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ORIGINAL RESEARCH

Inquiry, reasoning and the normativity of logic

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Abstract
According to the traditional view in the philosophy of logic facts of logic bear normative authority regarding how one ought to reason. Usually this is to mean that the relation of logical consequence between statements has some special relevance for how one's beliefs should cohere. However, as I will argue in this article, this is just one way in which logic is normative for reasoning. For one thing, belief is not the only kind of mental state involved in reasoning. Besides adopting and revising beliefs, rational agents pose and resolve questions. For another thing, the consequence relation of classical logic can be conservatively extended such that it includes logical relations between questions as well. Therefore, there is an argument to be made that not only the inference of new beliefs from extant beliefs but also the process of raising additional questions falls under the normative authority of logic. Accordingly, a nuanced account of the normativity of logic presents itself, which convincingly deals with problems such as clutter-avoidance and the paradox of the preface.

Keywords Inquiry · Reasoning · Normativity of logic · Logic of questions

1 Introduction

Beginning with Harman's seminal work *Change in View* (1986), the modern discourse on the normativity of logic is characterised by a widespread agreement that norms for how rational agents ought to reason should be sensitive to a variety of mitigating factors, in particular an agent's cognitive limitations. As I will argue in this article, there is another feature of human reasoning that should be taken into account when it comes to the relation between logic and reasoning: an agent's epistemic goals. Theoretical reasoning, I submit, is best understood in the context of inquiry. As such, there are two interdependent parts to reasoning: raising and resolving questions as well as inferring, adopting and revising question-answering beliefs. Both of these sets

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of activities are governed by the same logic, namely inquisitive logic. Therefore, the goal-directedness of reasoning, as opposed to an agent's cognitive limitations, is not a confining factor diminishing the normative authority of logic but rather another facet of logical normativity. As I will show, this perspective lends itself to a well-motivated account of the normativity of inquisitive logic which provides convincing solutions to problems such as mind cluttering and the paradox of the preface.

I will proceed as follows. To begin with, Sect. 2 introduces the view that an explication of the normative relationship between logic and reasoning should conform to the general schema of a bridge principle, as it is widely held since MacFarlane (2004). In Sect. 3, I will motivate and develop a perspective on reasoning which takes into account the questions an agent entertains. I will argue that an agent's beliefs are not only subject to norms induced by facts of logical consequence between statements, but furthermore by logical relations between questions and statements. In Sect. 4, I will then show how all of the normatively relevant logical relations between statements and questions can be subsumed under a uniform notion of entailment in inquisitive logic as developed by Ciardelli (2015). Sections 5 and 6 are dedicated to fully developing an account of the normativity of inquisitive logic for reasoning. In particular, I will provide possible bridge principles for the relationship between facts of inquisitive entailment and normative claims about reasoning. I will conclude the paper by a critical discussion of the adequacy of these bridge principles.

2 Bridges between logic and reasoning

According to the traditional view, usually associated with Frege (1893), logic provides norms which delineate correct from faulty reasoning. On an influential account (Beall & Restall, 2006, pp. 16–18) facts of logical consequence are even taken to essentially express normative claims about how one should reason (see Russell, 2017 for a critique of this view).

Harman (1986) eminently argues against this traditional view and contends that, in fact, there is a wide gap between deductive logic and the principles of rational reasoning: Logic, on the one hand, is concerned with the static relations between truth-bearers. Reasoning, on the other hand, in the sense of Harman's slogan, *reasoned change in view*, pertains to the dynamic mental processes by which rational agents form and revise their beliefs. Therefore, the kind of theory of reasoning which Harman is after should provide guidelines for how rational agents ought to expand, contract and revise their beliefs. As he argues, it is rather doubtful whether logic occupies any substantial normative role in this regard (Harman, 1986, pp. 3–8). In particular, an unrestricted normative authority of logic demands excessively much: most certainly it is beyond the limited cognitive capacities of humans to grasp all the logical consequences of one's beliefs or to become aware of possible intricate inconsistencies between one's beliefs. In fact, it appears to be perfectly reasonable not to believe irrelevant or uninteresting consequences of one's beliefs.

Much of the contemporary philosophical debate about the normativity of logic focuses on responding to Harman's sceptical challenge and bridging the gap between logic and reasoning via so-called bridge-principles as eminently proposed by MacFar-

lane (2004). To this effect, Harman's arguments are taken as desiderata for an adequate account of the normativity of logic.[1] The general form of a bridge-principle can be stated as follows[2]:

$$\text{If } A_1, \ldots A_n \models B, \text{ then } \Delta(\alpha(A_1), \ldots, \alpha(A_n), \beta(B)). \tag{BP-G}$$

where Δ expresses some (possibly conditional) normative claim, for instance a strict obligation, a mere permission or *pro tanto* reasons to act (or think) in a certain way. The operators α and β embedded under Δ denote (possibly distinct) doxastic attitudes, like belief and disbelief. Alternatively, if one is interested in an account of the normativity of logic for linguistic or communicative practices, α and β should refer to speech acts like assertion and denial.

Furthermore, one may consider bridge principles, where the antecedent is attitudinally restricted to known or recognised implications. This yields additional bridge principles of the following form:

$$\text{If } \epsilon(A_1, \ldots A_n \text{ imply } B), \text{ then } \Delta(\alpha(A_1), \ldots, \alpha(A_n), \beta(B)). \tag{BP-GR}$$

where ϵ denotes some doxastic attitude.

Since MacFarlanes's seminal work, a variety of sophisticated bridge principles have been proposed (Dutilh Novaes, 2015; Field, 2009; Steinberger, 2017). Undoubtedly, these contributions have, in various ways, expanded the ordinary understanding of reasoning and allow for a more fine-grained analysis of the issue of the normativity of logic.[3] However, they share a crucial, limiting assumption: that the content of the mental attitudes or in some cases speech acts involved in reasoning is declarative (or at least in so far as this content falls under the purported normative authority of logic). Consequently, the kind of logical relation which are assumed to bear normative import for reasoning are restricted to relations between statements.

In the next sections I want to question this tacit assumption behind the most prominent recent accounts of the normativity of logic by arguing for two interdependent ideas. First, I submit that theoretical reasoning is not limited to reasoning with and about statements but extends to posing, refining and resolving questions. Second, I will endorse the notion of logical consequence in inquisitive logic, which conservatively extends the notion of classical entailment such that it encompasses logical relations between questions and other questions as well as relations between statements and questions rather than just relations between statements.

[1] I will discuss various desiderata with respect to the bridge principles I will propose in Sect. 7.

[2] The formal representation is due to Steinberger (2017, pp. 312–313).

[3] I take these three examples as paradigmatic ways by which a novel perspective on the normativity of logic has been proposed. Field proposes bridge principles connecting classical logic with norms for how degrees of beliefs (rather than binary beliefs) should cohere. Dutilh Novaes develops an account of the normativity of logic for multi-agent dialogical practices rather than mono-agent mental processes. While Steinberger's work is more orthodox with regard to his understanding of reasoning and belief, he distinguishes between different kinds of normative functions which may come to logic: qua evaluations, appraisals or directives.

3 Reasoning and inquiry

Theoretical reasoning as it is usually understood comprises the deliberate dynamic processes of updating and revising one's doxastic states (MacFarlane, 2004, pp. 4–6; Steinberger, 2017, p. 309). Reasoning in this sense is traditionally assumed to aim at a correct mental representation of the world. However, without further qualification, this would trivialise human reasoning. If, in the words of Boghossian (2014, p. 5), the aim of reasoning consists in "figuring out what follows or is supported by other things one believes", someone who just randomly carries out logically valid inferences would engage in an activity true to this aim. However, this kind of behaviour could hardly be considered rational, as vindicated by Harman's (1986, pp. 12–15) well-known objection from clutter avoidance: not every inference, such as arbitrary disjunction introduction, produces interesting or useful truths. It is thus much more plausible that a rational reasoner aims at arriving at an accurate representation of only a small part of the world, namely the part which is of interest to them. I therefore submit that reasoning is an inherently goal-directed activity, aiming at figuring out what follows from one's beliefs with regard to certain issues of interest.

What these issues are about is arguably determined by the overarching objective of the respective activity a particular reasoning process is part of. Next to communicative interactions with other reasoners, human agents most naturally make use of their reasoning faculties when they are *inquiring*.[4] The general idea that reasoning and inquiry are closely related is far from new. In fact, the overall view that the primary subject of epistemology are inquiries rather than merely the beliefs resulting from them goes as far back as Peirce (1877). More recently, Olsson and Westlund (2006) argued that a theory of reasoning should not only consider an agent's beliefs, but furthermore the questions they want to resolve in order to account for certain patterns of belief revision. Relatedly, Koralus and Mascarenhas (2013) showed how the genuine question-directedness of reasoning explains a variety of well-known biases and fallacies in human reasoning.

3.1 Inquisitive reasoning

Inquiry can be characterised as a type of goal-directed activity a subject S engages in across some interval of time, throughout which they entertain an *inquisitive* attitude towards some well-posed question Q.[5] Being in such an inquisitive mental state, S

[4] Dutilh Novaes (2015) proposes an account of the normativity of logic specifically for dialogical practices. As she puts forth, there are two parts to dialogical argumentation: on the one hand there is an adversial dimension as each interlocutor aims at winning the argument. On the other hand, however, the interlocutors "are cooperating in a common inquiry to establish what follows from the premises, and thus to further investigate the topic in question" (Dutilh Novaes, 2015, pp. 599–600). Hence, one could argue along Stalnakerian lines (1984, 1998, 2002) that, given certain idealisations regarding possible interfering non-epistemic goals of the agents, dialogical argumentation is simply a kind of public inquiry about the truth of a given subject matter, starting from commonly believed premises.

