Theoretical and Mathematical Physics, **161**(3): 1647–1662 (2009)

FORMATION OF TRAPPED SURFACES IN THE COLLISION OF NONEXPANDING GRAVITATIONAL SHOCK WAVES IN AN AdS$_4$ SPACE–TIME

© **I. Ya. Aref'eva* and A. A. Bagrov***

We study the formation of marginally trapped surfaces in the head-on collision of two shock waves in anti-de Sitter space–time. We compare the obtained results with the corresponding results for de Sitter space–time. To clarify this comparison, we use coordinates that allow studying AdS/dS cases in a universal way. We also analyze the dependence of the area of the trapped surface on the choice of the regularization of the shock wave metric.

Keywords: anti-de Sitter, trapped surfaces, shock wave collision, black hole

1. Introduction

The possibility of black hole formation during the collision of ultrarelativistic point particles in the Minkowski space–time is a generally accepted theoretical fact [1]–[3]. In connection with the appearance of the TeV gravity paradigm [4], black hole formation during the collision of particles with a center-of-mass energy of a few TeV and the possible experimental manifestation of this process [5] recently became the subject of numerous comprehensive investigations [6]–[10]. We also note the proposed possibility of creating other more exotic objects at the LHC [11]–[13]. Black hole creation during collisions of ultrarelativistic particles in anti-de Sitter (AdS) and de Sitter (dS) space–times is also a subject actively discussed [14], [15].

A negative cosmological constant leads to a collapse of matter, while a positive cosmological constant facilitates its repulsion. It is therefore natural to expect that a negative cosmological constant promotes black hole formation and a positive cosmological constant weakens this process.

Here, we study the formation of marginally trapped surfaces during the head-on collision of two shock waves in an AdS space–time in detail. In particular, we analyze the dependence of physical parameters on the type of regularization of the δ-function appearing in the shock wave metric. We also study the area of the trapped surface as a function of the ratio of the cosmological radius and square of the shock wave energy. We note that if the space–time is assumed to be asymptotically flat, then the presence of a trapped surface usually guarantees the existence of the event horizon [16]–[19]. A similar theorem has not been proved for an AdS space–time, but it is thought that the existence of a trapped surface can indicate the formation of a black hole.

Our main motivation is related to the AdS/CFT correspondence. Despite the absence of a holographic dual description of QCD, describing heavy-ion collisions in terms of gravitational shock wave collisions in an AdS space–time was suggested in [20], [21]. Black hole formation in the bulk during collisions of objects dual to the nuclei was interpreted in the space of one lower dimension as formation of a quark–gluon plasma [22]–[24].

*Steklov Mathematical Institute, RAS, Moscow, Russia, e-mail: arefeva@mi.ras.ru, bagrov.andrey@googlemail.com.

Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol. 161, No. 3, pp. 382–399, December, 2009. Original article submitted May 12, 2009.

In AdS, a dimension-dependent critical effect appearing in the collision of smoothed gravitational shock waves was recently found [14]. For $D = 4$ and $D = 5$, there exists a critical value of the "blur" of the shock waves in the direction of their motion, above which formation of a trapped surface during the collision becomes impossible. We note that the results obtained in the AdS case agree qualitatively with the results obtained in a flat space–time.

This paper is organized as follows. We begin by introducing the basic mathematical objects and recall some basic facts about generalizing the Aichelburg–Sexl shock wave geometry [25]–[27] to nonexpanding shock waves propagating in D-dimensional space–times with a nontrivial cosmological constant [28]–[36]. In Sec. 3, we present the trapped-surface equations in the complex coordinates. These coordinates are especially convenient for analyzing the four-dimensional case. In the general case, the trapped surface can be found as the solution of the particular Dirichlet problem (see Sec. 3 for the details) for the Beltrami–Laplace operator on the hyperboloid where the shock wave is located. We conclude by comparing our results with results in [14], [37] obtained in a different coordinate system.

2. The shock wave in complex coordinates

2.1. The shock wave in plane coordinates. To begin, we allow ourself to briefly recall the results in [28], [32], [35] devoted to the geometry of shock waves propagating in the four-dimensional AdS space–time (see [38] for gravitational waves). In terms of the dependent plane coordinates $(U, V, \vec{Z}, Z^4)$, $\vec{Z} = (Z^2, Z^3)$, satisfying

$$-2UV + \vec{Z}^2 - {Z^4}^2 = -a^2, \tag{2.1}$$

the line element of the shock wave space–time is

$$ds^2 = -2\,dU\,dV + d\vec{Z}^2 - (dZ^4)^2 + F(Z^4)\delta(U)dU^2, \tag{2.2}$$

where U and V are related to Z_0 and Z_1 by $U = (Z_0 + Z_1)/\sqrt{2}$ and $V = (Z_0 - Z_1)/\sqrt{2}$.

The shock wave shape function F is a fundamental solution of the equation

$$\left(\triangle_{\mathbb{H}^2} - \frac{2}{a^2}\right)F = -16\sqrt{2}\,\pi G_4 \bar{p}\delta(\vec{n}, \vec{n}_0), \tag{2.3}$$

where $\Delta_{\mathbb{H}^2}$ is the Laplace–Beltrami operator on the two-dimensional hyperboloid $\mathbb{H}^2$, $\vec{Z}^2 - {Z^4}^2 = -a^2$, $\vec{n}_0$ is the location of the particle on the hyperboloid, $\bar{p}$ is the shock wave energy, and G_4 is the four-dimensional gravitational constant. The factor $\sqrt{2}$ in the right-hand side results from our choice of the coordinate system (that factor disappears if we set $-dU\,dV$ instead $-2\,dU\,dV$ in the metric for the AdS space and the shock wave). This metric is a solution of the Hilbert–Einstein equations with an energy–momentum tensor with the single nontrivial component $T_{UU} \sim \bar{p}\delta(U)$.

For the parameterization $Z^4 = a\xi$, we obtain

$$F(\xi) = 4\sqrt{2}\,p\left(-2 + \xi\log\left(\frac{\xi+1}{\xi-1}\right)\right), \tag{2.4}$$

where $p = \bar{p}G_4$ is the rescaled energy and $\xi > 1$.

Figure 1a schematically presents a single shock wave in a D-dimensional AdS space–time. The latter is represented as a hyperboloid embedded in the $(D+1)$-dimensional Minkowski space–time. The coordinates Z_2 and Z_3 are suppressed in this figure. The shock wave is located at the intersection of the hyperboloid with hyperplane $Z^0 - Z^1 = 0$, $U = (Z^0 + Z^1)/\sqrt{2}$, $V = (Z^0 - Z^1)/\sqrt{2}$.

Two shock waves colliding at $U = V = 0$ are presented in Fig. 1b.

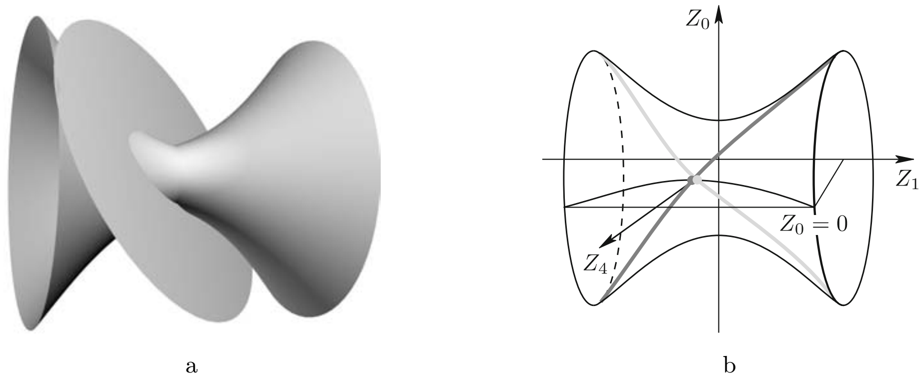

Fig. 1. (a) A single shock wave in AdS space. (b) Two shock waves collide at Z: $0 = 0$.

