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ORIGINAL RESEARCH

Powered properties, modal continuity, and the patchwork principle

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Abstract
The principle of modal continuity has become an increasingly popular bit of modal epistemology, featuring prominently in debates about mereology, value, causation, and theism. It claims, roughly, that degreed properties are modally unified. So, if the property of being three inches tall is exemplifiable, so is the property of being four inches tall, and five inches tall, etc. Despite its plausibility, in this paper I show that there is a class of counterexamples to modal continuity: what I call 'powered properties.' More surprisingly, I show that an instance of these powered properties is entailed by another widely popular family of modal principles: the Lewisian patchwork principles, also known as cut-and-paste, or recombination, principles. Thus, despite appearing to be similar, and motivated by plenitudinous intuitions about the nature of modality, it turns out that the continuity and recombination approaches to modality rely on crucially different pictures of plenitude.

Keywords Modal epistemology · Modal continuity · Patchwork principle · Plenitude

1 Introduction

Much has been said about how to construct principles that extend our modal knowledge. Authors have proposed that we can develop guides for what is possible by appealing to conceivability (Yablo, 1993), counterfactuals (Williamson, 2007), essences (Lowe, 2012), and patchwork, or cut-and-paste, principles (Lewis, 1986). More recently, the principle of modal continuity—developed most notably by Rasmussen (2014)—has gained traction. Modal continuity is built on the intuition that differences in degree do not make for modal differences. For instance, if it is possible

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for a cup to be 4 inches tall, continuity tells us it is also possible for a cup to be 5 inches tall, and 6 inches tall, and 7 inches tall, etc. It would be surprising if it were possible for a cup to be 4 inches tall, but metaphysically *impossible* for it to be, say, 8 inches tall.

The intuitiveness of modal continuity has led to its application in a number of areas, including debates about the possibility of an infinite past (Schmid & Malpass, 2023), value (Rasmussen, 2018), causal essentialism (Gibbs, 2018), mereological universalism (Rasmussen, 2014), arguments for theism (McIntosh, 2022), and much more.

In this paper, I show that there is a rather large class of properties to which modal continuity does not apply, which I have termed *powered properties*. Even more interestingly (and troublingly!), one of the instances of these powered properties is entailed by the Lewisian patchwork principles. Thus, in addition to constructing counterexamples to modal continuity, this paper evinces a surprising tension between two *prima facie* similar approaches to modal epistemology: the continuity and cut-and-paste approaches. While both are motivated by what might be called "plenitudinous" intuitions about the nature of modality, it turns out that these principles rely on subtly different conceptions of plenitude—i.e., of what counts as a 'gap' in modal space.

I will proceed as follows. In § 2, I explicate more formally the principle of modal continuity. In § 3, I develop the notion of a powered property and in § 4 I show how it operates as a counterexample to modal continuity. In § 5, I show how the patchwork principle leads to a powered property and thus conflicts with continuity. A few objections are considered in § 6, before a brief concluding remark in § 7.

2 Modal continuity

In a single sentence, modal continuity claims that there is no unified class of degreed properties with a modal gap. I take this definition from Rasmussen (2014: 528-9), who defines 'unified class of degreed properties' and 'modal gap' as such:

> *C* is a unified class of degreed properties iff there is a transitive and asymmetric relation R such that (i) for all properties $x, y \in C$, either Rxy or Ryx, and (ii) property x is a finite distance from property y.

G is a modal gap in *C* iff *C* is a unified class of degreed properties such that (i) at least one member of *C* is exemplifiable, and (ii) *G* is a finite, proper subset of *C* such that no member of *G* is exemplifiable.

To put it another way, modal continuity is a claim about how degreed properties (properties that differ in mere degree, such as the properties *being 3 inches tall* and *being 4 inches tall*) relate to one another. In particular, the claim is that any *class* of degreed properties cannot have a modal gap: if one of the properties is exemplifiable, all of them must be. So, suppose I weigh three pounds. Then, modal continuity tells us that it is also possible I weigh four pounds, or five, etc., because the class of properties of the form *being n-pounds* is such that all of its members are exemplifiable.

It is important to note that Rasmussen develops continuity as a *defeasible* principle of modal epistemology, as it does not apply to *all* degreed properties. For instance, the principle, as stated herein, has to be refined to exclude blatantly inconsistent properties (e.g., *being 3 pounds and 4 pounds*) or properties that violate essential limitations (e.g., *being human and eating n pounds of food*). Thus, continuity comes with certain provisos, limiting what domains the principle applies to. However, once so-refined, continuity seems to be on firm ground *qua* modal principle, as evidenced by (i) its application in far-reaching metaphysical debates, and (ii) Rasmussen's (2014: 531) remark that the refined continuity principle "is a continuity principle that has no clear-cut exceptions."

However, the fact that continuity might still be regarded by some as defeasible means that we will have to contend with the possibility of the following challenge[1]: even if I am successful in arguing that certain classes of properties—what I shall call 'powered properties'—have modal gaps (and thus do not exemplify continuity), all this means is that modal continuity requires a *further* proviso, i.e., a proviso restricting the continuity principle's application to *non*-powered properties. Naturally, one might then wonder: what, exactly, is the upshot of the ensuing investigation between modal continuity and powered properties?

This is a valuable question, for it helps clarify the purpose of our investigation. Let us suppose that such a proviso would be well-motivated, not ad hoc, and not detract from the all-things-considered plausibility of the modal continuity principle. First, it is important to recognize that, even still, the value of the cases to be explored herein would be that they *reveal* us to a novel domain in which continuity does not apply. As we will see, whether a property counts as 'powered' depends on the *structure* of the entities in question. Thus, powered properties reveal that, when doing modal epistemology, we cannot freely apply modal continuity to any property *simply* because it is coherent (and does not involve the essence of particular individuals). We must pay attention to an additional factor as well, namely, whether the entities in the property's extension have a certain *structure*—a way of interacting with the world that is more common than one might initially realize. Thus, our investigation here is meant to shed *light* on the nature and applicability of modal continuity, irrespective of whether one takes the examples developed herein to be *counterexamples* to the truth of continuity, or instead as *indicating* a further domain of inapplicability. Indeed, I must confess that I myself am quite sympathetic to the application of modal continuity to certain classes of properties, such as *weighing n pounds*—my purpose is not to undermine *those* particular inferences. My goal is to show that there is an additional dimension, or factor, we must be attentive to in these contexts; one common enough that it is present in a range of extant metaphysical views, such as mereological universalism, and a particular finitism about space. This factor is even present in the application of *other* principles of modality, e.g., Lewisian patchwork principles.