[5] As an agent may hold inquisitive attitudes towards distinct unrelated questions at a time, different inquires may very well overlap in time. Admittedly, it is an important question how to prioritise these parallel inquiries, which of these issues to attend to first or how and where to look for evidence. However, these questions pertain to procedural, zetetic normativity as discussed by Friedman (2020) and Thorstad (2021)

desires to resolve the question Q and therefore performs certain mental or physical actions conducive towards that goal. Inquisitive attitudes are thereby action-guiding in the same sense as desires (Friedman, 2019, pp. 297–303). Carruthers (2018) furthermore argues that such inquisitive mental attitudes exist on a very fundamental, even prelinguistic level.

The actions an agent engages in for the sake of inquiring can be divided, so I claim, into two different kinds: actions in the pursuit and retrieval of information, e.g., observation and memory recall; and actions which structure and harness the available information, in particular adopting, inferring and revising beliefs as well as posing and answering additional questions.

Actions of the first kind set up and expand the informational basis for the inquiry, either by filtering and processing sensory stimuli or by recalling information from long-term memory. As far as these actions can be consciously and intentionally performed, they are arguably guided by zetetic norms in Friedman's (2020) sense, i.e., norms for how one should gather evidence (Flores & Woodard, 2023; Hall & Johnson, 1998) or what to turn one's attention to (Siegel 2017).

As a result of these actions, the gathered information becomes available to the inquiring agent's working memory and can be engaged with by the second kind of actions. Hereafter, I refer to the entirety of these processes as *inquisitive reasoning*. Rather than providing new information, actions of inquisitive reasoning put the already available information to use with regard to resolving the issue under consideration. This includes the adoption of doxastic attitudes like beliefs, inference of new beliefs from existing beliefs as well as belief revision—in short, what is generally understood by theoretical reasoning. Importantly however, all of these processes are genuinely goal-directed, aiming at answering the primary question Q. In order to make an inquiry more tractable, an agent with limited cognitive capacities may furthermore raise additional questions based on the entertained beliefs and the overall issue of the inquiry.

By successively raising new questions based on existing questions and beliefs and inferring new beliefs which provide partial or complete answers to these questions, the logical space demarcated by the available information and the entertained issue Q becomes more and more structured—a complex web of interdependent questions and question-answering beliefs emerges. Ideally, Q can thereby be resolved and the inquiry ends. Contrarily, if the gathered information proves to be insufficient in order to resolve Q, the partial answers to Q which the inquirer was able to infer at least restrict the original question Q to a smaller question Q'. Arguably, Q' then allows for a better targeted search for new evidence, facilitating processes of information retrieval such as observation and memory recall.

rather then purely epistemic and in particular logical normativity. For the purposes of this paper I therefore want to bracket these questions and, for the sake of simplicity, assume that at one point in time, an agent commits to exactly one inquiry.

3.2 Questions in reasoning

As I will discuss in the following, there are interesting systematic connections between an agent's beliefs and the questions they entertain, which bear normative relevance for an agent's inquisitive reasoning.[6]

First, the questions an agent wonders about presuppose certain pieces of information, which the agent is rationally committed to believe. These presuppositions are often tautological, as in the case of a question like 'Is Montréal the capital of Canada?', which simply presupposes that Montréal either is or is not the capital of Canada. In some cases, however, these presuppositions are contingent. Suppose that one wonders whether Montréal or Toronto is the capital of Canada. In that case, one believes the false proposition that the capital of Canada is either Montréal or Toronto.

Second, an agent's primary question may be closely related to other questions in such a way that considering these questions amounts to making progress on the issue under inquiry. Most importantly, the resolution conditions of one question, i.e., the possible ways in which it can be answered, can be entailed by the resolution conditions of another question. For example, resolving the questions "Is the capital of Canada Ottawa or Toronto?" and "Is the capital of Ontario Ottawa or Toronto?" would automatically resolve the question whether the capital of Canada and the capital of Ontario are the same city. In a sense, the first two questions therefore jointly *entail* the latter question.

Such a *question entailment* can also be relative to a certain body of further information. Consider the following illustrative example.

Example 1 (Criminal Investigation I) Suppose you are a detective and most recently you are investigating the case of a pawnbroker, Mr. Roberts, who was murdered in his office. After assessing all the available evidence, you conclude that if the murder happened during daytime, the murderer must have been Mr. Roberts' customer, Mr. Smith, whereas if the murder happened during the night, it must have been Mr. Roberts' Business Partner, Mrs. Taylor.

How should you proceed in your inquiry? Obviously, you should start to investigate the time of the crime. That is, you should consider the question of when the murder happened, because given your beliefs, resolving this question amounts to resolving the primary question of who the murderer is. The newly considered question regarding the time of the murder may open up a new perspective on the available evidence. Information which may have seemed irrelevant regarding the question of who the murderer is may appear relevant with respect to the question regarding the time of the crime. Arguably, asking further questions can induce progress on an inquiry even if the resolution conditions of the primary question are not entailed by, but merely overlap with the resolution conditions of the additional question, i.e., if there are some possible states of affairs where resolving the additional question amounts to an answer to the primary question.

[6] The logical relations between questions and statements have first been studied by Aqvist (1965), Belnap (1966) and Hintikka (1983). I will formally discuss the general ideas behind a logic for questions as they are discussed in the newer literature in Sect. 4.

This leads us to the third point: an agent's beliefs are genuinely question-directed. The content of beliefs as they feature in reasoning are not just disparate pieces of mentally represented information—they contribute to partial or complete answers to the entertained issues.[7] Hence, it is in relation to the questions an agent asks themselves that they adopt and change their beliefs. As a result, an agent's belief states are subject to rationality requirements, such as certain closure and consistency conditions, only insofar the agent engages with these beliefs in reasoning. Suppose that one entertains a belief in course of some inquiry. This belief will have an infinitude of logical consequences which are of no relevance whatsoever with regard to the considered questions, whence there is no reason for the agent to believe these consequences. Similarly, some implicit inconsistencies in one's information become only reprehensible if they amount to blatantly inconsistent beliefs in course of the same inquiry.

Similar question-relative views of the content of belief states is becoming increasingly popular in epistemology and solves a large part of the problems related to ideal belief states (Hawke, 2016; Hoek, 2023; Schaffer, 2007; Yalcin, 2021).[8] It is a central aim of the present article to bring this perspective to the debate on the normativity of logic.

4 Logic beyond statements

Subsequently, I present the basic formal tools for an adequate treatment of inquisitive reasoning as sketched in the previous section. The main ideas stem from the research program of *Inquisitive Logic*, as developed in a streamline of publications by Ciardelli, Groenendijk and Roelofsen (for a comprehensive exposition see Ciardelli et al., 2018). Significant preceding work has been done by Belnap (1966), Hintikka (1983) and Groenendijk and Stokhof (1984). Most importantly, relying on Ciardelli (2015, 2022) I will discuss how the notion of entailment in inquisitive logic subsumes all of the normatively relevant logical relations between statements and questions.

4.1 Inquisitive semantics

For the purpose of this paper, it is sufficient to formalise the content of an inquirer's doxastic and inquisitive attitudes in a simple propositional language with the following basic connectives: negation ($\neg$), conjunction ($\wedge$) and the inquisitive disjunction ($\veebar$). Disjunction ($\vee$), material implication ($\rightarrow$) and equivalence ($\leftrightarrow$) are defined in the usual way. A model of a set of atomic sentences P is a pair $\langle W, V\rangle$ where W is a

[7] This does not mean that all of the beliefs which one may entertain are necessarily directed at the questions one wonders about. Arguably, one may implicitly hold certain beliefs, which do not have any relevance with respect one's issues. However, as I will argue, such beliefs do not feature in reasoning and therefore do not fall under the normative authority of logic.

[8] It is duly noted that the idea that beliefs are relative to questions is corroborated by research in the cognitive sciences: when being confronted with a particular question, humans are quite capable of knowing whether the information available to them is sufficient to resolve the question. In other words, relative to a given question, a human agent's belief state allows for certain cases of positive and negative introspection (Egré & Bonnay, 2012; Glucksberg & McCloskey, 1981; Hampton et al., 2011).

non-empty set of possible worlds and $V : P \times W \to \{0, 1\}$ is a valuation function that specifies the truth values of the atomic sentences at each possible world.

Semantics for questions are given in terms of *support conditions* rather than truth conditions. The basic idea is to switch from states of affairs to states of information as the entity at which the meaning of a sentence is evaluated. While possible worlds represent complete descriptions of possible states of affairs, information states represent abstract bodies of information which usually do not amount to a complete description of the world. As such, an information state is represented as the set of those possible worlds that are compatible with the respective information. We can now give semantics in terms of support for both, declarative sentences, i.e., statements, and interrogative sentences, i.e., questions.[9] A declarative sentence α is supported in an information state s if and only if α is established given the information in s; and an interrogative sentence μ is supported in s if and only if the information in s is sufficient to establish some complete answer to μ.[10]

Formally, the notion of support of a sentence φ in an information state s relative to a model M, written as $M, s \models \varphi$, is inductively defined as follows (Ciardelli, 2015, pp. 46–47; 2022, pp. 56–61).

Definition 1 (*Semantics*)
Let $M = \langle W, V \rangle$ be a model for a set of atomic sentences P and $s \subseteq W$ an information state.

