{(n)})$. If $a \geq 4$ and $(r,g) \in G_+(a)$, then $r^{(n)} \geq ((\lambda+1)/2)^n a$ and $g^{(n)} < n^{-1}(r^{(n)}+\lambda)^2$. Because $r^{(n)} \to \infty$ as $n \to \infty$, $c_1(r^{(n)}, g^{(n)}) \to 0$ and $c_2(r^{(n)}, g^{(n)}) \to 1$ as $n \to \infty$.

We obtain the following corollary from Theorem 2.

Corollary 1. *The regions $T_+(a)$ for $a \geq 4$ and $T_-(a)$ for $a \leq -4$ are invariant under the RG transformation and belong to the attraction area of the fixed point δ.*

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