Let's get on, then, with our investigation.

[1] I am indebted to an anonymous referee for revealing to me the importance of addressing this sort of challenge.

3 Powered properties

I begin first with a few preliminaries about the notion of a *powerset*. Let S be a set with *n*-many members. The powerset of S, written as *P*(S), is the set of all subsets of S. For instance, let S be the following set: {a, b, c}. *P*(S), then, is: {{}, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}}. Since, in most standard set theories, the empty set is a subset of every set (though not necessarily an element), *P*(S) includes the empty set. However, since this will be orthogonal to our purposes, I propose we work with a slightly non-standard notion of powerset: let *P**(S) designate the set of all *non-empty* subsets of S.

An interesting feature of powersets is that the number of members they have is a function of the number of members in the original set. Where *n* is the number of members in S, and *N* the number of members in *P**(S), we have: $2^n - 1 = N$. This entails that the number of members in a powerset has many "jumps," or "gaps". There are certain natural numbers *x* such that, necessarily, if *Q* is a powerset, then *Q* can never have *x* members. For instance, the number of elements in a powerset can *never* be even: for all naturals *n*, there is no even natural *N* which is equivalent to $2^n - 1$. Similarly, powersets can never have 5, 11, 13, or 29 members (since, for any natural *n*, $2^n - 1$ cannot ever be equal to any of these values). Thus, if *Q* is a powerset, then, necessarily, there are only a certain number of members *Q* can have.

It is because powersets cannot have certain numbers of members that they are ripe for a source of counterexamples to modal continuity.[2] Roughly, the idea will be that if the extension of some possible property *P* can be put into one-to-one correspondence with a powerset, then the class of degreed properties *being such that there are n-many x's such that P(x)* is riddled with modal gaps—for instance, any even instance of *n*. Thus, continuity fails (to apply).

To see how this style of counterexample works, it helps to consider a direct application. A wonderful example comes from Comesaña (2008), who uses this feature of powersets to mount an argument against mereological universalism. For our purposes, we can gloss mereological universalism as the view that, for any two (distinct) material objects, there is a third material object (namely, the fusion of the first two objects). If we assume that there are atoms, then mereological universalism entails that the number of objects in a world *w* is just the cardinality of the powerset of the set of the atoms in *w*.[3] To see this, suppose that *O* is the set of objects in *w* and *A* is the set of atoms. Let $A = \{atom_1, atom_2, atom_3\}$. Because atoms are material objects, $atom_1 \in O$. The same applies for $atom_2$ and $atom_3$. Additionally, by our statement of universalism, we have that the fusion of $atom_1$ and $atom_2$—denoted $\oplus(atom_1, atom_2)$—is itself a material object, and thus a member of *O*. But if $\oplus(atom_1, atom_2)$

2 Or, per our discussion in § 2, we might say they are ripe for a source of *domains of inapplicability*, rather than counterexamples *per se*. For concision's sake, I'll henceforth omit this qualification when describing powered properties' relation to Rasmussen's formulation of the principle of modal continuity, calling them simply 'counterexamples'.

3 As Comesaña (2008: 31 – 2) notes, this is a bit too quick. A few additional mereological assumptions will be needed: (i) that no *x*'s have more than one fusion, (ii) that any *x*'s have a fusion, (iii) that each object is a fusion of atoms, and (iv) the transitivity of the parthood relation. Since the purpose of this example is to clarify how to develop powered counterexamples to continuity, I omit the full derivation.

is a material object, and so is $atom_3$, then, by universalism, $\oplus(atom_1, atom_2, atom_3)$ is also a material object, and thus also in O.[4] By repeated application of universalism, we will find that every subset of A can be put in one-to-one correspondence with a member of O. Thus, assuming universalism and atomism, the number of objects at any world cannot take on certain values (e.g., it is metaphysically impossible for there to be a world with exactly 5 material objects). So, if one accepts these positions, then they ought to reject the application of modal continuity, as the class of properties of the form *being such that there are n-many objects* has a modal gap (assuming there could be some n of objects).

Another way to see how this works is to see that universalism is claiming something about the property *being an object*: namely, universalism is claiming that its extension can be put into one-to-one correspondence with a powerset—i.e., that *being an object* is a *powered* property. And (exemplifiable) powered properties violate modal continuity.

Of course, mereological universalism is a controversial view, but the fact that it entails that modal continuity cannot be applied to objecthood is both interesting and suggestive. Depending on our views about how a certain property interacts with the world—in particular, whether its extension can be put into one-to-one correspondence with a powerset—that property may turn out to be riddled with modal gaps. And there are, I hope to show, many such cases, supplied by views less controversial than mereological universalism.

To develop a wider blueprint for constructing powered properties, let us get a better grip on the nature of powered properties. Where $_{\mathrm{e}}(\mathrm{P})_w$ designates the extension of a property P in a world w, I propose the following account of a powered property[5,6]:

> A property P is a powered property iff for any metaphysically possible world w such that $|_{\mathrm{e}}(\mathrm{P})_w| < \aleph_0$, there is a set S such that there is a bijective function f: $_{\mathrm{e}}(\mathrm{P})_w \rightarrow P^*(\mathrm{S})$.

However, to develop a useful *blueprint* for constructing powered properties, it would be helpful to know what sorts of conditions on P could *obtain* for the above to be satisfied. In other words, it would be illuminating to find a sufficient condition for powered properties. To find this condition, let us consider what it was about the combination of universalism and the existence of mereological atoms that made *being an object* a powered property. First, the existence of mereological atoms supplied us with a 'base' set, the set of atoms, A, from which the set of objects, O, was generated. Second, universalism supplied us with a certain relation on O, *fusion*, that worked in a particular way. Namely, *any* non-empty subset of atoms from A corresponded with

[4] Or, more explicitly (taking fusion to be a two-place relation), $\oplus(\oplus(atom_1, atom_2), atom_3) \in O$.