1. For $p \in P$: $M, s \models p$ iff $V(p, w) = 1$ for all worlds $w \in s$
2. $M, s \models \neg\varphi$ iff for all $t \subseteq s$ s.t. $M, t \models \varphi$ it holds that $t = \emptyset$
3. $M, s \models \varphi \wedge \psi$ iff $M, s \models \varphi$ and $M, s \models \psi$
4. $M, s \models \varphi \veebar \psi$ iff $M, s \models \varphi$ or $M, s \models \psi$

The derived clause for the material conditional and disjunction come out as follows:

5. $M, s \models \varphi \to \psi$ iff for all information states $t \subseteq s$ it holds: if $M, t \models \varphi$, then $M, t \models \psi$
6. $M, s \models \varphi \vee \psi$ iff there are information states t_1, t_2 with $s = t_1 \cup t_2$ s.t. $M, t_1 \models \varphi$ and $M, t_2 \models \psi$.

Let us briefly consider these semantic clauses in more detail. First, the classical connectives: An atomic sentence is supported in an information state s just in case V assigns to it the value 1 at every possible world in s. A negated sentence $\neg\varphi$ is supported in a state s if any equally or more informed state $t \subseteq s$ which supports φ is the empty state $\emptyset$. Note that $\emptyset$, being more informed than any information state, supports every statement and is thus inconsistent (compare Proposition 3.1.4 in Ciardelli, 2022, p. 57). Naturally, a conjunction is supported when both of the conjuncts are supported and an implication $\varphi \to \psi$ is supported in a state s just in case in any more or equally informed state $t \subseteq s$, it holds that if the antecedent is supported, the consequent is supported as well. Finally, a disjunction is supported in a state s if s is the union of equally or more informed states t_1, t_2 which each support one of the disjuncts respectively. The

[9] As it will be clarified below, the declarative sentences in the language are exactly those sentences whose semantics can be given in terms of truth conditions (see Definition 3.4.1 in Ciardelli, 2022, p. 64).

[10] Here and henceforth, I stick to the following convention: letters α, β and γ stand for declarative sentences, μ, ν and ξ for interrogative sentences, and ϕ and ψ can stand for sentences of either kind.

clause for $\veebar$ states that $\varphi \veebar \psi$, which can be read as the question 'whether φ or ψ', is resolved if and only if s contains enough information to settle either φ or ψ. As a shorthand we write $?\{\alpha_1, \ldots, \alpha_n\}$ for sentences of the form $\alpha_1 \veebar \cdots \veebar \alpha_n$ and $?\alpha$ for polar questions $\alpha \veebar \neg\alpha$.

Crucially, inquisitive logic preserves classical logic as a fragment. This is because the support conditions for a declarative sentence can easily be translated into its classical truth conditions and vice versa: truth in a world, written as $M, w \models \varphi$, can simply be defined as support in the singleton state $M, \{w\} \models \varphi$. In that way, a sentence φ which is supported in a state s is true in all worlds $w \in s$ and a sentence which is true at a world w is supported at the maximally-consistent information state $\{w\}$ (Ciardelli, 2015, pp. 47–48). While it does not immediately make sense to speak of truth conditions for interrogative sentences, there is a corresponding declarative π_μ for each interrogative sentence μ such that the truth set $|\pi_\mu|_M = \{w \in W \ : \ M, w \models \pi_\mu\}$ is equal to $\bigcup_{s \in [\mu]_M} s$. Hence, the truth set of π_μ consists in the maximal information state in which μ can be truthfully resolved. We thus call π_μ the presupposition of μ.

As compared to propositions in truth-conditional semantics propositions in support-conditional semantics have more structure: the proposition $[\varphi]_M$ expressed by a sentence φ is identified with the set of information states in which the sentence is supported, rather than the set of worlds in which φ is true. As such, propositions in support-conditional semantics are downward closed in the following sense: if $s \in [\varphi]_M$ for an information state s and a sentence φ, then $t \in [\varphi]_M$ for every more informed state $t \subseteq s$. The maximal elements of a proposition $[\varphi]_M$ are then called the alternatives of $[\varphi]_M$ (Ciardelli, 2015, pp. 49–50; 2022, pp. 23–24). It is straightforward to see that the propositions expressed by declarative sentences have a single alternative, whereas the propositions expressed by questions have at least two alternatives.

For an important class of questions, the alternatives of the propositions expressed by it can be directly read off their syntactic form: Let us say that a question $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ is in normal form if for any equivalent question $\nu = ?\{\beta_1, \ldots, \beta_m\}$ it holds that $n \leq m$.[11] In this case, the alternatives of $[\mu]_M$ are simply given by the truth sets $|\alpha_i|_M = \{w \in W \ : \ M, w \models \alpha_i\}$ for $i = 1, \ldots, n$.[12] For the sake of simplicity, I will henceforth assume that questions are given in normal form.

4.2 Entailment in inquisitive logic

In Sect. 3, I already discussed the various ways in which statements and questions can relate to each other: Questions can presuppose statements, the resolution conditions of one question can entail the resolution conditions of another question, and, of course, statements can partially or completely settle a question. In inquisitive logic, all of

[11] Two sentences φ and ψ are equivalent if and only if they are supported in exactly the same information states. See below for a definition of equivalence in terms of logical consequence.

[12] As an example for a question which is not in normal form consider $\mu = p \veebar q \veebar (p \wedge q)$. The third inquisitive disjunct $(p \wedge q)$ is redundant, because the corresponding maximal state in which $(p \wedge q)$ is supported is a subset of the maximal states supporting the other disjuncts, namely $\{w \in W \ M, w \models p\}$ and $\{w \in W \ M, w \models q\}$, which therefore exhaustively list the alternatives of $[\mu]_M$. Accordingly, the equivalent question $p \veebar q$ is in normal form.

these can be captured by a single notion of entailment. Let us start with single premise entailment (Ciardelli, 2015, pp. 7–21):

Definition 2 (*Single premise entailment*) For arbitrary sentences φ and ψ: $\varphi \models \psi$ iff for all models $M = \langle W, V \rangle$ and information states $s \subseteq W$, if $s \models \varphi$, then $s \models \psi$.

If φ and ψ are declarative sentences, the classical notion of entailment is obtained: in these cases preservation of support simply boils down to preservation of truth. If φ is a question and ψ is a declarative sentence, entailment captures the fact that the presupposition of φ entails ψ, i.e., ψ is implicitly presupposed by φ as well. Let now φ be a declarative sentence and ψ a question. Then $\varphi \models \psi$ captures the fact that φ resolves ψ. All the information states where φ is supported also support ψ, precisely because the single alternative of the proposition $[\phi]_M$ is a subset of one of the alternatives of $[\psi]_M$. The last, and perhaps most interesting case pertains to when both the premise and the conclusion are questions. A question μ entails a question ν just in case every information state s in which μ is resolved, i.e., where s is a subset of one alternative of μ, also resolves ν. This is the case if and only if each alternative of ν is a union of alternatives of μ. If $\mu \models \nu$ but it is not the case that $\nu \models \mu$, the information needed to resolve ν is included in the information, which is needed to resolve μ, but not the other way round, i.e., more information is needed to resolve μ than to resolve ν. This is the case iff all alternatives of ν are a union of alternatives of μ and there is an alternative of μ which is a proper subset of an alternative of ν. We thus say that ν is more coarse-grained than μ and that μ is a refinement of ν.

Consider now the general case (Ciardelli, 2015, pp. 60–65; 2022, pp. 77–84).

Definition 3 (*Entailment*) For a (possibly empty) set of declarative sentences Γ, a (possibly empty) set of questions Ξ, and an arbitrary sentence ψ:
$\Gamma, \Xi \models \psi$ iff for all models $M = \langle W, V \rangle$ and information states $s \subseteq W$ it holds that if $s \models \gamma$ for all $\gamma \in \Gamma$ and $s \models \xi$ for all $\xi \in \Xi$, then $s \models \psi$.

If ψ is a declarative sentence and Ξ is empty, the support-conditional entailment $\Gamma \models \psi$ holds just in case the analogous classical, truth-conditional entailment obtains. Suppose now that Ξ is not empty. Then, $\Gamma, \Xi \models \psi$ holds exactly in those cases where the presuppositions of the questions in Ξ jointly with the statements in Γ entail ψ. One may contend here that a question should not entail a genuinely informative sentence, but rather presuppose it. Arguably, this is a pragmatic distinction which does not need to be taken care of in a purely formal system. However, when it comes to norms for inquisitive reasoning, the questions one entertains in course of an inquiry should not presuppose information which is not already believed.

Consider now the case where ψ is a question. By definition, $\Gamma, \Xi \models \psi$ holds if and only if for all models M and all information states s: If $s \models \Gamma$ and $s \models \Xi$, then $s \models \psi$. But this just means that $s \models \Xi$ implies $s \models \psi$ for all information states $s \subseteq |\Gamma|_M$, where $|\Gamma|_M$ is just the intersection of the truth sets of all formulas in Γ. Hence $\Gamma, \Xi \models \psi$ expresses the fact that the question ψ is entailed by the questions in Ξ relative to the context specified by the formulas in Γ (Ciardelli, 2015, p. 11).

The example *Criminal Investigation I* from Sect. 3.2 describes such a situation: Let s be a propositional variable standing for "Mr. Smith is the murderer", t stand for "Mrs.

Taylor is the murderer" and d for "the murder happened during daytime". Then the following holds: $s \vee t, d \rightarrow s, \neg d \rightarrow t, d \veebar \neg d \models s \veebar t$. That is, relative to a context where it is established that if the murder happened during daytime, the murderer must have been Mr. Smith and that if the murder did not happen during the day, it must have been Mrs. Taylor, resolving the question whether the murder happened during the day amounts to resolving the question regarding the identity of the murderer.