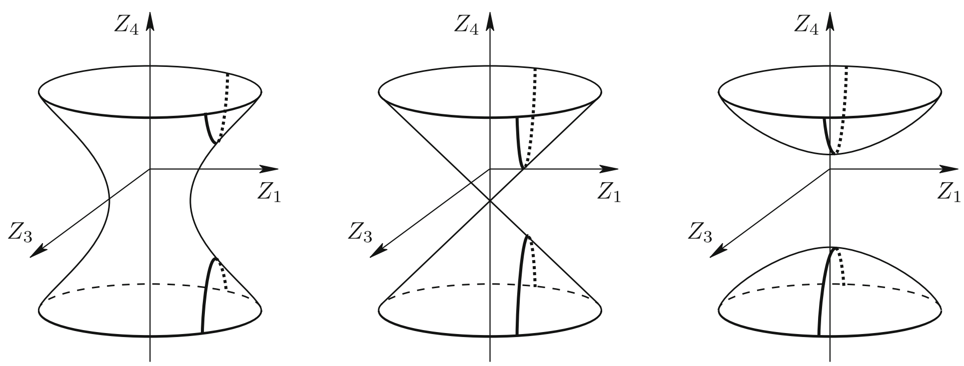

Fig. 2. A shock wave in AdS at discrete Z^0-time instants.

We consider a collision of two waves of the type described above. We suppose that in the region $\{U < 0\} \cup \{V < 0\}$, i.e., the part of the space–time before the collision, the metric is given by

$$ds^2 = -2\,dU\,dV + d\vec{Z}^2 - (dZ^4)^2 + F(\xi, \xi_1)\delta(U)dU^2 + F(\xi, \xi_2)\delta(V)dV^2. \tag{2.5}$$

Here, ξ_1 and ξ_2 are the locations of the two colliding particles (see [28] for the explicit formula for $F(\xi, \xi_i)$).

We can also draw the position of a single shock at discrete instants. In Fig. 2, we can easily see that our shock wave is nonexpanding.

In what follows, we use independent complex coordinates, which are convenient for a universal description of the dS and AdS spaces.

2.2. The shock wave metric in the independent coordinates. To study the structure of the space–time in terms of independent four-dimensional coordinates, it is convenient to use the complex conformal flat coordinates

$$w = \frac{2aU}{Z^4 + a}, \qquad \sigma = \frac{2aV}{Z^4 + a}, \qquad \zeta = \frac{\sqrt{2}\,a}{Z^4 + a}(Z^2 + iZ^3). \tag{2.6}$$

In these coordinates, the shock wave metric is

$$ds^2 = \frac{-2\,dw\,d\sigma + 2\,d\zeta\,d\bar{\zeta} + 2H(\zeta, \bar{\zeta})\delta(w)dw^2}{[1 \pm (w\sigma - \zeta\bar{\zeta})/(2a^2)]^2}, \tag{2.7}$$

where

$$H(\zeta, \bar{\zeta}) = \frac{1}{2}\left(1 \mp \frac{1}{2a^2}\zeta\bar{\zeta}\right) F\left(a\frac{1 \pm \zeta\bar{\zeta}/(2a^2)}{1 \mp \zeta\bar{\zeta}/(2a^2)}\right) \tag{2.8}$$

and F is given by (2.4). Hence,

$$H(\zeta,\bar\zeta)=2\sqrt{2}\,p\biggl(1\mp\frac{1}{2a^2}\zeta\bar\zeta\biggr)\biggl(-2+\frac{1\pm\zeta\bar\zeta/(2a^2)}{1\mp\zeta\bar\zeta/(2a^2)}\log\biggl(\frac{2a^2}{\zeta\bar\zeta}\biggr)\biggr), \tag{2.9}$$

where $\mp$ corresponds to the respective AdS or dS case.

The shock wave is located on the two-dimensional hyperboloid/sphere (we consider only the AdS case in what follows; see Fig. 2), and the induced metric on it is given by

$$ds^2=\frac{2\,d\zeta\,d\bar\zeta}{(1-\zeta\bar\zeta/(2a^2))^2}. \tag{2.10}$$

The rescaled shape function $H(\zeta,\bar\zeta)$ divided by $1\mp\zeta\bar\zeta/(2a^2)$ is the fundamental solution of the Beltrami–Laplace equation on this two-dimensional hyperboloid,

$$\biggl(\Delta_{\mathbb{H}^2}-\frac{2}{a^2}\biggr)\frac{H(\zeta,\bar\zeta)}{1-\zeta\bar\zeta/(2a^2)}=-8\sqrt{2}\,\pi\bar p G_4\delta(\zeta)\delta(\bar\zeta), \tag{2.11}$$

where

$$\Delta_{\mathbb{H}^2}=\frac{1}{2}\biggl(1-\frac{\zeta\bar\zeta}{2a^2}\biggr)^2\frac{\partial^2}{\partial\zeta\partial\bar\zeta} \tag{2.12}$$

or, in the $\rho\varphi$ coordinates, $\zeta=\rho e^{i\varphi}$,

$$\Delta_{\mathbb{H}^2}=\frac{1}{2}\biggl(1-\frac{\rho^2}{2a^2}\biggr)^2\biggl(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\biggr). \tag{2.13}$$

Because H given by (2.11) depends only on ρ, we have a fundamental solution of the ordinary differential operator

$$\frac{1}{2}\biggl(1-\frac{\rho^2}{2a^2}\biggr)^2\biggl(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}\biggr)\frac{H(\rho)}{1-\rho^2/(2a^2)}=-8\sqrt{2}\,\pi\bar p G_4\delta(\rho). \tag{2.14}$$

2.3. Geodesics in term of the independent coordinates. Explicit expressions for geodesics in terms of the dependent five-dimensional coordinates are known:

$$V=V_0+V_1U+Q(Z_0^j)\theta(U)+R(Z_0^j)\theta(U)U, \tag{2.15}$$

$$Z^i=Z_0^i+Z_1^iU+S_i(Z_0^j)\theta(U)U, \tag{2.16}$$

where

$$Q(Z_0^i)=\frac{1}{2}F(Z_0^i), \tag{2.17}$$

$$S^i(Z_0^i)=\frac{1}{2}F_{,i}+\frac{1}{2a^2}\,(F-Z_0^jF_{,j})Z_0^i, \tag{2.18}$$

$$R(Z_0^j)=\frac{1}{2}F_{,i}Z_1^i+\frac{1}{2a^2}\,(F-Z_0^iF_{,i})V_0+\frac{1}{8}\,F_{,i}^2+\frac{1}{8a^2}\,\bigl(F^2-(Z_0^iF_{,i})^2\bigr). \tag{2.19}$$

These formulas have been obtained by a straightforward analysis of the geodesic equations with constraints in [15] and also by an application of the theorem on embedding a manifold in [33]. Here and hereafter, we use the convention

$$\theta(0)=\frac{1}{2}. \tag{2.20}$$

Although the Heaviside function is a generalized function and its value at one point should not play a significant role in physically meaningful calculations, we are here obliged to deal with such an object as $\theta(0)$. We plan to discuss this aspect in detail in our next paper and perform an accurate regularization procedure. Here, we simply use convention (2.20).

Using coordinate change (2.6), we obtain an expression for geodesics in terms of the independent coordinates. In the first order of the parameter U, we obtain

$$\begin{aligned} w(U) &= w_1 U + \dots, \\ \sigma(U) &= \sigma_0 + \sigma_1 U + \dots \equiv \sigma_{0c} + \sigma_{0\theta}\theta(U) + \big(\sigma_{1c} + \sigma_{1\theta}\theta(U)\big)U + \dots, \\ \zeta(U) &= \zeta_0 + \zeta_1 U + \dots \equiv \zeta_{0c} + \zeta_{0\theta}\theta(U) + \big(\zeta_{1c} + \zeta_{1\theta}\theta(U)\big)U + \dots, \end{aligned} \tag{2.21}$$

where

$$w_1 = \frac{2}{1 + Z_0^4/a}, \tag{2.22}$$

$$\sigma_{0c} = \frac{2V_0}{1 + Z_0^4/a}, \tag{2.23}$$

$$\sigma_{0\theta} = \frac{2Q(Z_0^i)}{1 + Z_0^4/a}, \tag{2.24}$$

$$\sigma_{1c} = 2\bigg(\frac{V_1}{1 + Z_0^4/a} - \frac{Z_1^4}{a}\frac{V_0}{(1 + Z_0^4/a)^2}\bigg), \tag{2.25}$$

$$\sigma_{1\theta} = 2\bigg(\frac{R(Z_0^i)}{1 + Z_0^4/a} - \frac{Q(Z_0^i)Z_1^4}{a(1 + Z_0^4/a)^2} - \frac{QS^4(Z_0^i)}{a(1 + Z_0^4/a)^2} - \frac{S^4(Z_0^i)}{a}\frac{V_0}{(1 + Z_0^4/a)^2}\bigg), \tag{2.26}$$

$$\zeta_{0c} = \frac{\sqrt{2}\,z_0}{1 + Z_0^4/a}, \tag{2.27}$$

$$\zeta_{0\theta} = 0, \tag{2.28}$$

$$\zeta_{1c} = \frac{\sqrt{2}\,z_1}{1 + Z_0^4/a} - \frac{Z_1^4}{a}\frac{\sqrt{2}\,z_0}{(1 + Z_0^4/a)^2}, \tag{2.29}$$

$$\zeta_{1\theta} = \frac{\sqrt{2}\,\mathcal{S}(Z_0^i)}{1 + Z_0^4/a} - \frac{S^4}{a}\frac{\sqrt{2}\,z_0}{(1 + Z_0^4/a)^2}. \tag{2.30}$$