[5] It should be noted that this account arguably permits for a wider range of properties to count as powered properties than we may 'intuitively' accept. For instance, if one believes God necessarily exists, then *being God* is a powered property (since there is a powerset with just one member), albeit that *being God* doesn't quite exhibit the intuitive structure of *being an object* (on universalism). It also entails that unexemplifiable properties are powered, as well as properties with necessarily infinite extensions (vacuously).

[6] It also should be noted that I will be assuming, throughout, an *abundant* picture of properties—i.e., that any, or a great deal of, sets of particulars are, or correspond to, properties.

a fusion, which was *itself* a member of *O*. And it was the existence of this relation that allowed us to see that *O* could be put in one-to-one correspondence with the powerset of *A*—for any subset of *A* would have a corresponding member in *O*, given by fusion. Thus, the real key to identifying P as a powered property is finding a certain relation over $_{\text{e}}$(P).

To express this sufficient condition more precisely, let us make use of one piece of notation. Following the conventions of infinitary union notation, where $\bigcup_{x \in S}\{x\}$ designates the union of the singletons of each element *x* in a set S, let $R_S(x)$ designate a two-place relation, *R*, applied across each element *x* of some set S (and if $|S| = 1$, $R_S(x) = Rxx$). For instance, where $S = \{a, b, c\}$, $R_S(x) = R(a, R(b, c))$.[7] With this in hand, I propose the following sufficient condition for powered properties:

> P is powered if, for any metaphysically possible world *w* such that $|_{\text{e}}(P)_w| < \aleph_0$, there exists a two-place relation *R* on $_{\text{e}}(P)_w$ such that, for any non-empty set S $\subseteq \{m|\ (m = Rxy) \rightarrow (x = y)\}$, there is a $z \in {}_{\text{e}}(P)_w$ such that $z = R_S(x)$.

Very roughly, the claim here is that P is powered if its extension is such that any subset of the "atomic" instances of P corresponds to, vis-à-vis *R*, an instance of P—just as, on universalism, any set of objects *itself* corresponds to an object, vis-à-vis fusion. It is also important to add that, to mimic the operation of the fusion relation, the relation *R* must exemplify a few additional formal features:

Commutativity: If $Rxy = z$, then $z = Ryx$.

Associativity:
$R(Rxy, z) = R(x, Ryz)$.
Idempotence:
$Rxx = x$.

Atomic Equality: For any sets S_1 and S_2 such that (i) $S_1 \neq S_2$, and (ii) for any $m \in S_1 \cup S_2$, $m = Rxy \rightarrow x = y$, it is the case that $R_{S1}(x) \neq R_{S2}(x)$.

Unary Absorption[8]:
If $Rxy = z$, then $Rzx = Rzy = z$

[7] One might worry here that I have not at all *defined* how, in general, *R* should "apply across" each element of S. For instance, in what order should things be applied—e.g., why not $R(R(a, b), c)$—and, if S includes a fourth element, *d*, is $R_S(x) = R(a, R(b, R(c, d)))$, or instead, e.g., $R(R(a, b), R(c, d))$? I have neglected any specification in the text because, just like union notation, it is *strictly irrelevant*: since I will require that *R* is an associative and commutative binary relation, *any* specification will be equivalent. Thus, mainly for concision and clarity, I have elected to not detract our attention by articulating some specified manner of *R*'s "applying across" a set.

[8] I am extremely grateful for an anonymous referee's pointing out the importance of this feature of *R*—without it, the extension of a property at any world has an infinite cardinality, which, as we shall see, severs the existence of modal gaps. Strictly, the absorption condition on *R* is not *required* for a property's being *powered* (though that is no problem, since this is a sufficient condition), but I have included it for its importance in making a powered property a *counterexample* to modal continuity.

The attentive reader may have noticed by this point that the relation *R* defined here is simply (or, also) the formal analog of the *union* operation in set theory—indeed, the conditions we have imposed on *R* just are the paradigmatic features of the union.[9] And this should come as no surprise; for one way of *defining* powersets is as the set of all unions of all possible combinations of some set, in a structurally identical way to the present account.[10] Indeed, it is helpful to think of the above blueprint for powered properties in the following way: a property is powered if its extension is necessarily closed under something *like* a union operation. For the mereological universalist, there exists something like a union operation on the extension of *being an object*: the fusion relation.

4 Powered counterexamples to modal continuity

With this blueprint in place, we are now ready to start developing direct counterexamples to modal continuity vis-à-vis powered properties. Our first counterexample is the property *being an action I have performed*[11]:

> *(CX1)*: Let (ϕ_t) designate an action ϕ performed at *t*. For any two actions I have performed, (ϕ_t) and (ψ_{t*}), there is a third action I have performed: the action of ϕ-ing at *t* and ψ -ing at *t**, i.e., (ϕ_t and ψ_{t*}).

Here, the property *being an action I have performed* is powered—the set of actions I have performed can be put into one-to-one correspondence with the powerset of what we might call the "atomic" actions I have performed.[12] In terms of our sufficient condition for powered properties, we can see that what allows this to be a powered property is that there is a certain relation over actions, the *performing* relation, which is plausibly such that: (i) performing actions *x* at *t* and *y* at *t** is itself an action, (ii) performing actions *x* at *t* and *y* at *t** is the same as performing *y* at *t** and *x* at *t* [*Commutativity*], (iii) performing the performance of *x* at *t* and *y* at *t**, and *z* at *t*** is the same as performing *x* at *t* and the performance of *y* at *t** and *z* at *t*** [*Associativity*], (iv) performing *x* at *t* and *x* at *t* is the same as the action *x* at *t* [*Idempotence*], (v) performing distinct collections of atomic actions amounts to performing distinct actions [*Atomic Equivalence*], and (vi) performing the performance of *x* at *t* and *y* at *t** and

[9] Some might object that *Atomic Equality* is *not* a paradigmatic feature of the union, but note this is only because, in the set-theoretic context, this condition is trivially satisfied since sets cannot have duplicate elements. We must explicitly enforce this condition in our (more general) context.

[10] For the interested reader, the standard definition of a powerset in terms of the union operation is: $P(S) = \{A|\ A = U_{x \in F}\{x\}$ >for any $F \subseteq S\}$.

[11] It is worth remarking that, whilst I use the indexical 'I' in explicating this property (and throughout, for sake of simplicity and convenience), talk of any *particular* person can be replaced by talk of 'some agent' so as to meet Rasmussen's (2014: 531) haecceity proviso.