5 A framework for inquisitive reasoning

Being equipped with the formal tools of inquisitive logic, I will present a simple framework for an agent's beliefs and issues in the course of an inquiry. As posited in Sect. 3, the goal of reasoning consists in discerning whether the information one possesses is sufficient for resolving the issue of the inquiry and if so, to correctly infer the answer to this issue from the available information. To that end an agent S does not only infer new beliefs but also raises further questions in order to make the problem at hand more tractable and bring the available information to use with regard to resolving Q. At any rate, a model of the mental state of S while inquiring about Q should thus not only include S's beliefs but furthermore the questions S wonders about. This yields the following definition:

Definition 4 (*Inquisitive state*) A Q-inquisitive state of an agent S inquiring about an issue Q at a time t is given by a pair $\langle \Gamma_t^Q, \Xi_t^Q \rangle$, where Γ_t^Q is a set of declarative sentences and Ξ_t^Q is a set of questions, such that $Q \in \Xi_t^Q$.

Γ_t^Q is called S's doxastic state with respect to Q at time t and Ξ_t^Q their research agenda with respect to Q at time t.[13] For $\alpha \in \Gamma_t^Q$, we say that S believes α at time t with respect to Q and for $\mu \in \Xi_t^Q$, we say that S entertains μ at time t with respect to Q.[14] If time and primary question are clear from the context of the inquiry, we drop mentioning t and Q and just say that S believes α and entertains μ.

So far, Γ_t^Q and Ξ_t^Q are just arbitrary sets of sentences with no internal structure. In particular, Γ_t^Q and Ξ_t^Q are not closed under any meaningful notion of entailment. This is perfectly in line with the aim of this paper: human agents are by no means logically omniscient. Due to limited cognitive capacities regarding computing power, time and memory, humans frequently violate the most basic rationality and consistency requirements. Instead of modelling ideally rational reasoners as classical epistemic and doxastic logic does, a central objective of this paper is to articulate normative principles

[13] The general idea that the description of an agent's epistemic state should include their research agenda besides their beliefs is due to Olsson and Westlund (2006). However, as opposed to the concept of a research agenda as it occurs here, their notion of a research agenda is meant to encompass all of the questions which an agent wonders about at a given time, regardless of their subject matter.

[14] Note that I leave it largely open how exactly an agent mentally represents questions. This is at least partly an empirical questions pertaining to cognitive science. One possible account consistent with inquisitive logic is offered by mental model theory (Johnson-Laird, 1983, 2006; Johnson-Laird et al., 2004), which has been developed into a theory of reasoning involving questions by Koralus and Mascarenhas (2013).

for how non-ideal agents should reason. The requirements imposed on an agent by logical norms are therefore to be explicitly stated by bridge principles.[15]

Nevertheless, we want to assume some general adequacy constraints on an agent's inquisitive state which ensure that inferences of inquisitive reasoning are well-founded. They fix the general structure of an agent's representational mental state in course of inquiry. The bridge principles, which I will propose in the next section then specifically state norms for how beliefs and issues should cohere within this externally given structure. For one thing, it is hardly contestable that an agent should not wonder about questions which presuppose more than what is believed. Hence, we require that at any time t there is $\pi_\mu \in \Gamma_t^Q$ for all $\mu \in \Xi_t^Q$. For another thing, the available information Γ_t^Q should be consistent. This is arguably a rather strong assumption and may seem *ad hoc*. However, given the right interpretation for Γ_t^Q its consistency appears to be justified. Recall that in course of inquiring about Q an agent gathers evidence and accordingly adopts new beliefs. According to an influential strand of evidentialism (Kolodny, 2008) conflicting beliefs cannot be supported by the same evidence. Hence, if the evidence gathering agent obeys the relevant evidentialist norms, the beliefs which are thereby produced will naturally turn out to be consistent. At some point in time t during the inquiry about Q, before S starts to infer new beliefs, S's epistemic state w.r.t Q is therefore consistent. Similarly, if at some later point t' in course of the inquiry, S learns new evidence α, which is consistent with Γ_t^Q, they simply expand their doxastic state: $\Gamma_{t'}^Q = \Gamma_t^Q \cup \{\alpha\}$. If contrarily, the new evidence α is not consistent with Γ_t^Q, they ought to revise their doxastic state accordingly. However, how to do so is not a matter of logical norms, but rather pertains to questions of evidentialism or even pragmatism. Taken together, these two conditions amount to the following definition of a *well-founded* inquisitive state.

Definition 5 (*Well-founded Inquisitive state*) A Q-inquisitive state $\langle \Gamma_t^Q, \Xi_t^Q \rangle$ is well-founded at time t iff Γ_t^Q is consistent and for all $\mu \in \Xi_t^Q$ there is $\pi_\mu \in \Gamma_t^Q$.

It is important to note that well-foundedness does by no means require an agent to be globally consistent. An agent is only assumed to be locally consistent with regard to their beliefs relative to inquiring about some issue Q, i.e., Γ_t^Q is consistent. But

[15] Of course, there are formal models for the epistemic and doxastic states of non-ideal agents. A straightforward approach going back to Hintikka (1975) and Rantala (1982) is *impossible worlds semantics*. The general idea is to extend classical possible world models with worlds which are doxastically accessible but logically impossible, so-called impossible worlds. At possible worlds the semantics are recursively specified as usual. At impossible worlds, however, the semantics for all formulas is directly defined by a valuation function. Hence, at impossible worlds, any imaginable logical impossibility may be the case. For instance, both φ and $\neg\varphi$ may be true or $\varphi \wedge \psi$ may be false, even though both φ and ψ are true (Berto & Jago, 2019, pp. 112–113; Wansing, 1990, p. 526). Hence, impossible worlds just correspond to sets of arbitrary formulas without any internal structure, whereas possible worlds correspond maximally consistent sets of formulas. An agent's epistemic state can therefore be identified with the set of sentences they believe. But then, impossible world semantics does not provide a richer picture of an agent's epistemic state than the purely syntactic approach in the definition above. In fact, it has proven surprisingly difficult to construct impossible world models for the belief states of logically non-omniscient but still logically competent agents, i.e., what Cherniak (1986) calls minimally rational agents (Berto & Jago, 2019, pp. 115–123). I will therefore opt for a purely syntactic approach and stipulate the norms for governing the beliefs and questions an agent entertains explicitly via bridge principles.

they may very well inquire about another issue Q' (possibly at the same time) with Γ_t^Q and $\Gamma_t^{Q'}$ being jointly inconsistent. Nevertheless, there arguably are certain global coherence requirements in place, which however shall be of no primary concern here. I want to give one example for an important global coherence requirement: suppose that an agent S inquires into two distinct questions Q and Q' at the same time t where $Q \models Q'$, i.e., resolving Q amounts to resolving Q'. Hence, the information which is needed to resolve Q' is included in the information which is needed to resolve Q. Arguably, one's beliefs w.r.t. Q' should therefore be part of one's beliefs w.r.t Q, $\Gamma_t^{Q'} \subset \Gamma_t^Q$, and accordingly $\Xi_t^{Q'} \subset \Xi_t^Q$ (compare the recent work by Yalcin, 2018 and Hoek, 2023 on belief fragmentation). However, even this seemingly weak requirement appears to be problematic in situations akin to the preface paradox. I will discuss this issue in Sect. 7.

While the global consistency of all the agent's doxastic attitudes across various areas and contexts is highly implausible and arguably unattainable for human agents, the merely local consistency of all of the agent's beliefs which relate to the same subject matter or process of inquiry, as it is assumed here, is much more plausible and should be understood as a general adequacy constraint.

6 Inquisitive reasoning and the normativity of logic

The notion of an agent's Q-inquisitive state $\langle \Gamma_t^Q, \Xi_t^Q \rangle$ at some point in time t provides the basic framework for an account of the normativity of inquisitive logic for inquisitive reasoning. In accordance with the two-fold structure of an agent's inquisitive state, the bridge principles which I provide below come in pairs: one acting on an agent's doxastic state and one on their research agenda. In the ensuing section, I will motivate the central ideas behind these bridge principles. First, I will discuss the intuitions informing the norms governing which questions one is permitted or even obliged to ask relative to one's doxastic state and research agenda. Thereafter, I will discuss which relationship between a declarative sentence α and an agent's doxastic state Γ_t^Q and research agenda Ξ_t^Q commits them to believe α. This amounts to a pair of preliminary bridge principles, which I will then further develop in Sect. 7.

6.1 Raising further questions

On the perspective outlined in Sect. 3, it is a crucial part of reasoning to not only infer new beliefs but also raise further questions. Hence, both an agent's doxastic state and an agent's research agenda change in course of inquiring about Q. While the doxastic state may change as a consequence of evidence-gathering actions next to inferences, the research agenda only changes if new questions are consciously raised. This may happen for a couple of reasons: in particular, there may be external circumstances which prompt us to consider a certain question, for example, if an interlocutor raises a question or makes a statement which we do not immediately accept as being true. Furthermore, there may be questions one should raise given one's existing beliefs and

questions on purely logical grounds. Subsequently, I discuss which norms govern the change of an agent's research agenda.