Here, the complex variables $\mathcal{S}$, z_0, and z_1 are related to S^i, Z_0^i, and Z_1^i by

$$\mathcal{S}(Z_0^i) = S^2(Z_0^i) + iS^3(Z_0^i), \tag{2.31}$$

$$z_0 = Z_0^2 + iZ_0^3, \qquad z_1 = Z_1^2 + iZ_1^3. \tag{2.32}$$

We see that there is a discontinuity only for the σ variable.

2.4. Smooth coordinates. To eliminate $\delta(U)$ from the metric, we use a coordinate change analogous to one introduced in [39]:

$$\begin{aligned} w &= W, \\ \sigma &= \Sigma + H(\Upsilon, \overline{\Upsilon})\theta(W) + W\theta(W)H_\Upsilon H_{\overline{\Upsilon}}, \\ \zeta &= \Upsilon + W\theta(W)H_{\overline{\Upsilon}}. \end{aligned} \tag{2.33}$$

Here, $H_\Upsilon = \partial_\Upsilon H(\Upsilon, \overline{\Upsilon})$. In the new coordinates, we obtain the metric

$$ds^2 = \frac{-2\,dW\,d\Sigma + 2|d\Upsilon + W\theta(W)(H_{\Upsilon\overline{\Upsilon}}\,d\Upsilon + H_{\overline{\Upsilon}\,\overline{\Upsilon}}\,d\overline{\Upsilon})|^2}{[1 + \big(W\Sigma - \Upsilon\overline{\Upsilon} + W\theta(W)G\big)/(2a^2)]^2}, \tag{2.34}$$

where $H(\Upsilon, \overline{\Upsilon})$ depends on Υ and $\overline{\Upsilon}$ as $H(\zeta, \bar{\zeta})$ given by (2.8) depends on ζ and $\bar{\zeta}$ and

$$G = H - \Upsilon H_\Upsilon - \overline{\Upsilon} H_{\overline{\Upsilon}}. \tag{2.35}$$

We rewrite metric (2.34) as

$$ds^2 = 2g_{W\Sigma}\,dW\,d\Sigma + g_{\Upsilon\Upsilon}\,d\Upsilon\,d\Upsilon + 2g_{\Upsilon\overline{\Upsilon}}\,d\Upsilon\,d\overline{\Upsilon} + g_{\overline{\Upsilon}\,\overline{\Upsilon}}\,d\overline{\Upsilon}\,d\overline{\Upsilon}, \tag{2.36}$$

where

$$g_{W\Sigma} = -\frac{1}{\mathcal{N}}, \tag{2.37}$$

$$g_{\Upsilon\Upsilon} = \frac{2}{\mathcal{N}}\theta(W)WH_{\Upsilon\Upsilon}(1 + WH_{\Upsilon\overline{\Upsilon}}), \tag{2.38}$$

$$\begin{aligned} g_{\Upsilon\overline{\Upsilon}} &= \frac{1}{\mathcal{N}}\big[\big(1 + \theta(W)WH_{\Upsilon\overline{\Upsilon}}\big)\big(1 + \theta(W)WH_{\overline{\Upsilon}\Upsilon}\big) + \theta(W)W^2 H_{\Upsilon\Upsilon}H_{\overline{\Upsilon}\,\overline{\Upsilon}}\big] = \\ &= \frac{1}{\mathcal{N}}[1 + 2\theta(W)WH_{\Upsilon\overline{\Upsilon}} + \theta(W)W^2(H^2_{\overline{\Upsilon}\Upsilon} + H_{\Upsilon\Upsilon}H_{\overline{\Upsilon}\,\overline{\Upsilon}})], \end{aligned} \tag{2.39}$$

$$g_{\overline{\Upsilon}\,\overline{\Upsilon}} = \frac{2}{\mathcal{N}}\theta(W)WH_{\overline{\Upsilon}\,\overline{\Upsilon}}(1 + WH_{\overline{\Upsilon}\Upsilon}). \tag{2.40}$$

Here,

$$\mathcal{N} = \left[1 + \frac{1}{2a^2}\big(W\Sigma - \overline{\Upsilon}\Upsilon + \theta(W)W(H - H_\Upsilon\Upsilon - H_{\overline{\Upsilon}}\overline{\Upsilon}\,)\big)\right]^2. \tag{2.41}$$

2.5. Two shock waves. In the case of two shock waves, we have the metric

$$ds^2 = 2g^{(2)}_{W\Sigma}\,dW\,d\Sigma + g^{(2)}_{\Upsilon\Upsilon}\,d\Upsilon\,d\Upsilon + 2g^{(2)}_{\Upsilon\overline{\Upsilon}}\,d\Upsilon\,d\overline{\Upsilon} + g^{(2)}_{\overline{\Upsilon}\,\overline{\Upsilon}}\,d\overline{\Upsilon}\,d\overline{\Upsilon}, \tag{2.42}$$

where

$$g^{(2)}_{W\Sigma} = -\frac{1}{\mathcal{N}^{(2)}}, \tag{2.43}$$

$$g^{(2)}_{\Upsilon\Upsilon} = \frac{2H_{\Upsilon\Upsilon}}{\mathcal{N}^{(2)}}[\theta(W)W(1 + WH_{\Upsilon\overline{\Upsilon}}) + \theta(\Sigma)\Sigma(1 + \Sigma H_{\Upsilon\overline{\Upsilon}})], \tag{2.44}$$

$$g^{(2)}_{\Upsilon\overline{\Upsilon}} = \frac{1}{\mathcal{N}^{(2)}}\big[1 + 2\big(\theta(W)W + \theta(\Sigma)\Sigma\big)H_{\Upsilon\overline{\Upsilon}} + \big(\theta(W)W^2 + \theta(\Sigma)\Sigma^2\big)(H^2_{\overline{\Upsilon}\Upsilon} + H_{\Upsilon\Upsilon}H_{\overline{\Upsilon}\,\overline{\Upsilon}})\big], \tag{2.45}$$

$$g^{(2)}_{\overline{\Upsilon}\,\overline{\Upsilon}} = \frac{2H_{\overline{\Upsilon}\,\overline{\Upsilon}}}{\mathcal{N}^{(2)}}[\theta(W)W(1 + WH_{\overline{\Upsilon}\Upsilon}) + \theta(\Sigma)\Sigma(1 + \Sigma H_{\overline{\Upsilon}\Upsilon})]. \tag{2.46}$$

Here,

$$\mathcal{N}^{(2)} = \left[1 + \frac{1}{2a^2}\big(W\Sigma - \overline{\Upsilon}\Upsilon + \big(\theta(W)W + \theta(\Sigma)\Sigma\big)(H - H_\Upsilon\Upsilon - H_{\overline{\Upsilon}}\overline{\Upsilon})\big)\right]^2. \tag{2.47}$$

This form of the metric is correct for three space–time regions: $(\Sigma < 0, W < 0)$, $(\Sigma < 0, W > 0)$, and $(\Sigma > 0, W < 0)$. It should be modified for $(\Sigma > 0, W > 0)$ because of the gravitation interaction (finding a modification of the metric in the fourth region $(\Sigma > 0, W > 0)$ was recently attempted in [40]). In what follows, we present the main steps in deriving the expression for the trapped surface in the space–time region where formula (2.42) is applicable.