[12] Here, "atomic" actions should be understood as any action not composed of distinct actions. Recall the formalism introduced in § 3: in general, "atomic" *m*'s should be understood as *m*'s such that, if $m = Rxy$, then $x = y$. In developing powered counterexamples to continuity, we will often require the existence of such atomics for the relevant entity under discussion.

x at t is the same as simply performing x at t and y at t^* [*Unary Absorption*]. I won't spell out the relation in every example, but it is worth explicating R at least once to see that what makes, e.g., actions, powered is this relation.

Thus, the number of actions I have performed cannot take on certain values—e.g., it cannot be that I have performed an even number of actions. So, if we assume that it is possible that I perform a finite number of actions[13], then the class of properties of the form *being such that there are n-many actions I have performed* has a modal gap.

It is worth being explicit about this last point. We require the assumption that it is possible for me to only have performed a finite number of actions because, if this is *impossible*, then the class of properties of the form *being such that there are n-many actions I have performed* doesn't actually have a modal gap. For a class of properties only has a modal gap if one of the properties in the class is exemplifiable, *and* one of the properties in the class isn't. Thus, if there is *no* natural number n for which *being such that there are n-many actions I have performed* is exemplifiable, then we have not shown that there is any *gap* in the class. For any powered property P to be a counterexample to modal continuity, we will need this kind of assumption—call these possible-finitude assumptions. Of course, I do find the possible-finitude assumption for *being an action I have performed* rather plausible, as I did for *being an object*, but the requirement of this assumption is noteworthy.

Additionally, with this example on the table, note that, because actions have this structure (roughly, that any two actions compose an action), a similar example can be constructed with the actions I *will* perform. Several other examples can also plausibly be constructed using particular *kinds* of actions I have, or will, perform: e.g., all the actions I have performed related to cleaning my room, all the actions I have taken related to studying, and, perhaps, the *good* (and *bad*) actions I have taken. Again, at their core, what makes all such examples work as counterexamples to modal continuity is the fact that *actions* have a particular kind of structure: any two distinct actions "compose" a further action, and this "composition" relation is commutative, associative, idempotent, etc.

Let's walk through another example. Let us say that a *group* of people is any non-empty collection of persons (in the same world as one another). Thus, for any two groups of people, g_1 and g_2, there is another group, the people from g_1 and the people from g_2. So, *being a group of people* has a similar "composition" relation as actions, where any two distinct groups "composes" a further group, and the "composition" of groups of people is plausibly also commutative, associative, idempotent, etc. Thus, since *being a group of people* is a powered property, and we can construct all sorts of counterexamples concerning properties that distribute over groups of people. For instance:

(CX2): For any two groups of people that ought to be respected, there is a third group of people that also ought to be respected.

[13] This is, by my lights, eminently plausible—though it does require that actions (or, at least, some kind K of actions) not be *too* fine-grained.

Here, *being a group of people that ought to be respected* is powered, for its extension can be put into one-to-one correspondence with the powerset of the individual people that ought to be respected. And a myriad of other examples can be produced using other properties that distribute over groups of people: e.g., *being a malicious group of people*, *being tall*, *being in debt*, etc. And, since it is plausible to think that it is possible for finitely many groups of people to exist, all these properties could serve as properties with modal gaps. Furthermore, there is of course nothing particularly special about groups of *people*; many x's are such that properties centering around groups of x's will be powered. The key feature is simply that *groups*—whether groups of people, animals, cookies—have the compositional structure we have been stressing.

A few more examples are worth exploring. Let a 'coursed meal' be any meal with n courses, where $n > 0$.[14] Now consider:

(CX3): For any two coursed meals m_1 and m_2 I know how to cook, there is a third coursed meal that I also know how to cook—the courses of m_1 and the courses of m_2.

Thus, *being a coursed meal I know how to cook* is powered. The coursed meals I know how to cook can be put into one-to-one correspondence with the powerset of the individual meals I know how to cook—there is no possible world where I know how to cook, e.g., an even number of coursed meals. As such, assuming there is a possible world where I know how to cook finitely-many coursed meals, we have yet another counterexample to modal continuity.

Here is a similarly spirited example: let a fleet of ships be any non-empty collection of boats (in the same world). Then, we have:

(CX4): For any two fleets of ships f_1 and f_2 I could deploy, there is a third fleet I could deploy—the ships of f_1 and the ships of f_2.

And, of course, a similar point can be made about squadrons of aircraft, or packs of animals.[15] In all such cases, I find the relevant possible-finitude assumptions quite plausible.

One last example is worth discussing, given that it involves more contentious metaphysics (similar to mereological universalism). Consider the familiar notion of

[14] Assume also that coursed meals are distinguished only by the courses that compose them; order does not matter in determining the identity of a coursed meal. Tofu stir-fry and chocolate cake is the same coursed meal as chocolate cake and tofu stir-fry.

[15] I would like to point out an (arguably) intriguing point about these kinds of examples. Suppose, e.g., *naval strength* is a function of how many various fleets one can deploy, whereby each additional fleet that can be deployed by one entails an increased degree of naval strength (which I find to be at least somewhat *prima facie* plausible). Then, the class of properties *being of naval strength degree n* is *also* filled with modal gaps. I bring up this sort of case to merely highlight the possibility of degreed properties—which seem like more *paradigmatic* instances of properties which should be modally continuous—that might have modal gaps because they *rely* on powered entities.

a *spatial region*. Following the spatial logics of, *inter alia*, Randell et al. (1992) and Aiello et al. (2007), we can claim:

(CX5): For any two spatial regions s_1 and s_2, there exists a unique spatial region, the *sum* of s_1 and s_2, which is just the region such that it is connected to all and only those regions connected to either of s_1 or s_2.

Thus, *being a spatial region* is a powered property—the set of all spatial regions in a world *w* can be put into one-to-one correspondence with the powerset of the set of all "atomic" spatial regions (those spatial regions with no proper parts). Now, assuming there are such atomic spatial regions, and that there is a possible world with finitely many spatial regions, it follows that there are modal gaps in the number of spatial regions that could exist. Of course, the possible-finitude assumption this time around is by no means trivial: the atomic spatial regions at a world *w* are standardly taken to be (infinitely many) *spatial points*, and it strikes me as somewhat plausible to think that this constraint on space holds *necessarily*. Nevertheless, there are detractors to this standard (see, e.g., Pratt-Hartmann, 2007: 13–14), and it is at least defensible to maintain that finitism (about spatial regions) is metaphysically *possible*. At any rate, the relevant, and intriguing, point is that *if* one has metaphysical views of this stripe, then we have on our hands yet another powered property that violates modal continuity.[16] Additionally, this sort of example plausibly has a temporal analog, using *spans of times*.