In order to ensure that S's inquisitive state remains well-founded when asking new questions, S should not be permitted to raise a question which presupposes non-trivial information which is not already presupposed. In this context, I want to establish the following piece of terminology: a question μ is called appropriate at time t with respect to Q iff $\Gamma_t^Q \models \pi_\mu$. Hence, an agent S in an inquisitive state $\langle \Gamma_t^Q, \Xi_t^Q \rangle$ is not permitted to entertain a question μ which is not appropriate. Conversely, if μ is appropriate and S adds μ to their research agenda, I assume that S automatically adds π_μ to Γ_t^Q. Given the close syntactic relationship between a question and its presupposition, it is plausible that this is an act without additional cognitive cost.

Next to questions which one is not permitted to entertain in a given inquisitive state, there arguably are questions one ought to entertain in course of inquiry. This is because there may be questions $\mu_1, \ldots, \mu_m$ whose resolution conditions, given one's beliefs $\alpha_1, \ldots, \alpha_n$, entail the resolution conditions of another, possibly more difficult question μ on one's agenda (for instance the primary question Q). While a direct answer to μ may be out of reach, answers to some or all of the $\mu_1, \ldots, \mu_m$ may be inferred with relative ease. Consider the following example:

Example 2 (Criminal Investigation II) Again, you are a detective investigating the case of Mr. Roberts, who was murdered in his office. It is already established that the murderer is either Mr. Roberts' customer, Mr. Smith, or Mr. Roberts' business Partner, Mrs. Taylor. After your preliminary investigation you ascertain that Mrs. Taylor was on a business trip during the whole day, whence if she is the murderer, the crime must have been committed at night. Additionally, you have the testimony of Mr. Smith's wife that he was at home during the night. So if Mrs. Smith is speaking the truth and Mr. Robert's was murdered during night-time, it could not have been Mr. Smith. However, it could be that Mrs. Smith is lying, covering up for her husband, in which case he would be the murderer.

Let s stand for "Mr. Smith is the murderer", t for "Mrs. Taylor is the murderer", d for "the murder happened during the day" and l for "Mrs. Smith is lying". Accordingly, $\neg d$ stands for "the murder happened at night" and $\neg l$ for "Mrs. Smith is speaking the truth". The information learned during the primary investigation can therefore be represented as follows: $t \to \neg d$, $(\neg l \wedge \neg d) \to \neg s$, $l \to s$. It is presupposed that $s \vee t$, whence the primary question on the research agenda regarding the identity of the murderer can be narrowed down to $s \veebar t$. Consequently, your inquisitive state after the preliminary investigation is given by $\langle \{s \vee t, t \to \neg d, \neg l \wedge \neg d \to \neg s, l \to s\}, \{s \veebar t\} \rangle$, which is obviously well-founded.

A natural way to proceed with the inquiry in this situation is to start investigating the time of the crime as well as whether Mrs. Smith is lying. This intuition is well-justified in light of the entailment $s \vee t, t \to \neg d, (\neg l \wedge \neg d) \to \neg s, l \to s, d \veebar \neg d, l \veebar \neg l \models s \veebar t$, which expresses the fact that, relative to an information state where $s \vee t$, $t \to \neg d$, $\neg l \wedge \neg d \to \neg s$ and $l \to s$ are established, settling the questions $d \veebar \neg d$ and $l \veebar \neg l$ amounts to settling the primary question $s \veebar t$. Hence, relative to your doxastic state after the preliminary investigation, making progress on the questions $?d$ and $?l$ induces

progress on the question regarding the identity of the murderer. Raising these questions may therefore open up new possibilities for the inquiry: for instance, new actions in the pursuit of additional evidence may present themselves, like separate interrogation of Mr. and Mrs. Smith to compare their accounts of the events at night. Or, implicitly given evidence which may have seemed irrelevant regarding the primary question of who the murderer is, may turn out to be relevant with respect to these additional questions.

Generalising from this example, if $\Gamma \subseteq \Gamma_t^Q$ is a set of believed sentences and Ξ is a set of appropriate questions, the entailment $\Gamma, \Xi \models \mu$ states that relative to Γ, resolving all of the questions in Ξ amounts to resolving μ. Hence, if μ is on S's research agenda, they may have good reasons to add the questions in Ξ to the research agenda as well. The additional structure of the resulting inquisitive state then allows for harnessing the available information in an efficient, question-directed manner, thereby making the objective of resolving the question μ more tractable. Note that μ does not need to be the primary question Q, but may itself be a secondary question raised at a previous point in time. Of course, finding such questions can be a highly non-trivial, creative task. But this is precisely what a good deal of reasoning consists in. Good reasoning scarcely ever only encompasses straightforward inferences which immediately present themselves as yielding the desired information. Reasoning typically proceeds from two directions: from the already established information by inferring new beliefs and from one's epistemic goal by creatively raising questions which if answered provide parts of the information one wants to obtain.

There is another crucial point of consideration when it comes to the normative importance of (conditional) question entailments: on its own, the fact that $\Gamma, \Xi \models \mu$ for a set of sentences $\Gamma \subseteq \Gamma_t^Q$ and a question μ on S's research agenda hardly constitutes sufficient reasons for adding the questions in Ξ to the research agenda. This is because entailment in inquisitive logic is monotonic, whence the following holds: if $\Gamma, \Xi \models \mu$, then $\Gamma, \Xi, \xi \models \mu$ for arbitrary questions ξ. But this should not result in S having reasons to add ξ to their research agenda, as resolving ξ in addition to the questions in Ξ is absolutely irrelevant w.r.t. resolving μ. More generally, the problem can also be put the following way: the information needed for resolving Ξ and ξ goes beyond the information needed for resolving μ. Adding Ξ and ξ to one's research agenda could therefore unnecessarily complicate the inquiry. For instance, the ultimately fine-grained question "What is the case?" in the sense of "Which one of all possible worlds is the actual world?" would entail all other questions.

In order to avoid this problem, the entailments $\Gamma, \Xi \models \mu$ which bear reasons to add Ξ to one's research agenda should be restricted to those Ξ which are *maximally coarse-grained* while still entailing μ conditional on Γ. Hence, an agent should only have reasons to add a set of questions Ξ to their research agenda if Ξ entails μ relative to Γ and any more coarse-grained set of questions fails to entail μ relative to Γ. This yields the the following definition:

Definition 6 (*Maximally coarse-grained questions*) A set of questions Ξ is maximally coarse-grained regarding an entailment $\Gamma, \Xi \models \mu$ if for all φ with $\Xi \models \varphi$ and $\Gamma, \varphi \models \mu$ it holds that $\varphi \dashv\models \bigwedge_{\xi \in \Xi} \xi$.

Note that φ can stand for a question or a statement here. This is necessary because Π_t^Q and Γ on their own may be sufficient to resolve μ, in which case no set of questions Ξ would be maximally coarse-grained regarding Γ, $\Xi \models \mu$. In the example above, this means that the question one actually has reasons to add to one's agenda is whether it is the case that Mrs. Smith is not lying and the murder happened at night, i.e., $?(\neg d \wedge \neg l)$. Obviously this question is coarser than the conjunctive question $?d \wedge ?l$ and furthermore it holds that $s \vee t, t \rightarrow \neg d, (\neg l \wedge \neg d) \rightarrow \neg s, l \rightarrow s, ?(\neg d \wedge \neg l) \models s \trianglerighteq t$ because if $\neg d \wedge \neg l$ were true, the murderer is Mrs. Taylor whereas if either d or l is true, the murderer is Mr. Smith.

Taken together these considerations allow for the construction of a variety of possible bridge principles which connect facts of inquisitive entailment with norms for an agent's research agenda. Instead of considering a whole taxonomy as MacFarlane does, I will take the following bridge principle as a starting point and then fine-tune its constituent parts accordingly.

> If $\Gamma, \Xi \models \mu$ for a set of sentences Γ and a set of maximally coarse-grained appropriate questions Ξ, then S ought to see to it that (S adds Ξ to their research agenda if S believes Γ and entertains μ). (IBP-Q)

6.2 Inferring new beliefs

Let us now turn to the second part of inquisitive reasoning, besides raising new questions: the inference of new beliefs from existing beliefs. The problem at hand is at the core of the debate on the normativity of logic: which consequences of one's beliefs ought to be believed as well? Here, the inquisitive perspective lends itself to an elegant answer: only those consequences of one's beliefs ought to be believed which provide partial or complete answers to the questions one entertains. Let us consider this idea in more detail.

Assuming that an agent S raises new questions only in accordance with the general rules discussed in the previous section, relative to the presupposed information and S's doxastic state, all of the questions on their research agenda have resolution conditions which contribute to the resolution conditions of the primary question Q. Hence, if at any given point in time t, the information which is explicitly available to S, i.e., represented in Γ_t^Q licenses an inference to a possible partial or complete answer to one of these questions, S should arguably recognise and carry out this inference and thereby resolve that question (under certain performance idealisations). In order to couch this idea in the form of a bridge principle, one first needs to suitably generalise the question-answer relationship. Consider the following definition:

Definition 7 (*Pertinence*) Let $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ be a question in normal form. A declarative sentence α is pertinent w.r.t. μ if and only if there is a subset $\{\alpha_1^*, \ldots, \alpha_k^*\} \subsetneq \{\alpha_1, \ldots, \alpha_n\}$ such that $\alpha \models \bigvee_{i=1}^{k} \alpha_i^*$.

Thereby, a declarative sentence α is called pertinent regarding some question μ if α entails a partial or complete answer to μ. Given this definition, we can capture the idea that entertaining certain beliefs is conducive towards resolving the issue Q of an inquiry whereas other beliefs may be irrelevant with respect to this inquiry. We say that a declarative sentence α is Q-relevant for S at time t if there is a question μ on S's research agenda Ξ_t^Q such that α is pertinent w.r.t. μ. Hence, whether a given sentence is epistemically relevant for an inquiry depends on whether its informational content contributes to answering a question on the research agenda.