2.6. Derivation of the equation for the trapped surface. The trapped surface in the four-dimensional space–time is a two-dimensional closed spacelike surface determined by the condition that the convergence of null geodesics on it is equal to zero. We recall that the convergence of geodesics passing through a given surface is given by [17], [41], [42]

$$\theta = h^{MN}\nabla_M \xi_N, \tag{2.48}$$

where ξ_N is a tangent vector to a geodesic, ∇_M is the covariant derivative, and h^{MN} is a four-dimensional tensor related to the two-dimensional metric induced on the surface (the exact definition is given below).

In coordinate system (2.33), the trapped surface whose explicit expression we seek has two parts. We let $\mathcal{S}_1$ and $\mathcal{S}_2$ denote those two parts of the trapped surface in the respective regions $\Sigma < 0$ and $W < 0$. They are defined in terms of the two functions $\Psi_1(\Upsilon, \overline{\Upsilon})$ and $\Psi_2(\Upsilon, \overline{\Upsilon})$ as

$$\mathcal{S}_1\colon \begin{cases} W = 0, \\ \Sigma = -\Psi_1(\Upsilon, \overline{\Upsilon}), \end{cases} \qquad \mathcal{S}_2\colon \begin{cases} \Sigma = 0, \\ W = -\Psi_2(\Upsilon, \overline{\Upsilon}), \end{cases} \tag{2.49}$$

with the additional boundary conditions at the shock wave intersection $\mathcal{C} \subset \{W = \Sigma = 0\}$

$$\mathcal{S}_1|_{\mathcal{C}} = \mathcal{S}_2|_{\mathcal{C}} \tag{2.50}$$

and

$$\partial_{\vec{n}}\mathcal{S}_1|_{\mathcal{C}} = \partial_{\vec{n}}\mathcal{S}_2|_{\mathcal{C}}, \tag{2.51}$$

where $\partial_{\vec{n}}$ denotes the normal derivative to the surface.

Condition (2.50) means that

$$\Psi_1(\Upsilon, \overline{\Upsilon})\big|_{\mathcal{C}} = 0, \qquad \Psi_2(\Upsilon, \overline{\Upsilon})\big|_{\mathcal{C}} = 0, \tag{2.52}$$

and a consequence of (2.51) is given in explicit form below (see (2.73)).

Because $\mathcal{S}_1$ and $\mathcal{S}_2$ are in the respective regions $W < 0$ and $\Sigma < 0$, we also have $\Psi_1(\Upsilon, \overline{\Upsilon}) > 0$ and $\Psi_2(\Upsilon, \overline{\Upsilon}) > 0$.

We now derive equations for the two functions $\Psi_1(\Upsilon, \overline{\Upsilon})$ and $\Psi_2(\Upsilon, \overline{\Upsilon})$ determined by the condition that the surface they define is marginally trapped [17], [41], [42], i.e., that the convergence of outgoing null geodesics orthogonal to the surface has zero expansion. Our calculations are very close to the corresponding calculations in the review section in [21].

The null geodesics passing through the trapped surface can be specified by the tangent vectors ξ with the components for $W < 0$

$$\xi^W = w_1 = 1 - \frac{\Upsilon\overline{\Upsilon}}{2a^2}, \qquad \xi^\Sigma = \sigma_1, \qquad \xi^\Upsilon = \zeta_1, \qquad \xi^{\overline{\Upsilon}} = \bar{\zeta}_1. \tag{2.53}$$

Because of the isotropy condition $g_{MN}\xi^M\xi^N = 0$ for the geodesics, we obtain

$$2g_{W\Sigma}\sigma_1 w_1 + g_{\Upsilon\Upsilon}\zeta_1^2 + 2g_{\Upsilon\overline{\Upsilon}}\zeta_1\bar{\zeta}_1 + g_{\overline{\Upsilon}\,\overline{\Upsilon}}\bar{\zeta}_1^2 = 0. \tag{2.54}$$

This condition fixes the parameter σ_1:

$$\sigma_1 = -\frac{1}{2g_{W\Sigma}w_1}(g_{\Upsilon\Upsilon}\zeta_1^2 + 2g_{\Upsilon\overline{\Upsilon}}\zeta_1\bar{\zeta}_1 + g_{\overline{\Upsilon}\,\overline{\Upsilon}}\bar{\zeta}_1^2). \tag{2.55}$$

We also suppose that ξ is orthogonal to the surface $\mathcal{S}_1$, i.e.,

$$(\xi, K_a) = 0, \qquad a = \zeta, \bar{\zeta}, \tag{2.56}$$

where K_a^M are two independent four-component vectors tangent to the surface,

$$K_a^M = (0, -\partial_a \Psi, \delta_a^b). \tag{2.57}$$

This leads to

$$-g_{W\Sigma} \partial_a \Psi w_1 + g_{ab} \zeta^b = 0. \tag{2.58}$$

We hence easily obtain

$$\zeta^a = w_1 g^{ab} g_{W\Sigma} \partial_b \Psi. \tag{2.59}$$

For ξ_M, we have

$$\xi_M = g_{MN} \xi^N, \tag{2.60}$$

where

$$\xi^W = w_1, \tag{2.61}$$

$$\xi^\Sigma = \sigma_1 = -\frac{1}{2} g_{W\Sigma} w_1 (\partial_\Upsilon \Psi g^{\Upsilon\Upsilon} \partial_\Upsilon \Psi + 2 \partial_{\overline{\Upsilon}} \Psi g^{\overline{\Upsilon}\Upsilon} \partial_\Upsilon \Psi + \partial_{\overline{\Upsilon}} \Psi g^{\overline{\Upsilon}\,\overline{\Upsilon}} \partial_{\overline{\Upsilon}} \Psi), \tag{2.62}$$

$$\xi^\Upsilon = \zeta_1 = w_1 g_{W\Sigma} (g^{\Upsilon\Upsilon} \partial_\Upsilon \Psi + g^{\Upsilon\overline{\Upsilon}} \partial_{\overline{\Upsilon}} \Psi), \tag{2.63}$$

$$\xi^{\overline{\Upsilon}} = \bar{\zeta}_1 = w_1 g_{W\Sigma} (g^{\overline{\Upsilon}\Upsilon} \partial_\Upsilon \Psi + g^{\overline{\Upsilon}\,\overline{\Upsilon}} \partial_{\overline{\Upsilon}} \Psi). \tag{2.64}$$

Here, we can use an approximate expression for the metric. In the matrix notation up to the first order in W, we have

$$\begin{pmatrix} 0 & g_{W\Sigma} & 0 & 0 \\ g_{\Sigma W} & 0 & 0 & 0 \\ 0 & 0 & g_{\Upsilon\Upsilon} & g_{\Upsilon\overline{\Upsilon}} \\ 0 & 0 & g_{\overline{\Upsilon}\Upsilon} & g_{\overline{\Upsilon}\,\overline{\Upsilon}} \end{pmatrix} \approx \frac{1}{\mathcal{N}} \begin{pmatrix} 0 & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 2H_{\Upsilon\Upsilon}\theta(W)W & 1 + 2H_{\Upsilon\overline{\Upsilon}}\theta(W)W \\ 0 & 0 & 1 + 2H_{\Upsilon\overline{\Upsilon}}\theta(W)W & 2H_{\overline{\Upsilon}\,\overline{\Upsilon}}\theta(W)W \end{pmatrix}.$$

Hence, at the point $W = 0$, we obtain

$$\xi_M = -\left(\frac{\partial_\Upsilon \Psi \partial_{\overline{\Upsilon}} \Psi}{1 - \Upsilon\overline{\Upsilon}/(2a^2)}, \frac{1}{1 - \Upsilon\overline{\Upsilon}/(2a^2)}, \frac{\partial_\Upsilon \Psi}{1 - \Upsilon\overline{\Upsilon}/(2a^2)}, \frac{\partial_{\overline{\Upsilon}} \Psi}{1 - \Upsilon\overline{\Upsilon}/(2a^2)} \right), \tag{2.65}$$

$$h^{MN} = K_a^M K_b^N g^{ab} = \mathcal{N} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -2\partial_\Upsilon \Psi \partial_{\overline{\Upsilon}} \Psi & \partial_{\overline{\Upsilon}} \Psi & \partial_\Upsilon \Psi \\ 0 & \partial_{\overline{\Upsilon}} \Psi & 0 & -1 \\ 0 & \partial_\Upsilon \Psi & -1 & 0 \end{pmatrix}. \tag{2.66}$$