Hopefully, the structure of the counterexamples here is clear: find an entity *E* such that, for any two *E*'s that have *P*, by some suitable relation *R*, there is another *E* which itself exemplifies *P*. As long as it is possible for finitely many *E*'s to exemplify *P*, we have a property that is modally discontinuous.

Before moving on, I will offer here a few more counterexamples to modal continuity using powered properties. It is worth noting that it may well be controversial in some of these cases whether the entities in question really form a third, whether the property always extends to the third, whether the relevant relation *R* is really commutative, associative, idempotent, atomically equivalent, and absorptive, or whether the possible-finitude assumption is plausible. I leave this open and offer multiple examples in hopes of appealing to a wider crowd.

(CX6) Let an '*s*-bundle' be a bundle (non-empty collection) of item(s) from a store *s*. For any two *s*-bundles, there is a third *s*-bundle—the bundle of the items from the first two. Thus, the *s*-bundles can be put into one-to-one correspondence with the powerset of the items in *s*. Plausibly, *being an s-bundle my son would desire* is powered.[17]

[16] And, naturally, if one has these views about spatial regions, plenty of other powered properties may promise to be in the vicinity, e.g., perhaps *being a spatial region I have fully stepped in*.

[17] I would like to thank an anonymous referee for pointing out the following, which leads to an important lesson: many properties will not work (as powered properties) for *s*-bundles, e.g., *being a good s-bundle to buy*. For there are possible worlds such that the store in question only permits, e.g., bundles of *one* item to be bought, in which case the number of (good) bundles to buy can be any number. This problem arises because whether or not there exists a bundle *available for purchase* is not guaranteed by the existence

(CX7) Suppose sets exist, such that for any material object(s) $o_1,\ldots, o_n$ in a world *w*, there is a set $\{o_1,\ldots, o_n\}$ *belonging to* world *w*. Call such sets "material sets." It follows that the set of material sets belonging to a world *w* is the powerset of the objects *o* in *w*.

(CX8) For any two events, e_1 and e_2, that have occurred at *t* and *t**, there is a third event, the event of e_1 occurring at *t* and e_2 occurring at *t**. [And similarly for 'will occur'].

(CX9) Let a 'real-world artifact-collection' be a collection of artifacts, all of which are from the actual world (non-rigidly designated). For any two real-world artifact-collections such that I would profit from selling them, there is a third collection I would profit from selling—the artifacts of the first and the artifacts of the second.

(CX10) Let a 'TV binge-list at *w*' be a collection of TV shows from *w*. For any two TV-binge lists at *w* that I couldn't finish today, there is a third binge-list I couldn't finish today—the shows of the first list alongside the shows of the second list. More generally, the set of TV binge-lists at a world *w* can be put into one-to-one correspondence with the shows in *w*.

I doubt that these are near the total number of properties that, as applied to certain entities, become powered. Indeed, *many* entities exhibit the sort of identity conditions conducive to producing powered properties, whereby any plurality of *e*'s is itself an *e*.[18] This, in many ways, is one of the central insights I take powered properties to provide: the application of modal continuity to a given property P depends not merely on whether P is, e.g., *coherent*, but also on whether the world *interacts* with P in a certain way (e.g., whether there exists a certain *relation* over which P distributes), which makes P discontinuous.

5 Patchwork principles as a powered property

A particularly interesting feature of powered properties is that, in addition to shedding light on another factor which we must pay attention to in our applications of modal continuity, powered properties reveal that another popular family of modal principles—*recombination*, or *patchwork*, principles—entails the existence of a powered property, and is thus in tension with modal continuity. I leave it open just how strong my argument for the tension is, as the inference that what follows is a powered property is not, by my lights, as secure as our previous examples, as it requires a few

of two smaller bundles available for purchase: it also depends on the nature of the store rules. Unlike, e.g., *groups of people*, where the existence of a larger group of people *is* guaranteed by the existence of a smaller group of people.

[18] One entity with this structure that I have not explored much (because of certain difficulties with meeting the possible-finitude requirement) are *states of affairs*. Following Pollock (1984), we can define *conjunction* over states of affairs, such that for any two states of affairs there exists another—the two obtaining. Other entities that might have this structure come to mind too, e.g., *abilities*, *desires*, *duties*, and *reasons*.

additional assumptions and a contrived proof. My hope is to signal that powered properties might be a good source for developing this unforeseen tension.

Patchwork principles, adapted from Lewis (1986: 88–91) and endorsed by, *inter alia*, Koons (2014: 258), are Humean modal principles meant to capture the *plenitudinous* nature of modal space. Patchwork principles are supposed to do justice to the modal intuition that "anything can coexist with anything else, at least provided they occupy distinct spatiotemporal positions…if there could be a dragon, and there could be a unicorn, but there couldn't be a dragon and a unicorn side by side, that would be an unacceptable gap in logical space, a failure of plenitude" (Lewis, 1986: 88). The motivation here is that there are no *gaps* in modal space (*very* much like modal continuity!). As such, patchwork principles make claims about what sorts of spatiotemporal arrangements of objects are possible—in particular, that we should be able to *recombine* certain arrangements to produce others. Simplifying a bit, one version of the patchwork principles often claims something like the following[19]:

(PP) For any possible worlds w_1 and w_2 with spatiotemporal regions R_1 and R_2 respectively, containing contents c_1 and c_2 respectively, if there is a possible world w_3 with spatiotemporal region R_3 with enough 'room'[20] to contain R_1 and R_2 and their contents without overlap, then there exists a possible world w_4, with spatiotemporal region R_4, that has parts p_1 and p_2 that exactly resemble c_1 and c_2, in some arrangement a.[21]