We can now formulate a first, preliminary bridge principle governing an agent's doxastic state in course of inquiry. The central idea is to relativise the normative authority of logic to only those consequences which provide partial or complete answers to questions on the research agenda, i.e., those consequences which are Q-relevant:

> If $\Gamma \models \alpha$ and α is pertinent regarding some question μ on S's research agenda, then S ought to see to it that (S believes α if S believes Γ). (IBP)

Note that this bridge principle conforms to the general schema BP-G, where the consequent consists in the conditional 'If α is Q-relevant for S, then S ought to see to it that S believes α if S believes Γ'.

Before discussing the pair of bridge principles IBP and IBP-Q in detail in the next section, a few remarks about the notions of pertinence and Q-relevance are in order. Q-relevance as defined above is a dynamic concept: whether some sentence α is Q-relevant does not solely depend on α and Q but furthermore on other questions on the research agenda. Hence, a sentence α may not be Q-relevant at some point t but become Q-relevant at a later point in time t' after a certain question is added to the research agenda w.r.t. Q. Crucially, however, whether a question is on an agent's research agenda is governed by logical norms as well, for instance IBP-Q. The normative consequences of pairs of bridge principles for inquisitive reasoning are thus highly interdependent: which questions to be asked and which beliefs to adopt are not two separate questions, governed by two separate norms, but rather two interwoven parts of the same question—how to reason when inquiring about Q. Nevertheless, as Q is always on the research agenda, answers to Q are always Q-relevant; only other pieces of information which may serve as intermediate conclusions are contingently Q-relevant depending on the agent's inquisitive state.

7 Discussion of the desiderata

While the literature is rich with various proposals regarding adequacy criteria for bridge principles, it is questionable whether there is a canonical list of desiderata, which any account of the normativity of logic must meet. In fact, logic appears to have normative force on different levels: on a first-person level as a directive for how to think and on an intersubjective level as an evaluative standard. Qua a *directive*, logical norms serve as a subjective guide for how to manage one's doxastic states

in a given situation. Contrarily, as an *evaluation*, logical norms serve as external standards for assessing beliefs as correct or incorrect and inferences as valid or invalid. While directives should be in principle transparent and followable and thus sensitive to individual cognitive limitations, evaluations provide criteria of correctness which may be completely detached from what an agent's cognitive capacities actually can afford. Steinberger (2017) considers yet another way in which logic can be normative for reasoning—qua an *appraisal*. *Appraisals* occupy an intermediate role between evaluations and directives. They are intersubjective, third-person standards for praise or blame of other agents and thus generally similar to evaluations. However, more akin to directives, these standards should be responsive to differences in cognitive capacities or prior knowledge. The demands an appraisal puts on the appraisee may therefore vary and are highly dependent on a variety of contextual factors (Steinberger, 2019, pp. 12–19). Thereby, appraisals occupy a whole spectrum of different normative standards in between evaluations and directives.

7.1 Ideal inquiry

With regard to the present paper, I aim at an account of the normativity of logic for inquisitive reasoning qua appraisals and accordingly motivate suitable desiderata which I will take as a basis of the ensuing discussion. To begin with, I will discuss the proposed bridge principles without considering the cognitive limitations of non-ideal agents. Thereafter, I will de-idealise these bridge principles in order to make them sensitive to what an agent's cognitive capacities can be reasonably expected to afford.

Belief revision

As argued by Harman (1986), reasoning is a dynamic process of revising one's beliefs. Logic, however, deals with static structural relations between truth-bearers. It is thus not entirely clear how a relation between facts of logics and an agent's beliefs allows for the possibility of belief revision. Since MacFarlane (2004) it is well-known that this prima facie difficulty can easily be avoided by opting for a wide scope of the deontic operator. In particular, IBP simply states that if an agent's doxastic state licenses an inference to a sentence α which entails a partial or complete answer to one of the questions on the agent's research agenda, they ought to carry out this inference and adopt a belief in α or revise their doxastic state accordingly. Similarly, the wide scope of deontic operator blocks the bootstrapping objection by Broome (1999, pp. 405–406) i.e., the odd consequence of small-scope norms that anything which is actually believed ought to be believed.

Connectedness

On its own, IBP yields merely a wide-scope obligation to believe those consequences of one's beliefs which constitute answers to Q, because leaving aside IBP-Q, Q is the only question which necessarily is present on one's research agenda when inquiring

about Q.[16] From a purely instrumentalist point of view this may appear to be sufficient, assuming that the goal of inquiry consists solely in answering the primary question Q. Nevertheless, a key part of human rationality consists in the capacity to not only infer new beliefs from existing beliefs but doing so in a transparent, introspectable manner. In particular, there seems to be something wrong with an agent who, while believing the consequences of their beliefs which answer their questions, fails to believe certain intermediate conclusions towards that end.[17]

The belief set of an agent should therefore not only be closed under those implications which are pertinent to their primary questions. Furthermore, an agent should believe the consequences of their beliefs which connect premises and question-answering conclusions via simple inferences like modus ponens. Consider an agent S who believes p, $p \to q$ and $q \to r$ while inquiring about $?r$. As r is an answer to the question $?r$, IBP commits S to believe r. But, assuming that $?r$ is the only question on S's research agenda, IBP says nothing about whether S should believe q or $p \to r$. However, beliefs in q or $p \to r$ may be the results of important intermediate inferences which S carries out in order to infer r from p, $p \to q$ and $q \to r$. If S is capable of inferring r from the premises p, $p \to q$ and $q \to r$, they should be able to infer q and $p \to r$ as well. Otherwise, their belief in r may not be justifiable in a sufficiently transparent manner. However, in isolation, neither q nor $p \to r$ are Q-relevant, because there is no question on S's research agenda such that q or $p \to r$ are pertinent w.r.t. this question.

Hence, in order to ensure that the provided bridge principles are sufficiently strong, it needs to be guaranteed that an agent's research agenda includes all of the questions they should entertain for the sake of inquiring about Q. The preliminary candidate for a bridge principle governing an agent's research agenda IBP-Q states that if relative to the presupposed information Π_t^Q and S's belief in Γ, resolving the maximally coarse-grained questions in Ξ amounts to resolving μ, this obliges S to entertain the questions in Ξ if they entertain μ. Coming back to the example, the following two inquisitive entailments hold $q \to r, ?(q \vee r) \models ?r$ and $p, ?(p \to r) \models ?r$. Therefore, according to IBP-Q, S ought to entertain the questions $?(q \vee r)$ and $?(p \to r)$ if they maintain to believe $q \to r$ and p, respectively, and entertain $?r$. Whence, because q is a partial answer to $?(q \vee r)$ and $p \to r$ answers $?(p \to r)$, IBP amounts to the wide-scope obligation to believe q if S believes p and $p \to q$, as well as to believe $p \to r$ if S believes $p \to q$ and $q \to r$. Hence, IBP and IBP-Q jointly guarantee that an agent's beliefs are properly connected.

Note that the standard of ideal rationality, i.e., full logical closure, demands much more. Suppose that one believes sentences p and $p \to q$ and inquires about the question $?q$. As it should be, both the bridge principles IBP and IBP-Q as well as the standard of full logical closure amount to an obligation to believe q, given one maintains believing p and $p \to q$. But, as opposed to these evaluative bridge principles,

[16] Note that IBP thereby amounts to a kind of restricted intensionality constraint on an agent's beliefs: if $\alpha \dashv\models \beta$ and α is Q-relevant, the agent S ought to see to it that they believe α if and only if they believe β.

[17] Of course an agent may forget such intermediate conclusions after they use them in further inferences or never consciously represent them in the first place. But this may only be the case due to certain cognitive limitations of the agent regarding long-term and working memory, which are not considered as far as evaluations are concerned.

the demands of logical closure are unreasonably high: for instance, the wide-scope obligation to believe $p \vee r$ for arbitrary r, even though a belief in $p \vee r$ is completely irrelevant with regard to resolving the question $?q$. I therefore submit that the demands IBP and IBP-Q put on an agent qua evaluation are appropriately strong in light of an agent's epistemic goals but not overly petty as compared to the standards of ideal rationality.

Clutter-avoidance

One of the key-problems for an unrestricted normative authority of logic is the objection from clutter-avoidance. If one's beliefs should be closed under some notion of logical implication, every proposition which is logically implied by one's beliefs ought to be believed as well. This would not only be impossible to achieve, which would be a problem for directive norms but not so much for evaluative ones, but furthermore by no means appropriate. There is an infinitude of trivial consequences of one's belief, which one has no good reason to believe—for instance, arbitrary disjunctions, where one of the disjuncts is believed in the first place. In fact, wasting one's limited cognitive resources on inferring irrelevant consequences of one's beliefs appears to be plainly irrational (Harman, 1986, pp. 12–14; Steinberger, 2017, p. 310).

IBP solves this problem by the relativisation of the wide-scope obligation to believe a logical consequence α of one's beliefs Γ to only those α which are pertinent regarding a question on the research agenda.[18] Suppose that $\Gamma \models \alpha$ and α is pertinent w.r.t. a question $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ on S's research agenda, i.e., there is a subset $\{\alpha_1^*, \ldots, \alpha_k^*\} \subsetneq \{\alpha_1, \ldots, \alpha_n\}$ such that $\alpha \models \bigvee_{i=1}^{k} \alpha_i^*$. Obviously, given that S can be expected to infer α from Γ, they ought to believe α if they believe Γ. Let now β be an arbitrary sentence. Then it holds that $\Gamma \models \alpha \vee \beta$ but generally $\alpha \vee \beta$ is not pertinent w.r.t. μ (or another question on the research agenda), whence no obligation regarding believing $\alpha \vee \beta$ arises.