The components of the corresponding connection are

$$\Gamma^1_{11} = -\frac{\partial_\Sigma \mathcal{N}}{\mathcal{N}}, \qquad \Gamma^1_{12} = -\frac{\partial_\Upsilon \mathcal{N}}{2\mathcal{N}}, \qquad \Gamma^2_{12} = -\frac{\partial_\Sigma \mathcal{N}}{2\mathcal{N}}, \tag{2.67}$$

$$\Gamma^1_{22} = \frac{H_{\Upsilon\Upsilon}}{2}, \qquad \Gamma^2_{22} = -\frac{\partial_\Upsilon \mathcal{N}}{\mathcal{N}}, \qquad \Gamma^1_{13} = -\frac{\partial_{\overline{\Upsilon}} \mathcal{N}}{2\mathcal{N}}, \tag{2.68}$$

$$\Gamma^3_{13} = -\frac{\partial_\Sigma \mathcal{N}}{2\mathcal{N}}, \qquad \Gamma^1_{23} = -\frac{1}{2\mathcal{N}} (\partial_W \mathcal{N} - H_{\Upsilon\overline{\Upsilon}} \mathcal{N}), \qquad \Gamma^0_{23} = -\frac{\partial_\Sigma \mathcal{N}}{2\mathcal{N}}, \tag{2.69}$$

$$\Gamma^1_{33} = \frac{H_{\overline{\Upsilon}\,\overline{\Upsilon}}}{2}, \qquad \Gamma^3_{33} = -\frac{\partial_{\overline{\Upsilon}} \mathcal{N}}{\mathcal{N}}. \tag{2.70}$$

Substituting them in (2.48) and taking (2.66) into account, we obtain the zero-convergence equation (the equation for the trapped surface):

$$\left(1-\frac{\Upsilon\overline{\Upsilon}}{2a^2}\right)\partial^2_{\Upsilon\overline{\Upsilon}}(2\Psi-H)+\left(\frac{\Upsilon}{2a^2}\partial_\Upsilon+\frac{\overline{\Upsilon}}{2a^2}\partial_{\overline{\Upsilon}}\right)(2\Psi-H)-\frac{1}{2a^2}(2\Psi-H)=0. \tag{2.71}$$

The geometric meaning of this equation becomes clear if we rewrite (2.71) in the form

$$\left(\Delta_{\mathbb{H}^2}-\frac{2}{a^2}\right)\left(\frac{\Psi-\kappa H}{1-\rho^2/(2a^2)}\right)=0, \tag{2.72}$$

where $\rho^2=\Upsilon\overline{\Upsilon}$, $\Delta_{\mathbb{H}^2}$ on the shock wave is given by (2.12) ($\Upsilon=\zeta$ and $\overline{\Upsilon}=\bar{\zeta}$ on the shock wave), and $\kappa=1/2$ (this particular value of κ is related to our choice $\theta(0)=1/2$).

We recall that the shape function H of the gravitational shock wave divided by the factor $1-\rho^2/(2a^2)$, $F=2H/\bigl(1-\rho^2/(2a^2)\bigr)$, is the fundamental solution of Eq. (2.11). This means that the trapped-surface function Ψ also divided by the same factor $1-\rho^2/(2a^2)$ up to a solution to the homogeneous version of Eq. (2.4) is a fundamental solution of this equation.

We can easily see that condition (2.51) can be represented in the form

$$\partial_\Upsilon\Psi_1\partial_{\overline{\Upsilon}}\Psi_2\big|_{\mathcal{C}}=1. \tag{2.73}$$

Indeed, this boundary condition can be written in terms of the vectors $\vec{\xi}$ as

$$\vec{\xi}_1=\vec{\xi}_2 \tag{2.74}$$

because the vector $\vec{\xi}$ tangent to the geodesics is also normal to the trapped surface and

$$\xi_1^M=(1,\partial_\Upsilon\Psi\partial_{\overline{\Upsilon}}\Psi,-\partial_\Upsilon\Psi,-\partial_{\overline{\Upsilon}}\Psi)\left(1-\frac{\Upsilon\overline{\Upsilon}}{2a^2}\right), \tag{2.75}$$

$$\xi_2^M=(\partial_\Upsilon\Psi\partial_{\overline{\Upsilon}}\Psi,1,-\partial_\Upsilon\Psi,-\partial_{\overline{\Upsilon}}\Psi)\left(1-\frac{\Upsilon\overline{\Upsilon}}{2a^2}\right). \tag{2.76}$$

A sufficient condition for black hole formation in an asymptotically flat space–time is the existence of a marginally closed trapped surface at the hypersurface $\{W\le 0,\Sigma=0\}\cup\{W=0,\Sigma\le 0\}$ [9], [37], [36], [39], [43]. We note that there are no general theorems in non-asymptotically flat cases, but there is a widespread opinion that the existence of the marginally trapped surface can indicate creation of a a black hole.

3. The trapped surface in AdS_4 for head-on collisions

3.1. Solution of the trapped-surface equation. Here, we consider a head-on collision preserving the rotational symmetry around the axis of motion of massless particles, i.e., $O(2)$ symmetry in the D=4 case. Because of the $O(2)$ symmetry of the head-on collision, the functions $\Psi_1(\Upsilon,\overline{\Upsilon})$ and $\Psi_2(\Upsilon,\overline{\Upsilon})$ describing the trapped surface are identical and depend only on the parameter $\rho^2=\Upsilon\overline{\Upsilon}$: $\Psi_1(\Upsilon,\overline{\Upsilon})=\Psi_2(\Upsilon,\overline{\Upsilon})=\Psi(\rho^2)$. In this case, it is convenient to introduce the new function

$$\phi(\rho)=\frac{2\Psi(\rho)-H(\rho)}{1-\rho^2/(2a^2)}, \tag{3.1}$$

which depends only on ρ. Equation (2.71) then transforms into the ordinary differential equation

$$\left(1-\frac{\rho^2}{2a^2}\right)^2\left(\frac{\partial^2\phi}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial\phi}{\partial\rho}\right)-\frac{4\phi}{a^2}=0, \tag{3.2}$$

whose solution has the general form

$$\phi(\rho)=\frac{A(\rho^2+2a^2)+B\big((\rho^2+2a^2)\log\rho+4a^2\big)}{\rho^2-2a^2}. \tag{3.3}$$

Because we are interested in the regular solution of the homogenous equation, we must set $B=0$. We then obtain the expression

$$\Psi(\rho)=\sqrt{2}\,p\left(1-\frac{\rho^2}{2a^2}\right)\left(-2+\frac{2a^2+\rho^2}{2a^2-\rho^2}\log\left(\frac{2a^2}{\rho^2}\right)\right)-\frac{1}{2a^2}A(\rho^2+2a^2) \tag{3.4}$$

for the trapped surface function. As mentioned above, in the head-on case, we must solve the boundary problem

$$\Psi|_{\mathcal{C}}=0, \tag{3.5}$$

$$\partial_\Upsilon\Psi\partial_{\overline{\Upsilon}}\Psi|_{\mathcal{C}}=1, \tag{3.6}$$

for one function Ψ, which is a solution of the equation for the trapped surface.