Roughly, the intuition behind (PP) is that, for any two possible spatiotemporal regions (and their contents), there should be a *third* possible spatiotemporal region—the region containing exact duplicates of the contents of the first two, as long as there is some possible size and shape of spacetime permitting for the relevant arrangement of the exact duplicates. Let us say, for concision, that a spatiotemporal *patch* is a spatiotemporal region and its contents. Then, patchwork principles claim that, for any two possible spatiotemporal patches, there is a third possible spatiotemporal patch, the "combination" of the first two, as long as there is a possible spacetime that can accommodate it. At first blush, one might be tempted to immediately claim that the powered property is obvious here: PP entails that *being a possible spatiotemporal patch* is a powered property. The structure is strikingly similar to the examples explored in § 4. But (at least one) problem with this approach is that it is very plausible that there are infinitely many possible spatiotemporal patches, such that the class

[19] PP often comes with various other restrictions like, e.g., preserving metrical and topological features, and certain provisos related to metaphysical theses (see Schmid and Malpass, Forthcoming for a discussion), but we can exclude these for simplicity. Additionally, it should be noted that this is but *one* version of a patchwork principle, adapted from Koons (2014)—many other variants can well differ in various respects, e.g., in strength, or in the entities being quantified over, or in the provisos attached.

[20] À la Koons (2014: 258), the notion of having enough 'room' can be spelled out in terms of the existence of a structure-preserving function f from the spatiotemporal regions of w_1 and w_2 to the spatiotemporal regions of w_3, with no overlapping values.

[21] It is worth remarking that *exact resemblance* here means sharing intrinsic properties—i.e., intrinsic duplication.

of properties *being such that there are n possible spatial patches* plausibly won't be a counterexample to modal continuity.

Thus, to generate a counterexample, we will need to hone in on *a specific kind* of possible spatiotemporal patch—such that it is plausible that there are only finitely many of this *kind* of patch.

First, it will be helpful to focus on the following (restricted) instance of PP, which concerns *finite* spatiotemporal regions, allowing us to remove Lewis' spacetime proviso[22]:

> (PP*): For any two finite possible spatiotemporal patches $_1$ and $_2$, there is a third finite possible spatiotemporal patch, $_3$, containing duplicates of $_1$ and $_2$ in some arbitrary arrangement *a*.

To build this example, we start first with the notion of a *batch*. A *batch* of *x*'s is the minimal[23] finite spatiotemporal patch of *x*'s such that (i) each of the *x*'s is spatially related to one another, (ii) there are no *x*'s outside of the patch, and (iii) none of the *x*'s are exact duplicates of one another. For instance, since all of the cookies in the actual world are spatially related (and, I assume, none are exact duplicates), they form a batch: the batch of actual cookies. Note that there can be no more than *one* batch of cookies in any world, as batches are defined to be maximal (no cookie can be outside it). Next, we define the notion of a *w*-cookie: a cookie is a *w*-cookie just in case it is an exact duplicate of one (and only one) cookie in *w*. Thus, note that there may be *w*-cookies in worlds other than *w*. Because batches are spatiotemporal patches, the following (independently plausible) proposition is entailed by PP (and PP*)[24]:

> (Cookies): For any two distinct[25] possible batches of *w*-cookies $_1$ and $_2$, there is a third possible batch of *w*-cookies, $_3$, containing exact duplicates of the *w*-cookies of $_1$ and $_2$.

[22] I am, of course, *assuming* that this allows us to remove the spacetime proviso, since I assume any arbitrarily large (finite) spatiotemporal region is possible.

[23] If one has concerns about the notion of a *minimal* spatiotemporal region (and thus patches), say, because of various mereotopological paradoxes concerning spatiotemporal *borders*, note that the role of 'minimal' here is merely to pick out a *particular* spatiotemporal region containing the *x*'s. Concerned readers should feel free to supplement minimality with *some* particular spatiotemporal region.

[24] We might worry that *Cookies* does not specify any *arrangement* for $?_3$. Let us stipulate that batches are insensitive to arrangements (much like sets are insensitive to duplication)—this will help stave off worries about infinitely many possible arrangements of one and the same batch. We might put things as follows: suppose *w** is a possible world with a finite spatiotemporal patch of *w*-cookies such that (i) each of the *w*-cookies is spatially related to one another, (ii) there are no *w*-cookies outside of the patch, and (iii) none of the *w*-cookies are exact duplicates of one another. Let S(*w**) designate the set of all possible spatiotemporal patches *p* containing any combination of (duplicates of) the *w*-cookies in *w**, in any arrangement, with no exact duplicates within *p*. There is a function *f* from S(*w**) to one particular spatiotemporal patch *p* ∈ S(*w**) [the "minimal" patch]. Let us say, then, that the *batch* of *w*-cookies at *w** is *f*(S(*w**)). And things generalize appropriately—in other words, batches are patches insensitive to arrangement, because they always pick out a *particular* patch (vis-à-vis a function like *f*).

[25] Two batches are *distinct* whenever no member of one is an exact duplicate of another.

For instance, if there is a possible world, w^*, with the batch of w-cookies {cookie_A, cookie_B}[26], and another world, w^{**}, with the batch {cookie_C}, then there is a possible world, w^{***}, with the batch {cookie_A, cookie_B, cookie_C}. To produce a counterexample to modal continuity, we will need two more assumptions. First, that there is a possible world with finitely many cookies, not all of which are intrinsic duplicates of one another (call this A1). This is plausible—and probably *actually* the case. Second, we will need a modal assumption: for any cookie in a world *w*, there is a possible world where (a duplicate of) that cookie is the *only* cookie (call this A2). This too, I think, is plausible. Note that this is also supported by modal continuity (if 2 cookies are possible, so is 1).

With these modest assumptions, we now have a derivation from PP to a modal gap. Informally, it is as follows. Let *w* be a world with the following batch of cookies: {cookie_A, cookie_B, cookie_C}. There is a possible world, w^*, with the batch of w-cookies: {cookie_{A*}} (by A2).[27] There is another possible world, w^{**}, with the batch of w-cookies: {cookie_{B*}} (by A2). By PP, for any two distinct possible batches of *w*-cookies, there is a third possible batch of *w*-cookies, containing exact duplicates of the *w*-cookies of the first two. Thus, by PP, there is a possible world, w^{***}, with the batch of w-cookies: {cookie_{A**}, cookie_{B**}}.[28] By repeated application of PP, we will have that for any subset of *w*-cookies, there is a possible world where (duplicates of) the *w*-cookies in that subset exist, corresponding to a possible batch of *w*-cookies. In other words, the set of possible batches of *w*-cookies can be put into one-to-one correspondence with the powerset of the cookies in *w*. But, because it is possible for there to be a world with finitely many cookies, the class of properties of the form *being such that there are n-many possible distinct batches of w-cookies* has a modal gap.[29] So, Lewisian patchwork principles supply us with a powered property, alongside a class of properties with a modal gap.