Obtuseness

As discussed by MacFarlane (2004, p. 12), an overly lenient stance on what an agent ought to believe runs into the risk of 'Obtuseness': suppose an agent believes a proposition A and B but neither believes nor disbelieves the conjunction $A \wedge B$. This kind of active agnosticism about an immediate consequence of one's beliefs seems to be rather odd, if not irrational.

Consider an agent S, who believes some sentences α and β. Suppose that at least one of these two sentences is Q-relevant, i.e., pertinent w.r.t. some question μ on S's research agenda. In this case, due to monotonicity, $\alpha \wedge \beta$ is pertinent as well and IBP commits S to believe $\alpha \wedge \beta$. The same holds if $\alpha \wedge \beta$ itself is Q-relevant. Suppose now that neither α nor β nor $\alpha \wedge \beta$ are Q-relevant, and S's research agenda is in

[18] It is important to note that IBP forbids mind-cluttering only in so far as S is not considering any irrelevant questions. Arguably, there are zetetic norms against raising *junk* questions in course of inquiry (see Balcerak Jackson, 2022, pp. 12–15). Fortunately, this is entirely consistent with the respective logical norm IBP-Q. As S is only obliged to raise a question if it is maximally coarse-grained w.r.t. a relevant question entailment, no obligation to consider junk questions arises.

accordance with IBP-Q. Hence, S does not even consider whether to believe $\alpha \wedge \beta$ nor do they have a genuine reason to do so. In this case, there is no merit in believing the conjunction $\alpha \wedge \beta$, as not even the belief in α and β has any relevance regarding the goal of the inquiry whatsoever. It should therefore be condonable to not actively believe the conjunction in this case.

The preface paradox

Finally, let me address how the proposed pair of bridge principles fares with regard to the problem of purportedly unavoidable inconsistencies (Harman, 1986, p. 16) as in the infamous paradox of the preface (Makinson, 1965). It is instructive to formulate the paradox within an inquisitive framework. Suppose that a scientist S inquires into a number of research questions $\mu_1, \ldots, \mu_n$ which pertain to the same, descriptive subject matter, say in botany or zoology. As a result of meticulous research, S has gathered bodies of information $\Gamma_1, \ldots, \Gamma_n$ such that for each $k = 1, \ldots, n$, Γ_k entails a sentence α_k which amounts to a complete answer to the question μ_k. We may model this situation as one big inquiry into the primary question $Q := \mu_1 \wedge \cdots \wedge \mu_n$, with the doxastic state Γ_t^Q of S being such that $\Gamma_1 \cup \cdots \cup \Gamma_n \subset \Gamma_t^Q$. Note that, according to IBP-Q, the individual questions $\mu_1, \ldots, \mu_n$ ought to be on S's research agenda as well. In accordance with the bridge principle IBP, S is committed to believe $\alpha_1, \ldots, \alpha_n$ if they maintain their belief in $\Gamma_1, \ldots, \Gamma_n$. Furthermore, as Γ^Q entails the conjunction of the $\alpha_1, \ldots, \alpha_n$, S ought to believe $\bigwedge_{i=1}^n \alpha_i$, which constitutes a complete answer to Q and thus results in the inquiry to end. Suppose now that there is a body of information Δ available to S, which pertains to the general fallibility of human agents and thus entails $\neg \bigwedge_{i=1}^n \alpha_i$. If S were to adopt a belief in Δ at time $t' > t$, their doxastic state would become blatantly inconsistent because $\neg \bigwedge_{i=1}^n \alpha_i$ constitutes a partial answer to Q and therefore ought to be believed by S if none of the beliefs in Δ are revised.

While in principle the preface paradox therefore can arise, it is very much doubtful whether a rational agent would and in fact should adopt a belief in Δ when inquiring about their research questions $\mu_1, \ldots, \mu_n$. Note that this is not a question about the normativity of logic for inquisitive reasoning but a question of zetetic normativity regarding which evidence one should gather for the sake of a given inquiry. The statistical information about the high probability of the occurrence of errors does not constitute any relevant scientific evidence for the subject matter of one's research. Hence, regarding the epistemic goal of S's research, i.e., answering the research questions $\mu_1, \ldots, \mu_n$, the information Δ has no relevance whatsoever and should therefore not play any role in the process of inquiring about these questions.

Consider now another inquiry. After bringing together all the results of their research, S aims to publish these results in a non-fiction book, with its descriptive content being given by $\bigwedge_{i=1}^n \alpha_i$. When writing the preface to this book, S starts to wonder about whether everything they say in the book is correct or whether there may be mistakes. Hence they start to inquire about the question $Q' :=? \bigwedge_{i=1}^n \alpha_i = \bigwedge_{i=1}^n \alpha_i \veebar \neg \bigwedge_{i=1}^n \alpha_i$. Note that this question is entirely different from and much more coarse-grained than the question $\bigwedge_{i=1}^n ?\alpha_i$, which may be asked in course of an inquiry for the sake of double-checking every single one of the statements made in the book. In particular, $\alpha_1, \ldots \alpha_n$ are pertinent w.r.t. to the latter question but not the former.

Furthermore, IBP does not commit S to entertain the questions $?\alpha_1, \ldots, ?\alpha_n$, when inquiring about Q': while it holds that $?\alpha_1, \ldots, ?\alpha_n$ entails the question $?\bigwedge_{i=1}^{n} \alpha_i$, the set of questions $?\alpha_1, \ldots, ?\alpha_n$ is not maximally coarse-grained in that respect. Hence, the available bodies of information $\Gamma_1, \ldots, \Gamma_n$, which constitute the evidence bases for the individual research questions $\mu_1, \ldots \mu_n$ seem irrelevant regarding the question Q' and should arguably not be part of S's respective doxastic state. After all, they already have meticulously inquired about all of the questions $\mu_1, \ldots, \mu_n$ and the relevant information w.r.t. these questions is purportedly far too specific regarding Q'. Contrarily, the inductively obtained information about the general fallibility of researchers and the probability of errors Δ appears to be highly relevant in that regard and should therefore be adopted as part of S's doxastic state when inquiring about Q'. As $\Delta \models \neg \bigwedge_{i=1}^{n} \alpha_i$ and $\neg \bigwedge_{i=1}^{n} \alpha_i$ constitutes an answer to Q', S ought to believe $\neg \bigwedge_{i=1}^{n} \alpha_i$ relative to the question Q'.

I argued before that general evidentialist norms should ensure the well-foundedness of an inquisitive state, in particular, the consistency of Γ_t^Q. Pertaining to the paradox of the preface this means that an inconsistency like $\bigwedge_{i=1}^{n} \alpha_i \wedge \neg \bigwedge_{i=1}^{n} \alpha_i$ never ought to be believed regarding the same inquiry. As seen above, the inquiry regarding the actual content of the book should result in a belief that $\bigwedge_{i=1}^{n} \alpha_i$ whereas the inquiry about whether the book possibly contains factual errors should result in a belief that $\neg \bigwedge_{i=1}^{n} \alpha_i$. However, this inconsistency is merely global and thus perfectly condonable: as far as there is no local inconsistency in an agent's beliefs relative to one and the same inquiry, global inconsistencies are thus enclosed and isolated, which prevents them from giving rise to the problem of explosion.[19]

7.2 Considering cognitive limitations

So far, the preliminary bridge principles IBP and IBP-Q do not take into account what an agent's cognitive capacities can or should be expected to afford. As such they merely constitute highly idealised normative standards. In the ensuing section I will de-idealise these bridge principles by introducing suitable attitudinal restrictions and thereby develop an account of the normativity of logic qua appraisals.

Appropriate demands

Suppose it were, as stated by IBP and IBP-Q, normatively required to believe all relevant logical consequences of one's beliefs. In order to comply with this norm, it would be necessary to know all the respective facts about inquisitive entailment. Such demands are therefore undeniably excessive and untenable. A straightforward way to circumvent this problem is to restrict the normative authority of logic only to those entailments an agent recognises to be valid (see Harman, 1986, pp. 17–19). However,

[19] Compare the solution of the preface paradox by Leitgeb (2014), which also maintains that rational belief is question-sensitive. This relates to the important insight in formal epistemology that in order for an agent's degrees of belief and binary beliefs to cohere, belief must be dependent on how the agent is partitioning the logical space of all possible states of affairs (see Lin & Kelly, 2012, Sects. 13–14 for a precise formal result and proof). For a thorough philosophical discussion see Leitgeb (2017, pp. 127–148).

this runs the risk of an orthogonal problem, which MacFarlane (2004, p. 12) calls the *Priority Question*: if it is merely an agent's present logical knowledge which tells them how they ought to manage their beliefs, they would be able to reason entirely as they ought to, no matter how logically ignorant they are. The need to suitably balance the desiderata of excessive demands and the priority question calls for an attitudinal restriction not merely to what an agent's relevant cognitive faculties actually manage to afford but rather what they can be reasonably expected to afford in a given context:

> If S can be expected to determine or determines that $\Gamma \models \alpha$ and α is $Q-$relevant, then S ought to see to it that (S believes α if S believes Γ). (IBPa)

> If S can be expected to determine or determines that $\Gamma, \Xi \models \mu$ for a set of sentences Γ and a set of maximally coarse-grained appropriate questions Ξ, then S ought to see to it that (S entertains Ξ if S believes Γ and entertains μ). (IBPa-Q)

Here, the notion of an agent S determining that some entailment holds is not to be understood in a sense which requires S to consciously entertain certain beliefs about logical consequences involving formal concepts a lot of people are arguably unaware of. Rather, the fact that S determines that $\Gamma \models \alpha$ for a set of declarative sentences Γ and a statement α should be understood as requiring that S is able to correctly infer α from the premises in Γ. Analogously, S determining that $\Gamma, \Xi \models \mu$ for a set of declarative sentences Γ, a set of questions Ξ and a question μ, should be understood as S being able to see that given their belief in the premises Γ, resolving all of the questions in Ξ amounts to resolving μ.