It is obvious that the bound $\mathcal{C}$ of the sewing of identical components of the surface is a circle,

$$\rho=\rho_0=\mathrm{const}\,.$$

We thus obtain a system of two equations for the two constants A and ρ_0:

$$\sqrt{2}\,p\left(1-\frac{\rho_0^2}{2a^2}\right)\left(-2+\frac{2a^2+\rho_0^2}{2a^2-\rho_0^2}\log\left(\frac{2a^2}{\rho_0^2}\right)\right)-\frac{1}{2a^2}A(\rho_0^2+2a^2)=0, \tag{3.7}$$

$$\frac{1}{a^4\rho_0^2}\left(2\sqrt{2}\,pa^2-\rho_0^2(\sqrt{2}\,p-A)-\sqrt{2}\,p\rho_0^2\log\left(\frac{2a^2}{\rho_0^2}\right)\right)^2=4. \tag{3.8}$$

Substituting A from (3.7) in expression (3.4), we obtain

$$\Psi(\rho)=\sqrt{2}\,p\left(4\frac{\rho^2-\rho_0^2}{2a^2+\rho_0^2}+\left(1+\frac{\rho^2}{2a^2}\right)\log\left(\frac{\rho_0^2}{\rho^2}\right)\right). \tag{3.9}$$

To connect with the results in [37], it is instructive to rewrite the condition imposed on ρ as an equation in terms of the initial shape function F in the plane coordinates. By (2.4) and (3.4), we have

$$\Psi=\frac{1}{4}\left(1-\frac{\rho^2}{2a^2}\right)F(\rho)-\frac{1}{2a^2}\,A(\rho^2+2a^2). \tag{3.10}$$

Because this Ψ depends only on ρ, condition (3.6) can be rewritten as $d\Psi/d\rho=\pm 2$ (to avoid unphysical roots, we choose the minus sign):

$$\Psi'(\rho)=\frac{1}{4}\left(1-\frac{\rho^2}{2a^2}\right)F'(\rho)-\frac{\rho}{4a^2}F(\rho)-\frac{\rho}{a^2}A. \tag{3.11}$$

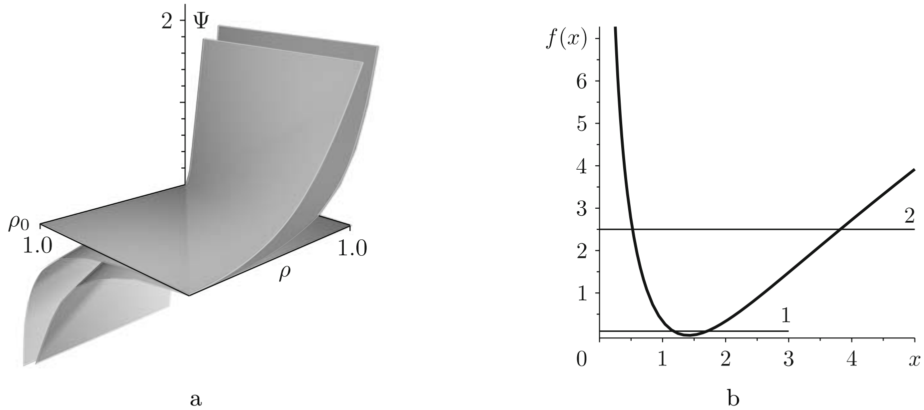

Fig. 3. (a) Plot of $\Psi(\rho)/(p\sqrt{2})$ for different values of ρ_0: this plot shows that $\Psi(\rho) > 0$ for $\rho < \rho_0$. (b) Plot of the function $f(x)$ defining the value of $\bar{\rho}_0$: line 1 corresponds to a small value of a/p, and line 2 corresponds to a large value of a/p.

On $\mathcal{C}$, we have

$$\Psi'(\rho)\big|_{\mathcal{C}} = \frac{1}{4}\left(1 - \frac{\rho_0^2}{2a^2}\right)F'(\rho_0) - \frac{\rho_0}{2a^2 + \rho_0^2}\,F(\rho_0), \tag{3.12}$$

and we obtain (3.6) in terms of the initial shape function,

$$\frac{1}{4}\left(1 - \frac{\rho_0^2}{2a^2}\right)F'(\rho_0) - \frac{\rho_0}{2a^2 + \rho_0^2}F(\rho_0) + 2 = 0. \tag{3.13}$$

We can define ρ_0 from requirement (3.13). We introduce a dimensionless parameter $\bar{\rho}_0 = \rho_0/a$. From Eqs. (3.9) and (3.13), we find that $\bar{\rho}_0$ satisfies the equation

$$f(\bar{\rho}_0) = \sqrt{2}\frac{a}{p}, \tag{3.14}$$

where

$$f(x) \equiv \frac{1}{x}\frac{(2 - x^2)^2}{2 + x^2}. \tag{3.15}$$

This equation has a solution for each value of the ratio a/p, and we therefore have no critical effect, unlike in a dS space–time. Because the point $\rho = \sqrt{2}\,a$ corresponds to spatial infinity (the dS horizon), we are interested in solutions in the interval $0 < \rho_0 < \sqrt{2}\,a$, and Eq. (3.14) has unique solution in this interval for a given a/p.

For a small ratio a/p (high-energy particles and/or a strongly curved space–time, which we call the high-energy limit), the solution of (3.14) is close to $\bar{\rho}_0 \sim \sqrt{2}$:

$$\rho_0^{\mathrm{HE}} = \sqrt{2}\,a - \sqrt{\frac{a^3}{p}}. \tag{3.16}$$

For a large ratio a/p (low-energy particles and/or a weakly curved space–time, which we call the low-energy limit), the solution of (3.14) is about $\bar{\rho}_0 \sim \sqrt{2}\,p/a$:

$$\rho_0^{\mathrm{LE}} \approx \sqrt{2}\,p. \tag{3.17}$$

3.2. Area of the trapped surface. We can now easily obtain the area of the trapped surface. Because the metric of the space–time with two shock waves is given by (2.42), the induced metric on half of the trapped surface (for definiteness, $W = 0$ and $\Sigma = -\Psi(\Upsilon, \overline{\Upsilon})$) is

$$g_{\alpha\beta} = \frac{1}{\mathcal{N}} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \tag{3.18}$$

Therefore, the whole area of the two parts of the trapped surface can be written in the form

$$\mathcal{A} = 2 \int_{\rho<\rho_0} 2\sqrt{|\det g_{\alpha\beta}|} 2\pi\rho\, d\rho = 2 \int_0^{\rho_0} \frac{4\pi\rho\, d\rho}{\mathcal{N}} = 8\pi a^2 \frac{\rho_0^2/(2a^2)}{1 - \rho_0^2/(2a^2)}. \tag{3.19}$$

For a large ratio a/p, as shown above, $\rho_0 \approx \sqrt{2}\, p$, i.e., ρ_0^2/a^2 is small, and

$$\mathcal{A} \approx 4\pi\rho_0^2 + 2\pi \frac{\rho_0^4}{a^2} \approx 8\pi p^2 \left(1 + \frac{p^2}{a^2} + \dots\right). \tag{3.20}$$

The first term here reproduces the result in the flat case.

For a small ratio a/p, we have $\rho_0 \approx \sqrt{2}\, a - \sqrt{a^3/p}$ and

$$\mathcal{A} \approx 4\pi a \sqrt{2pa}. \tag{3.21}$$

We can hence see that the presence of a small negative cosmological constant leads to a small increase in the area of the trapped surface formed during the collision of ultrarelativistic particles and correspondingly, perhaps, to an increase in the cross section of black hole creation.

3.3. Comparison with recent results. An equation describing the trapped surface in an AdS space–time was also recently presented in [14], [37], [44], where mainly the Poincaré coordinates were used. It is helpful to compare our result and the result in [37] and to demonstrate their equivalence. For this, we must show that the equations for the trapped surface and also the boundary conditions obtained here and in [37] coincide.

We recall the relation between plane coordinates (2.1) and the Poincaré coordinates:

$$Z^0 = a \frac{t}{\sqrt{2}\, z}, \tag{3.22}$$

$$Z^1 = a \frac{x^1}{\sqrt{2}\, z}, \tag{3.23}$$

$$Z^2 = a \frac{x^2}{z}, \tag{3.24}$$

$$Z^3 = \frac{z}{2} \left(-1 + \frac{a^2 - \vec{x}^2 + t^2}{z^2}\right), \tag{3.25}$$

$$Z^4 = \frac{z}{2} \left(1 + \frac{a^2 + \vec{x}^2 - t^2}{z^2}\right). \tag{3.26}$$

The shock wave located at an intersection of hyperboloid (2.1) and one of the two planes $Z^0 \pm Z^1 = 0$ is located at $x^\pm \equiv t \pm x^1 = 0$ in the Poincaré coordinates. Metric (2.2) can be rewritten in the new coordinates as

$$ds^2 = ds^2_{\mathrm{AdS}_4} + \frac{a}{z} \Phi_-(z, x^2) \delta(x^-) dx^{-2}, \tag{3.27}$$

where the function $\Phi_-(z, x^2)$ is the solution of the equation

$$\left(\Delta_{\mathbb{H}^2} - \frac{2}{a^2}\right)\Phi_-(z, x^2) = -16\pi G_4 p \delta(z-a)\delta(x^2) \tag{3.28}$$

and the Beltrami–Laplace operator $\Delta_{\mathbb{H}^2}$ on the hyperboloid H^2 in the coordinates (x^2, z) can be obtained from the metric on H^2,

$$ds^2_{\mathbb{H}^2} = \frac{a^2}{z^2}\big((dx^2)^2 + dz^2\big), \tag{3.29}$$

and has the form

$$\Delta_{\mathbb{H}^2} = \frac{z^2}{a^2}\frac{\partial^2}{\partial z^2} + \frac{z^2}{a^2}\left(\frac{\partial}{\partial x^2}\right)^2. \tag{3.30}$$