To my mind, this modal gap is an interesting discovery, not only because of how it interacts with modal continuity, but because Lewis' patchwork principle was meant to *prevent* the existence of "unacceptable gaps in logical space" (Lewis, 1986: 88). Taking this remark seriously, perhaps we ought to conclude that *discontinuities* do not make for unacceptable gaps in modal space: only failures of *combination* count as unacceptable gaps. While both Rasmussen's and Lewis' principles are based on vindicating the intuition of the plenitudinous nature of the modal landscape, it seems they are undergirded by different conceptions of plenitude—i.e., what counts as an unacceptable *gap* on these pictures is (crucially!) different.

Indeed, it is important to note here the nature of the conflict. There is *nothing* in the nature of batches, *qua* batches, preventing the following from being true: for any

[26] I, of course, do not mean here to suggest that the batch of *w*-cookies *is* the set {cookie_A, cookie_B}. I simply mean to *use* {cookie_A, cookie_B} to *pick out* the cookies contained in the batch of *w*-cookies at w^*.

[27] I use cookie_{A*} to designate that this is a *duplicate* of cookie_A.

[28] It is important to note that, to call this a batch of *w*-cookies, we also must assume that the 'duplicate of' relation is transitive.

[29] One might worry that, given S5, what is possible is necessarily possible, and as such, there can really only be *one* exemplifiable property in this class. However, there is no reason to restrict this class to the *w*-cookies of any *particular* world *w*. PP entails the much stronger result that, for *any w whatsoever*, the set of possible batches of *w*-cookies can be put in one-to-one correspondence with a powerset.

n, and some world *w*, there could be *n* possible batches of *w*-cookies. It is the truth (if it is a truth) of the *patchwork* principle that prevents this from being true. If one was an ardent defender of continuity, one could easily and freely believe that possible batches come in any number; it would simply require embracing the failure of modal recombination at at least one world. Thus, in at least some cases, those attracted to modal continuity and Lewisian patchwork principles on the grounds of the plenitudinous nature of modality must make a choice: is their commitment to plenitude based on a combinatorial intuition, or a continuousness intuition?

6 Objections

I'd like to close by considering a few potential objections—in particular, concerning whether modal continuity is really threatened by the existence of powered properties, or if, instead, powered properties fall under one of its provisos.

The first objection I'd like to consider is that modal continuity is not threatened by the existence of powered properties because powered properties fall under the *coherency* proviso. Recall that refined versions of modal continuity are standardly formulated to exclude narrowly logically inconsistent properties like *being 3-sided and 4-sided*—i.e., they come attached with a coherency proviso. Might it be objected that all of the cases of powered properties I've developed herein are not genuine counterexamples, as they are excluded by this proviso? After all, it is *logically impossible* for there to be, say, an even-number of actions I have performed, for purely *set-theoretic* reasons.

In response, I find it hard to see how the counterexamples I have given here are *narrowly logically* impossible. They all require notable metaphysical assumptions about how objects combine, their identity conditions, what the extensions of properties are like, and possible-finitude assumptions. For instance, take the case of the mereological universalist: it seems, indeed, that *if* one is convinced of universalism, and *if* it is metaphysically possible that (only) finitely many atoms exist, one has a direct counterexample to modal continuity. Crucially, then, whether *being an object* is a property which admits of a class with modal gaps is not a matter of the *definition* of being an object: it depends on, among other things, several contentious claims about the nature of the world. It is no matter of *logic* whether finitely many atoms could exist. This point is also evident in the case of the patchwork-inspired counterexample—PP is far from a *narrowly logical* truth. It is a deeply contentious view about the nature of possibility. Thus, while it is true that *if* a property is powered, it will follow as a matter of logic that the property is a counterexample to modal continuity, *whether* a property is powered is seldom, by my lights, a narrowly logical truth.

Another way to see this point is to compare powered properties with the kinds of examples Rasmussen (2014) seeks to exclude with his constraint against logically inconsistent degreed properties. For instance, take the class of properties of the form *being an n-sided triangle*. This class is riddled with modal gaps: indeed, only one property in this class is exemplifiable, because it follows from the *definition* of being a triangle that it can only have 3 sides. This class is no counterexample to modal continuity, however, precisely because of this feature. Compare, now, the class of properties of the form *being such that there are n material objects*. If the universalists are right, this class is also riddled with modal gaps. But surely it does not follow from the *definition* of what it is to be a material object that the property *being a material object* encodes various powerset-like features. The fact that there cannot be, e.g., 622 material objects—if it is a fact at all—is true in virtue of a particular metaphysical view, that is, in virtue of the particular way the world is structured and the identity conditions of objects. Indeed, if there are counterexamples to modal continuity, shouldn't they take precisely this form? Some metaphysical feature *F* of the world is such that *F* entails that, while a certain property is possible to a certain degree, it is not possible to other degrees. Surely, we cannot exclude such counterexamples merely because *F logically entails* this fact. Indeed, if the logical constraint on modal continuity is construed this broadly, I find it difficult for the principle to enjoy the wide metaphysical application it has experienced.

A second objection one might press is to claim that, while powered properties are not excluded on the basis of the coherency proviso, they *are* excluded on the basis of Rasmussen's *essential limits* proviso.[30] Recall that, in addition to the mandate that the properties within a unified class of degreed properties be coherent, Rasmussen (2014: 531) also requires that, in order for the class to have no modal gaps, the class must not have any properties involving the violation of the essential limits of some particular thing, *p*. For instance, consider the class of degreed properties of the form *being LeBron James and capable of eating n pounds of lentils* (a modified example from Rasmussen). Plausibly, there is some mass of lentils such that it is too large for LeBron James to possibly eat. This, however, is no counterexample to modal continuity because there is a property in this class that involves a violation of LeBron James' essential limitations, to wit, his capacity to eat lentils. More generally, Rasmussen proposes that modal continuity only apply to properties that do not involve the mentioning of *any particular individuals*, e.g., LeBron James and Keith Lehrer. With this proviso in hand, the following might be pressed: powered properties do not threaten the principle of modal continuity, for it plausibly lies in *the very essence* of the relevant entity that there cannot be, e.g., an even number of that entity. For instance, if mereological universalism is true, it plausibly lies in the *nature* of what it is to be an object that there cannot be 22 of them.[31] Powered properties, then, involve violations of essential limits.