As discussed by Steinberger (2017), the demands an appraisal puts on the appraisee are highly dependent on a variety of contextual factors, such as the stakes of the inquiry at hand, the available time and resources, and the logical competence of the agent. This is captured by the given bridge-principles. The higher an agent's logical competence, the more logical entailments they should be expected to recognise and thus the less restricted the bridge principles become.[20] In the extreme, an agent who is logically fully competent can be expected to recognise all valid logical entailments, whence the attitudinal restriction de facto disappears. Contrarily, a less ideal agent can be expected to determine the validity of much fewer entailments. In this case, the bridge principles IBPa and IBPa-Q therefore successfully deal with the objection from excessive demands. However, one might object that they become more susceptible to the priority question. But this is not quite right as the desideratum of the priority question merely demands that the normative force of a bridge principle is not unrea-

[20] Arguably, the appropriate standard for logical competence is determined in a social context. For instance, in a dialogical argument between epistemic peers, the standard of logical competence they expect from each other is roughly the same as the one they hold themselves to. Compare the bridge principles Dutilh Novaes (2015, pp. 599–606) proposes for dialogical interactions between two agents who argue about whether a given purported conclusion follows from the already granted premises.

sonably weakened below what it should afford in a given situation. This is guaranteed as IBPa and IBPa-Q are merely restricted to what an agent's cognitive capacities can be reasonably expected to afford in some situation and not what they actually manage to afford.

Clutter-avoidance

IBPa obliges an agent to believe only those consequences of their existing beliefs which are pertinent regarding a question on their research agenda. Therefore, the problem of mind-cluttering in its original form does not arise. Unfortunately, there is another looming problem. Let α and β be arbitrary sentences such that $\Gamma \models \alpha$ and α is pertinent w.r.t. a question $\mu = ?\{\alpha_1, \ldots, \alpha_n\}$ on S's research agenda and $\Gamma \models \beta$ but β is not Q-relevant. It then obviously holds that $\Gamma \models \alpha \wedge \beta$ and furthermore that $\alpha \wedge \beta$ is pertinent w.r.t. μ because if α entails some partial or complete answer to μ, so does $\alpha \wedge \beta$. Given that β may itself be a disjunction $\beta_0 \vee \beta_1$, where only β_0 follows from Γ and β_1 is arbitrary, this would constitute a worry similar to the original problem of mind-cluttering.

Fortunately, however, the attitudinal restriction in IBPa considerably mitigates this worry. According to IBPa, the entailment $\Gamma \models \alpha \wedge \beta$ results in a wide-scope obligation to believe $\alpha \wedge \beta$ only if S can be expected to determine or in fact determines this entailment. However, as I will now argue, an agent S can generally not be expected to recognise that $\Gamma \models \alpha \wedge \beta$; and if they do recognise that the conjunction $\alpha \wedge \beta$ is entailed by Π_t^Q, Γ, the resulting obligation to believe $\alpha \wedge \beta$ appears to be absolutely appropriate.

So, under which circumstances, can S be expected to recognise that $\Gamma \models \alpha \wedge \beta$? As opposed to α, β in itself is not Q-relevant, whence there are no good reasons for them to even consider β. Thus, if the appraisal is supposed to fulfil a less idealised function, there is no need to expect S to infer β from Γ. But if S cannot be expected to determine that $\Gamma \models \beta$, they should not be expected to determine that $\alpha \wedge \beta$ follows from Γ, just because they can be expected to determine or in fact determine that $\Gamma \models \alpha$. Be that as it may, if for whatever reason S determines that $\Gamma \models \alpha \wedge \beta$, where α is Q-relevant, it is perfectly fine for them to be obliged to adopt a belief in $\alpha \wedge \beta$ (if they maintain their beliefs in Γ). In fact, it is ubiquitous in our reasoning that we use more information than what is actually necessary to derive a certain conclusion: suppose, one believes that $\alpha \vee (\beta \wedge \gamma)$ and one inquires about whether β and nothing else. The gathered evidence suggests that α is not the case, whence one adopts a belief that $\neg\alpha$. If one now recognises the entailment $\neg\alpha, \alpha \vee (\beta \wedge \gamma) \models \beta \wedge \gamma$, it appears to be perfectly fine to infer $\beta \wedge \gamma$ and accordingly expand one's beliefs even though γ itself is not Q-relevant. In addition, it would even be a rational failure similar to obtuseness if one were to think about two sentences α and β, where at least one of these sentences is Q-relevant, recognise that $\Pi_t^Q, \Gamma \models \alpha \wedge \beta$ and not adopt a belief in $\alpha \wedge \beta$.

Obtuseness

As an agent S can arguably always be expected to determine the validity of entailments of the form $\alpha, \beta \models \alpha \wedge \beta$, the discussion of obtuseness from above carries over.

Connectedness

The way the attitudinally unrestricted bridge principles ensure that an agent's beliefs are properly connected does not immediately carry over. This is because the obligation to entertain questions which conditionally entail a question on the research agenda in IBPa-Q is restricted to those question-entailments which the agent can be expected to determine or determines. However, as I will now argue, connectedness is much less important as a desideratum when the limitations of the cognitive capacities of an agent are taken into account.

Recall the example from before: an agent S believes sentences p, $p \to q$ and $q \to r$ and inquires about $?r$. Obviously it holds that $p, p \to q, q \to r \models r$. Suppose now that S can be expected to be able to infer r from the premises p, $p \to q$ and $q \to r$ and complies with IBPa by adopting a belief in r. Furthermore, assume that S does not entertain a belief in q. From an evaluative point of view, S's doxastic state in this situation is not entirely as it ought to be. However, if one were to appraise S's doxastic state in light of their limited cognitive capacities as a human agent, there are several possible reasons for why it may be perfectly fine for them not to actively believe q. For one thing, S may have inferred r without explicitly inferring q along the way. They may just have inferred $p \to r$ from $p \to q$ and $q \to r$ and then proceeded to infer r with *modus ponens*, for instance. Or they may have inferred q from p and $p \to q$ and then inferred r but forgotten about q as it is of no direct interest to them. In both cases, S's reasoning appears to be rational and actually rather resource-efficient.

Nevertheless, there are circumstances in which S should believe that q. For, if one were to ask S which reasons prompt them to believe r, they should be able to state these reasons in a satisfactorily transparent manner. Of course, S may just state their beliefs in the premises and invoke the entailment $p, p \to q, q \to r \models r$; but suppose that one does not immediately recognise this entailment as valid and thus ask S to justify their belief in r on more transparent grounds. In this case, S would be expected to afford more than just being able to derive an answer to their primary question $?r$. A natural way for S to meet this request would be to refer to simpler entailments such as $p, p \to q \models q$ to justify a belief in q and $q, q \to r \models r$ to warrant their belief in r. If S can be expected to be able to infer r from the premises p, $p \to q$ and $q \to r$, they surely should be expected to determine these entailments as well. Furthermore, I submit, S should be expected to determine that $q \to r, ?(q \vee r) \models ?r$ in the respective situation: if S believes $q \to r$ and wonders whether r is the case, they are surely able to ascertain that if they were to establish $q \vee r$ they would be able to infer r (resolving $?(q \vee r)$ in the negative immediately yields the answer $\neg r$ via the entailment $\neg q \wedge \neg r \models \neg r$). Hence, other things being equal, according to IBPa-Q, S ought to entertain the question $?(q \vee r)$. Since the conclusion of $p, p \to q \models q$ answers this question, IBPa amounts to an obligation to believe q.

This concludes the discussion of the desiderata. The proposed pair of bridge principles accounts for the dynamic interplay between an agent's doxastic state and their research agenda in two main ways. For one thing, the rationality constraints on an agent's doxastic state such as closure requirements are sensitive to the questions on an agent's research agenda, whence the problems of mind cluttering and the paradox of the preface can be elegantly addressed. For another thing, the questions which ought

to be on the research agenda are determined by the agent's existing beliefs and the questions of ongoing inquiry. This ensures that an agent's beliefs and questions are reasonably coherent and connected via traceable inferences.

8 Conclusion

As I have argued, reasoning as part of rational inquiry encompasses the processes of raising and resolving questions besides the adoption and revision of beliefs. Hence, a normative theory of reasoning has to account not only for how an agent should manage their beliefs but furthermore for which questions an agent is permitted and obliged to ask in course of inquiry. I then showed that all of the normatively relevant logical relations between an agent's beliefs and the questions they ask themselves can be subsumed under a uniform notion of entailment in inquisitive logic. Therefore, I argued, the dynamics of an agent's belief as well as issues fall under the normative authority of inquisitive logic. Most importantly, I developed these ideas into a pair of bridge principles in the sense of MacFarlane, and discussed how these bridge principles fare with regard to the desiderata found in the literature, such as clutter-avoidance, excessive demands and the paradox of the preface.

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