The authors of [37], [44] used the chordal distance coordinate, i.e., the distance in the embedding space metric between a chosen point (x^2, z) and the fixed point $(0, a)$,

$$q \equiv \frac{(z-a)^2 + x_2^2}{4az}. \tag{3.31}$$

Metric (3.29) in terms of these coordinates is

$$ds^2 = a^2\left(\frac{dq^2}{q(q+1)} + 4q(q+1)d\varphi^2\right). \tag{3.32}$$

Here, we introduce the angle variable $\tan\varphi = -Z_3/Z_2$. The Beltrami–Laplace operator $\Delta_{\mathbb{H}^2}$ can be written as

$$\Delta_{\mathbb{H}^2} = \frac{1}{a^2}\left(q(q+1)\frac{\partial^2}{\partial q^2} + (1+2q)\frac{\partial}{\partial q} + \frac{1}{4q(q+1)}\frac{\partial^2}{\partial \varphi^2}\right). \tag{3.33}$$

The chordal distance q is related to the parameter ρ by

$$q = \frac{\rho^2/(2a^2)}{1-\rho^2/(2a^2)}. \tag{3.34}$$

Using (3.34), we can write boundary condition (3.13) in terms of q_0 corresponding to ρ_0:

$$F'(q_0) - \frac{2}{1+2q_0}F(q_0) + \frac{8a}{\sqrt{2q_0(1+q_0)}} = 0. \tag{3.35}$$

We can now easily see that this equation up to a difference in the coefficients coincides with the similar equation previously obtained in [37]. This difference results from a different convention for $\theta(0)$. If we set $\theta(0) = 1$ and rescale the shape function $F(q) = \sqrt{2}\,\Phi(q)$, which is required by the difference in the coordinate systems, then we obtain:

$$\Phi'(q_0) - \frac{2}{1+2q_0}\Phi(q_0) + \frac{2a}{\sqrt{q_0(1+q_0)}} = 0, \tag{3.36}$$

which exactly reproduces the result in [37].

We now note that if we use an arbitrary regularization $\theta(0) = \kappa$, then we obtain

$$\left(\Delta_{\mathbb{H}^2} - \frac{2}{a^2}\right)\left(\frac{\Psi - \kappa H}{1-\rho^2/(2a^2)}\right) = 0 \tag{3.37}$$

instead of Eq. (2.71). It is easy to understand how the area of the trapped surface depends on κ. If $\theta(0) = \kappa$, then Eq. (3.14) changes to

$$f(\rho_0) = \frac{a}{\sqrt{2}\, p\kappa}, \tag{3.38}$$

which leads to

$$\rho_0^{\mathrm{LE}} \approx 2\sqrt{2}\, p\kappa, \tag{3.39}$$

$$\rho_0^{\mathrm{HE}} \approx \sqrt{2}\, a - \sqrt{\frac{a^3}{2p\kappa}}\,. \tag{3.40}$$

For the area of the trapped surface, we obtain

$$\mathcal{A}_{\mathrm{LE}} = \frac{8\pi a^2 \rho_0^2}{2a^2 - \rho_0^2} \approx 4\pi\rho_0^2 + 2\pi\frac{\rho_0^4}{a^2} \approx 32\pi p^2\kappa^2\left(1 + \kappa^2\frac{p^2}{a^2}\right) \tag{3.41}$$

in the first case and

$$\mathcal{A}_{\mathrm{HE}} = \frac{8\pi a^2 \rho_0^2}{2a^2 - \rho_0^2} \approx \frac{4\sqrt{2}\,\pi a^3}{\sqrt{2}\, a - \rho_0} \approx 8\pi\sqrt{\kappa p a^3} \tag{3.42}$$

in the second case. We can hence see that in both energy regimes, the influence of the parameter κ, depending on the kind of regularization of the initial metric singularity, is very significant. Only a detailed analysis of this problem can indicate the physically reasonable choice.

4. Concluding remarks

We have studied the process of forming marginally trapped surfaces during the head-on collision of two shock waves in the AdS_4 space–time and established the dependence of the physical characteristics of the surface on the ratio of a and p. A multiplicative correction to the area of the trapped surface (compared with the flat case) depends on that ratio in a scaling way. In contrast to the dS_4 case, there is no criticality in the AdS_4 case. We noted that our result differs from the previous results by a certain numerical factor, which results from the choice of the type of regularization of the initial singularity of the wave metric. A more detailed study of regularization requires further consideration and will be the subject of a separate paper.

Acknowledgments. One of the authors (I. Ya. A.) is grateful to E. Guseva and I. Volovich for the fruitful discussions.

This work is supported in part by the Russian Foundation for Basic Research (Grant No. 08-01-00798), the Federal Agency for Science and Innovations (Contract No. 02.740.11.5057), the Program for Supporting Leading Scientific Schools (Grant No. NSh-795.2008.1, I. Ya. A.), and the Russian Academy of Sciences program "Nonlinear Dynamics" (I. Ya. A.).

REFERENCES

1. G. 't Hooft, "Gravitational collapse and particle physics," in: *6th Topological Workshop on Proton–Antiproton Collider Physics* (Aachen, Germany, June 30–July 4, 1986, K. Eggert, H. Faissner, and E. Radermacher, eds.), World Scientific, Singapore (1987), pp. 669–688; *Phys. Lett. B*, **198**, 61–63 (1987); *Nucl. Phys. B*, **304**, 867–876 (1988).
2. D. Amati, M. Ciafaloni, and G. Veneziano, *Phys. Lett. B*, **197**, 81–88 (1987); **216**, 41–47 (1989); **289**, 87–91 (1992); *Internat. J. Mod. Phys. A*, **3**, 1615–1661 (1988); *Nucl. Phys. B*, **347**, 550–580 (1990).