This is a wonderful objection, for it allows us to explore and unveil a number of important points concerning our investigation. As such, I have a few responses to offer.

First, it is not clear to me that powered properties actually violate Rasmussen's essential limits proviso. That proviso, as mentioned above, is formulated as a constraint against properties mentioning *particular individuals*, so as to rule out the possibility of modal continuity's applying to a property that involves some particular's

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[30] I am indebted to an anonymous referee for bringing up this objection.

[31] At the very least, it seems to lie in the very nature of some *plurality* of things, i.e., objects and classical extensional mereology. And similarly for our other examples. I'll simply restrict myself to talking about the relevant object, and not some plurality, because I do not here spend time challenging the claim that it is in the nature of the relevant property that it is powered.

essence. It is, to use Rasmussen's terminology, a requirement that the properties we apply continuity to be *non-haecceitous*. But it seems very clear that many powered properties do not involve any such haecceities. Or, at the very least, that they needn't. For instance, *Rasmussen himself* (2014: 531)[32] claims that the property *being such that there are n co-located objects* does not violate his requirement of not mentioning any particular individuals—indeed, it is a paradigmatic example of not violating the proviso. But, of course, if *being such that there are nco-locatedobjects* does not count as invoking a particular essence, surely neither does *being such that there are n material objects*. And I am tempted to say something similar for actions, groups, spatial regions, meals, sets, fleets, and the like.

Second, putting this last point aside, if we *do* take a reading of the essential limits proviso on which powered properties involve violations of some thing's essence, I am worried that this reading might seriously constrain the applicability of modal continuity, in a way that arguably severs continuity *more* than if we simply conceded that it does not apply to the domain of powered properties. Consider, first, that if it really lies in the *nature* of a material object that its extension is powered, and if this means that modal continuity does not apply to the number of material objects that could exist, plenty of judgements about *other* properties—which we originally wanted to apply modal continuity to—can no longer be supported by appeal to modal continuity. For instance, Rasmussen's opening example of modal-continuity-style reasoning—that, if two objects could be co-located, so could *n*—may turn out to not be an arena in which we can directly employ modal continuity. For, if mereological universalism is true, perhaps it turns out to lie in the very *nature* of (co-located) material objects that they are powered, and as suchcontinuitydoesnotapply (depending on whether co-located objects retain the mereological assumptions required, e.g., that they do not involve objects having multiple fusions).. And, as it turns out, there is a more general worry here. Enforcing an essential-limits proviso as wide as the one being pressed herein to exclude powered properties entails that, before applying modal continuity to some domain, we must *settle* the various modal questions about the natures of the concepts involved in the relevant domain. If, e.g., we are interested in questions about the possibility of certain kinds of causal and nomic relations and hope to apply modal continuity to make headway (Gibbs, 2018), or interested in questions about the possibility of various degrees of value (Rasmussen, 2018), it turns out we will have to first settle questions about the *nature* of these entities *before* we apply modal continuity. But this, we might worry, seems to affect the *purpose* of modal continuity, which is to be a *guide* to possibility. Of course, we could instead maintain that modal continuity always provides us with a *defeasible reason* to believe certain modal claims—rather than that it *does not apply* until we have settled certain questions—and that we must simply keep in mind that this reason is always defeated by concerns about the essences of the entities involved. But, aside from worries about strength, this version of the proviso seems to grant weight back to some of the results

[32] Strictly speaking, on page 531 Rasmussen claims that it is the property *being such that there are n co-locatedthings* that doesn't violate the proviso. But Rasmussen opens the example of that property (pp. 526-7) by talking about *objects*, and uses the two terms interchangeably therefrom. At any rate, the point made here is no less plausible if we talk instead of co-located *things*.

concerning powered properties. For instance, it seems to entail that modal continuity gives us a defeasible reason to disbelieve the patchwork principle (in at least some cases). In a nutshell, my worry for this objection is that limiting modal continuity in the way being proposed might inevitably "spill over" to limiting the principle from being applied in debates where it has enjoyed application, thereby restricting the principle *anyways*.

Lastly, even if (i) the essential limits proviso really does exclude powered properties, and (ii) there are no worries for such a reading of the proviso, I'd like to suggest that our investigation of powered properties remains worthwhile and provides insights concerning modal continuity. Firstly, investigating the existence of powered properties opened our eyes to the fact that modal continuity can fail to apply to properties with a certain structure—a structure notable enough that it is *imposed* by patchwork principles onto certain properties, revealing a difference between combination-based and continuity-based pictures of plenitude. Secondly, the existence of powered properties might still merit caution about how wide continuity applies. For not only might there be many more powered properties than we initially suspect, but the idea of a powered property which I have evinced herein is only a single instance of a more general fact about modal continuity: any property P with an extension E whose number of members is determined by some function f serves as a property to which we cannot apply modal continuity, as long as, for all naturals n, there is some natural n^* such that $f(n) \neq n^*$. In this paper, we focused on one such function, $f(n) = 2^n - 1$, but I find it hard to believe there is no other such function f which appropriately models the extension of some property P. Paying attention, then, to any *functioned* property is a relevant factor when applying modal continuity. And it is, at the very least, an epistemically live possibility that there are properties that are a function of something else—or, that certain principles *impose* this sort of structure on some properties.

7 Conclusion

In total, we have made two noteworthy advances with respect to modal continuity. First, we showed that there is a family of classes of properties with modal gaps—powered properties. More surprisingly, we found that another popular principle of modal epistemology, the patchwork principle, is likely in tension with modal continuity, as it supplies us with a direct powered property. I am happy to leave open the extent to which modal continuity applies as a guide to possibility in light of these findings. It may well be that, e.g., continuity and patchwork principles can be reconciled, and made plausible in tandem, with the right kinds of constraints in place. But that investigation is for another day.

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