3. I. Ya. Aref'eva, K. S. Viswanathan, and I. V. Volovich, *Nucl. Phys. B*, **452**, 346–366 (1995); Erratum, **462**, 613–617 (1996); arXiv:hep-th/9412157v1 (1994); *Internat. J. Mod. Phys. D*, **5**, 707–721 (1996); arXiv:hep-th/9512170v1 (1995).
4. N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys.Lett. B*, **429**, 263–272 (1998); arXiv:hep-ph/9803315v1 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. R. Dvali, *Phys. Lett. B*, **436**, 257–263 (1998); arXiv:hep-ph/9804398v1 (1998).
5. G. F. Giudice, R. Rattazzi, and J. D. Wells, *Nucl. Phys. B*, **544**, 3–38 (1999); arXiv:hep-ph/9811291v2 (1998); **630**, 293–325 (2002); arXiv:hep-ph/0112161v2 (2001).
6. T. Banks and W. Fischler, "A model for high energy scattering in quantum gravity," arXiv:hep-th/9906038v1 (1999).
7. I. Ya. Aref'eva, *Phys. Atomic Nuclei*, **31**, No. 7a, 169–180 (2000); arXiv:hep-th/9910269v1 (1999).
8. S. Dimopoulos and G. Landsberg, *Phys. Rev. Lett.*, **87**, 161602 (2001); arXiv:hep-ph/0106295v1 (2001).
9. S. B. Giddings and S. Thomas, *Phys. Rev. D*, **65**, 056010 (2002); arXiv:hep-ph/0106219v4 (2001); D. M. Eardley and S. B. Giddings, *Phys. Rev. D*, **66**, 044011 (2002); arXiv:gr-qc/0201034v2 (2002).
10. A. Ringwald and H. Tu, *Phys. Lett. B*, **525**, 135–142 (2002); arXiv:hep-ph/0111042v2 (2001); E.-J. Ahn, M. Cavaglià, and A. V. Olinto, *Phys. Lett. B*, **551**, 1–6 (2003); arXiv:hep-th/0201042v2 (2002); S. N. Solodukhin, *Phys. Lett. B*, **533**, 153–161 (2002); arXiv:hep-ph/0201248v2 (2002); E. Kohlprath and G. Veneziano, *JHEP*, **0206**, 057 (2002); arXiv:gr-qc/0203093v2 (2002); H. Yoshino and Ya. Nambu, *Phys. Rev. D*, **67**, 024009 (2003); arXiv:gr-qc/0209003v1 (2002); O. I. Vasilenko, "Trap surface formation in high-energy black holes collision," arXiv:hep-th/0305067v2 (2003); H. Yoshino and V. S. Rychkov, *Phys. Rev. D*, **71**, 104028 (2005); Erratum, **77**, 089905 (2008); arXiv:hep-th/0503171v2 (2005); M. Cavaglià, *Internat. J. Mod. Phys. A*, **18**, 1843–1882 (2003); arXiv:hep-ph/0210296v1 (2002); P. Kanti, *Internat. J. Mod. Phys. A*, **19**, 4899–4951 (2004); arXiv:hep-ph/0402168v2 (2004); S. B. Giddings and V. S. Rychkov, *Phys. Rev. D*, **70**, 104026 (2004); arXiv:hep-th/0409131v1 (2004); V. Cardoso, E. Berti, and M. Cavaglià, *Class. Q. Grav.*, **22**, L61–L69 (2005); arXiv:hep-ph/0505125v2 (2005); G. L. Landsberg, *J. Phys. G*, **32**, R337–R365 (2006); arXiv:hep-ph/0607297v1 (2006); D. M. Gingrich, *Internat. J. Mod. Phys. A*, **21**, 6653–6676 (2006); arXiv:hep-ph/0609055v2 (2006); H. Stöcker, *J. Phys. G*, **32**, S429–S437 (2006); B. Koch, M. Bleicher, and H. Stöcker, *J. Phys. G*, **34**, S535–S542 (2007); arXiv:hep-ph/0702187v2 (2007); N. Kaloper and J. Terning, *Internat. J. Mod. Phys. D*, **17**, 665–672 (2008); *Gen. Relativity Gravitation*, **39**, 1525–1532 (2007); arXiv:0705.0408v1 [hep-th] (2008); M. Cavaglià, R. Godang, L. M. Cremaldi, and D. J. Summers, *JHEP*, **0706**, 055 (2007); arXiv:0707.0317v1 [hep-ph] (2007); P. Meade and L. Randall, *JHEP*, **0805**, 003 (2008); arXiv:0708.3017v1 [hep-ph] (2007); S. B. Giddings, "High-energy black hole production," in: *Particles, Strings, and Cosmology* (AIP Conf. Proc., Vol. 957, A. Rajantie, P. Dauncey, C. Contaldi, and H. Stoica, eds.), Vol. 1, Amer. Inst. Phys., Melville, N. Y. (2007), pp. 69–78; arXiv:0709.1107v3 [hep-ph] (2007).
11. I. Ya. Aref'eva and I. V. Volovich, *Int. J. Geom. Meth. Mod. Phys.*, **5**, 641–651 (2008); arXiv:0710.2696v2 [hep-ph] (2007).
12. A. Mironov, A. Morozov, and T. N. Tomaras, *Facta Univ. Ser. Phys. Chem. Tech.*, **4**, 381–404 (2006); arXiv:0710.3395v1 [hep-th] (2007).
13. P. Nicolini and E. Spallucci, "Noncommutative geometry inspired wormholes and dirty black holes," arXiv:0902.4654v1 [gr-qc] (2009).
14. L. Alvarez-Gaume, C. Gomez, A. S. Vera, A. Tavanfar, and M. A. Vazquez-Mozo, *JHEP*, **0902**, 009 (2009); arXiv:0811.3969v1 [gr-qc] (2008).
15. I. Aref'eva, A. Bagrov, and E. Guseva, "Critical formation of trapped surfaces in the collision of non-expanding gravitational shock waves in de Sitter space–time," arXiv:0905.1087v3 [hep-th] (2009).
16. S. W. Hawking and R. Penrose, *Proc. Roy. Soc. London Ser. A*, **314**, 529–548 (1970).
17. S. W. Hawking and G. R. F. Ellis, *The Large Scale Structure of Space–Time* (Cambridge Monogr. Math. Phys., Vol. 1), Cambridge Univ. Press, London (1973).
18. G. W. Gibbons, *Class. Q. Grav.*, **16**, 1677–1687 (1999); arXiv:hep-th/9809167v1 (1998).
19. R. Penrose, *J. Astrophys. Astron.*, **20**, 233–248 (1999).
20. H. Nastase, "On high energy scattering inside gravitational backgrounds," arXiv:hep-th/0410124v2 (2004).

21. H. Nastase, “The RHIC fireball as a dual black hole,” arXiv:hep-th/0501068v3 (2005).
22. E. Shuryak, S.-J. Sin, and I. Zahed, *J. Korean Phys. Soc.*, **50**, 384–397 (2007); arXiv:hep-th/0511199v3 (2005).
23. A. J. Amsel, D. Marolf, and A. Virmani, *JHEP*, **0804**, 025 (2008); arXiv:0712.2221v1 [hep-th] (2007).
24. D. Grumiller and P. Romatschke, *JHEP*, **0808**, 027 (2008); arXiv:0803.3226v3 [hep-th] (2008).
25. P. C. Aichelburg and R. U. Sexl, *Gen. Relativity Gravitation*, **2**, 303–312 (1971).
26. T. Dray and G. ’t Hooft, *Nucl. Phys. B*, **253**, 173–188 (1985).
27. V. Ferrari, P. Pendenza, and G. Veneziano, *Gen. Relativity Gravitation*, **20**, 1185–1191 (1988).
28. M. Hotta and M. Tanaka, *Class. Q. Grav.*, **10**, 307–314 (1993).
29. K. Sfetsos, *Nucl. Phys. B*, **436**, 721–745 (1995); arXiv:hep-th/9408169v3 (1994).
30. J. Podolský and J. B. Griffiths, *Class. Q. Grav.*, **15**, 453–463 (1998); arXiv:gr-qc/9710049v1 (1997).
31. G. T. Horowitz and N. Itzhaki, *JHEP*, **9902**, 010 (1999); arXiv:hep-th/9901012v2 (1999).
32. J. Podolský and J. B. Griffiths, *Phys. Lett. A*, **261**, 1–4 (1999); arXiv:gr-qc/9908008v1 (1999); J. Podolský, “Exact impulsive gravitational waves in spacetimes of constant curvature,” arXiv:gr-qc/0201029v1 (2002).
33. J. Podolský and M. Ortaggio, *Class. Q. Grav.*, **18**, 2689–2706 (2001); arXiv:gr-qc/0105065v1 (2001).
34. R. Emparan, *Phys. Rev. D*, **64**, 024025 (2001); arXiv:hep-th/0104009v2 (2001).
35. G. Esposito, R. Pettorino, and P. Scudellaro, *Int. J. Geom. Meth. Mod. Phys*, **4**, 361–372 (2007); arXiv:gr-qc/0606126v3 (2006).
36. K. Kang and H. Nastase, *Phys. Rev. D*, **72**, 106003 (2005); arXiv:hep-th/0410173v2 (2004); **71**, 124035; arXiv:hep-th/0409099v2 (2004).
37. S. S. Gubser, S. S. Pufu, and A. Yarom, *Phys. Rev. D*, **78**, 066014 (2008); arXiv:0805.1551v1 [hep-th] (2008).
38. J. B. Griffiths, *Colliding Plane Waves in General Relativity*, Clarendon, New York (1991).
39. P. D. D’Eath and P. N. Payne, *Phys. Rev. D*, **46**, 658–674, 675–693, 694–701 (1992).
40. D. Grumiller and P. Romatschke, *JHEP*, **0808**, 027 (2008); arXiv:0803.3226v3 [hep-th] (2008).
41. R. M. Wald, *General Relativity*, Univ. of Chicago Press, Chicago (1984).
42. E. Poisson, *A Relativist’s Toolkit: The Mathematics of Black-Hole Mechanics*, Cambridge Univ. Press, Cambridge (2004).
43. R. Penrose, “Results presented at the Cambridge University seminar,” unpublished (1974).
44. S. S. Gubser, S. S. Pufu, and A. Yarom, “Off-center collisions in AdS_5 with applications to multiplicity estimates in heavy-ion collisions,” arXiv:0902.4062v1 [hep-th] (2009).